CSE 102 **Spring 2021**
Homework Assignment 1

1. (Problem 3.1-1) Let $f(n)$ and $g(n)$ asymptotically positive functions. Prove that $f(n) + g(n) = \Theta(\max(f(n),\ g(n)))$.

2. Prove or disprove: If $f(n) = \Theta(g(n))$, then $f(n)^2 = \Theta(g(n)^2)$.

3. Prove or disprove: If $f(n) = \Theta(g(n))$, then $2^{f(n)} = \Theta(2^{g(n)})$.

4. Let $f(n)$ and $g(n)$ be asymptotically positive functions, and assume that $\lim_{n\to\infty} g(n) = \infty$. Prove that if $f(n) = \Theta(g(n))$, then $\ln\big(f(n)\big) = \Theta\big(\ln\big(g(n)\big)\big)$.

5. (Problem 3.2-8) Show that if $f(n)\ln f(n) = \Theta(n)$, then $f(n) = \Theta(n/\ln n)$. Hint: use the result of the preceding problem.

6. Consider the statement: $f(cn) = \Theta(f(n))$.
   a. Determine a function $f(n)$ and a constant $c > 0$ for which the statement is false.
   b. Determine a function $f(n)$ for which the statement is true for all $c > 0$.

7. Determine the asymptotic order of the expression $\sum_{i=1}^{n} a^i$ where $a > 0$ is a constant, i.e. find a simple function $g(n)$ such that the expression is in the class $\Theta\big(g(n)\big)$. (Hint: consider the cases $a = 1$, $a > 1$, and $0 < a < 1$ separately.)

8. Use limits to prove the following:
   a. $n \ln(n) = o(n^2)$.
   b. $n^5 2^n = \omega(n^{10})$.
   c. If $P(n)$ is a polynomial of degree $k \geq 0$, then $P(n) = \Theta(n^k)$. State any assumptions you need to make for the above statement to be true.
9. Determine whether the first function is o, Θ, or ω of the second function.
   a. n^n compared to $2^{n \ln n}$
   b. $\sqrt{\ln n}$ compared to $\ln(\ln n)$.