

Investigation of Novel Lattice Designs for 3D Printing

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Abstract – Additive manufacturing technology, with its numerous advantageous, present superior means of lattice construction. A study was conducted to investigate the mechanical properties of novel lattice designs composed of two different type of unit cells. Lattice with no internal voids were studied as to mitigate the limitations of 3D printing not being able to print structures with internal voids. Sixteen different 3x3x3 symmetrical lattice compositions, consisted of truss simple cubic unit cell and plate simple cubic unit cell, was modelled, and simulated using finite element analysis. Variation of relative modulus of the lattice against relative density of the lattice showed consistent patterns despite variation in unit cell thickness. (1mm, 2mm, 3mm, 4mm) Furthermore, pattern revealed that lattice with 19 plate unit cell had the largest relative modulus among lattices with no internal voids. Optimization of unit cell thickness at given relative density has also showed lattice with 19 plate unit cell to have optimal mechanical properties among internal void free lattices. Further optimization on lattice with only 19 plate unit cell and no truss unit cell was done to discover that the specific configuration shows superior mechanical properties in some relative densities. Lattice with 7 plate unit cell, albeit its inferior mechanical properties compared to 19 plate unit cell lattice, showed exceptional properties compared to compositions with similar number of plate unit cell.

Although pure plate simple cubic lattice shows superior mechanical properties, Lattice configuration with 19 plate unit cell is able to substitute pure lattice with minimal loss in mechanical properties while allowing 3D printing due to its internal void free structure.

Keywords – Additive manufacturing, Lattice construction, Finite element analysis

1 INTRODUCTION

1.1 BACKGROUND

3D printing has many advantages compared to conventional manufacturing means, including but not limited to accessibility, faster speed, reduced cost, ease of prototyping, and feasibility of complex geometries. 3D printing technology is being utilized in wide range of fields, including

manufacturing, medicine, and construction [1]. However, one main disadvantage of 3D printing is its inability to print geometries with internal voids. This disadvantage influences many different fields, notably lattice construction. Many lattices, due to the unit cell geometries, have internal voids, rendering them infeasible for construction using 3D printing.

1.2 OBJECTIVES

The objective of the study is to investigate in a novel lattice consisted of two different unit cell types to obtain internal void free lattice with mechanical properties similar or superior to that of the pure lattice consisted of one unit cell type. Simulations on different lattice compositions were done to find the optimal configuration that could substitute pure lattices while allowing for 3D printing.

1.3 SCOPE

16 different 3x3x3 symmetrical lattices consisted of plate simple cubic unit cell and truss simple cubic unit cell is investigated using finite element analysis. The effect of unit cell thickness on mechanical properties of lattice is studied and unit cell thickness optimization is carried out to determine the optimal lattice configuration with superior mechanical properties.

2 LITERATURE REVIEW

Literature review was conducted to understand the basics of finite element analysis and different mechanical properties of lattices, such as modulus or relative density.

2.1 FINITE ELEMENT ANALYSIS (FEA)

FEA is a computed method to predict how a model will react to real life phenomenon like heat, force, or magnetism. It involves splitting the model of interest into small meshes to study. For this investigation, stress analysis on a model will be extensively used to evaluate the stress lattice receives with prescribed displacements. COMSOL Multiphysics will be used for the FEA of the investigation. [2]

2.2 ISO STRAIN AND ISO STRESS

Isostrain and isostress are two principles that appear in mechanics of composites. Since the

lattice investigated in this study is consisted of two different unit cell types, it can be treated as a composite of two different unit cell types. Isostrain is applicable if the layer of composite is parallel to the direction of stress applied, while isostress is applicable if the layer of composite is perpendicular to the direction of stress applied. The 3x3x3 lattice to be investigated will apply isostrain principle on each of its layers and apply isostress principle to combine the three layers to produce an estimation of mechanical property of composite lattice.

2.3 RULE OF MIXTURES

Rule of mixture states that the property of composites can be approximated by a weighted average of its constituent components. In this investigation, the mechanical property of composite lattice can be approximated using property of its constituent unit cells. Furthermore, it can be predicted that the property of composite lattice must be superior then property of pure unit cell consisted of truss simple cubic unit cell, unit cell of inferior property, and inferior then property of pure unit cell consisted of plate simple cubic unit cell, unit cell of superior property.

3 METHODOLOGY

Methods used to carry out this investigation is outlined in this section. This includes the production of lattice models and finite element analysis studies to investigate mechanical properties of the lattices.

3.1 LATTICE CONSTRUCTION

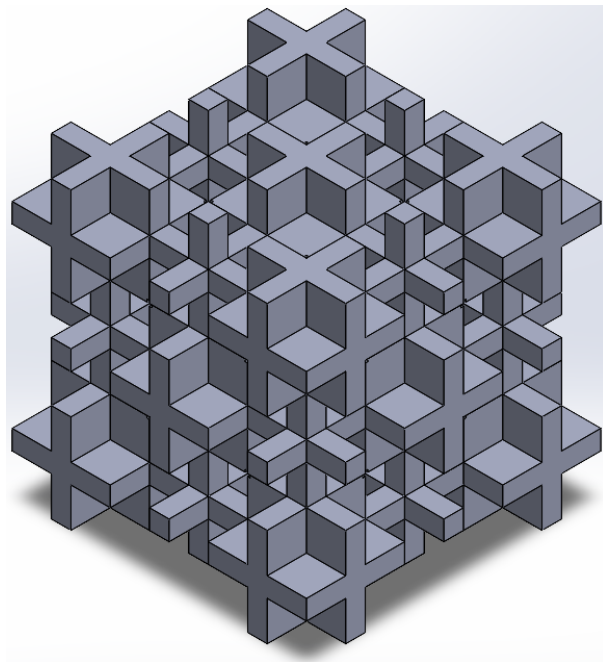


Figure 1: 14 plate unit cell lattice in solid works

Sixteen different 3x3x3 symmetrical lattice consisted of plate simple cubic unit cell and truss

simple cubic unit cell were developed using solid works. Each unit cell was 10mm in length, width, and height. The thickness of the unit cells was parameterized for altering purposes and optimization purposes.

3.2 STATIONARY STUDY FOR LATTICE ELASTIC MODULUS

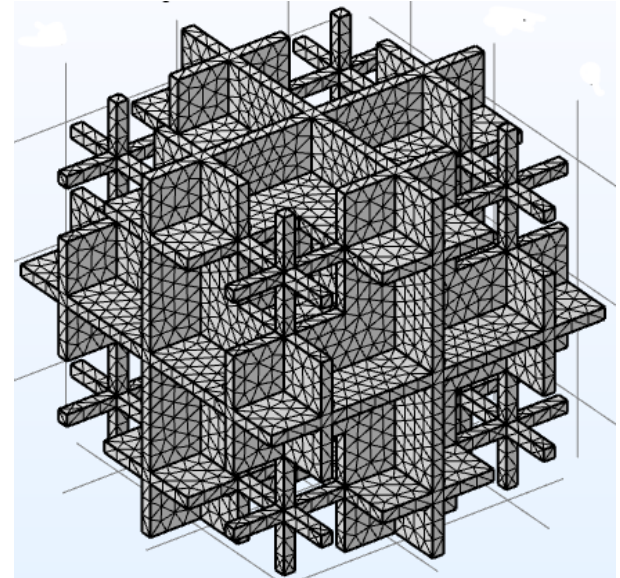


Figure 2: Meshed lattice in COMSOL

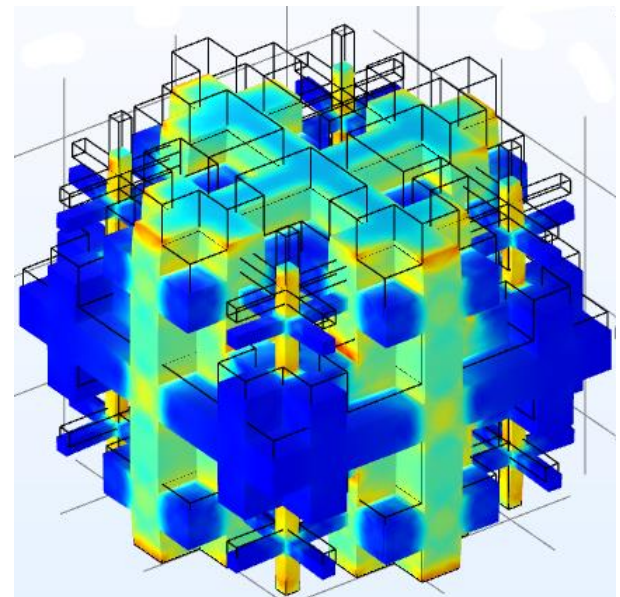


Figure 3: Stress Concentration of Lattice under prescribed displacement in COMSOL

Each lattice configuration was imported into COMSOL to undergo stationary finite element analysis to simulate mechanical stress under effective strain of 0.1% to 1% with 0.1% interval. The stress strain plot was obtained, and the slope of the plot was used as the elastic modulus of the lattice. Unit cell thickness of lattice was varied from 1mm to 4mm with 1mm interval to simulate mechanical stress with equal strain conditions.

3.3 LATTICE UNIT CELL THICKNESS OPTIMIZATION

Each lattice configuration with their unit cell thickness parameterized was imported into COMSOL to undergo thickness optimization to yield optimal stress at a 1 percent strain at given relative densities. Relative density computed range from 0.2 to 0.9, with 0.1 intervals. Configurations that showed exceptional properties were further tested by removing the truss simple cubic unit cell from the lattice, allowing COMSOL to alter only the thickness of plate simple cubic unit cell during the optimization.

4 RESULTS AND DISCUSSIONS

4.1 RESULTS OF STATIONARY STUDY FOR LATTICE ELASTIC MODULUS

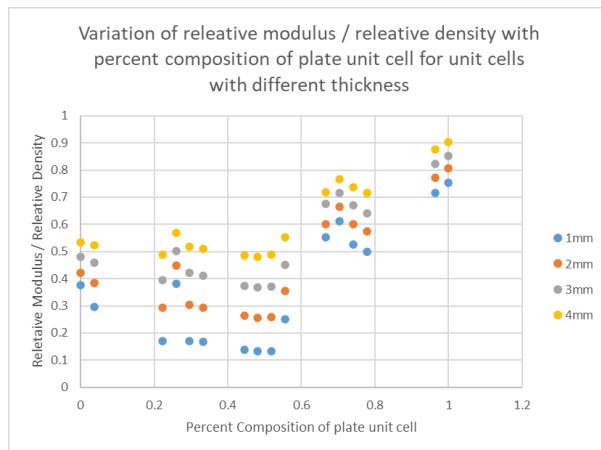


Figure 4: Variation of Specific Relative Modulus for Different Plate Simple Cubic Unit Cell Composition of Lattice with Different Unit Cell Thickness

The table used to generate figure 4 can be found in the appendix. From the results collected from stationary analysis shown in figure 4, variation trend of specific relative modulus shows consistency, regardless of different unit cell thickness. Furthermore, lattice with plate simple cubic unit cell composition of 25.93 percent and 70.37 percent, or lattice with 7 and 19 plate simple cubic unit cell, showed exceptional properties, as compared to its neighboring points, while not having internal voids in its structures. Since the exceptional property was evident in all unit cell thickness, the two configurations are to be investigated further in optimization studies to verify their properties. Furthermore, studies on mechanical property of lattice with only 7 and 19 plate simple cubic unit cell and no truss simple cubic unit cell will be studied to test if superior properties compared to its original configuration can be achieved.

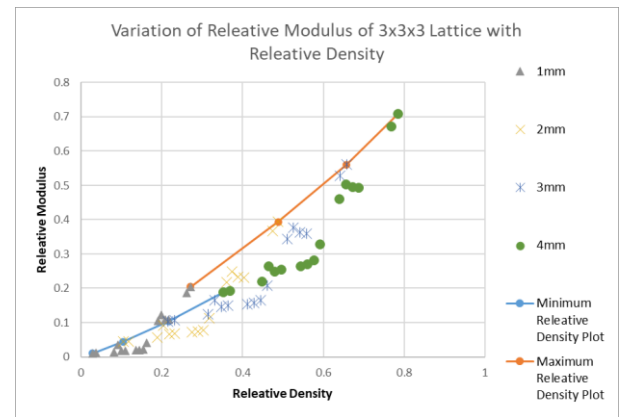


Figure 5: Variation of Relative Modulus of Lattice with Relative Density

Same data used to generate figure 4 was used to generate figure 5, which shows how the relative modulus of lattice varies with relative density of the lattice. The variation trend of specific relative modulus observed in figure 4 is observable in figure 5, with differences in magnitude of relative modulus variation, showing greater changes at higher relative densities. Furthermore, it can be observed that none of the points fall beneath the minimum relative modulus (0 plate simple cubic unit cell lattice) and exceed above the maximum relative modulus (27 plate simple cubic unit cell lattice), proving that the study follows the rule of mixtures.

4.2 RESULTS OF STATIONARY STUDY FOR PURE LATTICES

To compare the mechanical properties of optimized compound lattice, mechanical properties of pure lattices consisted of single unit cell type was obtained.

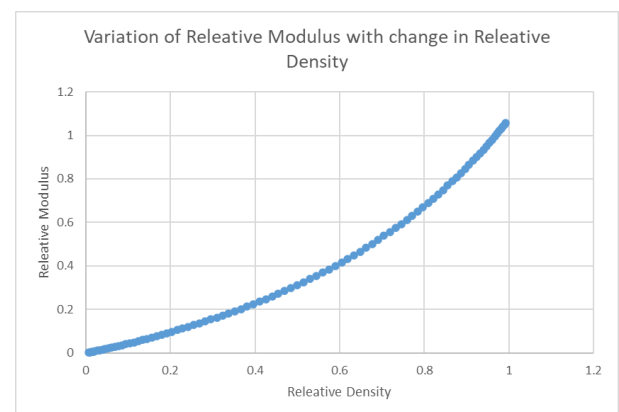


Figure 6: Variation of Relative Modulus of Truss Simple Cubic Lattice with Relative Density

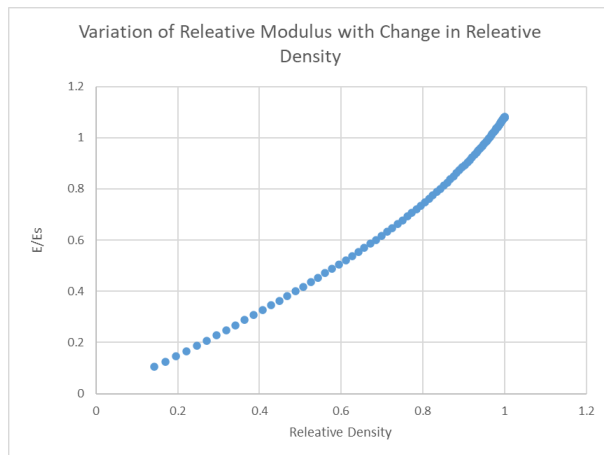


Figure 7: Variation of Relative Modulus of Plate Simple Cubic Lattice with Relative Density

4.3 RESULTS OF LATTICE UNIT CELL THICKNESS OPTIMIZATION

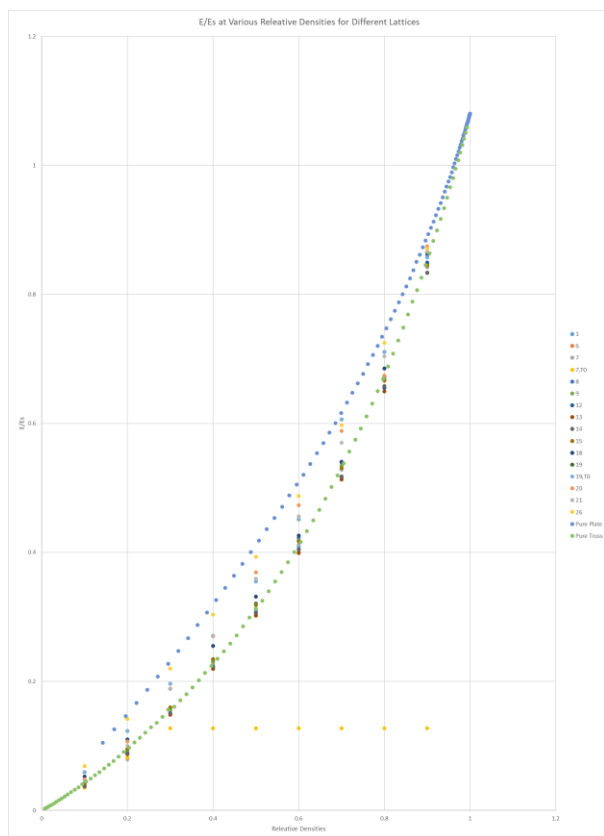


Figure 8: Variation of Optimized Relative Modulus Against Relative Densities of Different Lattice Configuration

With reference to figure 8, lattice with 19 plate simple cubic unit cell showed optimal mechanical properties at almost all relative densities, lattice with only 19 plate simple cubic unit cell showed optimal mechanical properties at other relative densities. It was also observed that at 0.5 and 0.6 relative densities, lattice with 20 plate cubic unit cell showed optimal mechanical properties among

all other configurations. Comparing the optimal mechanical properties of a lattice at given relative densities with the mechanical properties of pure plate simple cubic lattice, the composite lattice is able to attain relative modulus that is comparable to that of the pure lattice while having no internal voids in its structures.

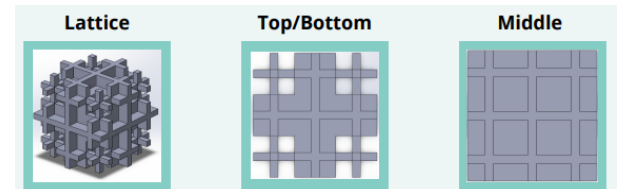


Figure 9: Structure of 19 plate unit cell lattice

6 CONCLUSION

In Conclusion, lattice consisted of 19 plate simple cubic unit cell showed optimal mechanical properties in both stationary study of lattice modulus and unit cell thickness optimization. Furthermore, its relative modulus was comparable to that of the relative modulus of pure plate simple cubic lattice. The lattice configuration does not contain internal voids, making it suitable for construction via 3D printing.

Moreover, lattice consisted of only 19 plate unit cell showed superior properties in certain relative densities, while lattice with 20 plate unit cell showed superior properties in specific relative density regions.

Further studies may be conducted to find the range of relative densities in which specific configuration shows optimal properties. Also, physical model of a lattice may be constructed and tested under loading to verify the simulated results. Finally, lattice consisted of other type of unit cells may be simulated to attain the optimal properties of compound lattice and compared with the properties of lattice configurations investigated in this research.

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- [2] “Finite Element Analysis Software | Autodesk,” *Autodesk.com*, Dec. 04, 2020. [https://www.autodesk.com/solutions/finite-element-analysis#:~:text=Finite%20element%20analysis%20\(FEA\)%20is,the%20way%20it%20was%20designed.](https://www.autodesk.com/solutions/finite-element-analysis#:~:text=Finite%20element%20analysis%20(FEA)%20is,the%20way%20it%20was%20designed.) (accessed May 06, 2022).

APPENDIX

Table used to generate figure 4:

| Percent Composition of Plate Unit Cell | 1mm | 2mm | 3mm | 4mm |
|---|---|---|---|---|
| Relative Modulus / Relative Density (Specific Relative Modulus) | Relative Modulus / Relative Density (Specific Relative Modulus) | Relative Modulus / Relative Density (Specific Relative Modulus) | Relative Modulus / Relative Density (Specific Relative Modulus) | Relative Modulus / Relative Density (Specific Relative Modulus) |
| 0 | 0.375079431 | 0.422828089 | 0.479785728 | 0.533483988 |
| 0.037037037 | 0.295872873 | 0.384840055 | 0.459208643 | 0.522035024 |
| 0.222222222 | 0.169447777 | 0.293606594 | 0.39395387 | 0.489358315 |
| 0.259259259 | 0.381121101 | 0.448108773 | 0.501453383 | 0.569587905 |
| 0.296296296 | 0.170435657 | 0.304934416 | 0.422816434 | 0.518886785 |
| 0.333333333 | 0.16905347 | 0.292780042 | 0.412298467 | 0.510960411 |
| 0.444444444 | 0.138503565 | 0.263116578 | 0.373625331 | 0.485988933 |
| 0.481481481 | 0.133096482 | 0.256151259 | 0.3677156 | 0.481780844 |
| 0.518518519 | 0.133176309 | 0.25772514 | 0.371838399 | 0.489741162 |
| 0.555555556 | 0.250188387 | 0.353679972 | 0.450400754 | 0.553961405 |
| 0.666666667 | 0.553203296 | 0.601690236 | 0.675097841 | 0.7167601641 |
| 0.703703704 | 0.610706653 | 0.664153261 | 0.715476807 | 0.766935206 |
| 0.740740741 | 0.525434149 | 0.600706002 | 0.670290001 | 0.736369799 |
| 0.777777778 | 0.498743006 | 0.574252709 | 0.641620137 | 0.716376556 |
| 0.962962963 | 0.714830365 | 0.77260852 | 0.822779617 | 0.875181503 |
| 1 | 0.752230422 | 0.80715226 | 0.852967729 | 0.903067151 |

Table used to generate figure 5:

| 1mm | 2mm | 3mm | 4mm |
|------------------|------------------|------------------|------------------|
| Relative Density | Relative Modulus | Relative Density | Relative Modulus |
| 0.028 | 0.010502224 | 0.104 | 0.043974121 |
| 0.037 | 0.010947296 | 0.118222222 | 0.045496646 |
| 0.082 | 0.013894717 | 0.189333333 | 0.055589515 |
| 0.091 | 0.03468202 | 0.203555556 | 0.09121503 |
| 0.1 | 0.017043566 | 0.217777778 | 0.066407939 |
| 0.109 | 0.018425828 | 0.232 | 0.06792497 |
| 0.136 | 0.018836485 | 0.274666667 | 0.072269354 |
| 0.145 | 0.01929899 | 0.288888889 | 0.073999253 |
| 0.154 | 0.020509152 | 0.303111111 | 0.078119354 |
| 0.163 | 0.040780707 | 0.317333333 | 0.112234444 |
| 0.19 | 0.105108626 | 0.36 | 0.216608485 |
| 0.199 | 0.121530606 | 0.374222222 | 0.248540909 |
| 0.208 | 0.109290303 | 0.388444444 | 0.233340909 |
| 0.217 | 0.108227232 | 0.402666667 | 0.231232424 |
| 0.262 | 0.187285556 | 0.473777778 | 0.366044747 |
| 0.271 | 0.203854444 | 0.488 | 0.393890303 |

Table used to generate figure 8:

| Mixed Lattice | E/E ₁ | | | | | | | | | | | | | | | | |
|------------------|------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|--|
| Relative Density | 1 | 6 | 7 | 7.70 | 8 | 9 | 12 | 13 | 14 | 15 | 18 | 19 | 19.70 | 20 | 21 | 26 Mix | |
| 0.1 | 0.04754 | 0.03907 | 0.03524 | 0.03543 | 0.04158 | 0.04049 | 0.03827 | 0.03747 | 0.03786 | 0.04262 | 0.05258 | 0.05004 | 0.05004 | 0.04847 | 0.04548 | 0.03605 | |
| 0.2 | 0.004386 | 0.00863 | 0.01703 | 0.00733 | 0.00433 | 0.00863 | 0.00717 | 0.00853 | 0.00948 | 0.01031 | 0.0131 | 0.01237 | 0.01071 | 0.00983 | 0.01478 | 0.01198 | |
| 0.3 | 0.155067 | 0.148678 | 0.158944 | 0.137733 | 0.157556 | 0.150478 | 0.150533 | 0.1483 | 0.15508 | 0.159978 | 0.188344 | 0.198322 | 0.194056 | 0.188922 | 0.188511 | 0.218944 | |
| 0.4 | 0.222044 | 0.218622 | 0.228567 | 0.173733 | 0.232376 | 0.202039 | 0.212256 | 0.215544 | 0.222111 | 0.249489 | 0.255089 | 0.270311 | 0.260144 | 0.270309 | 0.303452 | 0.303452 | |
| 0.5 | 0.300133 | 0.303489 | 0.313256 | 0.127333 | 0.321078 | 0.318078 | 0.306444 | 0.3005 | 0.3064 | 0.318378 | 0.333444 | 0.355056 | 0.355056 | 0.3589 | 0.350889 | 0.382956 | |
| 0.6 | 0.408444 | 0.398056 | 0.412322 | 0.127333 | 0.422511 | 0.418078 | 0.404433 | 0.399544 | 0.403556 | 0.412789 | 0.425022 | 0.451378 | 0.450811 | 0.473189 | 0.450033 | 0.48757 | |
| 0.7 | 0.528778 | 0.515011 | 0.528111 | 0.127333 | 0.546889 | 0.541784 | 0.5129 | 0.517256 | 0.538811 | 0.539867 | 0.600033 | 0.60005 | 0.60005 | 0.588333 | 0.569787 | 0.597447 | |
| 0.8 | 0.689533 | 0.655322 | 0.673478 | 0.127333 | 0.672556 | 0.6881 | 0.6547 | 0.649511 | 0.657556 | 0.688311 | 0.685233 | 0.71005 | 0.644556 | 0.678011 | 0.703712 | 0.734722 | |
| 0.9 | 0.848456 | 0.848511 | 0.842144 | 0.127333 | 0.861178 | 0.865167 | 0.849322 | 0.833456 | 0.833233 | 0.848867 | 0.864589 | 0.857411 | 0.854456 | 0.878011 | 0.862978 | 0.870678 | |

Table used to generate figure 6:

| Truss Thickness | Relative Density | Reaction Force | E/Es |
|-----------------|------------------|----------------|-------------|
| 0.5 | 0.00725 | 2.3238 | 0.002582 |
| 0.6 | 0.010368 | 3.3713 | 0.003745889 |
| 0.7 | 0.014014 | 4.6073 | 0.005119222 |
| 0.8 | 0.018176 | 6.0712 | 0.006745778 |
| 0.9 | 0.022842 | 7.7281 | 0.008586778 |
| 1 | 0.028 | 9.5742 | 0.010638 |
| 1.1 | 0.033638 | 11.752 | 0.013057778 |
| 1.2 | 0.039744 | 14.044 | 0.015604444 |
| 1.3 | 0.046306 | 16.518 | 0.018353333 |
| 1.4 | 0.053312 | 19.313 | 0.021458889 |
| 1.5 | 0.06075 | 22.269 | 0.024743333 |
| 1.6 | 0.068608 | 25.392 | 0.028213333 |
| 1.7 | 0.076874 | 28.779 | 0.031976667 |
| 1.8 | 0.085536 | 32.345 | 0.035938889 |
| 1.9 | 0.094582 | 36.177 | 0.040196667 |
| 2 | 0.104 | 40.184 | 0.044648889 |
| 2.1 | 0.11378 | 44.412 | 0.049346667 |
| 2.2 | 0.1239 | 48.905 | 0.054338889 |
| 2.3 | 0.13437 | 53.518 | 0.059464444 |
| 2.4 | 0.14515 | 58.38 | 0.064866667 |
| 2.5 | 0.15625 | 63.524 | 0.070582222 |
| 2.6 | 0.16765 | 68.877 | 0.07653 |
| 2.7 | 0.17933 | 74.94 | 0.083266667 |
| 2.8 | 0.1913 | 81.545 | 0.090605556 |
| 2.9 | 0.20352 | 87.937 | 0.097707778 |
| 3 | 0.216 | 94.642 | 0.105157778 |
| 3.1 | 0.22872 | 101.25 | 0.1125 |
| 3.2 | 0.24166 | 108.56 | 0.120622222 |
| 3.3 | 0.25483 | 115.76 | 0.128622222 |
| 3.4 | 0.26819 | 122.25 | 0.135833333 |
| 3.5 | 0.28175 | 130.55 | 0.145055556 |
| 3.6 | 0.29549 | 140.6 | 0.156222222 |
| 3.7 | 0.30939 | 144.84 | 0.160933333 |
| 3.8 | 0.32346 | 153.57 | 0.170633333 |
| 3.9 | 0.33766 | 162.24 | 0.180266667 |
| 4 | 0.352 | 171.75 | 0.190833333 |
| 4.1 | 0.36646 | 181.61 | 0.201788889 |
| 4.2 | 0.38102 | 191.7 | 0.213 |
| 4.3 | 0.39569 | 201.65 | 0.224055556 |
| 4.4 | 0.41043 | 211.64 | 0.235155556 |
| 4.5 | 0.42525 | 221.99 | 0.246655556 |

| | | | |
|-----|---------|--------|-------------|
| 4.6 | 0.44013 | 232.68 | 0.258533333 |
| 4.7 | 0.45505 | 244.11 | 0.271233333 |
| 4.8 | 0.47002 | 256.65 | 0.285166667 |
| 4.9 | 0.485 | 268.98 | 0.298866667 |
| 5 | 0.5 | 281.5 | 0.312777778 |
| 5.1 | 0.515 | 292.64 | 0.325155556 |
| 5.2 | 0.52998 | 305.99 | 0.339988889 |
| 5.3 | 0.54495 | 319.32 | 0.3548 |
| 5.4 | 0.55987 | 332.68 | 0.369644444 |
| 5.5 | 0.57475 | 346.16 | 0.384622222 |
| 5.6 | 0.58957 | 360.27 | 0.4003 |
| 5.7 | 0.60431 | 374.76 | 0.4164 |
| 5.8 | 0.61898 | 389.47 | 0.432744444 |
| 5.9 | 0.63354 | 404.7 | 0.449666667 |
| 6 | 0.648 | 419.43 | 0.466033333 |
| 6.1 | 0.66234 | 434.71 | 0.483011111 |
| 6.2 | 0.67654 | 451.16 | 0.501288889 |
| 6.3 | 0.69061 | 467.42 | 0.519355556 |
| 6.4 | 0.70451 | 484.1 | 0.537888889 |
| 6.5 | 0.71825 | 500.9 | 0.556555556 |
| 6.6 | 0.73181 | 517.35 | 0.574833333 |
| 6.7 | 0.74517 | 533 | 0.592222222 |
| 6.8 | 0.75834 | 549.74 | 0.610822222 |
| 6.9 | 0.77128 | 567.38 | 0.630422222 |
| 7 | 0.784 | 584.88 | 0.649866667 |
| 7.1 | 0.79648 | 601.92 | 0.6688 |
| 7.2 | 0.8087 | 619.57 | 0.688411111 |
| 7.3 | 0.82067 | 637.43 | 0.708255556 |
| 7.4 | 0.83235 | 655.61 | 0.728455556 |
| 7.5 | 0.84375 | 673.68 | 0.748533333 |
| 7.6 | 0.85485 | 692.09 | 0.768988889 |
| 7.7 | 0.86563 | 709.65 | 0.7885 |
| 7.8 | 0.8761 | 726.02 | 0.806888889 |
| 7.9 | 0.88622 | 743.11 | 0.825677778 |
| 8 | 0.896 | 760.91 | 0.845455556 |
| 8.1 | 0.90542 | 777.19 | 0.863544444 |
| 8.2 | 0.91446 | 794.62 | 0.882911111 |
| 8.3 | 0.92313 | 809.4 | 0.899333333 |
| 8.4 | 0.93139 | 825.19 | 0.916877778 |
| 8.5 | 0.93925 | 840.43 | 0.933811111 |
| 8.6 | 0.94669 | 854.68 | 0.949644444 |
| 8.7 | 0.95369 | 869.57 | 0.966188889 |
| 8.8 | 0.96026 | 881.99 | 0.979988889 |
| 8.9 | 0.96636 | 895.33 | 0.994811111 |
| 9 | 0.972 | 907.33 | 1.008144444 |
| 9.1 | 0.97716 | 918.11 | 1.020122222 |
| 9.2 | 0.98182 | 928.48 | 1.031644444 |
| 9.3 | 0.98599 | 937.42 | 1.041577778 |
| 9.4 | 0.98963 | 945.49 | 1.050544444 |
| 9.5 | 0.99275 | 952.68 | 1.058533333 |

Table used to generate figure 7:

| Plate Thickness | Relative Density | Reaction Force | E/Es |
|-----------------|------------------|----------------|-------------|
| 0.5 | 0.14262 | 94.368 | 0.104853333 |
| 0.6 | 0.16942 | 113.01 | 0.125566667 |
| 0.7 | 0.19564 | 131.56 | 0.146177778 |
| 0.8 | 0.22131 | 149.95 | 0.166611111 |
| 0.9 | 0.24643 | 168.28 | 0.186977778 |
| 1 | 0.271 | 186.49 | 0.207211111 |
| 1.1 | 0.29503 | 204.75 | 0.2275 |
| 1.2 | 0.31853 | 222.58 | 0.247311111 |
| 1.3 | 0.3415 | 240.54 | 0.267266667 |
| 1.4 | 0.36394 | 258.55 | 0.287277778 |
| 1.5 | 0.38588 | 276.3 | 0.307 |
| 1.6 | 0.4073 | 293.5 | 0.326111111 |
| 1.7 | 0.42821 | 310.38 | 0.344866667 |
| 1.8 | 0.44863 | 327.24 | 0.3636 |
| 1.9 | 0.46856 | 343.85 | 0.382055556 |
| 2 | 0.488 | 360.27 | 0.4003 |
| 2.1 | 0.50696 | 376.41 | 0.418233333 |
| 2.2 | 0.52545 | 392.33 | 0.435922222 |
| 2.3 | 0.54347 | 408.18 | 0.453533333 |
| 2.4 | 0.56102 | 423.55 | 0.470611111 |
| 2.5 | 0.57813 | 439.41 | 0.488233333 |
| 2.6 | 0.59478 | 454.36 | 0.504844444 |
| 2.7 | 0.61098 | 468.46 | 0.520511111 |
| 2.8 | 0.62675 | 483.53 | 0.537255556 |
| 2.9 | 0.64209 | 498.42 | 0.5538 |
| 3 | 0.657 | 512.63 | 0.569588889 |
| 3.1 | 0.67149 | 527.05 | 0.585611111 |
| 3.2 | 0.68557 | 540.46 | 0.600511111 |
| 3.3 | 0.69924 | 554.6 | 0.616222222 |
| 3.4 | 0.7125 | 568.91 | 0.632122222 |
| 3.5 | 0.72537 | 582.66 | 0.6474 |
| 3.6 | 0.73786 | 595.77 | 0.661966667 |
| 3.7 | 0.74995 | 608.97 | 0.676633333 |
| 3.8 | 0.76167 | 622.7 | 0.691888889 |
| 3.9 | 0.77302 | 635.48 | 0.706088889 |
| 4 | 0.784 | 648.03 | 0.720033333 |
| 4.1 | 0.79462 | 660.67 | 0.734077778 |
| 4.2 | 0.80489 | 672.86 | 0.747622222 |
| 4.3 | 0.81481 | 685.16 | 0.761288889 |

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|-----|---------|--------|-------------|
| 4.4 | 0.82438 | 696.95 | 0.774388889 |
| 4.5 | 0.83362 | 708.83 | 0.787588889 |
| 4.6 | 0.84254 | 720.07 | 0.800077778 |
| 4.7 | 0.85112 | 731.2 | 0.812444444 |
| 4.8 | 0.85939 | 742.34 | 0.824822222 |
| 4.9 | 0.86735 | 753.78 | 0.837533333 |
| 5 | 0.875 | 765.23 | 0.850255556 |
| 5.1 | 0.88235 | 775.49 | 0.861655556 |
| 5.2 | 0.88941 | 785.67 | 0.872966667 |
| 5.3 | 0.89618 | 795.07 | 0.883411111 |
| 5.4 | 0.90266 | 804.14 | 0.893488889 |
| 5.5 | 0.90888 | 812.79 | 0.9031 |
| 5.6 | 0.91482 | 821.62 | 0.912911111 |
| 5.7 | 0.92049 | 830.21 | 0.922455556 |
| 5.8 | 0.92591 | 839.25 | 0.9325 |
| 5.9 | 0.93108 | 847.48 | 0.941644444 |
| 6 | 0.936 | 855.56 | 0.950622222 |
| 6.1 | 0.94068 | 863.23 | 0.959144444 |
| 6.2 | 0.94513 | 870.62 | 0.967355556 |
| 6.3 | 0.94935 | 877.6 | 0.975111111 |
| 6.4 | 0.95334 | 883.64 | 0.981822222 |
| 6.5 | 0.95712 | 890.16 | 0.989066667 |
| 6.6 | 0.9607 | 897.41 | 0.997122222 |
| 6.7 | 0.96406 | 903.04 | 1.003377778 |
| 6.8 | 0.96723 | 909.16 | 1.010177778 |
| 6.9 | 0.97021 | 914.43 | 1.016033333 |
| 7 | 0.973 | 919.57 | 1.021744444 |
| 7.1 | 0.97561 | 924.44 | 1.027155556 |
| 7.2 | 0.97805 | 928.99 | 1.032211111 |
| 7.3 | 0.98032 | 933.34 | 1.037044444 |
| 7.4 | 0.98242 | 937.31 | 1.041455556 |
| 7.5 | 0.98437 | 941.43 | 1.046033333 |
| 7.6 | 0.98618 | 944.81 | 1.049788889 |
| 7.7 | 0.98783 | 947.97 | 1.0533 |
| 7.8 | 0.98935 | 950.93 | 1.056588889 |
| 7.9 | 0.99074 | 953.67 | 1.059633333 |
| 8 | 0.992 | 956.28 | 1.062533333 |
| 8.1 | 0.99314 | 958.56 | 1.065066667 |
| 8.2 | 0.99417 | 960.57 | 1.0673 |
| 8.3 | 0.99509 | 962.34 | 1.069266667 |
| 8.4 | 0.9959 | 963.93 | 1.071033333 |
| 8.5 | 0.99662 | 965.47 | 1.072744444 |
| 8.6 | 0.99726 | 966.81 | 1.074233333 |
| 8.7 | 0.9978 | 967.96 | 1.075511111 |
| 8.8 | 0.99827 | 968.76 | 1.0764 |
| 8.9 | 0.99867 | 969.68 | 1.077422222 |
| 9 | 0.999 | 970.38 | 1.0782 |
| 9.1 | 0.99927 | 970.93 | 1.078811111 |
| 9.2 | 0.99949 | 971.42 | 1.079355556 |
| 9.3 | 0.99966 | 971.78 | 1.079755556 |
| 9.4 | 0.99978 | 972.03 | 1.080033333 |
| 9.5 | 0.99987 | 972.19 | 1.080211111 |