# Predator prey interaction with MERA III

Jade, Oct 31

## 1 Growth functions for one predator feeding on one prey

With the MERA III framework (write-up Oct 13) I have proved that the growth function for one species without competitors or predators is

$$\begin{aligned} g &= \frac{r(1-\frac{N}{\hat{N}})}{1+(r-1)\frac{N}{\hat{N}}} \qquad \text{(partial redistribution)} \\ &= \frac{r(1-\frac{N}{\hat{N}})}{1+r\frac{N}{\hat{N}}} \qquad \text{(complete redistribution)} \end{aligned} \tag{1}$$

For the predator, it has to be a complete redistribution scenario (please refer to Oct 13 write up for definitions; as opposed to the complete redistribution scenario, partial redistribution does not allow abundance to decrease since all resources obtained by the existing population cannot be lost). In this scenario, $\hat{N} = \frac{rR_0}{(r+1)\theta}$, therefore

$$g = \frac{rR_0 - (r+1)\theta N}{R_0 + (r+1)\theta N} \tag{2}$$

By convention of predator prey models, in all following equations, $X$ represents the abundance of the the prey while $Y$ represents the abundance of the predator; subscript $x$ indicates prey variables while subscript $y$ indicates predator variables.

First I will derive the growth function for the predator species. $R_{0,y}$ is the total amount of prey biomass available for the predator,

$$R_{0,y} = \theta_x X \tag{3}$$

Substituting into Eq. 2, the per capita net growth rate of the predator $g_y$ can be expressed as:

$$g_y = \frac{r_y \theta_x X - (r_y + 1)\theta_y Y}{\theta_x X + (r_y + 1)\theta_y Y} \tag{4}$$

For the prey, here we assume that the prey also follows a complete redistribution scenario (but not necessarily, see Appendix for the partial scenario). Before predation happens, the per capita net growth rate of the prey $g_{x,before}$ is

$$g_{x,before} = \frac{r_x R_0 - (r_x + 1)\theta_x X}{R_0 + (r_x + 1)\theta_x X} \tag{5}$$

The prey biomass consumed by the predator $R_{consumed}$ is

$$R_{consumed} = \theta_y Y \tag{6}$$

Therefore the growth function of the prey after predation should take this into account

$$
\begin{aligned}
\theta_x X(g_x + 1) &= \theta_x X(g_{x,before} + 1) - R_{consumed} \\
&=> g_x = g_{x,before} - \frac{R_{consumed}}{\theta_x X} \\
&= \frac{r_x R_0 - (r_x + 1)\theta_x X}{R_0 + (r_x + 1)\theta_x X} - \frac{\theta_y Y}{\theta_x X}
\end{aligned} \tag{7}
$$

Eqs 4 and 7 can be simplified into

$$
g_y = \frac{r_y[1 - \theta\frac{(r_y+1)Y}{r_y X}]}{1 + (r_y + 1)\theta\frac{Y}{X}} \tag{8}
$$

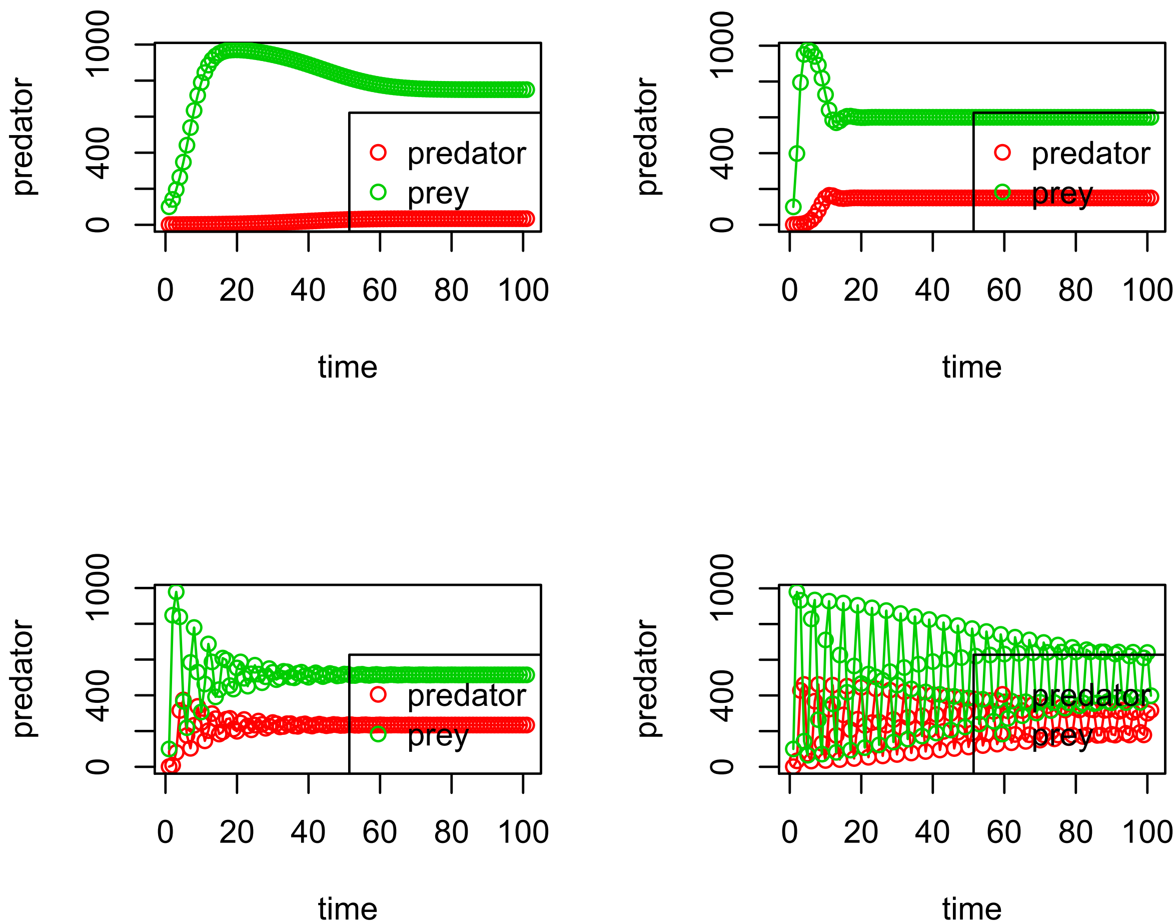
$$
g_x = \frac{r_x[1 - \frac{(r_x+1)X\theta_x}{r_x R_0}]}{1 + \frac{(r_x+1)X\theta_x}{R_0}} - \theta\frac{Y}{X} \tag{9}
$$

Where $\theta = \theta_P/\theta_p$. Since by definition $g_y = \frac{dY}{Ydt}$ and $g_x = \frac{dX}{Xdt}$, they each correspond to:

$$
\frac{dY}{dt} = g_y \times Y = \frac{r_y Y[1 - \theta\frac{(r_y+1)Y}{r_y X}]}{1 + (r_y + 1)\theta\frac{Y}{X}} \tag{10}
$$

$$
\frac{dX}{dt} = g_x \times X = \frac{r_x X[1 - \frac{(r_x+1)X\theta_x}{r_x R_0}]}{1 + \frac{(r_x+1)X\theta_x}{R_0}} - \theta Y \tag{11}
$$

With Eqs. 10 - 11, population dynamics for both the prey and the predator can be modeled. The following graph shows how the pattern varies when the magnitude of the intrinsic growth rates $r_x$ and $r_y$ increases.




($\theta = 2, X_0 = 100, Y_0 = 1$, top left: $r_1 = 0.1, r_2 = 0.5$, topright: $r_1 = 1, r_2 = 5$, bottomleft: $r_1 = 10, r_2 = 50$, bottomright: $r_1 = 100, r_2 = 500$; environment carrying capacity for the prey $\frac{r_x R_0}{(r_x+1)\theta_x}$ controlled to be 1000.)

# 2 Future directions: what to do from here

## 2.1 Building on the current model

First, I can add more trophic levels and try to reveal the dynamics of multiple species along a food chain. This might be a must if the data contains multiple ($> 2$) trophic levels.

Second, I can also reveal the dynamics for multiple competitors at one trophic level (with or without predator). Potentially this will yield equations that resemble the generalized Lotka-Volterra equations under certain circumstances (possibly depending on $D_r$). However, this is less testable due to the open definition and lack of measurement for $D_r$. Before we can more precisely pin $D_r$ down, the best we can do is make general inferences like compare when $D_r = 0$ vs when $D_r = 1$.

## 2.2 Data test

The Lotka-Volterra predator-prey equations are

$$\frac{dY}{dt} = \alpha X - \beta XY \tag{12}$$

$$\frac{dX}{dt} = \sigma XY - \gamma Y \tag{13}$$

Eqs. 10 - 11 have the same amount of parameters as The Lotka-Volterra predator-prey equations ($r_x$, $r_y$, $\theta$, $R_0$). However, the physical meanings of MERA parameters are much clearer: $r_x$ and $r_y$ are each the intrinsic growth rates for the prey or the predator when food is unlimited, $\theta$ is the ratio between their metabolic rates, $R_0$ is amount of total fundamental resource for the prey. They can be measured and compared between systems with different species composition (e.g. a prey-only system vs a predator-prey system). This way the MERA solutions can be explicitly tested. In summary, it will be ideal if we can find dataset(s) that satisfy the following requirements:

1. Time series of predator and prey population sizes;

2. Body sizes or metabolic rates of both species can be estimated;

3. For the same environment, prey dynamics when there is no predator (to acquire $r_x$ and $R_0$);

4. For the same environment, predator dynamics when prey is abundant (to acquire $r_y$ and $\theta$).

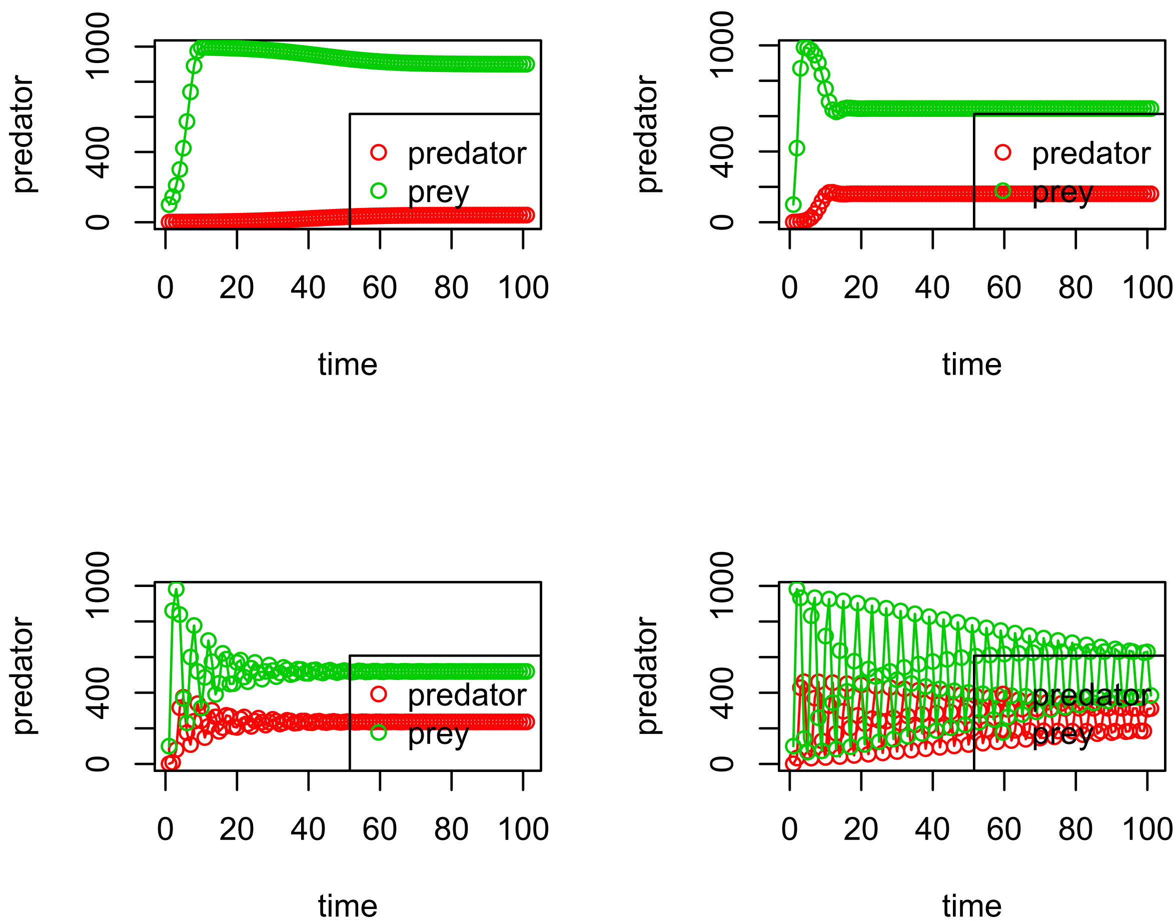
With only 1, it is probably hard to distinguish MERA solutions from the Lotka-Volterra equations. But with 2-4 in addition, it will be a much stronger test. Given the above, I think the microcosm experiments are my best bet. Please advise me if you have information and/or thoughts on this.

# 3 Appendix: When the prey follows a partial redistribution scenario

The per capita net growth rate of the prey will be different:

$$g_x = \frac{r_x(1 - \frac{X\theta_x}{R_0})}{1 + \frac{(r_x - 1)X\theta_x}{R_0}} - \theta\frac{Y}{X} \tag{14}$$

$$\frac{dX}{dt} = g_x \times X = \frac{r_x X(1 - \frac{X\theta_x}{R_0})}{1 + \frac{(r_x - 1)X\theta_x}{R_0}} - \theta Y \tag{15}$$



Same parameter setting as the first graph.