Practice problems for Continuous Probability Distributions, Cumulative Distributions and Bivariate Distributions.

Problem 1.

(a) $P(0 < x < 1) = \int_0^1 f(x)\, dx = \int_0^1 \frac{2(x+2)}{5}\, dx = \frac{2}{5} \times \frac{5}{2} = 1$

(b) $P(0.25 < x < 0.5) = \int_{0.25}^{0.5} \frac{2(x+2)}{5}\, dx = \frac{2}{5} \times \frac{19}{32} = \frac{19}{80}$

Problem 2.

(a) $\int_{-\infty}^{+\infty} f(x)\, dx = 1 \Leftrightarrow \int_0^1 k\sqrt{x}\, dx = 1 \Leftrightarrow k \cdot \frac{2}{3} = 1 \Leftrightarrow k = \frac{3}{2}$

(b) $F(x) = \int_{-\infty}^{x} f(z)\, dz = x^{\frac{3}{2}}$. $P(0.3 < x < 0.6) = F(0.6) - F(0.3) = 0.4648 - 0.1643 = 0.3005$

Problem 3.

$$F(x) = \begin{cases} 0, & x < 0, \\ 0.41, & 0 \leq x < 1, \\ 0.78, & 1 \leq x < 2, \\ 0.94, & 2 \leq x < 3, \\ 0.99, & 3 \leq x < 4, \\ 1, & x \geq 4 \end{cases}$$

Problem 4.

(a) $P(X \leq 2, Y = 1) = P(X = 0, Y = 1) + P(X = 1, Y = 1) + P(X = 2, Y = 1) = \frac{1}{30} + \frac{2}{30} + \frac{3}{30} = \frac{1}{5}$.

(b) $P(X > 2, Y \leq 1) = P(X = 3, Y = 0) + P(X = 3, Y = 1) = \frac{3}{30} + \frac{4}{30} = \frac{7}{30}$.

(c) $P(X > Y) = P(X > 0, Y = 0) + P(X > 1, Y = 1) + P(X > 2, Y = 2) = \frac{9}{15}$.

(d) $P(X + Y = 4) = P(X = 3, Y = 1) + P(X = 2, Y = 2) = \frac{4}{30} + \frac{4}{30} = \frac{4}{15}$.

Problem 5.

(a)

$$f(x) = \int_{-\infty}^{+\infty} f(x,y)\ dy = \int_0^1 \frac{2}{3}(x+2y)\ dy = \frac{2}{3}(x+1),\ 0 \leq x \leq 1$$

and 0 otherwise.

(b)

$$f(y) = \int_{-\infty}^{+\infty} f(x,y)\ dx = \int_0^1 \frac{2}{3}(x+2y)\ dy = \frac{2}{3}(2y+\frac{1}{2}),\ 0 \leq y \leq 1$$

and 0 otherwise.

(c) $P\,(X < 0.5) = \int_0^{0.5} \frac{2}{3}(x+1)\ dx = \frac{5}{12}$

Problem 6.

(a) $P\,(X+Y < 0.5) = \int_0^{0.5} \int_0^{0.5-x} 24\ xy\ dx\ dy = 12[\frac{1}{8}x^2 - \frac{1}{3}x^3 + \frac{1}{4}x^4]_0^{0.5} = \frac{1}{16}$

(b) $f(x) = \int_{-\infty}^{+\infty} f(x,y)\ dy = \int_0^{1-x} 24\ xy\ dy = 12\ x(1-x)^2,\ 0 \leq x \leq 1$

Problem 7.

(a)

x	1	2	3
g(x)	0.1	0.35	0.55

(b)

y	1	2	3
h(y)	0.2	0.5	0.3

(c) $P\,(Y = 3|X = 2) = \frac{f(y=3,x=2)}{g(x=2)} = \frac{0.2}{0.05+0.1+0.2} = 0.571$

(d) Since $f(y=3, x=2) = 0.2 \neq h(y=3)g(x=2) = 0.35 \cdot 0.3 = 0.105$, the two variables are dependent of each other.