

Quiz 1 Reflection

3. ~~$f(n) = \dots$~~

$$f(n) = o(g(n)) \not\Rightarrow \ln f(n) = o(\ln g(n))$$

There exist a counter-example that $f(n) = n$ $g(n) = n^2$

$$f(n) = o(g(n)) \text{ — obvious}$$

$$n = o(n^2)$$

$$\ln f(n) = \ln n$$

$$\ln(g(n)) = \ln n^2 = 2 \ln n.$$

$$\text{Thus } \ln f(n) = \theta(g(n)).$$

6. Compute the $\lim_{n \rightarrow \infty} \frac{f(n)/g(n)}{h(n)}$, guess $h(n) = n$

$$\lim_{n \rightarrow \infty} \frac{f(n)/g(n)}{h(n)} = \lim_{n \rightarrow \infty} \frac{a_0 + a_1 n + \dots + a_k n^k}{n^{k-1} \cdot n}$$

$$= \lim_{n \rightarrow \infty} \frac{a_0}{n^k} + \frac{a_1}{n^{k-1}} + \dots + a_k$$

$$= a_k.$$

$$\text{Since } a_k > 0, \frac{f(n)}{g(n)} = \theta(h(n))$$

3: Since we've proved that $f(n) = \theta(g(n)) \Rightarrow \ln f(n) = \theta(\ln g(n))$,

I thought it would similar to prove "o" case. But I forgot $o \neq \theta$.

6. I first compute $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)}$, then using the result directly to compute

$\lim_{n \rightarrow \infty} \frac{f(n)/g(n)}{h(n)}$. But I forget I've used (k-1) times L'Hopital's theorem on first result. So I get the final result wrong.