Hints

Homework 3

Problem 4

Multiplication of n-bit integers

Divide into $n/3$ bit-numbers $\Rightarrow b = 3$

number of multiplications needed $\Rightarrow a = ?$

combine work $\Rightarrow f(n) = cn$

$a = 9 \Rightarrow \log_3 9 = 2 \Rightarrow T(n) = \Theta(n^2)$

not better

what a would improve over $n^{\log_2 3}$?

Problem 6 Round-Robin Tournament

Assume $n = 2^k$

2 Teams

S_1 T_1 $(1, S_1, T_1)$ 1 day

4 Teams

$S_1 S_2$ | $T_1 T_2$ 1 day + 3 days

$S_1 T_1$	$S_1 T_2$
$S_2 T_2$	$S_2 T_1$

2 days

8 Teams

$S_1 S_2 S_3 S_4$ | $T_1 T_2 T_3 T_4$ 3 days + 7 days

$S_1 T_1$			
$S_1 T_2$			
$S_1 T_3$			
$S_1 T_4$			

4 days

⋮

Generalize

☆ Find Appropriate Mathematical Expression to describe the final solution in pseudo-code

☆ Make sure you understand that expression

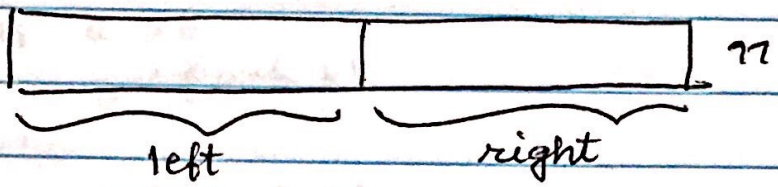
Problem A

Finding Majority element

$> \frac{n}{2}$ occurrences

$n = 6 \Rightarrow$ occurs ≥ 4 times

$n = 7 \Rightarrow$ occurs ≥ 4 times



$b = 2$ (2 subproblems of $1/2$ size)

Case 1: Left has no majority, right has no majority — Prove there is no majority

Case 2: Left has majority, right has no majority } What to do ? How to combine

Case 3: opposite of case 2 $\leftrightarrow$ opposite of case 2

Case 4: Both have majority $\leftrightarrow$ what to do ? How to combine

How many subproblems ?

What is the amount of work involved in combine = "what-to-do" ?

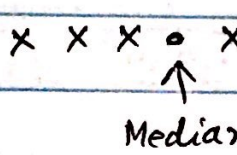
Problem 1: Median of Two Sorted Arrays of Size n

What is median?

Median = "Middle Value" "Half-way Point"

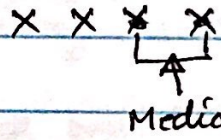
"$\frac{1}{2}$ Array values" $\leq$ Median $\leq$ "$\frac{1}{2}$ Array values"

n = odd sorted Array



$\frac{n+1}{2}^{th}$

n = even sorted Array



Average of n^{th} & $\frac{n+1}{2}^{th}$

For odd sorted arrays, median = $\left(\frac{n+1}{2}\right)^{th}$ element of array

For even sorted arrays, median = Average of n^{th} & $\left(\frac{n+1}{2}\right)^{th}$ element of array

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NAe: "Definitions" & Medians vary slightly between books

Brassard & Bratley: $\lceil \frac{n}{2} \rceil$-th smallest element

CLRS3 : Medians < lower median / upper median

Median of Two Sorted Arrays of Same Size

Base Cases: $n = 1$ $\boxed{a}$ $\boxed{b}$

$$\text{Median} = \frac{a+b}{2}$$

Base cases: $n = 2$ $\boxed{a_1\ a_2}$ $\boxed{b_1\ b_2}$

☆ Prove: $\text{Median} = \dfrac{\text{Max}(a_1, b_1) + \text{Min}(a_2, b_2)}{2}$

Proof Outline : Consider all possible cases

How many arrangements possible

Theoretically ~~24 = 16 cases~~ $4 \cdot 3 \cdot 2 \cdot 1 = 24$ cases

without loss of generality assume a_1 is the least (otherwise switch two arrays) $\Rightarrow$ $\boxed{a_1 \cdot \cdot \cdot}$ $3 \cdot 2 \cdot 1 = 6$ cases

~~$2^3 = 8$ cases~~

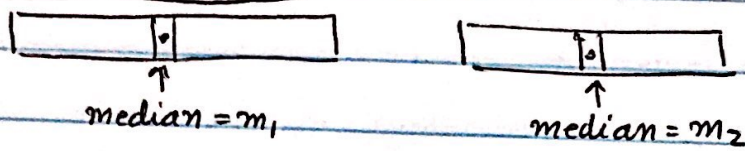
$a_1\ a_2\ b_1\ b_2$
$a_1\ a_2\ b_2\ b_1$ X
$a_1\ b_1\ b_2\ a_2$
$a_1\ b_1\ a_2\ b_2$
$a_1\ b_2\ a_2\ b_1$ X
$a_1\ b_2\ b_1\ a_2$ X

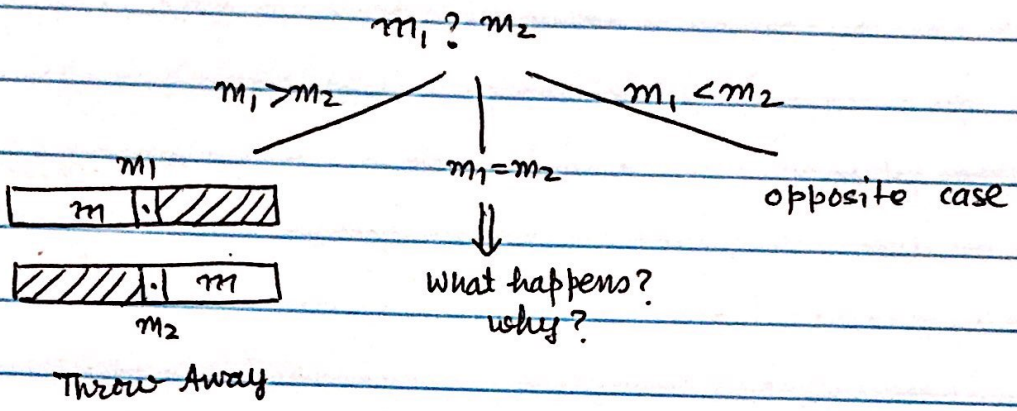
} only 3 cases left

If you use ☆, you need to prove this!

Median of Two Sorted Arrays of Same size

Case: n is odd $= 2m+1$



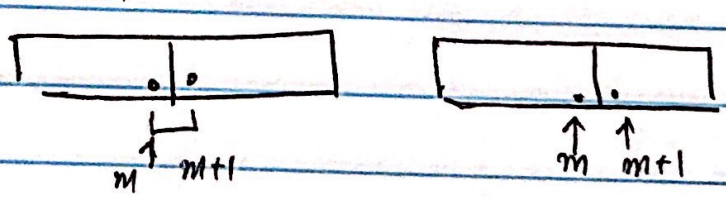


☆ What is the EXACT size of subproblem
Are two subarrays of same size?
Original size of problem = $4m+2$
What is the size of subproblem?
How many subproblems?
How much "combine" work required?

☆ why will be the solution to subproblems be the same as solution to the original problem?

Need Understanding / Answers to ☆

Case n is even $= 2m$



similar analysis needed