

Let

$$P(x) = p_0 + p_1x + p_2x^2 + \cdots + p_nx^n$$

and extend x to a dual number $x + \dot{x}\mathbf{d}$.

Then,

$$\begin{aligned}P(x + \dot{x}\mathbf{d}) &= p_0 + p_1(x + \dot{x}\mathbf{d}) + \cdots + p_n(x + \dot{x}\mathbf{d})^n \\&= p_0 + p_1x + p_2x^2 + \cdots + p_nx^n \\&\quad + p_1\dot{x}\mathbf{d} + 2p_2x\dot{x}\mathbf{d} + \cdots + np_nx^{n-1}\dot{x}\mathbf{d} \\&= P(x) + P'(x)\dot{x}\mathbf{d}\end{aligned}$$

- ▶ \dot{x} may be chosen arbitrarily, so choose $\dot{x} = 1$ (currently).
- ▶ *The second component is the derivative of $P(x)$ at x*