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软件学院本科生 2019——2020 学年第 2 学期算法导论课程期末考试试卷（A 卷）

专业： 年级： 学号： 姓名： 成绩:

得分

一、选择题（本题共 30 分，每小题 3 分）

1. Which one indicates polynomial (多项式) time complexity in terms of big-O notation? ( )

A. $O(n!)$ B. $O(1)$ C. $O(n^2)$ D. $O(\log n)$

2. The number of executions grows extremely quickly as the size of the input increases when it has ( )

A. Exponential Time B. Linear Time C. Polynomial Time D. Constant Time

3. Let $W(n)$ and $A(n)$ denote the worst case and average case running time of an algorithm with an input of size n, respectively. Which of the following is ALWAYS TRUE? ( )

A. $A(n) = \Omega(W(n))$ B. $A(n) = O(W(n))$ C. $A(n) = \Theta(W(n))$ D. $A(n) = o(W(n))$

4. Which of the following is not $O(n^2)$? ( )

A. $15^{10}n + 12099$ B. $n^{1.98}$ C. n^2 D. $n^3/\sqrt{n}$

5. Consider a situation where you don't have any function to calculate power (e.g., pow() function in C), and you need to calculate x^n where x can be any number and n is a positive integer. What is the best possible time complexity of your power function? ( )

A. $O(\log n)$ B. $O(\log\log n)$ C. $O(n\log n)$ D. $O(n)$

6. An undirected graph G has n nodes. Its adjacency matrix (邻接矩阵) is given by an $n \times n$ square matrix whose (i)

diagonal (对角线) elements are 0 and (ii) non-diagonal elements are 1. Which one of the following is TRUE? (　　)

A. Graph G has no minimum spanning tree (MST)

B. Graph G has a unique MST of cost $n-1$

C. Graph G has multiple distinct MSTs, each costs $n-1$

D. Graph G has multiple spanning trees of different costs

7. If one uses straight two-way merge sort algorithm to sort the following elements in ascending order (升序): 20, 47, 15, 8, 9, 4, 40, 30, 12, 17, then the order of these elements after the second pass of the algorithm is (　　)

A. 8, 15, 20, 47, 4, 9, 30, 40, 12, 17

B. 8, 9, 15, 20, 47, 4, 12, 17, 30, 40

C. 15, 20, 47, 4, 8, 9, 12, 30, 40, 17

D. 4, 8, 9, 15, 20, 47, 12, 17, 30, 40

8. The main difference between greedy algorithms and dynamic programming algorithms is (　　)

A. Optimal substructure

B. Greedy choice

C. Construct optimal solution

D. Define the optimal solution

9. What happens when a top-down approach (without memorization) of dynamic programming is applied to a problem? (　　)

A. It increases both time complexity and space complexity

B. It increases time complexity only

C. It increases time complexity and decreases space complexity

D. It decreases both time complexity and space complexity

10. Consider the undirected graph below. Using Prim's algorithm to construct a minimum spanning tree starting with node A, which one of the following sequences of edges represents a possible order of the edges added to construct the minimum spanning tree? (　　)

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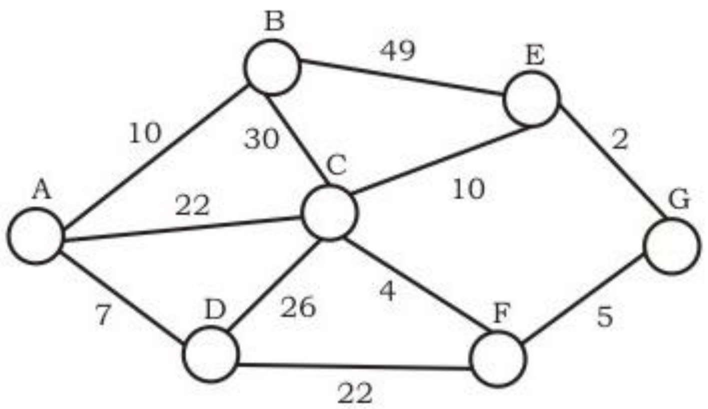

A. (E, G), (C, F), (F, G), (A, D), (A, B), (A, C)

B. (A, D), (A, B), (A, C), (C, F), (G, E), (F, G)

C. (A, B), (A, D), (D, F), (F, G), (G, E), (F, C)

D. (A, D), (A, B), (D, F), (F, C), (F, G), (G, E)

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二、填空题（本题共 20 分，每空 2 分）

1. Rank the following functions: $2n+1,\ 3\log n,\ 5n^4,\ 8,\ 4n!,\ 7^n$ in ascending order (升序) based on their asymptotic (渐进的) expression ________________________________

2. The time complexity of the following code is ______________, and the space complexity of the following code is ______________.

```
int i, j, k = 0;
for (i = n / 2; i <= n; i++) {
        for (j = 2; j <= n; j = j * 2) {
                k = k + n / 2;
        }
}
```

3. In terms of merge sort, the average time complexity is ______________.

4. The recurrence relation (递归关系) $T(1) = 2,\ T(n) = 3T\left(\frac{n}{4}\right) + n$ has the solution $T(n) = O($______________$)$.

5. Steps to design a dynamic programming algorithm:

a) Break up a problem into a series of ________________.

b) Combine solutions to ________________ to form solution to ________________.

6. The earliest-________________(start/finish)-time-first algorithm is optimal for interval scheduling problems. The earliest-________________(start/finish)-time-first algorithm is optimal for interval partitioning problems.

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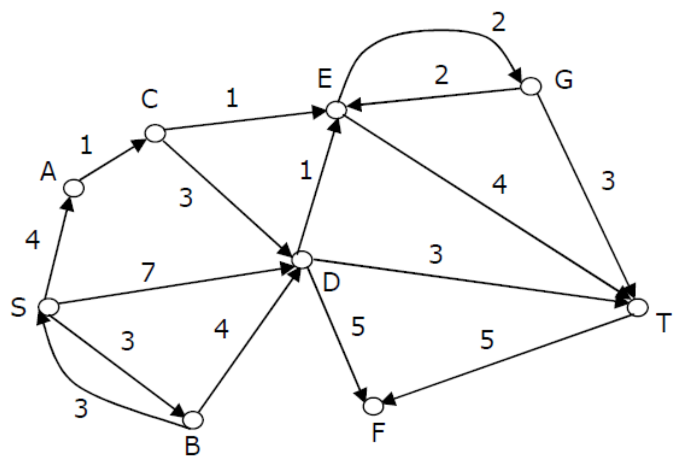
三、简答题（本题共 20 分）

1. Consider the directed graph shown in the figure below. Please use Dijkstra algorithm to find out the shortest path from S to T. Suppose that in any iteration (迭代) the shortest path to a vertex v is updated only when a strictly shorter path to v is discovered. Note that there may be multiple shortest paths from S to T, but only one of them is output by Dijkstra algorithm. Please briefly show the process of Dijkstra algorithm. （本小题 10 分）



2. Table 1 shows men’s preference ranking for women, and table 2 shows women’s preference ranking for men. We call it a stable matching if no matched man and woman both prefer each other to their current spouses (配偶). Please give an example of a stable matching and briefly describe the core idea of your algorithm. （本小题 10 分）

Table 1

Man	1st	2nd	3rd
Albert	Diane	Emily	Fergie
Bradley	Emily	Diane	Fergie
Charles	Diane	Emily	Fergie

Table 2

Woman	1st	2nd	3rd
Diane	Bradley	Albert	Charles
Emily	Albert	Bradley	Charles
Fergie	Albert	Bradley	Charles

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四 、综合题（本题共 30 分）（注：凡是要求设计算法的题目，请写出详细的伪代码）

1. You are given n real numbers $x_1, \cdots, x_n$. Design an efficient algorithm that uses the minimum number (m) of intervals $[i, i+1)$ $(1 \le i \le x_n)$ to cover all the input numbers. A number x_j is covered by an interval $[i, i+1)$ if $i \le x_j < i+1$. For example, consider the input with $n = 4$: 0.1, 0.9, 1.1, 1.555. The two intervals $[0.1,1.1)$ and $[1.1,2.1)$ cover all the input numbers (*i.e.,* in this case $m = 2$).

   (a) Describe your algorithm with pseudo code (伪代码).

   (b) Prove why your algorithm is correct.

   （本小题 12 分）

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2. Consider the coin changing problem: Given coin denominations (面值), devise a method to pay an amount to a customer using the fewest number of coins.

Input:

- An array denomination array $[1..n]$ containing the n coin denominations $d_1, \cdots, d_n$ that you can use (for example, $[1, 10, 21, 34, 70, 100]$, and thus $n = 6$). Suppose that this array is already sorted in ascending order (with no repetitions (重复)), and you have unlimited number of coins for each denomination.
- The amount M that you need to pay (e.g., $M = 140$).

Note that all the coin denominations and M are positive integer numbers.

Output: The optimal (minimum) number of coins needed to pay amount to M (in the above example, the result is 2).

Answer the following two questions:

(a) Which type of algorithm should be used to solve this problem, and what is the core idea of this type of algorithm?

(b) Give the pseudo code of solving this problem.

（本小题 18 分）

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