# Predator prey interaction with new MERA

Jade, Oct 30

Partial redistribution does not apply here!!! Population size has to be able to drop!!!

## 1 Growth functions for one predator preying on one prey

With the new MERA framework we have proved that the growth function for one species without competitors or predators is

$$\begin{aligned} g &= \frac{r(1-\frac{N}{\hat{N}})}{1+(r-1)\frac{N}{\hat{N}}} \qquad \text{(partial redistribution)} \\ &= \frac{r(1-\frac{N}{\hat{N}})}{1+r\frac{N}{\hat{N}}} \qquad \text{(complete redistribution)} \end{aligned} \tag{1}$$

We will look at partial redistribution scenario first. In this scenario, $\hat{N} = R_0/\theta$, therefore

$$g = \frac{r(R_0 - \theta N)}{R_0 + (r-1)\theta N} \tag{2}$$

In all following equations, subscript $p$ indicates prey variables while subscript $P$ indicates predator variables.

For the predator species, $R_0$ is the total amount of prey biomass available,

$$R_{0,P} = \theta_p N_p \tag{3}$$

$$g_P = \frac{r_P(\theta_p N_p - \theta_P N_P)}{\theta_p N_p + (r_P - 1)\theta_P N_P} \tag{4}$$

The prey biomass actually consumed by the predator $C_p$ is

$$\begin{aligned} C_p &= \theta_P N_P g_P \\ &= \frac{r_P \theta_P N_P(\theta_p N_p - \theta_P N_P)}{\theta_p N_p + (r_P - 1)\theta_P N_P} \end{aligned} \tag{5}$$

Before predation happens, the amount of resource the prey species obtains for its net growth is

$$R_p = \frac{r_p \theta_p N_p (R_0 - \theta_p N_p)}{R_0 + (r_p - 1)\theta_p N_p} \tag{6}$$

After predation, the amount of prey biomass consumed by the predator $C_p$ has to be subtracted :

$$\begin{aligned} g_p &= \frac{R_p - C_p}{\theta_p N_p} \\ &= \frac{r_p(R_0 - \theta_p N_p)}{R_0 + (r_p - 1)\theta_p N_p} - \frac{r_P \theta_P N_P(\theta_p N_p - \theta_P N_P)}{\theta_p N_p[\theta_p N_p + (r_P - 1)\theta_P N_P]} \end{aligned} \tag{7}$$

Eqs 4 and 7 can be simplified into

$$g_P = \frac{r_P(1 - \theta \frac{N_P}{N_p})}{1 + (r_P - 1)\theta \frac{N_P}{N_p}} \tag{8}$$

$$g_p = \frac{r_p(1-\frac{N_p}{\hat{N}_p})}{1+(r_p-1)\frac{N_p}{\hat{N}_p}} - \frac{r_P\theta(1-\theta\frac{N_P}{N_p})}{\frac{N_p}{N_P}+\theta(r_P-1)} \tag{9}$$

Where $\theta = \theta_P/\theta_p$, $\hat{N}_p = R_0/\theta_p$. Since $g_P = \frac{dN_P}{N_P dt}$ and $g_p = \frac{dN_p}{N_p dt}$, they each correspond to:

$$\frac{dN_P}{dt} = \frac{r_P N_P(1-\theta\frac{N_P}{N_p})}{1+(r_P-1)\theta\frac{N_P}{N_p}} \tag{10}$$

$$\frac{dN_p}{dt} = \frac{r_p N_p(1-\frac{N_p}{\hat{N}_p})}{1+(r_p-1)\frac{N_p}{\hat{N}_p}} - \frac{r_P N_p\theta(1-\theta\frac{N_P}{N_p})}{\frac{N_p}{N_P}+\theta(r_P-1)} \tag{11}$$

Similarly for the complete redistribution scenario,

Prey has to minus not just the growth but the total consumptions!!

$$\frac{dN_P}{dt} = \frac{r_P N_P(1-\theta\frac{(r_P+1)N_P}{r_P N_p})}{1+(r_P+1)\theta\frac{N_P}{N_p}} \tag{12}$$

$$\frac{dN_p}{dt} = \frac{r_p N_p(1-\frac{N_p}{\hat{N}_p})}{1+r_p\frac{N_p}{\hat{N}_p}} - N_P\theta \tag{13}$$

At the special case where $r_P = r_p = 1$, Eqs. 10 and 11 are simplified into

$$\frac{dN_P}{dt} = N_P(1-\theta\frac{N_P}{N_p}) \tag{14}$$

$$\frac{dN_p}{dt} = N_p(1-\frac{N_p}{\hat{N}_p}) - N_P\theta(1-\theta\frac{N_P}{N_p}) \tag{15}$$