5. $\sum_{j=1}^{n} j a^n = a^n + 2a^n + 3a^n + \dots + na^n$

$= \frac{n^2+n}{2} a^n \geq \frac{1}{2} n^2 a^n$, so $c_1 = \frac{1}{2}$, $k=2$, $f(n) = \Omega(g(n))$

$\cancel{\frac{n^2+n}{2} a^n \cdot \frac{n+n^2}{2} a^n}$

$\cancel{\geq n^2 a^n}$

$\frac{n^2+n}{2} a^n \leq \frac{n^2+n^2}{2} a^n$

$\leq n^2 a^n$, so $c_2 = 1$, so $f(n) = O(g(n))$

we find that $\frac{1}{2} n^2 a^n \leq \sum_{j=1}^{n} j a^n \leq n^2 a^n$. Thus $\sum_{j=1}^{n} j a^n = \theta(n^2 a^n)$, $k=2$

6. (a) $T(n) = T(n-2) + n-2$

$= T(n-4) + n-2 + n-4$

$= \dots$

$\cancel{= T(n-k) + \frac{k}{2} n - \frac{(2+k)\cdot\frac{k}{2}}{2}}$ $\rightarrow T(n-2k) + kn - \frac{(2+2k)k}{2}$

it reaches the end when $n-k \leq 1$ ~~Therefore, w~~

(b): ① case 1 : n is even, $n-2k=0$

$T(n) = T(0) + \frac{n^2}{2} - \frac{n^2+2n}{4}$

$= \cancel{0+} \frac{n^2-2n}{4}$

② Case 2 : n is odd. $n-2k=1$

$T(n) = T(1) + \frac{n^2-n}{2} - \frac{(n-1)\cdot(n+1)}{4}$

$= 1 + \frac{n^2-n}{2} - \frac{n^2-1}{4}$

$= \cancel{\frac{n^2-2n+3}{4}}$ $\frac{n^2-2n+3}{4}$