

# Updated SLR definition and additional metrics on herbivory

Jade, Jul 16

To make our metrics comparable to Neo's, from now on I will focus on subsets of the food web where all species are connected to at least one other species. In other words, plant species that are not eaten by any other species will be left out in calculating SLR. In addition, metrics on herbivory are calculated: 1) the fraction in number of plant species that are eaten by other species (herbivore or omnivores); 2) the fraction in plant biomass that is eaten by other species. The effect of the key parameters ( $D_r$ ,  $\gamma$  and  $\tau_u$ ) on the metrics will be tested.

## 1 Result

The algorithm has been improved so that it takes 1/4 the time to simulate a community of the same size. A wider range for  $\gamma$  and  $D_r$  as well as a larger community size can thus be used (previously it takes  $> 3$  hours to simulate a 30 species community with a  $\gamma < 0.1$  and  $D_r < 0.3$ ).

## 1.1 $D_r$ and $\gamma$ effect

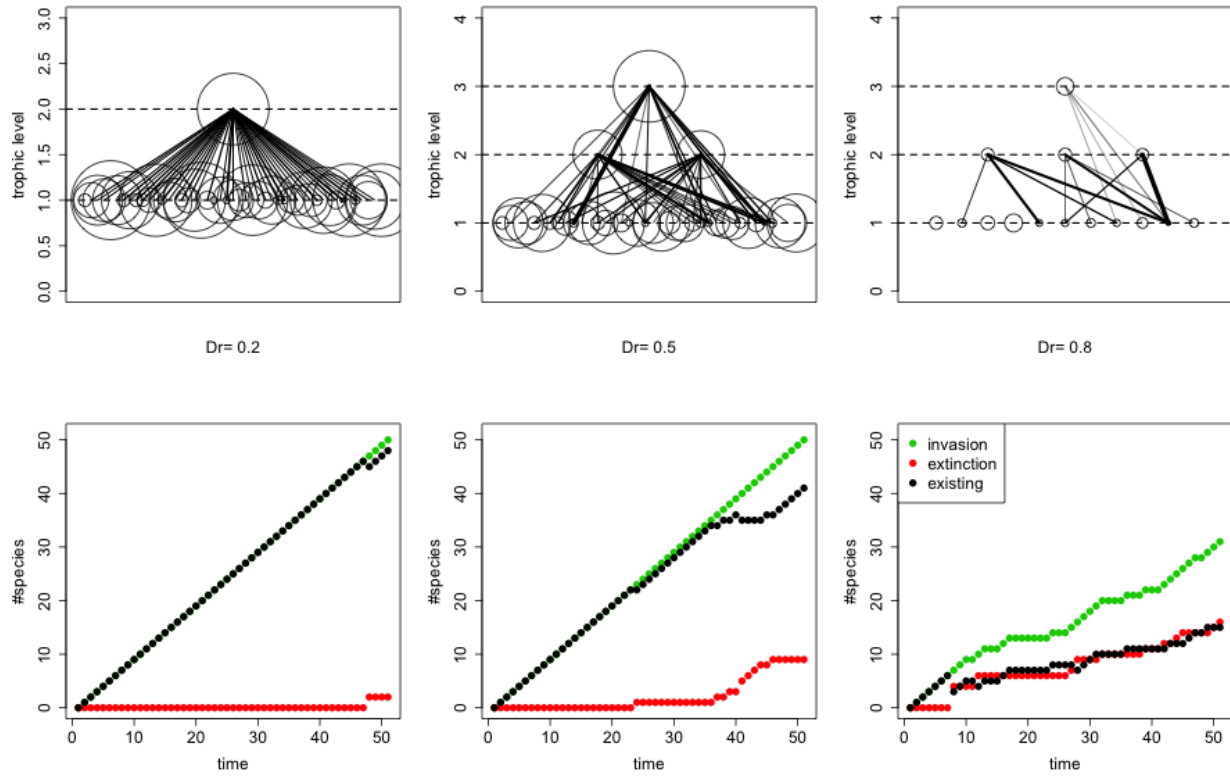
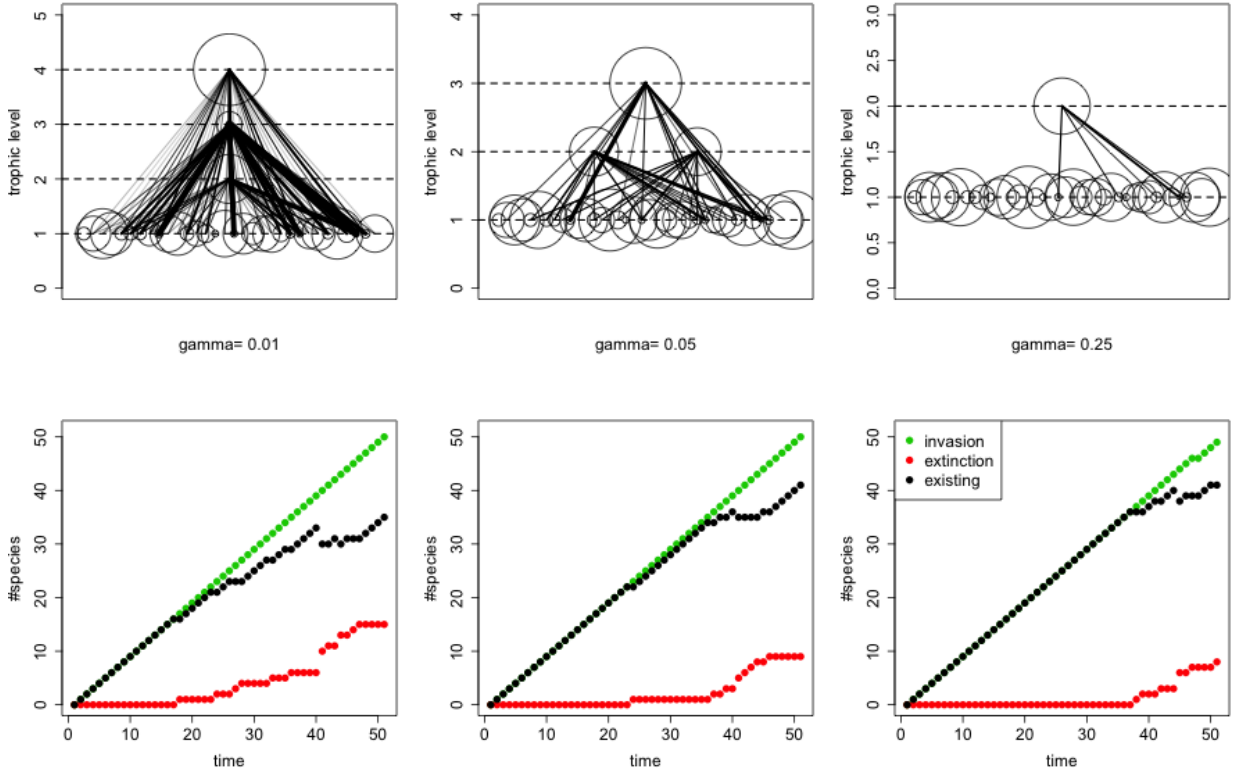


Fig. 1 The effect of  $D_r$  on the food web structure



**Fig. 2** The effect of  $D_r$  on the food web structure

With a wider range of  $\gamma$  for comparison we can see a pattern that was not clear before: the higher the  $\gamma$ , the higher the number of species eventually included in the community. Since a higher  $\gamma$  also leads to fewer trophic links, this result suggests a bigger contrast in the slope of the link-species relationship between different  $\gamma$ s than previously inferred.

## 1.2 Link-Species Relationship

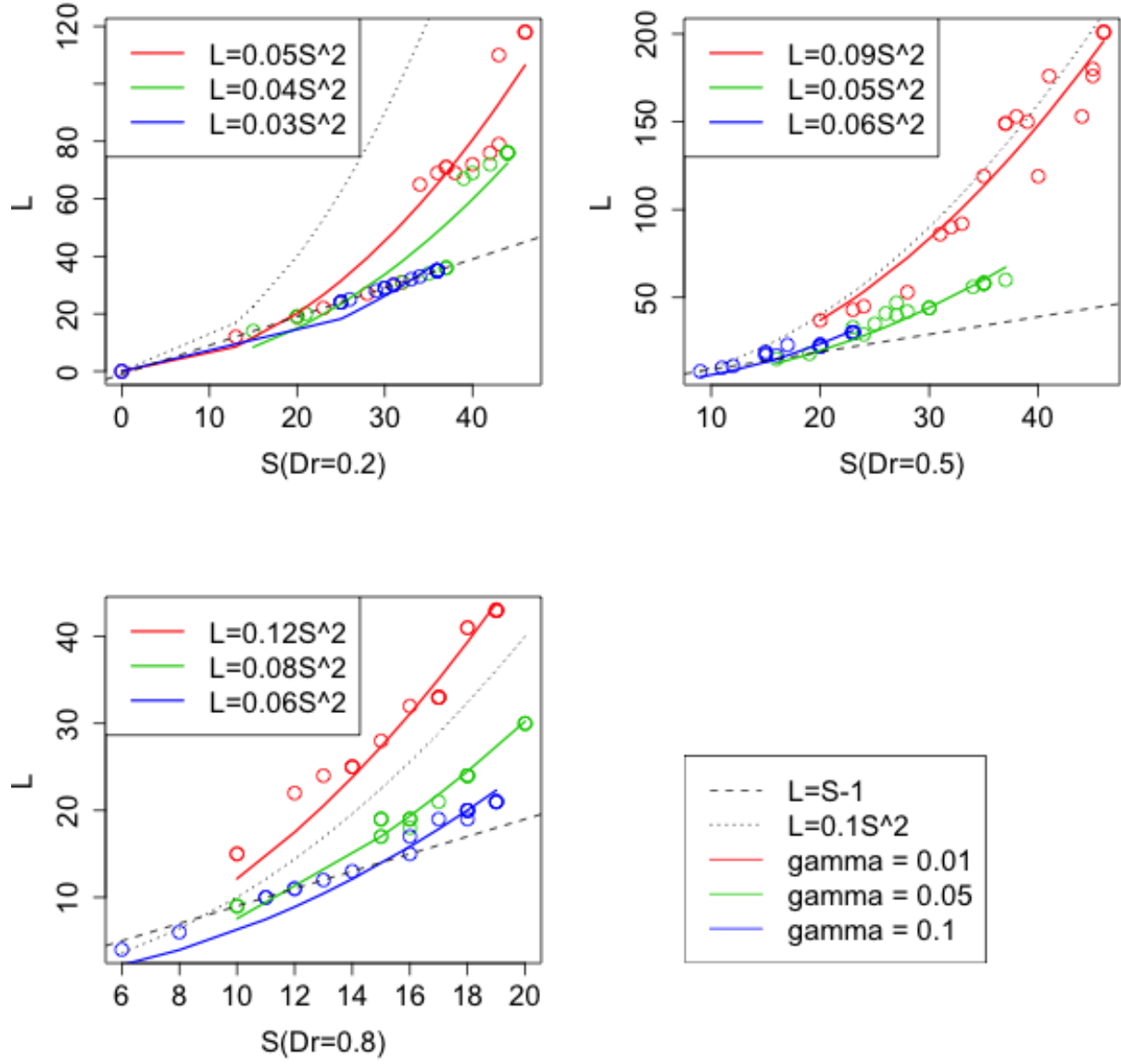
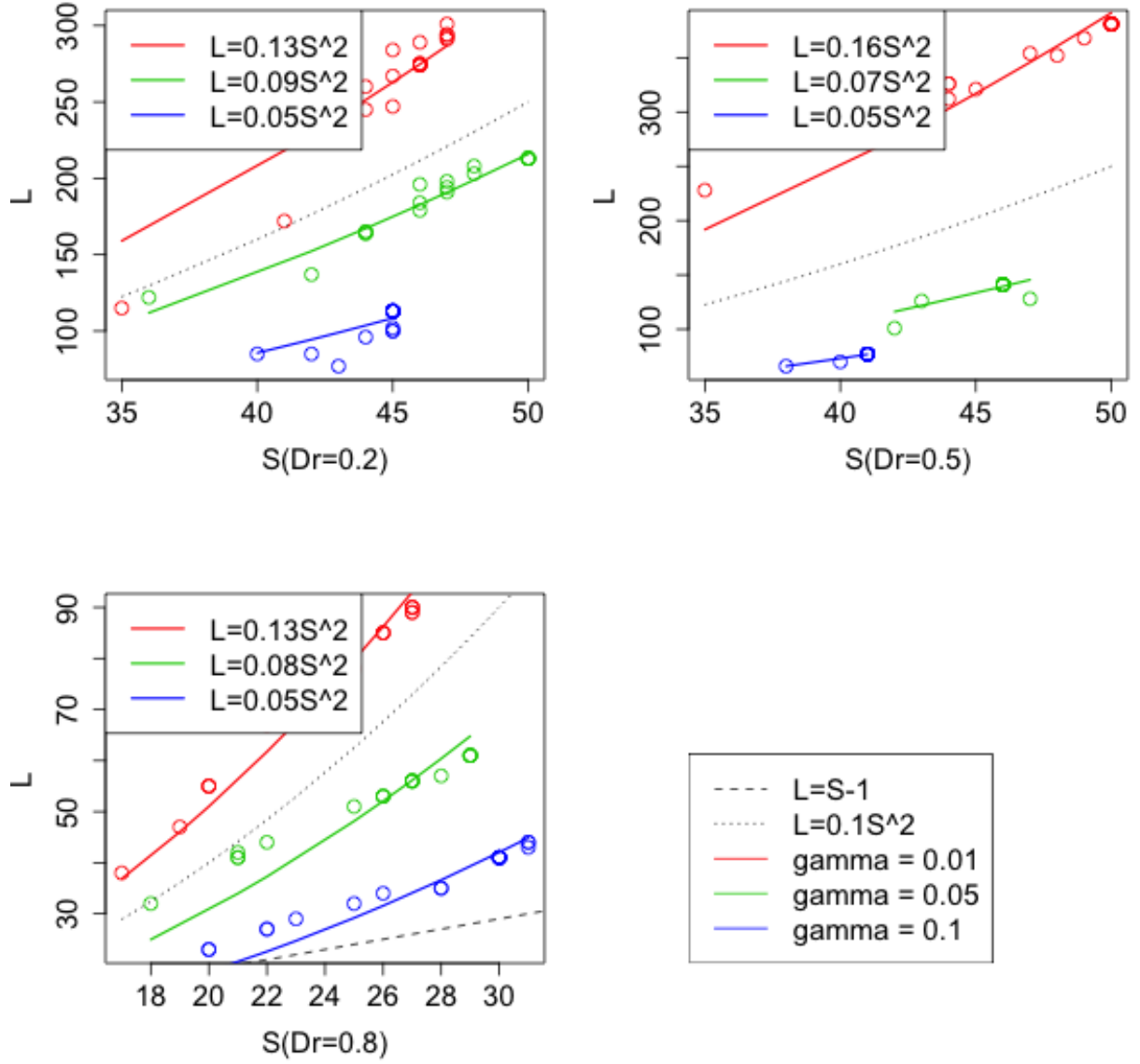


Fig. 3 Link-species relationship ( $\tau_u = 0.1$ )



**Fig. 4 Link-species relationship** ( $\tau_u = 0.15$ )

With the updated definition of LSR (not counting plant species that are not eaten by any species), the slope is lower (compared to Scenario 1 in July 3 write-up) and in general more quadratic ( $L = aS^2$ ) than linear. As before, the higher the  $\tau_u$ , the steeper the LSR (bigger  $a$ ). When  $\tau_u = 0.1$ , the

effects of  $\gamma$  and  $D_r$  are similar as before: higher  $\gamma$  leads to slower (smaller  $a$ ) while higher  $D_r$  leads to steeper LSR (bigger  $a$ ). When  $\tau_u = 0.15$ , while the effect of  $\gamma$  is the same, the effect of  $D_r$  is not significant.

Notice that for  $\tau_u = 0.1$  and  $\gamma = 0.1$ , some of the communities have dropped below the lower boundary, or  $L < S - 1$ . This is because for these communities, there are independent components that are not connected to each other. For example, if there is a herbivore not preyed by any predator and feeds on a plant species that is not consumed by any other herbivores, these two species form a component that is not connected to any other species in the food web. Allowing this kind of situation, the minimum number of links a food web with  $S$  species has is  $L_{min} = S/2$ , in which case the food web is separated into  $S/2$  independent components each with two species connected to each other but not to any other species. But as we can see, that seldom happens.

### 1.3 Herbivory fractions

Here I have calculated two fractions about herbivory: first, the fraction in number of plant species that are preyed on by other species (herbivore or omnivores); second, the fraction in plant biomass that is eaten by other species. In the following I will address the former as  $F_1$  while the latter as  $F_2$ .

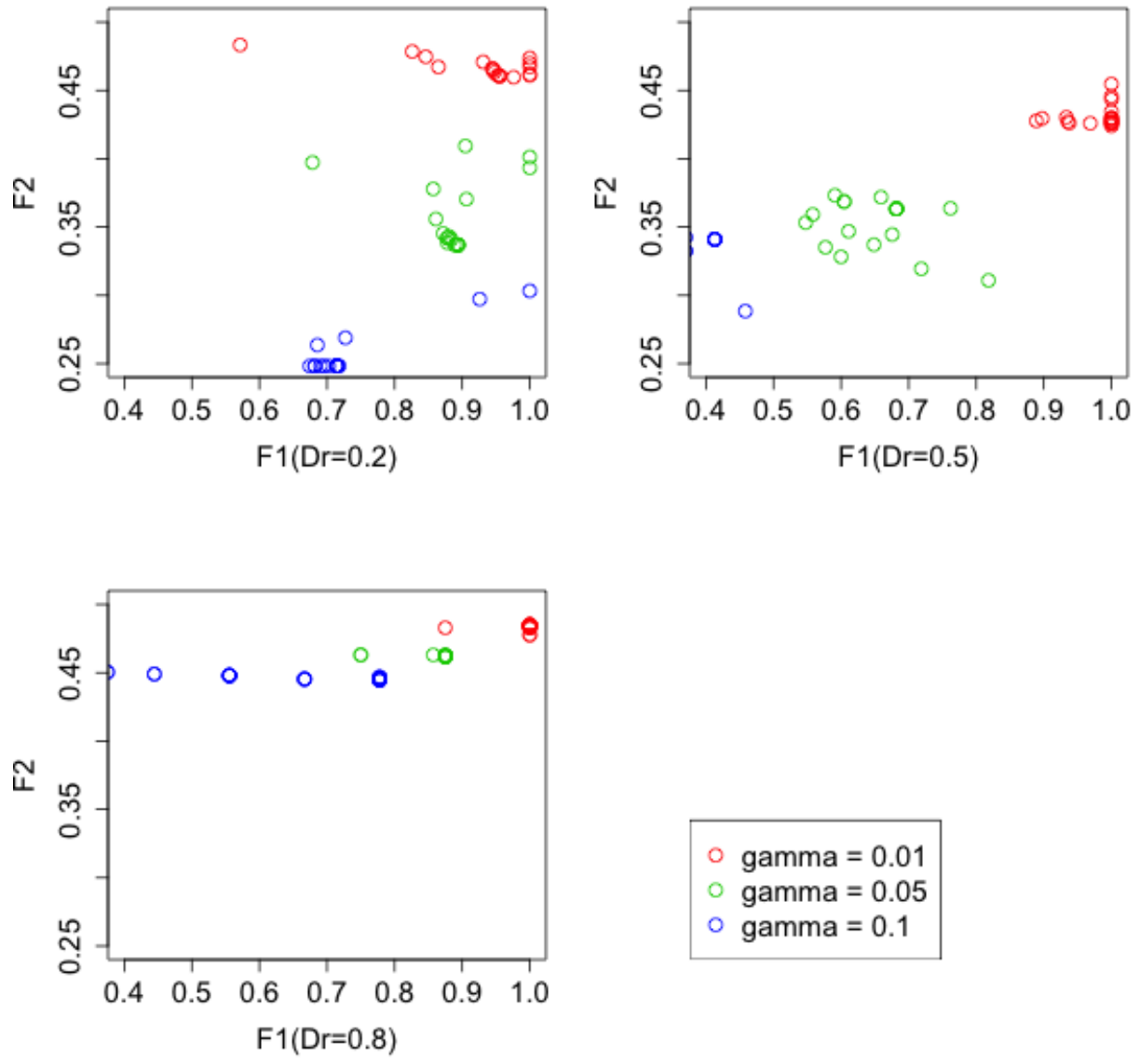
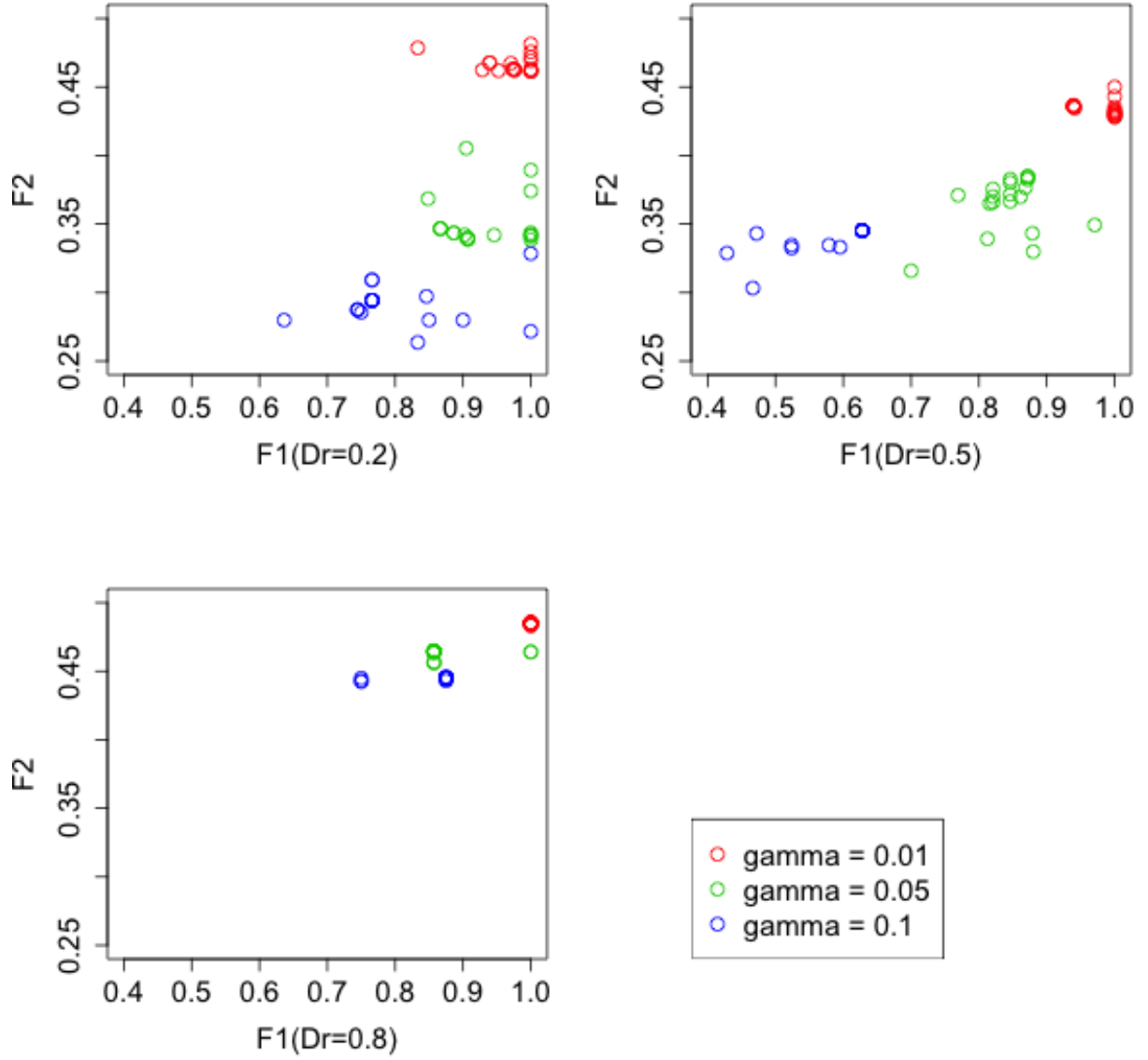


Fig. 5 Herbivory fraction mapping ( $\tau_u = 0.1$ )



**Fig. 6 Herbivory fraction mapping** ( $\tau_u = 0.15$ )

From the graphs we can see that  $F_1$  and  $F_2$  are in generally positive correlation (although not significant when  $D_r = 0.2$ ). The most striking pattern is that the bigger the  $\gamma$ , the lower the  $F_2$ , which is indicated by the clear vertical separation of dots in different colors (different  $\gamma$ s). The effect



of  $\gamma$  on  $F_1$  is less significant: bigger  $\gamma$  leads to significantly lower  $F_1$  only when  $D_r$  is 0.5 or 0.8 but not when  $D_r = 0.2$ . The effect of  $D_r$  is in general less straightforward: it depends on the value of  $\gamma$  and is not always linear, e.g. when  $\gamma = 0.01$ , the lowest  $F_2$  appears for  $D_r = 0.5$  instead of when  $D_r$  is the lowest ( $= 0.2$ ) or the highest ( $= 0.8$ ). One less ambiguous pattern is that  $F_1$  and  $F_2$  are both the highest when  $D_r$  is high.

With this result, the high  $F_2$  (fraction being eaten in total plant biomass) in the tropics can be associated with a low  $\gamma$  or non-intermediate  $D_r$  (high or low). The issue is complicated by the in conceptual ambiguity of  $\gamma$  and  $D_r$  (please see discussion 2.2 below).

## 2 Discussion

### 2.1 Analytical approximation of the mean number of prey for each species

The analytical solution of MERA to the steady state abundance is:

$$SSN_i = \sum_p^{P_i} (C'_p \theta_{c,i})^{\frac{1}{D_{r,i}-1}} \quad (1)$$

where  $P_i$  is the total number of prey species for species  $i$ ,  $\theta_{c,i}$  is the capture-adjusted resource requirement, defined as

$$\theta_{c,i} = \frac{\theta_i}{\tau_{c,i}} = \theta_i e^{\gamma \sum_p^{P_i} D_{r,p}} \quad (2)$$

Assuming all species have the same  $\theta$  and  $D_r$  (and therefore the  $i$  subscript will be suppressed), from Eqs. 1 and 2 we get:

$$SSN = P(C'\theta_i e^{\gamma P D_r})^{\frac{1}{D_r-1}} \quad (3)$$

To maximize  $SSN$ , I equal the derivative of  $SSN$  over  $P$  to zero:

$$\frac{\partial SSN}{\partial P} = \frac{P\gamma D_r}{D_r - 1} (C'\theta_i e^{\gamma P D_r})^{\frac{1}{D_r-1}} + (C'\theta_i e^{\gamma P D_r})^{\frac{1}{D_r-1}} = 0 \quad (4)$$

From which we can solve for the  $P$  that maximizes  $SSN_i$ :

$$P_{optimal} = \frac{1 - D_r}{\gamma D_r} \quad (5)$$

We can see that, although derived from an approximation that all species have the same  $\theta$  and  $D_r$ , Eq. 5 is consistent with the general patterns demonstrated in the simulation: the smaller the  $\gamma$  and  $D_r$ , the more preys a species tends to have and therefore better connected the food web is.

The more accurate measure of connectedness should be the average number of prey per species. From the result section 1.1 we can see that a higher  $\gamma$  leads to a higher total number of species. This means that the effect of  $\gamma$  on the overall connectedness (indicated by the number of prey per species) of the food web is unambiguously negative: a smaller  $\gamma$  will always result in a better connected food web because there are fewer species with a higher optimal number of prey; a bigger  $\gamma$  will result in a poorly connected food web because there are more species with a lower optimal number of prey.

In contrast, the effect of  $D_r$  on the overall connectedness of the food web is unclear. With  $D_r$  increasing, both the number of species and the optimal number of prey decreases, generating opposing effect on the number of prey per species. This might be the reason why the metrics respond to  $D_r$  in less straightforward and non-linear ways.

## 2.2 Are $\gamma$ and $D_r$ independent?

Here is a conceptual conundrum: if we accept the interpretation of  $D_r$  as the level of generalization (so that generalists have higher  $D_r$  than specialists), it seems that  $\gamma$  is not independent from  $D_r$  but rather correlated. To see this, recall that by definition, a higher  $\gamma$  means higher cost in extending food source to include one additional species. The question is, should this cost be higher, lower or the same for specialists compared to generalists?

Naturally we would think it's higher, since specialists are less diverse or flexible to prey on different species. If that is the case, why is it that not all species are generalists with higher  $D_r$  when it means all gains, i.e. higher competitiveness, less chance to be preyed on (since it causes higher cost in capture efficiency for the predator) and less cost in having more prey species (and therefore access to more resource) and no downsides at all? The upside of being a specialists with lower  $D_r$  has not been reflected in the current formula anywhere.

Whatever the answer might be, clarifying the interpretations and

underlying connections between  $\gamma$  and  $D_r$  will enormously help reduce the uncertainty in model assumptions and facilitate future empirical tests.