

The optimal model will have enough components to accurately fit data and be predictive, but remain simple enough for interpretation. Additionally, the model is subject to over-fitting constraints.

Three metrics are used to evaluate the utility of adding a new component (a):

$R^2X$ : sum of squares for the variation in the  $\mathbf{X}$  matrix

$$R^2X = 1 - \frac{\sum (X_{\text{model},a} - X_{\text{obs}})^2}{\sum (X_{\text{obs}}^2)}$$

$R^2Y$ : sum of squares for the variation in the  $\mathbf{Y}$  matrix

$$R^2Y = 1 - \frac{\sum (Y_{\text{model},a} - Y_{\text{obs}})^2}{\sum (Y_{\text{obs}}^2)}$$

$Q^2Y$ : fraction of the total variation in the  $\mathbf{Y}$  matrix that can be predicted

$$Q^2Y = [1.0 - \Pi(\text{PRESS}/\text{SS})_a]$$

PRESS = Prediction Error Sum of Squares

1) Remove an individual data element (i,k)

2) Fit model

3) Predict the element i,k that was withheld

$$(\text{observed}_{i,k} - \text{predicted}_{i,k})^2$$

4) Repeat until each element has been withheld once and only once