Extend all numbers by adding a second component,

$$x \mapsto x + \dot{x}\mathbf{d}$$

- $\mathbf{d}$ is just a symbol distinguishing the second component,
- analogous to the imaginary unit $\mathbf{i} = \sqrt{-1}$.
- But, let $\mathbf{d}^2 = 0$, as opposed to $\mathbf{i}^2 = -1$.

Arithmetic on dual numbers:

$$(x + \dot{x}\mathbf{d}) + (y + \dot{y}\mathbf{d}) = x + y + (\dot{x} + \dot{y})\mathbf{d}$$

$$\begin{aligned}(x + \dot{x}\mathbf{d}) \cdot (y + \dot{y}\mathbf{d}) &= xy + x\dot{y}\mathbf{d} + \dot{x}y\mathbf{d} + \overbrace{\dot{x}\dot{y}\mathbf{d}^2}^{=0} \\ &= xy + (x\dot{y} + \dot{x}y)\mathbf{d}\end{aligned}$$

$$-(x + \dot{x}\mathbf{d}) = -x - \dot{x}\mathbf{d}, \qquad \frac{1}{x + \dot{x}\mathbf{d}} = \frac{1}{x} - \frac{\dot{x}}{x^2}\mathbf{d} \quad (x \neq 0)$$