

Let $\begin{cases} \dot{x} = f(x, y) \\ \dot{y} = g(x, y) \end{cases}$ be a non - linear system with fixed point (x^*, y^*)

$$0 = f(x^*, y^*) = g(x^*, y^*)$$

Let $\begin{cases} u = x - x^* \\ v = y - y^* \end{cases}$ be deviations from fixed point

↓ Change of variable

$$\begin{aligned} \dot{u} &= \dot{x} \quad (x^* \text{ is constant}) \\ &= f(u + x^*, v + y^*) \\ &= f(x^*, y^*) + u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y} + O(u^2, v^2, uv) \end{aligned} \quad \begin{array}{l} \text{linear} \\ \text{Taylor series expansion} \end{array}$$

Likewise, $\dot{v} = u \frac{\partial g}{\partial x} + v \frac{\partial g}{\partial y} + O(u^2, v^2, uv)$