

## Practice problems for Bivariate Distributions.

**Problem 1.** If the joint probability distribution of  $X$  and  $Y$  is given by

$$f(x, y) = \frac{x + y}{30}, \text{ for } x=0,1,2,3 \text{ and } y=0,1,2$$

find

- (a)  $P(X \leq 2, Y = 1)$ .
- (b)  $P(X > 2, Y \leq 1)$ .
- (c)  $P(X > Y)$ .
- (d)  $P(X + Y = 4)$ .

**Problem 2.** A privately owned liquor store operates both a drive-in facility and a walk-in facility. On a randomly selected day, let  $X$  and  $Y$ , respectively, be the proportions of the time that the drive-in and walk-in facilities are in use, and suppose that the joint density function of these random variables is

$$f(x, y) = \begin{cases} \frac{2}{3}(x + 2y), & 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Find the marginal density of  $X$ .
- (b) Find the marginal density of  $Y$ .
- (c) Find the probability that the drive-in facility is busy less than one-half of the time.

**Problem 3.** A candy company distributes boxes of chocolates with a mixture of creams, toffees, and cordials. Suppose that the weight of each box is 1 kilogram, but the individual weights of the creams, toffees, and cordials vary from box to box. For a randomly selected box, let  $X$  and  $Y$

represent the weights of the creams and the toffees, respectively, and suppose that the joint density function of these variables is

$$f(x, y) = \begin{cases} 24xy, & 0 \leq x \leq 1, 0 \leq y \leq 1 \text{ and } x + y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Find the probability that in a given box the cordials account for more than  $1/2$  of the weight.
- (b) Find the marginal density for the weight of the creams.
- (c) Find the probability that the weight of the toffees in a box is less than  $1/8$  of kilogram if it is known that creams constitute  $3/4$  of the weight.  
[Note: this is a problem for the next chapter: Conditional Distributions]

**Problem 4.** Let  $X$  denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let  $Y$  denote the number of times a technician is called on an emergency call. Their joint probability distribution is given as

		<b>x</b>		
		1	2	3
<b>y</b>	1	0.05	0.05	0.1
	2	0.05	0.1	0.35
	3	0	0.2	0.1

- (a) Evaluate the marginal distribution of  $X$ .
- (b) Evaluate the marginal distribution of  $Y$ .
- (c) Find  $P(Y = 3|X = 2)$ . [Note: this is a problem for the next chapter: Conditional Distributions].
- (d) Determine if the two random variables  $X$  and  $Y$  are dependent or independent.