

- **E-step** – Fix parameters, find expected state assignments

$$\gamma_i(t) = p(S_t = i | O, \theta) = \frac{\alpha_t^i \beta_t^i}{\sum_j \alpha_t^j \beta_t^j}$$

**Forward-Backward algorithm**

$$\begin{aligned} \xi_{ij}(t) &= p(S_{t-1} = i, S_t = j | O, \theta) \\ &= \frac{p(S_{t-1} = i | O, \theta) p(S_t = j, O_t, \dots, O_T | S_{t-1} = i, \theta)}{p(O_t, \dots, O_T | S_{t-1} = i, \theta)} \\ &= \frac{\gamma_i(t-1) p_{ij} q_j^{O_t} \beta_t^j}{\beta_{t-1}^i} \end{aligned}$$