ज्ञानेन प्रकाशते जगत्
INDUS UNIVERSITY

Institute of Sciences, Humanities and Liberal Studies

Department of Mathematics

Subject: Probability, Statistics & Numerical Analysis		
Program: B. Tech. (CE, Civil, EC, Meta,CSE,IT,ICT)	**Subject Code: MA0314**	**Semester: III**
QUESTION BANK (TUTORIALS OF ALL UNITS-1,2,3,4)		

Unit-1 Basics of Probability & Probability Distributions.

Tutorial-1

1) Three unbiased coins are tossed. Find the probability of getting (i) exactly two heads, (ii) at least one tail, (iii) at most two heads, (iv) a head on the second coin, and (v) exactly two heads in succession.

2) From a collection of 10 bulbs, of which 4 are defective, 3 bulbs are selected at random and fitted into lamps. Find the probability that (i) all three bulbs glow, and (ii) the room is lit.

3) A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

4) A bag contains 3 red and 4 white balls. Two draws are made without replacement. What is the probability that both the balls are red?

5) A businessman goes to hotels X, Y, Z for 20%, 50%, 30% of the time respectively. It is known that 5%, 4%, 8% of the rooms in X, Y, Z hotels have faulty plumbings. What is the probability that the businessman's room having faulty plumbing is assigned to Hotel Z?

6) A fair die is tossed once. If the random variable is getting an even number, find the probability distribution of X.

7) A fair dice is tossed. Let the random variable X denote the twice the number appearing on the dice. Write the probability distribution of X.

8) Two unbiased dice are thrown at random. Find the probability distribution of the sum of the numbers on them.

9) Show that the function f(x) defined by

$$f(x)=\frac{1}{7} \quad 1<x<8$$
$$=0 \quad otherwise$$

is a Probability Density Function for a random variable. Hence, find P(3<x<10)

10) An unbiased coin is tosses 6 times. Find the probability of getting (i) exactly 4 tails; (ii) at least 4 tails.

11) The probability that a pen manufactured by a company will be defective is 1/10. If 12 such pens are manufactured, find the probability that

1) Exactly two will be defective
2) At least two will be defective
3) None will be defective

12) Let X be a Poisson variate with $P(X=2)=0.25$ and $P(X=3)=0.125$. Find $P(X=0)$, $P(X=1)$ and $P(X<3)$.

13) 100 electric bulbs are found to be defective in a lot of 500 bulbs. Use Poisson distribution to find the probability that at most 3 bulbs are defective in a box of 100 bulbs. [Use $e^{-2}=0.1353$]

Tutorial –Unit-II-Statistics

1.

Find the arithmetic mean from the following frequency distribution:

x	5	6	7	8	9	10	11	12	13	14
f	25	45	90	165	112	96	81	26	18	12

2.

Find the arithmetic mean of the marks from the following data:

Marks	0–10	10–20	20–30	30–40	40–50	50–60
Number of students	12	18	27	20	15	8

3.

The daily earnings (in rupees) of employees working on a daily basis in a firm are

Daily earnings (₹)	100	120	140	160	180	200	220
Number of employees	3	6	10	15	24	42	75

Calculate the mean of daily earnings.

4.

Calculate the arithmetic mean of the following distribution:

Class Interval	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Frequency	3	8	12	15	18	16	11	5

5.

Calculate the arithmetic mean of the following marks obtained by students in mathematics:

Marks (x)	5	10	15	20	25	30	35	40	45	50
Number of students (f)	20	43	75	67	72	45	39	9	8	6

6.

Obtain the median for the following frequency distribution.

x	0	1	2	3	4	5	6	7
f	7	14	18	36	51	54	52	18

7.

Find the median of the following distribution:

x	5	7	9	12	14	17	19	21
f	6	5	3	6	5	3	2	3

8.

Calculate the geometric mean of the following data:

10, 110, 120, 50, 52, 80

9.

Calculate the harmonic mean of the following data:

x	20	21	22	23	24	25
f	4	2	7	1	3	1

10.

Calculate the standard deviation of the weights of ten persons.

Weight (in kg)	45	49	55	50	41	44	60	58	53	55

11.

Calculate the standard deviation of the following data:

x	10	11	12	13	14	15	16	17	18
f	2	7	10	12	15	11	10	6	3

12.

Find the standard deviation from the following data:

Size of the item	10	11	12	13	14	15	16
Frequency	2	7	11	15	10	4	1

13.

Find the standard deviation for the following distribution:

Marks	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Number of Students	5	12	15	20	10	4	2

14.

The runs scored by two batsmen A and B in 9 consecutive matches are given below:

A	85	20	62	28	74	5	69	4	13
B	72	4	15	30	59	15	49	27	26

Which of the batsmen is more consistent?

15.

Calculate the correlation coefficient between x and y using the following data:

x	2	4	5	6	8	11
y	18	12	10	8	7	5

16.

Calculate the coefficient of correlation from the following data:

x	12	9	8	10	11	13	7
y	14	8	6	9	11	12	3

17.

Calculate the coefficient of correlation for the following data:

x	9	8	7	6	5	4	3	2	1
y	15	16	14	13	11	12	10	8	9

18.

Calculate the correlation coefficient between the following data:

x	5	9	13	17	21
y	12	20	25	33	35

19.

Calculate the correlation coefficient between for the following values of demand and the corresponding price of a commodity:

Demand in Quintals	65	66	67	67	68	69	70	72
Price in rupees per kg	67	68	65	68	72	72	69	71

20.

Calculate the coefficient of correlation for the following pairs of x and y:

x	17	19	21	26	20	28	26	27
y	23	27	25	26	27	25	30	33

21.

Ten participants in a contest are ranked by two judges as follows:

x	1	3	7	5	4	6	2	10	9	8
y	3	1	4	5	6	9	7	8	10	2

Calculate the rank correlation coefficient.

22.

Ten competitors in a musical test were ranked by the three judges A, B, and C in the following order:

Rank by A	1	6	5	10	3	2	4	9	7	8
Rank by B	3	5	8	4	7	10	2	1	6	9
Rank by C	6	4	9	8	1	2	3	10	5	7

Using the rank correlation method, find which pair of judges has the nearest approach to common liking in music. **[Summer 2015]**

23.

Find the regression coefficients b_{yx} and b_{xy} and hence, find the correlation coefficient between x and y for the following data:

x	4	2	3	4	2
y	2	3	2	4	4

24.

The following data give the experience of machine operators and their performance rating as given by the number of good parts turned out per 100 pieces.

Operator	1	2	3	4	5	6
Performance rating (x)	23	43	53	63	73	83
Experience (y)	5	6	7	8	9	10

Calculate the regression line of performance rating on experience and also estimate the probable performance if an operator has 11 *years of experience.* **[Summer 2015]**

25.

The number of bacterial cells (y) per unit volume in a culture at different hours (x) is given below:

x	0	1	2	3	4	5	6	7	8	9
y	43	46	82	98	123	167	199	213	245	272

Fit lines of regression of y on x and x on y. Also, estimate the number of bacterial cells after 15 *hours.*

Tutorial- UNIT-3 Interpolation

Q.(1) Use **newton's forward interpolation formula** find the approximate value of f(2.3) from the following data:

x	2	4	6	8
f(x)	4.2	8.2	12.2	16.2

Answer:- 4.8

Q.(2) Use **newton's forward interpolation formula** find the value of(218) from the following data:

x	100	150	200	250	300	350	400
f(x)	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Answer:- 15.6993

Q.(3) The population of a town is given below. Estimate the population for the year 1895 and 1930 using **suitable interpolation.**

Year x	1891	1901	1911	1921	1931
Population(in thousand) y	46	66	81	93	101

Answer:- 54.8528, 100.4705

Q.(4) Using **Stirling's formula,** estimate the value of tan 16°.

x	0°	5°	10°	15°	20°	25°	30°
y = tanx	0	0.0875	0.1763	0.2679	0.3640	0.4663	0.5774

Answer:- 0.2867

Q.(5) Compute f(4) form the tabular values given:

x	2	3	5	7
f(x)	0.1506	0.3001	0.4517	0.6259

Using **Lagrange's interpolation formula.**

Answer:- 0.3896

Q.(6) Find the **Lagrange interpolation polynomial** form the following data:

x	0	1	4	5
f(x)	1	3	24	39

Answer:- 1/20 ($3x^3 + 10x^2 + 27x + 20$)

Q.(7) Using **Newton's divided difference interpolation** evaluate f(9) using the following table:

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

Answer:- 810

Q.(8) Using **Newton's divided difference interpolation find a polynomial** from the given data:

x	1	2	4	7
f(x)	10	15	67	430

Answer:- $\boldsymbol{2x^3 - 7x^2 + 12x + 3}$

Q.(9) Evaluate $\int_1^2 \frac{dx}{1+x^2}$ taking h = 0.2 using **trapezoidal rule.**

Answer:- 0.3228

Q.(10) Calculate $\int_0^1 2e^x dx$ with n=10 using **trapezoidal rule.**

Answer:- 3.4394

Q.(11) Evaluate $\int_0^5 \frac{dx}{4x+5}$ by **using Simpson's 1/3 rule,** taking n=10.

Answer:- 0.4026

Q.(12) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using (i) **trapezoidal rule (ii) Simpson's 1/3 rule**

(iii) Simpson's 3/8 rule

Answer:- (i) 1.4108 (ii) 1.3662 (iii) 1.3571

Q.(13) Given the following table of x and y

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
y	1.000	1.025	1.049	1.072	1.095	1.118	1.140

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ **at x = 1.05; Applying Newton forward differentiation formula**

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Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ **at x = 1.25; Applying Newton Backward differentiation formula**

Tutorial- unit-4 NUMERICAL METHODS

Q. (1) Find a real root of the following equations using bisection method correct up to mentioned decimal places.

(1) $x^3 - x - 1 = 0$ (up to 5th iteration)

Answer:- 1.3438

(2) $x^3 - 5x + 3 = 0$ (correct up to 4 decimal places.)

Answer:- 0.6566

(3) $xe^x - 1 = 0$

Answer:- 0.5671

Q. (2) Find a real root of the following equations using Regula-Falsi method correct up to mentioned decimal places.

(1) $\log_{10} x - cosx = 0$ (correct up to 4 decimal places.)

Answer:- 1.41840

(2) $x^3 - 4x - 9 = 0$ (correct up to 3 decimal places.)

Answer:- 2.707

(3) $\mathrm{x} - \mathrm{e}^{-\mathrm{x}} = 0$ (correct up to 3 decimal places.)

Answer:- 0.5672

Q.(3) Find a real root of the following equations using Newton-Raphson method correct up to mentioned decimal places.

$(1)\ \mathrm{x} - \mathrm{cosx} = 0$ (correct up to 3 decimal places.)

Answer:- 0.7391

(2) $x^3 - 5x + 3 = 0$

Answer:- 0.6566

(3) $\mathrm{x} - 2\mathrm{sinx} = 0$

Answer:- 1.8955

Q.(4) Taylor's series Method.

(1) Using the Taylor's series method, find root correct up to four decimal places, the value of y(0.1), given $\frac{dy}{dx} = x^2 + y^2$ and y(0) = 1.

Answer:- 1.1115

(2) Evaluate y(0.1) correct to four decimal places using the Taylor's series method if $\frac{dy}{dx} = y^2 + x, y(0) = 1.$

Answer:- 1.1165

Q.(5) Using Euler's method, find the approximate value of y at x=1.5 taking h=0.1 . Given $\frac{dy}{dx} = \frac{y-x}{\sqrt{xy}}$ and y(1) = 2.

Answer:- 2.2940

Q.(6) Using Euler's method, find the approximate value of y at x=1 taking h=0.2 . Given $\frac{dy}{dx} = x^2 + y^2$ and y(0) = 1.

Answer:- 4.5559

Q.(7) Apply the modified Euler's method to solve the initial value problem $\frac{dy}{dx} = x + y$ with y(0) = 0 choosing h = 0.2 and compute y for x = 0.2, x = 0.4.

Answer:- 0.0938

Q.(8) Apply the modified Euler's method to solve the initial value problem $\frac{dy}{dx} = x + 3y$ with y(0) = 1 choosing h = 0.05 and compute y for x = 0.1.

Answer:- 1.3548

Q.(9) Use the fourth order Runge-Kutta method to find the value of y at x = 1 given that $\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1$ with h = 0.5.

Answer:- 1.3399, 1.4991

Q.(10) Use the fourth order Runge-Kutta method to find the value of y at x = 0.1 given that $\frac{dy}{dx} = 3e^x + 2y, y(0) = 0$ with h = 0.1.

Answer:- 0.3487

Q.(11) Use the second order Runge-Kutta method to find the value of y at x = 1.2, x = 1.4 given that $\frac{dy}{dx} = xy, y(1) = 2$.

Answer:- 2.4921, 3.2311

Q.(12) Solve the following system of equations using Gauss-Jacobi Method:-

(1) 8x - y + 2z = 13

x - 10y + 3z = 17

3x + 2y + 12z = 25

Answer:- (x = 1, y = -1, z = 2)

(2) 4x + y + 3z = 17

x + 5y + z = 14

2x - y + 8z = 12

Answer:- (x = 3, y = 2, z = 1)

Q.(12) Solve the following system of equations using Gauss-Siedel Method:-

(1) 2x + y + 6z = 9

8x + 3y + 2z = 13

x + 5y + z = 7

Answer:- (x = 1, y = 1, z = 1)

(2) 28x + 4y - z = 32

2x + 17y + 4z = 35

x +3y + 10z = 24

Answer:- (x = 0.99, y = 1.51, z = 1.85)

Q. (13) Find the Dominant Eigen Value correct up to 2 decimal place and its corresponding Eigen Vector by Power Method -

(1) $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$

Answer:- $\lambda = 5.372, X=\begin{bmatrix}0.457\\ 1.000\end{bmatrix}$

(2) $A=\begin{bmatrix}5 & 4\\ 1 & 2\end{bmatrix}$

Answer:- $\lambda = 6, X=\begin{bmatrix}1.000\\ 0.250\end{bmatrix}$

(3) $A=\begin{bmatrix}2 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 2\end{bmatrix}$

Answer:- $\lambda = 3.41, X=\begin{bmatrix}0.74\\ -1\\ 0.67\end{bmatrix}$

----------All the Best----------