$$V_{iu} \approx (WH)_{iu} = \sum_{a=1}^{r} W_{ia} H_{au}$$

One possible objective function

Reconstruction error:

$$\arg\min_{W,H} E_r = \|V - WH\|^2, \text{s.t. } W, H \geq 0$$

Update rule:

$$H_{au} \leftarrow H_{au} \sum_i W_{ia} \frac{V_{iu}}{(WH)_{iu}}$$

- H_{au}: update a^{th} coefficient for the u^{th} face
- W_{ia}: a^{th} basis
- $\sum_i$: sum over all pixels
- V_{iu}: projection for i^{th} pixel
- $\frac{V_{iu}}{(WH)_{iu}}$: ratio of actual to reconstructed pixel value for the u^{th} face

$$W_{ia} \leftarrow W_{ia} \sum_u \frac{V_{iu}}{(WH)_{iu}} H_{au}$$

$$W_{ia} \leftarrow \frac{W_{ia}}{\sum_j W_{ja}}$$

Normalize

$$\arg\min_{W,H} E_r = \|V - WH\|^2, \text{s.t. } W, H \geq 0$$

$$V_{iu} \approx (WH)_{iu} = \sum_{a=1}^{r} W_{ia} H_{au}$$

One possible objective function

Update rule:

$$H_{au} \leftarrow H_{au} \sum_i W_{ia} \frac{V_{iu}}{(WH)_{iu}}$$

$$W_{ia} \leftarrow W_{ia} \sum_u \frac{V_{iu}}{(WH)_{iu}} H_{au}$$

$$W_{ia} \leftarrow \frac{W_{ia}}{\sum_j W_{ja}}$$

Normalize

a^{th} basis projection for i^{th} pixel

update a^{th} coefficient for the u^{th} face

sum over all pixels

ratio of actual to reconstructed pixel value for the u^{th} face

Basic idea: multiply current value by a factor depending on the quality of the approximation.

If ratio > 1, then we need to increase denominator.

If ratio < 1, then we need to decrease denominator.

If ratio = 1, do nothing.