Quiz 3

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(1) (a) number increase too fast.

(b) No. $\lceil n^{\log_2 3} \rceil$ is

(c) Clever brain to choose an appropiate Algrithm like Divide and Conqueor, Greedy, DP, etc.

(2) 23×12, 54×65, $(23+54)\cdot(65+12)$

(3) (a) $\log_4 n$

(b) Let $A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$ $B = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$ $C = \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix}$

$c_1 = a_1 \cdot b_1 + a_2 \cdot b_3$

$c_2 = a_1 \cdot b_2 + a_2 \cdot b_4$

$c_3 = a_3 \cdot b_1 + a_4 \cdot b_3$

$c_4 = a_4 \cdot b_2 + a_4 \cdot b_4$

8 multiplications

Assume multiplication need time a, adding need b.

$$T(n) = \begin{cases} a, & n = 1 \\ 8T(\frac{n}{2}) + 4(\frac{n}{2})^2 b \end{cases}$$

(c)

$T(n) = 7T(\frac{n}{2}) + f(n)$

Improve to 6 mutiplications

(4) (a) Let $A = [0, 1, 1, 2, 2, 3, 4, 4, 5]$, $n = 9$

Then each sub-part $[0, 1, 1]$, $[2, 2, 2]$ $[4, 4, 5]$ has a majority 1, 2, 4 for each

However, for A, there is no majority.

(b) Assume $n = 3m$, then each part contains m element

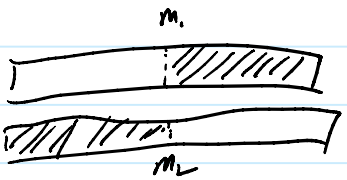
Since there is no majority in each part, then most often element count would be $a_1 \le \frac{m}{2}$, $a_2 \le \frac{m}{2}$, $a_3 \le \frac{m}{2}$

Add these inequality. $a_1 + a_2 + a_3 \le \frac{3}{2}m = \frac{n}{2}$

Therefore, the array has no majority

(5) (a) $\frac{24+24}{2} = 24$.

One case



$m_1 = \frac{a_{12} + a_{13}}{2}$

$m_2 = \frac{b_{12} + b_{13}}{2}$

$m_1 < m_2$

Divide the two array, add one of their part together.

(b) $[4, 5, 9, 11, 12]$
$[3, 6, 7, 8, 10]$ $\quad 9 > 7$

$[4, 5, 9]$
$[7, 8, 10]$ $\quad 5 < 8$

$[5, 9]$ $\quad$ median $= \frac{7+8}{2}$

[5, 9)
[7, 8)

median = $\frac{7+8}{2}$

= 7.5

6.