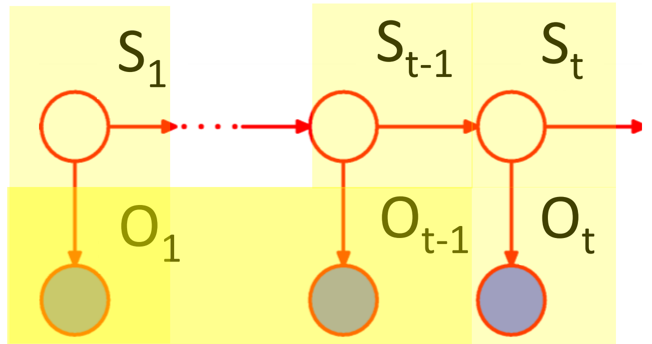
$$p(\{O_t\}_{t=1}^T) = \sum_k p(\{O_t\}_{t=1}^T, S_T = k) = \sum_k \alpha_T^k$$

Compute forward probability α_t^k recursively over t



$$\alpha_t^k \quad := \quad p(O_1, \ldots, O_t, S_t = k)$$

Introduce S_{t-1}

Chain rule

Markov assumption

$$= \quad p(O_t | S_t = k) \sum_i \alpha_{t-1}^i p(S_t = k | S_{t-1} = i)$$