Practice problems for Continuous Probability Distributions, Cumulative Distributions and Bivariate Distributions.

Problem 1. The proportion of people who respond to a certain mail-order solicitation is a continuous random variable X that has the density function

$$f(x) = \begin{cases} \frac{2(x+2)}{5}, & 0 < x < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Show that $P(0 < X < 1) = 1$.

(b) Find the probability that more than 1/4 but fewer than 1/2 of the people contacted will respond to this type of solicitation.

Problem 2. Consider the density function

$$f(x) = \begin{cases} k\sqrt{x}, & 0 < x < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the value of k.

(b) Find F(x) and use it to calculate $P(0.3 < X < 0.6)$.

Problem 3. The probability distribution of X, the number of imperfections per 10 meters of synthetic fabric in continuous rolls of uniform width, is given by

x	0	1	2	3	4
f(x)	0.41	0.37	0.16	0.05	0.01

Construct the cumulative distribution of X.

Problem 4. If the joint probability distribution of X and Y is given by

$$f(x,y) = \frac{x+y}{30}, \text{for x=0,1,2,3 and y=0,1,2}$$

find

(a) $P(X \leq 2, Y = 1)$.

(b) $P(X > 2, Y \leq 1)$.

(c) $P(X > Y)$.

(d) $P(X + Y = 4)$.

Problem 5. A privately owned liquor store operates both a drive-in facility and a walk-in facility. On a randomly selected day, let X and Y, respectively, be the proportions of the time that the drive-in and walk-in facilities are in use, and suppose that the joint density function of these random variables is

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y), & 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the marginal density of X.

(b) Find the marginal density of Y.

(c) Find the probability that the drive-in facility is busy less than one-half of the time.

Problem 6. A candy company distributes boxes of chocolates with a mixture of creams, toffees, and cordials. Suppose that the weight of each box is 1 kilogram, but the individual weights of the creams, toffees, and cordials vary from box to box. For a randomly selected box, let X and Y represent the weights of the creams and the toffees, respectively, and suppose that the joint density function of these variables is

$$f(x,y) = \begin{cases} 24\ xy, & 0 \le x \le 1,\ 0 \le y \le 1 \text{ and } x + y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the probability that in a given box the cordials account for more than 1/2 of the weight.

(b) Find the marginal density for the weight of the creams.

Problem 7. Let X denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let Y denote the number of times a technician is called on an emergency call. Their joint probability distribution is given as

			x	
	f(x,y)	1	2	3
	1	0.05	0.05	0.1
y	2	0.05	0.1	0.35
	3	0	0.2	0.1

(a) Evaluate the marginal distribution of X.

(b) Evaluate the marginal distribution of Y.

(c) Find $P(Y = 3|X = 2)$. [Note: this is a problem for the next chapter: Conditional Distributions].

(d) Determine if the two random variables X and Y are dependent or independent.