1

a) 99 ← 1 Point

b) 99 ← 1 Point

c) 1 ← 0.5 Point

d) 0 ← 0.5 Point

2

K_{10} 10 vertices

Minimal spanning Tree => 9 edges ← 1 Point

a) 8 edges of weight 5 = 40
1 edge of weight 10 = 10 } Total cost 50 ← 1 Point

b) To reach v_1 we will have to choose an edge of weight 10
all other edges will have weight 5. ← 1 Point

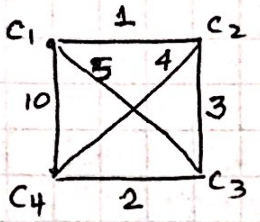
3

a) $\underbrace{S_1, S_5, S_6, S_7}_{\text{set chosen}}$ # of sets chosen = 4 ← 1 Point

1 Point

b) Not an optimal solution
optimal solution is S_2, S_3 ; # of sets chosen = 2 ← 1 Point

(4) counterexample



Greedy strategy C_1, C_2, C_3, C_4, C_1
total cost $= 1+3+2+10 = 16$ } 1 Point for correct greedy solution

optimal strategy C_1, C_2, C_4, C_3, C_1
total cost $= 1+4+2+5 = 12$

or optimal strategy C_1, C_3, C_4, C_2, C_1
total cost $= 5+2+4+1 = 12$ } 2 Points for correct optimal solution

many other variations exist.

(5) Let $(g_1\ g_2\ g_3\ g_4)$ be greedy solution
Let $(n_1\ n_2\ n_3\ n_4)$ be optimal solution

$$N = g_1 + 5g_2 + 10g_3 + 25g_4$$
$$= n_1 + 5n_2 + 10n_3 + 25n_4$$
$$\Rightarrow (g_1 - n_1) + 5(g_2 - n_2) + 10(g_3 - n_3) + 25(g_4 - n_4) = 0$$
} 1 Point

$$\Rightarrow g_1 - n_1 = 0 \pmod 5.$$

$0 \le g_1 < 5$ because greedy algorithm can always choose a coin of denomination 5 if $g_1 \ge 5$. } 1 Point

$0 \le n_1 < 5$ because if $n_1 \ge 5$ then optimal algorithm can do better by reducing n_1 by 5 & replacing it with a coin of denomination 5. } 1 Point

Because of the two inequalities above & because $g_1 - n_1 = 0 \pmod 5$ } $\Rightarrow g_1 - n_1 = 0$. } 1 Point

(6)

(a)

		Implication satisfied	X	Y	New Assignment?	check	
	Initial		F	F			
Row 1	$\Rightarrow X$	NO	T		Y		
ROW 2	$X \land Y \Rightarrow Y$	Yes			No		← 1 Point
ROW 3	$\neg Y$					T	← 1 Point
Row 4:	Final Assignment		T	F			← 1 Point

(b)

<u>only</u> ← 1 Point

Assignment(X, Y, Z) = (T, T, F) ← 1 Point

implication is <u>not</u> satisfied