Given HMM parameters $p(S_1), p(S_t|S_{t-1}), p(O_t|S_t)$ & observation sequence $\{O_t\}_{t=1}^T$

find probability that hidden state at time t was k $p(S_t = k|\{O_t\}_{t=1}^T)$

$$
\begin{aligned}
p(S_t = k, \{O_t\}_{t=1}^T) &= p(O_1, \ldots, O_t, S_t = k, O_{t+1}, \ldots, O_T) \\
&= p(O_1, \ldots, O_t, S_t = k)p(O_{t+1}, \ldots, O_T|S_t = k)
\end{aligned}
$$

Compute recursively

α_t^k $\qquad$ β_t^k