软件学院本科生 2020——2021 学年第 2 学期算法导论课程期末考试试卷（A 卷）

专业：　　　　　　　　　年级：　　　　　学号：　　　　　姓名：　　　　　成绩:

草　稿　区

得分

一 、选择题（本题共 30 分，每小题 3 分）

1. In terms of asymptotic analysis, which function grows the fastest? (　　)

A. $2n+3$　B. $2n^3+5n^2+n$　C. 10^n　D. $n\log n$

2. The average time complexity of quick sort is (　　)

A. $O(n)$　B. $O(n\log n)$　C. $O(n^2)$　D. $O(\log n)$

3. If m denotes the number of edges in a graph, and n denotes the number of nodes. The time complexity of Prim is (　　)

A．$O(nm)$　B. $O(n\log m)$　C. $O(m\log n)$　D. $O(n+m)$

4. Which of the following problems can be solved by dynamic programming (　　)

A. Minimum Spanning Tree　B. Stable Matching　C. Merge two ordered array　D. Weighted Interval Schedule

5. Considering the following functions $f(n)$, $g(n)$, which satisfies that $f(n)=O\big(g(n)\big)$ and $g(n)\neq O\big(f(n)\big)$? (　　)

A．$f(\mathrm{n})=n^3, g(\mathrm{n})=n^2\log(n^2)$

B. $f(\mathrm{n})=\log\sqrt{n}, g(\mathrm{n})=n\log\sqrt{n}$

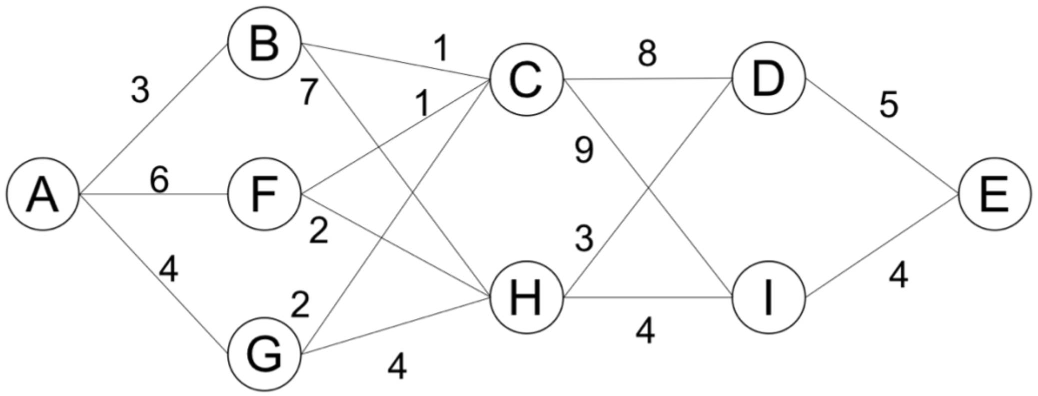
C. $f(\mathrm{n})=\sqrt{n}+1, g(\mathrm{n})=10\sqrt{n}$

D. $f(\mathrm{n})=\log n+1.001^n, g(\mathrm{n})=1000\log\sqrt{n}$

6. Given a graph G. All of its edges are positively weighted. Suppose that you change the length of every edge of G as follows. For which is every shortest path in G also the shortest path in G'? (　　)

A. Multiply by 10　　B. Add 10　　C. Take its square (平方)　　D. Take its square root (开平方)

7. What is the length of the shortest path from A to E in the following graph? (　　)



A. 14　　B. 15　　C. 16　　D. 17

8. The main difference between greedy algorithms and the dynamic programming algorithms is (　　)

A. Optimal substructure　　B. Define the optimal solution　　C. Construct optimal solution　　D. Greedy choice

9. What is the time complexity of the following code? (　　)

```
int count = 0;
for (int k = 1; k <= n; k *= 2)
    for (j = 1; j <=n; j++)
        count++;
```

A. $O(n\log n)$　　B. $O(\log n)$　　C. $O(n)$　　D. $O(n^2)$

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10. Given the following favorite order, the current matching is {Atlanta-Xavier, Boston-Zeus, Chicago-Yolanda}.

Which pair is an unstable pair? (   )

	1st	2nd	3rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1st	2nd	3rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

A. Atlanta-Yolanda   B. Boston-Yolanda   C. Chicago-Zeus   D. Boston-Xavier

得分

二 、填空题（本题共 20 分，每空 2 分）

1. Steps to design a dynamic programming algorithm：
   a) Break up a problem into a series of________________;
   b) Combine____________to____________to form solution to____________.
2. Rank the following functions: $6n+1$, $\log\sqrt{n}$, $\sqrt{2n}$, $(2n+1)!$, 3^n, n^3 in ascending order in terms of computational complexity: ____
3. The running time complexity of the following code is _______, and its space complexity is _______.

```
for (int k = 0; k < n; k++)
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            d[i][j] = min(d[i][j], d[i][k] + d[k][j])
```

4. The asymptotic expression Θ indicates ________. (上界/下界/紧的界)

草 稿 区

5. Given the sequence X = [3, 2, 1, 5, 3, 2, 5] and Y = [1, 2, 3, 1, 3, 5, 2, 5], please give one of the longest common subsequence of X and Y ____________________.
6. In the problem of interval scheduling, we sort the schedules by their finish-time. When two schedules share the same finish-time, the one with __________ (earlier/later) start-time should have higher priority (should be scheduled).

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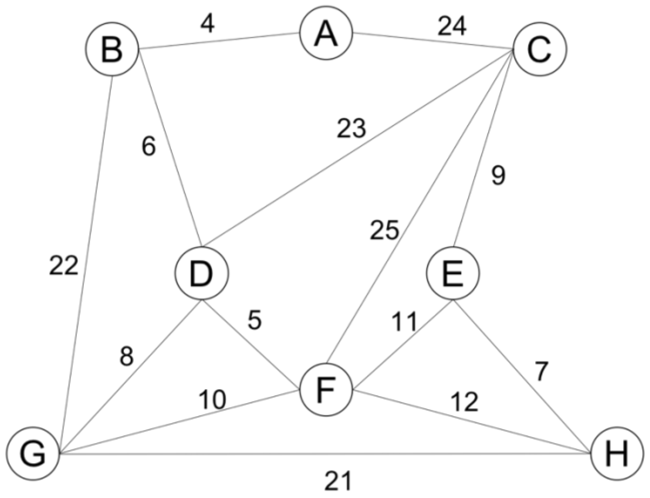
三 、简答题（本题共 20 分）

1. Answer questions according to the following graph. **(本小题 12 分)**



a) How many edges are there in the minimum spanning tree (MST) of this graph?
b) What is the total weight of the minimum spanning tree of this graph?
c) Describe the process of Kruskal's algorithm in an undirected graph with no isolated nodes.

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2. $f(n) = 17n^2 \log_2 n + 5n + 3$. Prove that $f(n)$ is $\Theta(n^2 \log_2 n)$. **(本小题 8 分)**

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四 、综合题（本题共 30 分）（注：凡是要求设计算法的题目，请写出详细的伪代码）

1. You're given n tasks, each of which is described by a pair of integer (t_i, d_i). t_i is the time you need to finish the $task_i$, and d_i is the due time (deadline) of $task_i$. For a specific schedule, the finish time of $task_i$ is denoted by f_i. The latency of each task is $L_i = \max(0, f_i - d_i)$. In other words, if a task is finished before its due time, its latency is 0. Otherwise, its latency is equal to its finish time subtracted (减去) by its due time.
You want to schedule your tasks so that the maximum latency is as small as possible (minimize maximum latency).
Question a) and b) are based on the table given below, while c) and d) are general questions. **(本小题 15 分)**

Task id	1	2	3
t_i	3	2	1
d_i	1	2	3

a) If you schedule the tasks in the order 2-1-3, the maximum latency is 4. What's the maximum latency if you schedule the tasks in the order 2-3-1?
b) For the tasks in the table, please give the optimal schedule and its corresponding maximum latency. (Without explanation)
c) Describe the strategy to solve this problem.
d) Show the pseudocode of this strategy.

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2. There are three campuses of Nankai University: the Balitai campus, the Jinnan campus, and the TEDA campus. The freshman(大一) and sophomore(大二) students of College of Software study at Jinnan campus, while the junior(大三) and senior(大四) students study at TEDA campus. So there are offices for the professor both at Jinnan and TEDA campus.
The professor answers questions of the students. When he goes from one campus to another, he spends one hour on transportation. As a result, his time on answering questions will be reduced. The number of students having questions on Jinnan campus each day is denoted as J_1, J_2, J_3, ... J_n, and the number of students having questions on TEDA campus each day is denoted as T_1, T_2, T_3, ... T_n. If the professor goes from one campus to another, there will be 3 students at most whose questions cannot be answered. For example, if there are 5 students having questions in the destination campus, the professor can only answer 2 of them. If there are 1, 2, or 3 students who have questions, the professor cannot answer the questions of anyone. The professor wants to answer the questions of as many students as possible. The following table gives an example of the number of students having questions each day. The maximum number of students that the professor can answer is $10 + 6 + (11 - 3) + 15 = 39$. The plan is that the professor stays at TEDA for the 1st and 2nd days. On the 3rd day, the professor goes to Jinnan, then he stays at Jinnan for the 3rd and 4th days. (The professor can choose TEDA or Jinnan for the 1st day at will, which does not impact on the number of questions he can answer.) **(本小题 15 分)**

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Table 1 The number of students having questions each day

i	1	2	3	4
J_i (Jinnan)	1	2	11	15
T_i (TEDA)	10	6	2	3

a) Show that the following algorithm does not correctly solve this problem by giving an instance where it does not return the correct answer.

```
for (int i = 1; i <= n; i++)
    if (T[i] > J[i])
        cout << "The" << i << "th day in TEDA campus." << endl;
    else
        cout << "The" << i << "th day in Jinnan campus." << endl;
```

b) Give the pseudocode of the solution to this problem. Your code should return the maximum number of students that the professor can answer their questions.

c) Suppose that there are n days in total. What is the time complexity of your algorithm in terms of big-O notation?