CSE 102
Homework Assignment 2

1. Use induction to prove that $\sum_{k=1}^{n} k^4 = \frac{n(n+1)(6n^3+9n^2+n-1)}{30}$ for all $n \geq 1$.

2. Let $T(n)$ be defined by the recurrence

$$T(n) = \begin{cases} 1 & \text{if } \ n = 1 \\ T(\lfloor n/2 \rfloor) + n^2 & \text{if } \ n \geq 2 \end{cases}$$

Use substitution method to Show that $\forall n \geq 1$: $T(n) \leq (4/3)n^2$, and hence $T(n) = O(n^2)$.

3. Recall the n^{th} harmonic number was defined to be $H_n = \sum_{k=1}^{n} \left(\frac{1}{k}\right) = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1} + \frac{1}{n}$. Use induction to prove that

$$\sum_{k=1}^{n} kH_k = \frac{1}{2}n(n+1)H_n - \frac{1}{4}n(n-1)$$

for all $n \geq 1$. (Hint: Use the fact that $H_n = H_{n-1} + \frac{1}{n}$.)

4. Define $T(n)$ by the recurrence

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ T(n-1) + n & \text{if } n \geq 2 \end{cases}$$

Use the iteration method to find the exact solution to this recurrence, then determine an asymptotic solution.

5. Define $T(n)$ by the recurrence

$$T(n) = \begin{cases} 9 & \text{if } 1 \leq n < 15 \\ T(\lfloor n/2 \rfloor) + 6 & \text{if } n \geq 15 \end{cases}$$

Use the iteration method to find the exact solution to this recurrence, then determine an asymptotic solution.

5. 6. Use the Master Theorem to find tight asymptotic bounds on the following recurrences.

a. $T(n) = 3T(2n/3) + n^3$
b. $T(n) = 2T(n/3) + \sqrt{n}$
c. $T(n) = 5T(n/4) + n^{\lg\sqrt{5}}$
d. $T(n) = 3T(2n/5) + n \log n$
e. $S(n) = aS(n/4) + n^2$ (your answer will depend on the parameter a.)