

Computer code for $f(x_1, x_2) = x_1 x_2 + \sin(x_1)$ might read

Original program

$$w_1 = x_1$$

$$w_2 = x_2$$

$$w_3 = w_1 w_2$$

$$w_4 = \sin(w_1)$$

$$w_5 = w_3 + w_4$$

Dual program

$$\dot{w}_1 = 0$$

$$\dot{w}_2 = 1$$

$$\dot{w}_3 = \dot{w}_1 w_2 + w_1 \dot{w}_2 = 0 \cdot x_2 + x_1 \cdot 1 = x_1$$

$$\dot{w}_4 = \cos(w_1) \dot{w}_1 = \cos(x_1) \cdot 0 = 0$$

$$\dot{w}_5 = \dot{w}_3 + \dot{w}_4 = x_1 + 0 = x_1$$

and

$$\frac{\partial f}{\partial x_2} = x_1$$

The chain rule

$$\frac{\partial f}{\partial x_2} = \frac{\partial f}{\partial w_5} \frac{\partial w_5}{\partial w_3} \frac{\partial w_3}{\partial w_2} \frac{\partial w_2}{\partial x_2}$$

ensures that we can *propagate* the dual components throughout the computation.