CLASSICAL MECHANICS

⇒ Fundamentals Of motion

Types of Motion

motion: phenomenon where object's position change with respect to time

1> Rectilinear (Linear) Motion:
"motion along a straight line"

2> Curvilinear Motion:
"move along a curved path, not neccessary a circle."

3> Circular Motion:
"curvilinear motion, but the path is perfect circle"

4> Rotational motion
"object rotates about it's own axis"

5> Translation motion
"when every point of a body moves the same direction, same distance in same time"

Scalars and Vectors

scalar: quantities with only magnitude
e.g distance, speed, mass, temperature, time

vector: quantities with magnitude and directi

Vector Operations

Extra Knowledge:

Q Can a body be rest & motion in same frame bu at rest in another?

Yes! Think of ~~this~~ sitting on a train. You at rest relative to seat, but is moving relative to the station.

" Motion is Relative, not absolute "

⇒ Linear Kinematics

"Physics of How Things move in a straight line"

Kinematics: branch of physics that studies motion

Displacement, Velocity, and Acceleration

• vector
• m
• vt

Displacement: (S, x)

- change in position of object, considering both distance and direction from the starting point
- "shortest distance between an object's initial and final position, regardless of the path taken, but considering direct"
- (S or x)
- formula: velocity × time
- unit m.
- can be +tive, -tive and zero

vector
m/s
s/t

Velocity: (v)

"rate of change of displacement."
tells you how fast and in which direction you're moving.

- formula: displacement / time
- types

1) average velocity: $v_{avg} = \frac{\Delta x}{\Delta t}$

• vector
• m/s^2
• v/t
($\Delta v / \Delta t$)

for the whole journey.
2) instaneous velocity :
velocity at a specific moment

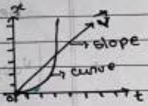
• vector
• m/s^2
• v/t
($\Delta v/\Delta t$)

Acceleration : (a)

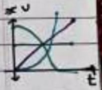
"Rate of change of velocity with time"
• can be +tive -tive or zero
• types:
1) uniform : constant increase in velocity
2) non-uniform : Δ in v is not constant
3) centripetal : points towards the center

Graphical Analysis of Motion :

i) Displacement (x-t) graph.
Slope: velocity
Straight line: constant velocity
curve = change in velocity



ii) Velocity - Time
Slope : acceleration
area under the curve : displacement



iii) Acceleration - Time
area under curve = change in velocity.

Equations of Motion (Uniform Acceleration)

⇒ SUVAT equations
(only when "uniform acceleration")

s = displacement u = initial velocity t : time
a = acceleration v = final velocity

- $v = at + u$

(use when distance is missing)

derivation :

we know that $a = \frac{v-u}{t}$

so $\boxed{at + u = v}$

- $s = ut + \frac{1}{2}at^2$

(use when final velocity is missing)

derivation :

∵ here you can't directly apply $s = vt$ because v changes, so find avg.

$$\boxed{v_{avg} = \frac{u+v}{2}}$$

replacing this in $s = v.t$

$$s = \left(\frac{u+v}{2}\right)t \quad (\because \text{replacing } v = at+u)$$

$$s = \left(\frac{u + (at+u)}{2}\right)t$$

$$s = \left(\frac{2u + at}{2}\right)t$$

$$\boxed{s = ut + \frac{1}{2}at^2}$$

- $v^2 = u^2 + 2as$ — $\frac{v^2 - u^2}{2a} = s$ — $\frac{v^2 - u^2}{2s} = a$

3rd eq: (use this when no time)

derivation:

(here we need to eliminate t, so)

$t = \frac{v-u}{a}$

replace this in $s = ut + \frac{1}{2}at^2$

$$s = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2$$

$$s = \frac{vu - u^2}{a} + \frac{1}{2a}(v-u)^2$$

$$= \frac{vu - u^2}{a} + \left(\frac{v^2 + u^2 - 2vu}{2a}\right)$$

$$= \frac{2vu - 2u^2 + v^2 + u^2 - 2vu}{2a}$$

$$\boxed{s = \frac{v^2 - u^2}{2a}}$$

Summary Box

1> $s = vt$, 4> $a = \frac{\Delta v}{t}$, 6> $a = \frac{v-u}{t}$,

2> $v = \frac{s}{t}$,

5> $v_{avg} = \frac{v+u}{2}$, 7> $v = u + at$,

3> $t = \frac{s}{v}$,

8> $s = ut + \frac{1}{2}at^2$, 9> $s = \frac{v^2 - u^2}{2a}$, 10> $a = \frac{v^2 - u^2}{2s}$,

11> $2as = v^2 - u^2$, 12> $2as + u^2 = v^2$, 13> $u^2 = v^2 - 2as$

FREE FALL AND VERTICAL MOTION UNDER GRAVITY

Free Fall: object falls under gravity's influence only (no air resistance)

- acceleration: 9.8 m/s^2
- $u = 0$, $a = g = 9.8$
- time to rise = time to fall
  velocity upwards = velocity downwards
- e.g. A ball thrown vertically upwards with 20 m/s

given: $g = a = 9.8\ m/s^2$ (−, because upwards)
$v = 0$ m/s
$u = 20$ m/s

Find = s and t

$$s = \frac{v^2 - u^2}{2a} = \frac{400}{-19.6} = |-20.40| \text{ m}$$

$$t = \frac{v - u}{a} = \frac{20}{9.8} = \boxed{2.04\ s}$$

∵ here $v = 20$ m/s too if we consider from the

⇒ Dynamics of Linear Motion

The world forces, motion and Newton's brilliance (maybe yes!)

Newtons Law of Motion

1) First Law: LAW OF INERTIA

"A body remains in its state of rest or motion (uniform) in a straight line unless acted upon by an external force"

Objects are LAZY, They resist CHANGE

- INERTIA: The tendency of a body to resist change in motion
  "More mass ∝ More Inertia"
  (kick heavy can and light ball)
  (Shape can matter too, like throwing a square (same weight) object and a circular one)

2) Second Law: $F = ma$

"Force acting upon a body is directly proportional to the rate of change of momentum and takes place in the direction of the applied force."

- Momentum: tendency of a moving object to stay in motion

  $\boxed{p = mv}$ (mass × velocity)

  unit: kg s/m   vector

- Force is "RATE OF CHANGE OF MOMENTUM"

  $F = \frac{mv}{t}$ (momentum / time)
  $(v/t = a)$

  $\boxed{F = ma}$ (technically its $F \propto ma$)

3) Third Law: Action = - Reaction

"For every action there is an equal and opposite reaction"

You can't push without being pushed back

Inertial Frames: physics law only work in correct inertial frames (places where there is no unbalanced acceleration)

Forces And Types

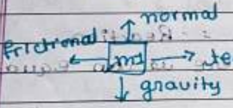
.vector
.N
.ma

(F)
Force "a push/pull upon an object resulting from object's interaction with another object"
Newton (N), vector $F = ma$

Types:

1) Contact (need physical touch)
    - a) Normal ⊥ to the surface
    - b) Tension: in string/ropes pulling tight
    - c) Frictional: Opposes motion
    - d) Spring: force in a spring
2) Non Contact
    - a) gravitational: acts between masses
    - b) electrostatic: between charges
    - c) magnetic: between magnets

· Friction: "force resisting the relative motion of solid surfaces, liquids and materials.



(Free Body Diagram)

⇒ Momentum And Impulse

"Physics of Push and Punch"

Studying how much "oomph" something has? (oomph = momentum)

Linear Momentum

- vector
- $\frac{kg\,m}{s}$
- p
- $p = mv$

Momentum : "positive energy and pogress that builds over time as you work towards your goal"

product of mass and velocity

$p = m\vec{v}$

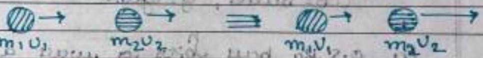
How stubborn a moving object is

can be +, 0, - (if velocity -)

→ Law of Conservation of Linear Momentum

"in an isolated system, the total momentum before interaction = total momentum after

$$\boxed{m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2}$$



before collision   after collision

In frictionless air track, two gliders colliding demonstrate this law beautifully

Impulse

"effect of force acting over time"

"change in momentum"

- vector
- impulse $= \Delta p$ $= F.\Delta t$
- Ns

Impulse $= F.\Delta t = \Delta p$

unit : Ns

e.g. Catching a ball, you move your hands back, (this increase time, reduces force momentum.

∴ FORCE - TIME GRAPH (F-t)
slope: impulse

e.g force on obj for time 0.2 sec changing its momentum by 10 kg m/s

given: $\Delta t = 0.2 sec$

$\Delta p = 10$ kg m/s

impulse = 10 ~~N~~

$F = \frac{impulse}{t} = \frac{10}{0.2} = 50N$

∴ ~~given~~ force = 50N

Special Cases

1) explosion : initial momentum = 0
after explosion, parts fly in opposite directions, but total momentum = 0

2) Recoil : $\boxed{m_b v_b = -m_g v_g}$

(b = bullet, g = gun)

Eg 0.5 kg ball hits a wall at 10 m/s and bounces back at 8 m/s. what's the impulse delivered.

A: $\Delta p = \boxed{mv - mu}$

$= 0.5(-8 - 10)$

$= -9$ kg m/s

⇒ Circular Motion
"Physics in Circle"

Angular Variables

variables, terms, relations ~~with~~ in angular motion

1) Angular Displacement (θ)
"angle swept by an obj in circular motion"
unit: Radians (R)

2) Angular Velocity (ω)
"rate of change of angular displacement, direction given by right hand rule"
unit: Radians/s $\omega = \frac{d\theta}{dt}$ or $\frac{\theta}{t}$

3) Angular Acceleration (α)
"rate at which angular velocity changes."
unit: Radian/s^2 $\alpha = \frac{d\omega}{dt}$ or $\frac{\omega}{t}$

* Linear Angular Velocity
- displacement (s) $= r\theta$
- velocity (v) $= r\theta/t = r\omega$
- acceleration (a) $= r\theta/t^2 = r.\alpha$

Why does the outer edge of a spinning CD move faster than the inner part?
$v = r\omega$ (larger radius, ↑ speed).

Uniform Circular Motion
"object moving at constant speed in circular path"
NOTE: Speed can be constant, but velocity won't be, because direction changes.

1) Centripetal Acceleration (and C Force):
· centripetal force is UFM force which always directs force towards center

$\boxed{a_c = \frac{v^2}{r} = r\omega^2}$ $F_c = \boxed{\frac{mv^2}{r}}$

2) Centrifugal Force and Acceleration
· centrifugal force is UFM force which is the apparent outward force on a mass when rotated.
(tries to force out, when left)

$\boxed{a_c = \frac{v^2}{r} = r\omega^2}$ $\boxed{F_c = \frac{mv^2}{r}}$

Non Uniform Circular Motion
What if angular speed is changing?
Now, there are two accelerations:
1. Radial (centripetal) → TOWARDS CENTRE
2. Tangential (due to changing speed) → ALONG PATH

Total acceleration = $\sqrt{a_c^2 + a_t^2}$

Centrifugal Force
"Why you feel thrown outward in a turning car?"

This is a fictitious or pseudo force - it shows up only in a rotating or non-inertial frame.

· Fictitious force: a.k.a pseudo / inertial is a force that appears to act on an object when its motion is described or experienced from a non inertial frame.

e.g. Car accelerates force → you feel pulled back.

Imaginary force that appears when you observe motion from an accelerating (non-interial) frame
- doesn't rise from any physical interaction between the bodies

Swing a bucket upside down in a circle and water stays inside, why?
∴ The bucket is accelerating downwards faster enough to match gravity.
The centrifugal force pushes the water against the bottom of bucket, it won't spill.

⇒ Work, Energy, and Power
"chapter where physics explain effort, fuel, and performance."

$\boxed{W = \vec{F}\cdot\vec{d}} = \boxed{Fd\cos\theta}$ → angle between F & d
work, force, displacement
unit J (Joules), J = N.m

- scalar
- J / Nm
- F.d

$\boxed{W = Fd}$

Work is only done when displacement happens in the force's direction.
No moment = no movement work.

Work is dot product of 2 vectors, so it is scalar. the +, -, and 0 indicates towards the ~~direct~~ angle of force applied nothing to do with direction

Types

1) Positive : F and d in same direction
e.g. lifting a book

2) Negative : F ~~and d~~ opposes d
e.g. friction, braking car

3) Zero : No displacement or ⊥ Force
e.g. Holding suitcase circular motion

∵ Lifting something upward? you do positive work, but gravity does negative.

Work by Variable Force

If force is not constant (spring), we use area under F-x(d) graph

$$\boxed{W = \int F(x)\,dx}$$

Work Energy Theorem

The net work done by all forces on a body equals the change in its kinetic energy.

$\boxed{W_{net} = \Delta K.E.}$ (KE is Kinetic energy)

ENERGY

"capacity of doing work"
unit : Joule scalar $W = E$

Types

1) Kinetic Energy :
"energy due to motion"

$$\boxed{KE = \frac{1}{2} mv^2}$$

Eg. moving car, bullet etc.

2) Potential Energy
"Stored energy due to position and configurations"
(Height mostly)
- Gravity PE = mgh (mass. gravity. height)
- Elastic $\boxed{PE = \frac{1}{2} kx^2}$ (spring constant, x (displacement)^2)

3) Mechanical Energy
$\boxed{ME = KE + PE}$
"total is what care about in systems without friction"

Laws of Conservation of Mechanical Energy
"In absence of non-conservative forces total mechanical energy remains constant"
Energy is never produced / destroyed its form is changed.

Conservative v/s Non-Conservative forces
→ energy conserved — → energy lost as heat / sound
→ gravity, spring — → friction, air resistance

Eg Bow and Arrow: Elastic PE become KE on release
Dam water: PE turns to KE in turbines

Power
rate at which work is done or energy is transferred

$\boxed{P = \frac{W}{t}}$; unit : J/s   scalar

- Instantaneous Power
  $\boxed{P = \vec{F}\cdot\vec{v}}$ $(= \vec{F}\cdot\frac{\vec{s}}{t})$ $(\frac{s}{t} = v)$

- Electric Unit
  1kWH = 1000W × 3600s = $\boxed{3.6 \times 10^{6}\ J}$

- Efficiency
  "How much input energy is converted into useful output?"
  Efficiency = $\left(\frac{\text{Used power output}}{\text{Used power input}} \times 100\right)$
  e.g. if machine uses 10 J, but does 7 J work, 70% efficiency.

⇒ Laws of Friction

What is Friction?
"force that opposes relative motion or the tendency of motion between two surfaces in contact."
Friction always resists motion or the tendency to move.

- when two objects touch their irregularities interlock, creating resistance to move.

Types of Friction

Type	Mean	When	E.g.
Static	prevent motion	obj at rest	push heavy box
Kinetic (Sliding)	Oppose motion	when obj is sliding	box sliding on floor.
Rolling	Resist rolling	obj rolls	ball rolling on ground
Fluid (Drag)	resistive force in fluid	in liquid/gas	Skydivers/ falling.

Friction Hierarchy (Magnitude):

$$f_{static} > f_{kinetic} > f_{rolling}$$

It's harder to start moving than keep it moving.
Balls roll easily than blocks slides.

Static Friction

"friction where there is no motion - but there's a tendency".

Range of Static Friction

- $0 \le f_s \le \mu_s N$

f_s : static friction ; μ_s : coefficient of static friction (depends on surface)
N : normal reaction force ($\perp$ to surface)

e.g. · Push a Box with 5N → Friction 5N
· Push harder → friction increases
· At 20N, suddenly the box moves → you've exceeded $\mu_s N$

"Static friction adjusts itself to match the applied force up to a limit ($\mu_s N$)"

Kinetic (Sliding) Friction

"Friction during motion always oppose direction of velocity."

$$f_k = \mu_k N$$

f_k: kinetic force, μ_k: coefficient of kinetic friction
N: normal force reaction

· constant once obj is sliding
· $\mu_k < \mu_s$
· opposes direction of velocity

Rolling Friction

"friction that opposes rolling motion"
· smaller than sliding and static
· reduces ball ~~hube~~ cants and bearings

Laws of Friction

1. Friction is indepent of contact area
(friction doesn't affect until normal force is friction)

(normal force: force exerted by a surface that is perpendicular to the surface and acts to prevent object from passing through each other.)

2. Friction is independent of speed (approx) (until very high speeds or special materials)

3. Friction depends on surface nature
Rough → ↑ μ, Smooth → ↓ μ

4. Friction ∝ Normal Reaction

$\boxed{f \propto N}$

Angle of Friction

"max θ at which a surface can incline before an obj starts sliding (moving)"

$\boxed{\tan\theta = \mu_s}$ It's about the forces acting at the point of contact, not about the tilt of the obj

Angle of Repose

"~~the θ between the normal reaction and the resultant contact force~~ to which a surface is inclined with the horizontal such that an object just begins to slide down on its own due to gravity

$\tan\theta_R = \mu_s$

It's about the surface tilt, not the obj

Angle of Repose: How steep the slope gets before ... and you slide

· Angle of Friction : The limit of your friend's strength to keep shaking your back before stopping

Applications & Important Situations

1) Ladder leaning on wall
   - friction on both top & bottom
   - tendency to slide → friction opposes it

2) Inclined Planes
   - friction acts up the incline when block tends to slide down, vice versa

3) Rope and pulley system
   - friction helps or resists tension change

4) Car turns on roads
   - centripetal force = frictional force

System of Particles and Rigid Body motion

System of Particles

is simply a collection of particles that may interact with each other through internal forces and are also affected by external forces

Centre of Mass

we define one magical point that represents the entire system's motion.

.... Other Topics doesn't seem important