

Advanced Homework 1: Asymptotic Growth of Functions (5 points)

Attempt **one** of the following 5 problems/exercises: problems do NOT carry equal weights.

1. (1point:easy) Problem 0.2 on Page 9 of DPV with proofs.
2. (2 points: medium) Problem 4 on Page 67-68 of KT with proofs

Exercise: Prove that $\binom{2n}{n} = \theta\left(\frac{4^n}{\sqrt{n}}\right)$, where $\binom{m}{k}$ denotes the binomial coefficient $\binom{m}{k} = \frac{m!}{k!(m-k)!}$, for $0 \leq k \leq m$.

Exercise: Determine a number $a > 0$ such that $\binom{3n}{n} = \Theta\left(\frac{a^n}{\sqrt{n}}\right)$.

Exercise: For each $k \geq 2$, determine a number $a_k > 0$ such that $\binom{kn}{n} = \Theta\left(\frac{a_k^n}{\sqrt{n}}\right)$.