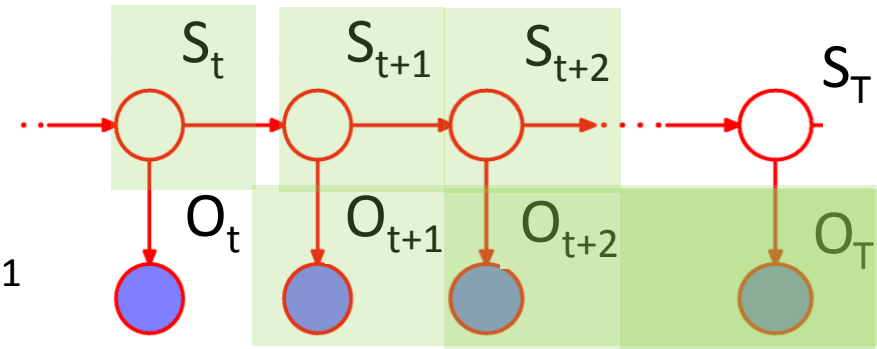
$$p(S_t = k, \{O_t\}_{t=1}^T) = p(O_1, \ldots, O_t, S_t = k) p(O_{t+1}, \ldots, O_T | S_t = k) = \alpha_t^k \beta_t^k$$

Compute forward probability β_t^k recursively over t

$$\beta_t^k := p(O_{t+1}, \ldots, O_T | S_t = k)$$



Introduce S_{t+1}

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Chain rule

Markov assumption

$$= \sum_i p(S_{t+1} = i | S_t = k) p(O_{t+1} | S_{t+1} = i) \beta_{t+1}^i$$