Quiz 4

2021年4月29日 8:23

(1) (a) 99

(b) 1

(c) 1

(d) 0

(2) (a) $5\times 8+10=50$

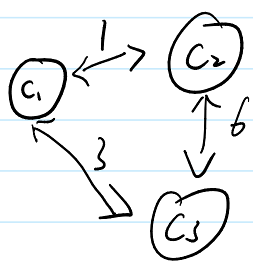
(b) Starting from V_{10}, we can find each edge betwae. (V_i, V_{10}) $i\in[2,9]$ is smaller than (V_1, V_{10}), which is 10. Apply this algorithm, to other edge until thast edge. (V_2, V_1) which is 10.

(3) (a) $S_1 \to S_5 \to S_6 \to S_7$

(b) No.

$S_2 \to S_3$

(4)



Apply the Greedy algorithm: $C_1 \to C_2 \to C_3$, cost $=1+6=7$

Optimal: $C_1 \to C_2 \to C_1 \to C_3$, Cost $=1+1+3=5$

(5) Let $C=\{C_1=1,\ C_2=5,\ C_3=5\times 2,\ C_4=5^2\}$

Suppose $X=\{x_1, x_2, x_3, x_4\}$ be the optimal solution.

Let $g=\{g_1, g_2, g_3, g_4\}$ be the solution to the greedy algorithm

We need to prove $\sum_{i=1}^{4} x_i c_i = \sum_{i=1}^{4} g_i c_i$

$$x_1+5x_2+2.5x_3+5^2x_4 = g_1+5g_2+2.5g_3+5^2g_4$$

$$x_1 + 5x_2 + 2\cdot 5x_3 + 5^2x_4 = g_1 + 5g_2 + 2\cdot 5g_3 + 5^2g_4$$

Reducing the equation by mod 5 yields $x_1 \equiv g_1 \pmod 5$

Notice that $0 \le x_1 < 5$, $0 \le g_1 < 5$, which implies $x_1 = g_1$

Then we can eliminate x_1 and g_1, divide 5 we get equation

$$x_2 + 2x_3 + 5x_4 = g_2 + 2g_3 + 5g_4$$

we can get $x_2 + 2x_3 = (g_2 + 2g_3) \pmod 5$

we cand notice $0 \le x_2 < 2$, $0 \le x_3 < 2.5$, so as g_2 and g_3

$x_2 = 0, 1$ $\quad$ $x_3 = 0, 1, 2$

Result = 0, $x_2 = 0$ $x_3 = 0$, Unique

Result = 1, $x_2 = 1$, $x_3 = 0$, unique

Result = 2, $x_2 = 0$, $x_3 = 1$, unique

Result = 3, $x_2 = 1$, $x_3 = 1$, unique

Result = 4, $x_2 = 0$, $x_3 = 2$, unique

For every result, the solution is unique. Thus $x_2 = g_2$, $x_3 = g_3$.

Use the similar approach, we got $x_4 = g_4$

Thus we prove every element in greedy solution is the same as the optimal solution.

(6)

	Imp sat	X	Y	New	Check
Initial		F	F		
$\Rightarrow X$	No	T		Y	
$X \wedge Y \Rightarrow Y$	No			T	
$\neg Y$					F
Final		N/A	N/A		

(b) The only case that it's not satified is that Z is false, $X \wedge Y$ is True.

X	Y	Z
T	T	F