Quiz 3

(1) (a) 'Decimal war' is an attempt to improve asymptotic complexity of a problem by a tiny decimal amount. ← 1 Point

(b) No ← 1 Point

(c) The algorithm may not be practical. ← 1 Point

(2) 2354 * 1265

Three multiplications are 23 * 12, 54 * 65 ← 1 Point

$$\begin{array}{r} 23 \\ +\,54 \\ \hline 77 \end{array} \qquad \begin{array}{r} 12 \\ +\,65 \\ \hline 77 \end{array} \quad \leftarrow \text{2 Points}$$

lose .5 points if numbers are not added above

lose 2.5 points if only $(X_0+X_1) * (Y_0+Y_1)$ are written with no numbers } ⇒ only 0.5 credit

Variations: 3254 * 1265

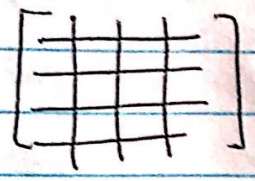
Three multiplications are 32 * 12, 54 * 65

$$\begin{array}{r} 32 \\ +\,54 \\ \hline 86 \end{array} \qquad \begin{array}{r} 12 \\ +\,65 \\ \hline 77 \end{array}$$

Variation : 1254 * 6534

Three multiplications are 12 * 65, 54 * 34

$$\begin{array}{r} 12 \\ +\,54 \\ \hline 66 \end{array} \qquad \begin{array}{r} 65 \\ 34 \\ \hline 99 \end{array}$$

③ (a)  4×4 = 16 Blocks ← 1 Point

(b) Number of Multiplications Required = $4^3 = 64$ ← 0.5 Point

$T(n) = 64T\left(\frac{n}{4}\right) + c \cdot n^2$ ← 0.5 Point

(c) # of multiplications = M

Looking for maximum M ⇒ $\log_4^M < \log_2^7 = \log_4^{49}$ ← 1 Point

Therefore $\boxed{M = 48}$ ← 2 Points

Variation 8×8 partition

(a) 8×8 partitions ⇒ 64 Blocks

(b) $T(n) = 8^3 T\left(\frac{n}{8}\right) + cn^2$

(c) $\log_8^M < \log_2^7 = \log_8^{343} \Rightarrow \boxed{M = 342}$

Variation 5×5 partition

(a) 5×5 partitions ⇒ ~~825 Block~~ 25 Blocks

(b) $T(n) = 5^3 T\left(\frac{n}{5}\right) + cn^2$

(c) $\log_5^M < \log_2^7$ $\log_5^{90} = 2.7958$, $\log_5^{91} = 2.802$ $\log_5^{92} = 2.809$

$\Rightarrow M = 91$

④ (a) Not True.

Counter-example

[111 222 333] ← 2 Points

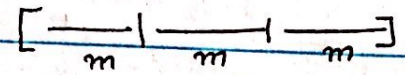
Each of the three subparts have a majority element but the array has no majority element

(b) True.

Proof by contradiction.

suppose an element E is a majority element for the array.

3 Points



$n = 3m$

E appears $< \frac{m}{2}$ $< \frac{m}{2}$ $< \frac{m}{2}$ $\Rightarrow$ E appears $< \frac{3m}{2} = \frac{n}{2}$

$\Rightarrow$ E is not a majority element ☒

Variations with partitions of 4 & 5 partitions are similar

Note: (a) NO credit if stated not true but no counter-example is provided

(b) NO credit if stated true but only an example is provided without proof.

If proof simply asserts the truth, but does not present arguments using inequalities (or something similar) partial credit

⑤ (a) 24 → 13 → 7 → 4 → (3) → (2) — Important, Important; Base case — 1 Point

even case [. . . . .] must include $\frac{n}{2}+1$ at the next level — 1 Point

odd case [. . . . .] must include $\frac{n+1}{2}$ at the next level — 1 Point

(b) [4 5 (9) 11 12] [3 6 (7) 8 10]

⟹ [4 (5) 9] [7 (8) 10] — 1 Point

⟹ [5 9] [7 8] — 1 Point

⟹ $\frac{\text{Max}(5,7) + \text{Min}(9,8)}{2}$ — 1 Point

$= \frac{7+8}{2} = \frac{15}{2} = (7.5)$

Note (a) must state clearly by text or diagram that median element is included. Base case must be 2. otherwise very little credit

(b) must write down the subarrays explicitly

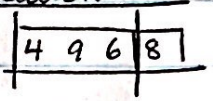
variations: 45 → 23 → 12 → (7) → 4 → 3 → (2) — Important, Important

23 → 12 → (7) → 4 → 3 → (2) — Important, Important

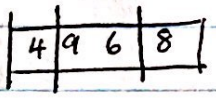
similar variations for (b)

Quicksort

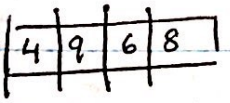
⑥



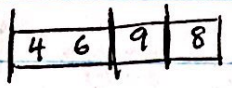
1 Point



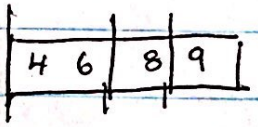
compare 4 with 8; no swap but a partition introduced between 4 and 9



compare 9 with 8; no swap but a partition introduced between 9 and 6



compare 6 with 8 and swap 9 & 6 ☆ 2 Points



Insert the pivot in place by swapping 8 and 9

} 1 Point

Variations · similar

different methodologies may have been used by students

critical elements to look at

→ sorting is "in-place"

→ comparing only with pivot element 8

→ only two elements swapped at any time

→ pivot does not move till the end

→ ☒ 4 distinct steps as outlined above