UNITS AND MEASURE-MENTS

⇒ Quantities
(property of a phenomenon, body that can be measured and expressed numerically)
- Physical Quantities
- Non Physical Quantities
(amount of something that can't be measured.)

⇒ Physical Quantities
- made up of 2 parts
1) unit 2) magnitude
- types
1) Fundamental
"Basic and independent quanti[ties]
i.e can't be broken

Length	L	meter (m)
Mass	m	Kilogram (kg)
Time	t	seconds (s)
Electric Current	I	ampere (A)
Thermodynamic temperature	T	kelvin (K)
amount of substance	n	mole (mol)
Luminous Intensity	I_v	candela (cd) (S)

∵ Electric Current = $\frac{\text{charge}}{\text{time}}$

charge isn't accurately measured, but Electric current is simpler to measure

2) ~~Derived~~

"Quantities which are derived (dependent) upon fundamental units"

- $Q = IT$ (charge = current × Time)
- $A = \frac{v}{t} = \frac{L}{T^2}$ (acceleration = $\frac{\text{Length}}{\text{Time}^2}$)

⇒ System of units

complete set of units for fundamental and derived ~~physical~~ quantities.

1) CGS (Gaussian)
cm (Length), g (gram (mass))
s (Time)

2) MKS (Georgian) (Standard (SI))
meter (Length), kilogram (Mass)
Second (Time)

3) FPS (Retarded)
~~for~~ FOOT (Length), Pound (mass)
second (Time)

⇒ Unitless Quantities (Dimensionless)
"unit's having no quantities"
· Ratio (number only):
when it is a ratio of same physical quantities.
e.g: Refractive Index
: Relative Density
exception: "Angle"
angle = $\frac{l}{r}$ (length of arc / radius of circle) Radians
: angles are inherently unitless because they are ratios (technically)
· Radians (rad) are only added to reflect the measurement
· Degrees are scaled version of radians *($1° = \frac{\pi}{180}$ rad)* so they aren't unitless, "so degree is a true unit, but radians is a representational unit."

"Solid Angles" : angle of ~~three~~ 3D angle

⇒ Dimension:
"when derived quantity is written in terms of fundamental ~~units~~ quantities, it is written as a product of different powers of the fundamental quantities"

1) speed: distance/time $= L/T = L^1T^{-1}$
(∵ 1 in length and -1 in time)

2) force: mass × acceleration $= M.LT^{-2}$
3) energy: force × displacement $= M.L.T^{-2}.L = ML^2T^{-2}$

:- Dimensions remain same in all system of units

- CONVERSION
1> write demisions
2> writting into units
3> divide
e.g:
i. FOR velocity
1> $v = LT^{-1}$
2> $v = 1\ m/s$
$v = 1\ Km/h = 1000m/3600h$
3> $\frac{1\ m/s}{1000\ m/3600h} = \frac{18}{5}\ m/s$

So $\cancel{1\ m/s}\ 1Km/h = \frac{18}{5}\ m/s$

(CGS) (MKS)
FOR dyne → newton
1> $F = MLT^{-2}$
2> $F = 1cm\ 1s^{-2}\ 1g$ (dyne)
3> $F = 1m\ 1^{-2}s\ 1kg$ (newton) $(100cm\ 1^{-2}s\ 1000g)$
3> $\frac{1cm\ 1g\ 1^{-2}s}{100cm\ 1000g\ 1^{-2}s} = \frac{1}{100000}$ (newton)
So 1 dyne $= 1^{-5}$ newton

⇒ Applications

1) find unit of constants
2) find dimensions of constants
3) conversion of units
4) To check the correctness of an equation
(Based on principle of Homogenity)
convert into dimension (both LHS and RHS)
Principle of Homogenity
"only quantities of same dimensions can be added or subtracted"
"Dimensions of LHS = RHS"
"powers raised to quantities are always dimensionless"
5) derive new formulas
Suppose we don't know a formula, but dimension of few units, and atleast know some relation to other known units

e.g. F ⇒ mass, acceleration

$F \propto m^x a^y$

$F = k\, m^x a^y$

(placing dimensions)

$(MLT^{-2}) = k\; M^x (LT^{-2})^y$

(comparing powers)

$x = 1 \quad x = 1$

$y = 1$

so $F = kma$

∵ problem : we will always get some constant, unknown value.

Changing the Base Dimension
"asked to change fundamental quantities"

e.g If Speed V, area A and force F are chosen as fundamental units, then the dimension of Young's modulus will be

(∵ using "deriving formula" thing)
so $Y \propto V^x A^y F^z$

(Young's modulus is $\frac{F}{A}$)

so $Y = k V^x A^y F^z$

$\frac{F}{A} = k V^x A^y F^z$

so comparing exponents
$x = 0$, $y = -1$, $z = 1$
so $\boxed{Y = F A^{-1} V^0}$

e.g If momentum [P], area [A], and time [T] are taken as fundamentals quantities, then the dimensional formula for coefficient of viscosity is:

(∵ using "deriving formula" thing)
$\eta \propto P^x A^y T^z$

$(\eta = ML^{-1}T^{-1})$

$ML^{-1}T^{-1} = k [MLT^{-1}]^x [L^2]^y [T]^z$
$ML^{-1}T^{-1} = k [M^x][L^{x+2y}][T^{-x+z}]$

comparing exponents
$x = 1$, so $y = -1$, ~~z = -2~~ $z = 0$
$\eta = k P A^{-1} T^0$

Limitations of Dimensional Analysis

1> Two different quantities can have same dimensions.
Eg Energy $[ML^2T^{-2}]$ and Torque
(both are different but have same unit (Nm) and dimension

2> ! 100% correct, due to constants

⇒ Rounding Off

1> if digit to be dropped, is less than 5, then preciding left unchanged
(2.74 → 2.7)

2> if digit to be dropped is more than 5, then preciding ++
(2.78 → 2.8)

3> if digit to be dropped is 5, ~~the~~
a) look at digit after 5, if != 0 than ++
(2.753 → 2.8)

b) if digit after 5 == 0, than return
↳ if number before (2.750 → 2.7)
4> ~~confusing~~ 5 is odd than increase it (2.35 → 2.4)
↳ if number before
5 is even than unchanged (2.25 → 2.2)
** (2.205 → 2.2 (0 is even))

e.g you have vernier readings like
5.55mm, 5.45mm, 5.65mm, 5.50mm
avg : 5.5375 mm, standard deviation
0.07395

∴ if readings are in 2 dec than answer should be in 2 dec

So ans is 5.54 ± 0.07

⇒ Order of Magnitude

"order of magnitude of a given number is nearest power of ten to which it is approximated"

Find : order of magnitude (x) = $n \times 10^x$, where x should lie between 0.5 & 5

so for 2×10^5, order of magnitude = 5

6×10^7, order of magnitude = 8

↳ (0.6×10^8)

7 ⇒ 1 ~~(bef~~ (0.7×10)

0.07 = −1 (0.7×10^{-1})

Exceptions

0.5×10^3 or 5×10^2

consider the whole number one

so OFM → 2

are about how accurate we calculated (measured)

⇒ Significant Figures

"tells you the number digits we have surity in measurement, about"

- Larger the number of significant figures greater the accuracy.

1) all non zeros are significant

e.g 2.345 , 3.8

(4) (2)

2) Sandwitch zeros are significant

e.g 2.305 , 2.054 2.004

(4) (4) (4)

3) leading (left) zeros are insignificant

e.g. 0.063 , 0.534 , 05.347
(2) (3) (4)

4) trailing zeros are significant

a) e.g after decimal, trailing zeros are significant, because they tell the precision of the measurement

0.63400 , ~~06~~ 6.340
(5) (4)

c ∵ for measurements 0.63400 != 0.634

b) ~~dec~~ before decimal, trailing zeros aren't significant

eg 6000 6700
(1) (2)

arg: • ~~leading~~ trailing zeros are impacted by unit conversion, so changing units can change significant numbers too, Thats wrong.

• rounding can also result in zero, so these zeros aren't measured right?

1234, when round to 1 significant digit gives 1000 here 000 aren't measured and accurate...

c) if trailing zeros before and after dec, than all are significant (~~de~~ arg) zeros after decimal represent precision and accuracy)

06.340 600.00 00.06
(4) (5) (1)

Here decimals matters a lot, like

"number with decimal is more accurate significant than a number without it"
technically
+600.00m has 5 significant
+60000 cm has 1
than also 600.00 is more accurate because it has decimal because in measurements representation also matters (format)

d) in exponential notations, only the numeric one portion represents significant figures
1.23×10^{-2} (0.0123), 1.23×10^{2}
(3) (3)

OPERATIONS
a) Add/Subtract (round to least dec)
- count only significant digits
- add/subtract
- answer's decimal is approximated to least,

e.g: 2.320 + 4.26 = 6.580 = 6.58 (app)

B) multiplication / division (round to least
- multiple / divide significant digits
- round off according to significant digits (least)

e.g 4.56 × 3.1 = 14.136 = 14

for a experiment if true not provided mean value is considered true value ⇒

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⇒ Error Analysis

1) True ~~mean~~ value: theoretical (accepted) standard value of a quantity (assumed to be exact or closest to reality)
- not known exactly in real world
- taken from standard references
- use to calculate absolute and relative errors.

2) mean value: average of multiple measured values of quantity. $\frac{\sum_{i=1}^{n} x_i}{n}$

2) absolute value: difference between measured value and true/mean value

$\Delta a_1 = |a_m - a_1|$ (can be $a_1 - a_m$ because '| |')

it's purpose is to calculate difference of MAGNITUDE, between measured and mean values, so mode is taken, so it's always '+tive'

3) Mean absolute value: average of all absolute values

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \dots |\Delta a_n|}{n}$$

So for real value representation.

real value = "$a_m \pm \overline{\Delta a}$"

4) relative (fractional error); ratio of absolute error to the true value (shows how large the error is in comparison)

Relative Error : $\frac{\Delta a}{a_m}$ (∵ when 1 value)

: $\frac{\overline{\Delta a}}{a_m}$ (∵ when many values)

5) percentage error: percentage representation of fractional / relative error.

$$\text{Percentage} : \frac{\overline{\Delta a}}{a_m} \times 100\%$$

:: Extra Note ::

Errors

Natural (measurement) | Quantify (Basic math) (mentioned above) | Other

6. systematic error
(by the machine, always getting errors in one direction like + or - in error)

• random errors
(unpredictable, due to uncontrollable variations)

4. Gross error
(by the human while reading)

• instrumental (System) ; observation,
• environmental, • theoritical (Subcategories)

e.g measurements for a pendulum experiment 90, 91, 95, 92 (s). minimum division : 1s, reported mean time (real value)

$a_m = (91+92+90+95)/4 = 92$

$\overline{\Delta a} = (|(92-91)| + |92-90| + |92-92| + |92-95|)/4$
$= 6/4 = 1.5$

real value : 92 ± 1.5 sec ✗
(minimum division is 1)

(after rounding)
real value : 92 ± 2 sec

PROPOGATION OF ERRORS

"process of tracking how errors spread or combine is called error propogation"

1) Addition / Subtraction (errors add hote hai)

If $Q = A \pm B$

$Q \pm \Delta Q = (A \pm \Delta A) \pm (B \pm \Delta B)$

$Q \pm \Delta Q = (A \pm B) \pm (\cancel{A B} \pm \Delta A \pm \Delta B)$

$\pm \Delta Q = \pm(\Delta A \pm \Delta B)$ If humne jahan $\pm$ le liya hai toh order doesn't matter...

$\Delta Q = \Delta A + \Delta B$ (when addition)

$\Delta Q = \Delta A + \Delta B$ (when subtraction)

Simply : uncertainities ($\Delta a, \Delta b$) could both increase and decrease the result, doesn't matter they are added / subtracted

so $\Delta c = \Delta a + \Delta b$ for $Q = A \pm B$

2) Multiplication / Division (fractional errors add hoke fractional error deta hai)

If $Q = AB$

$Q \pm \Delta Q = (A \pm \Delta A)(B \pm \Delta B)$

$Q \pm \Delta Q = AB \pm A\Delta B \pm B\Delta A \pm \Delta A \Delta B$

(here $\Delta A \Delta B$, will give a negligible value, cuz they are too small)

$Q \pm \Delta Q = AB \pm A\Delta B \pm B\Delta A$

(dividing by Q or AB)

$1 \pm \frac{\Delta Q}{Q} = 1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A}$

$$\frac{\pm\Delta Q}{Q} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$$ (remove '±')

$$\boxed{\frac{\Delta Q}{Q} = \frac{\Delta A}{A} + \frac{\Delta B}{B}}$$ or $\% \Delta x = \% \Delta a + \% \Delta b$

Same for divide (test it) (try differentiation)

- Eg calculate speed and it's percentage error

$d = \cancel{20}\ 40.0 \pm 0.4$, $t = 5.0 \pm 0.6$

$s = \frac{d}{t} = \frac{40.0}{5.0} = \cancel{20}\ 8$

$\frac{\Delta s}{s} = \frac{\Delta d}{d} + \frac{\Delta t}{t}$

$\frac{\Delta s}{\cancel{20}8} = \frac{0.4}{40.0} + \frac{\cancel{5.0}\ 0.6}{\cancel{0.75}\ 5.0}$

$\frac{\Delta s}{\cancel{20}8} = \frac{1}{100} + \frac{6}{50}$

$\Delta s = \cancel{20}^{8}(0.13)$

$\Delta s = \cancel{2.6}\ 1.04$

rounded to 1.0

(Because given in 1 dec)

So speed $= 8.0 \pm 1.0$ m/s

$\% \Delta s = 12.5\%$

(without rounding)

$s = 8 \pm 1.04$ m/s

$\Delta s\% = 13\%$

$\% \Delta s = \% d + \% t$

$\% \Delta s = (0.01 + 0.12) \times 100$

$\% \Delta s = 13\%$

so $\%13$ of 8 is 1.04

wanna round?? than $\Delta s = 1.0$, and $\Delta s\%$ turns 12.5%

$s = 8 \pm 1.0$ m/s (rounded)

$s = 8 \pm 1.04$ m/s

$\Delta s\% = \cancel{13}\ 12.5\%$ (rounded)

$\Delta s\% = 13$

3) Quantity raised to some Power :-

If $x = \frac{a^l b^m}{c^n}$

than $\frac{\Delta x}{x} = |l| \left|\frac{\Delta a}{a}\right| + |m| \left|\frac{\Delta b}{b}\right| +$

$$\boxed{\frac{\Delta x}{x} = |l| \frac{\Delta a}{a} + |m| \frac{\Delta b}{b} + |n| \frac{\Delta c}{c}}$$

OR

$$\boxed{\% \Delta x = |l| \% \Delta a + |m| \% \Delta b + |n| \% \Delta c}$$

⇒ Vernier Caliper & Screw Guage

"precision instrument used to measure dimesions accurately,"

REFER A IMAGE (DIAGRAM) FOR CLARITY

COMPONENTS :-

1) main scale : graduated in cm, mm (inches)
2) vernier scale : Slides along main scale to give more precise measurements.
~~3) fixed jaw : fixed par~~
3) Outer jaws : jaws on vernier and body to hold.
4) inner jaws : generally on top to find inner measurements.
5) depth rod : at the end of the body to measure deapth
6) locking screw : above (top) in start, to lock (fix) the measurement.

TERMS:

1> Least Count : smallest value that can be measured by a instrument

2> Main Scale Division (MSD) (division (least measurable) on the main scale)

Vernier Scale Division (VSD) (smallest division on vernier Scale.

$\boxed{LC = 1 MSD - 1 VSD}$

∴ do see and add units while calculating

2> Zero error: error in the instrument when the jaws are closed, but the reading isn't zero. (+tive or -tives)

3> positive zero error: when vernier zero is at the right of the main zero

4> negetive zero error: when vernier zero is at the left of the main zero

READING:

1> make ~~the~~ sure that zero's of main and vernier align (if not, find (+tive) -tive error)

2> place obj

3> ~~read the~~ find where zero of vernier aligns in main scale, select significant (least) number. like if between 1.3 and 1.2 than select 1.2, THIS IS MSR

4> find where mark of main and vernier align perfectly, find (calc) ~~the~~ division of vernier ~~Scale~~ Scale, THIS IS VSR

5> Reading = MSR + (VSR × LC) - Positive error
+ Negetive error

or $\frac{MSD}{VSD}$ (like division on V matc 24 on main 25 on main)

or $\frac{MS\text{ Division}}{VSD\text{ Division}} \times MSD$

SCREW GUAGE (MICRO-METER)

"accurate than vernier calipers, instrument to calcute (measures) things"

REFER IMAGE FOR CLARITY

COMPONENTS:

1> Frame: the C shaped frame (body) that holds anvil and barrel

2> Anvil: the fixed mettalic surface against which the object is placed

3> Spindle: the movable cylindrical part that comes into contact with the object.

4> Sleeve / Barrel: A fixed cyclindrical scale with main scale markings (mm)

5> Thimble: A rotating sleeve attached to spindle; carrier circular scale.

6> Ratchet: A small knob at the end of the thimble that ensures uniform pressure.

7> Lock screw: locks measurement.

TERMS:

1> Pitch: distance moved by spindle on main scale a in a full 360° rotation

2> Least Count: Pitch / No. of division on circular scale

3> Positive zero error: when tightened, zero of vernier (circular) is on ~~right~~ left (~~above~~ below) zero of main

4> Negetive zero Error: when tightened, zero of circular is on ~~left~~ right (~~below~~ above) the zero of main

5> Zero error: when zero of circular and main doesn't align

READING

1> make sure you don't zero error, if there and positive/negative error

2> place the obj between anvil and spindle

3> read number of division (values) visible on main scale (MSR)

4> read division on circular scale which aligns with line on main scale (CVSR)

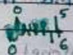
5> Reading = MSR + (CVSR × LC) - Positive error + Negative error

Eg. MSD: 0.1 cm , 10 VSD = 9 MSD

when no obj: mark of main scale (5th) aligns with 6th mark on vernier sclae

when obj: vernier zero between 3.1 and 3.2

1st division aligns



MSD = 0.1 cm

10 VSD = 9 MSD

1 VSD = 0.9 (0.1) cm = 0.09 cm

LC = 1 MSD - 1 VSD = 0.1 - 0.09 = 0.01 cm

Reading = 3.1 + (1 × 0.01) = 3.11 cm

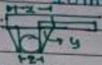
Error calculation: so technically error (difference between vernier zero and main zero is 6 VSD - 5 MSD)

so 6 (0.09) - 5 (0.1) = 0.04

so as this is negetive error so

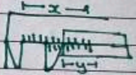
Final Reading = 3.11 + 0.04 = $\boxed{3.15 \text{ cm}}$

E.g. There are 2 vernier calipers
$MSD_1 = 0.1$ cm $MSD_2 = 0.1$ cm
for VC_1 (C_1) : 10 VSD = 9 MSD
VC_2 (C_2) : 11 VSD = 10 MSD
find readings respectively:



For VC_1
MSD = 0.1 cm
1 VSD = $\frac{10}{9}$ MSD = 0.09 cm

LC = 1 MSD − 1 VSD = 0.01 cm
~~LC=~~
MSR_1 = 2.8 VSR_1 = 7
Reading: 2.8 + 0.07
= 2.87 cm



For VC_2
MSD = 0.1 cm
1 VSD = $\frac{11}{10}$ MSD = 0.11 cm

LC = 1 MSD − 1 VSD = −0.01 cm

when LC is −ive
× formula can be applied, so
calculate by distance between jaws.

⇒ like its basically finding distance by gap between jaws
you basically know distance from start jaw (main zero)
to the mark of main that aligns
AND
distance from next jaw (vernier zero) to the mark on vernier ~~you~~ aligning
MSR_2 = 2.8 VSR_2 = 7 VSD
Division coinciding : 3.6 MSD, 7 VSD
so ~~x=~~ ~~2.8~~, y = 0.77 cm, x = 3.6 cm
Reading : x − y = 2.83 cm

so for $\boxed{VC1 : 2.87 \quad VC2 = 2.83}$