1. $c=1 \Rightarrow 2c\sqrt{n} = 2\sqrt{n}$

When $n=5$ $2\cdot\sqrt{5} \approx 2\times 2.236 = 4.472 < 5$

when $n=4$ $2\cdot\sqrt{4} = 4 = 4$

Therefore the smallest $n=5$

2. Proof. When $\lim_{n\to\infty} f(n) = L_1$ and $0<L_1<\infty$

also ~~f(n)~~ $\lim_{n\to\infty} g(n) = L_2$, $0<L_2<\infty$.

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{L_1}{L_2}$ $0<\frac{L_1}{L_2}<\infty$

By the ~~theorem~~ theorem we learned, ~~$f(n)=\Theta(g$~~ $f(n)=\Theta(g(n))$

This completes the proof □

3. ~~if~~ Proof.

If $f(n)=o(g(n))$ then ~~$0<c_1g(n)<f(n)<c_2(g$~~

$0\le f(n) < Cg(n)$

$\ln f(n) < \ln Cg(n)$

$\ln f(n) < \ln c + \ln g(n)$

We can easily find a c' that makes

$c'\ln g(n) > \ln c + \ln(g(n))$, e.g. $c' > \frac{\ln c}{\ln(g(n))} + 1$

Hence. we get when $f(n)=o(g(n))$, $\ln f(n) = o(\ln(g(n)))$ □

4. ~~$\frac{d}{dn} f(n)$~~:

~~$f(n) = (2n)^n$~~

~~$\ln f(n) = \ln(2n)^n$~~

$$\cancel{\frac{d}{dn} f(n) = \frac{d}{dn}(2n)^n = \frac{d}{dn} e^{n\ln(2n)} = \frac{d}{dn} e^{n\ln(2n)}\cdot\frac{d}{dn}(n\ln(2n)) = e^{n\ln(2n)}\cdot(\ln(2n)+1)}$$

$f(n) = \cancel{g(n)}\,(2n)^n$

$\ln f(n) = \ln(2n)^n$

$f(n) = e^{n\ln(2n)}$

$f(n)' = e^{n\ln(2n)}\cdot\frac{d}{dn}(n\ln(2n))$

$= e^{n\ln(2n)}\cdot(\ln(2n)+1)$

$= (2n)^n\cdot(\ln(2n)+1)$