"Common Functions"

① • Polynomials (exponent is a constant) : $n, n^2, n^3, \ldots$
• Exponentials (base is a constant) : $e^n, 2^n, a^n, \ldots$
• Logarithms (base is a constant) : $\ln n, \log n, \log_{10} n, \ldots$

② What do you need to compare "Asymptotic Growth of Functions"
→ How to take derivatives : Composition Rule : $(f(g(x)))' = f'(g(x)) \cdot g'(x)$
→ When / How to apply L'Hopital's Rule to compute $L = \lim_{n\to\infty} \frac{f(n)}{g(n)}$

Examples: $d(a^n) = \ln(a) \cdot a^n$; $d(\log_a(n)) = \frac{1}{n \cdot \ln(a)}$

③ Beware of "very important" differences :

• n^n or $f(n)^{g(n)}$: both base and exponents are functions of n
Method : Apply ln and then take derivative *

• Logarithms < polylogarithmic : $(\ln(n))^2 = \ln^2(n)$
iterated composition : $\ln(\ln(n)) = \ln^{(2)}(n)$

These are deceptively similar looking, but they are different!

Functions that require special care	"Advanced" Functions
• Step Functions $\lfloor x \rfloor$ $\lceil x \rceil$	• $n!$ (Sterling Approximation)
mod functions $a \bmod n = a - n \lfloor \frac{a}{n} \rfloor$	• Combinatorial fun $\binom{2n}{n}$
• Trigonometric Functions ($\sin x$, $\cos x$)	• Fibonacci Numbers
	. . .

* $f(n) = n^n \Rightarrow \ln f(n) = n \ln n$

Take Derivative $\Rightarrow \frac{1}{f(n)} f'(n) = \left(1 \cdot \ln n + n \cdot \frac{1}{n}\right) = (\ln n + 1)$

$\Rightarrow f'(n) = f(n)(\ln n + 1)$

$\Rightarrow (n^n)' = n^n (1 + \ln n)$ ☑