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AMS-131. Spring 2010

Name: ____________________

Final Exam (Type A)

The midterm is closed-book, you are only allowed to use one page of notes and a calculator. Please attach your formula sheet.

There are 5 problems, 100 points total. The point values for each problem are listed below. Some problems have multiple parts. Please provide a detailed calculation (not just a number) or an explanation (or both, as needed) to support your idea of the right answer in each problem. If you run out of space in any of the questions please use the last page and indicate that you are doing so.

REMEMBER: THIS TEST IS TO BE ENTIRELY YOUR OWN EFFORTS.

Problem	Possible Points	Your points
1	10	
2	50	
3	20	
4	10	
5	10	
Total	100	

1. (10 points) Suppose that a box contains three coins, and that for each coin there is a different probability that a head will be obtained when the coin is tossed. Let p_i be denote the probability that a head will be obtained when the ith coin is tossed $(i = 1, 2, 3)$ and suppose that $p_1 = 0$, $p_2 = 1/3$, $p_3 = 3/4$. Suppose that one coin is selected at random from the box and when it is tossed once, a head is obtained. What is the probability that the third coin was selected?

2. (50 points) Suppose that a person's score X on a mathematics aptitude test is a number in the interval $(0, 1)$ and that her score Y on a music test is also a number in the interval $(0, 1)$. Suppose also that in the population of all college students in the United States, the scores X and Y are distributed in accordance with the following joint p.d.f.:

$$f(x, y) = \begin{cases} \frac{2}{5}(2x + 3y) & \text{for } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) (4 points) Find the marginal p.d.f. of X.

   (b) (4 points) Find the marginal p.d.f. of Y.

   (c) (3 points) Are X and Y independent? Justify your answer.

(d) (3 points) Find $E(X)$.

(e) (3 points) Find $E(Y)$.

(f) (5 points) Find the p.d.f. of the random variable $W = e^{(X+1)}$.

(g) (5 points) Find the conditional p.d.f. of $X|Y$.

(h) (5 points) Compute $Pr(X > 1/2|Y = 1/3)$.

(i) (4 points) Find $E(X|Y = y)$.

(j) (7 points) Are X and Y positively correlated, negatively correlated, or uncorrelated? (Justify your answer).

(k) (7 points) Find the p.d.f. of $X + Y$.

3. (20 points) Let X_1, X_2, X_3, and X_4 be independent lifetimes of memory chips. Suppose that each X_i has a normal distribution with mean 400 hours and standard deviation 10 hours.

   (a) (8 points) Compute the probability that at least one of the four chips lasts at least 400 hours.

(b) (7 points) Let $Y_4 = \max\{X_1, X_2, X_3, X_4\}$. Compute $Pr(Y_4 \leq 400)$.

(c) (5 points) Let $Y_1 = \min\{X_1, X_2, X_3, X_4\}$. Compute $Pr(Y_1 \geq 400)$.

4. (10 points) Suppose that a random sample of 16 observations is drawn from a normal distribution with mean μ and standard deviation 12; and that independently another random sample of 25 observations is drawn from a normal distribution with the same mean μ and standard deviation 20. Let $\bar{X}$ and $\bar{Y}$ denote the sample means of the two samples (i.e., $\bar{X} = \frac{\sum_{i=1}^{16} X_i}{16}$ and $\bar{Y} = \frac{\sum_{i=1}^{25} Y_i}{25}$). Evaluate $Pr(|\bar{X} - \bar{Y}| \le 5)$.

5. (10 points) Suppose that a random sample of n independent measurements of the specific gravity of a certain body are to be taken by a physicist. It is assumed that these measurements follow certain distribution with mean μ and standard deviation σ. Determine the smallest number of items n that must be taken in order to satisfy the following relation:

$$Pr\left(|\bar{X}_n - \mu| \leq \frac{\sigma}{4}\right) \geq 0.99$$

Hint: Use Central Limit Theorem