$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{pmatrix} \cdot \begin{pmatrix} u \\ v \end{pmatrix}$$

$$\dot{\vec{x}} = \overleftrightarrow{A}\vec{x} \qquad \overleftrightarrow{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad \text{let } \vec{x}(t) = e^{\lambda t}\vec{v}$$

$$\lambda e^{\lambda t}\vec{v} = \overleftrightarrow{A} e^{\lambda t}\vec{v} \qquad \lambda\vec{v} = \overleftrightarrow{A}\vec{v}$$

$$(\overleftrightarrow{A} - \lambda I)\vec{v} = 0$$

$$\begin{pmatrix} a - \lambda & b \\ c & d - \lambda \end{pmatrix}\vec{v} = 0$$

$$\det\begin{pmatrix} a - \lambda & b \\ c & d - \lambda \end{pmatrix} = 0$$

$$(a - \lambda)(d - \lambda) - bc = 0$$

$$\lambda^2 - \tau\lambda + \Delta = 0$$

$$\lambda_1 = \frac{\tau + \sqrt{\tau^2 - 4\Delta}}{2} \qquad \lambda_2 = \frac{\tau - \sqrt{\tau^2 - 4\Delta}}{2}$$

τ=trace

Δ=determinant

If $\lambda_1 \neq \lambda_2$, then v_1 & v_2 are linearly independent and solutions of the following form are valid.

$$\vec{x}(t) = c_1 e^{\lambda_1 t}\vec{v}_1 + c_2 e^{\lambda_2 t}\vec{v}_2$$