(1)

(a) False ← 0.5 Point

(b) n = # of denominations

c = change needed = <u>value</u> of input

$= 2^b$, where b = # of bits needed to represent C ← 1 Point

= input size

$O(nC) = O(n 2^b)$

↑ Exponential

(c) True ← 0.5 Point

(d) True

(because $P \subseteq NP$) ← 0.5 Point

(e) Integer Factorization

Given a positive integer N,

are there integers $j, k > 1$ such that $N = j * k$ ← 0.5 Point

(f) Counterexample

Denomination 1, 5, 12 ← 1 Point

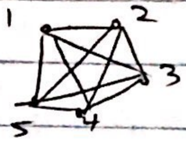
Greedy Algorithm will use 8 coins of 12 and 4 coins of 1 ← 1 Point

2 Points

(other solutions exist)

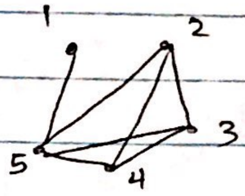
② Connected Graph: Adversary Argument

Start with complete Graph K_5 =



Q	Response
1,2	No
1,3	No
1,4	No
1,5	Yes

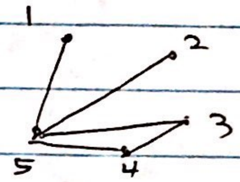
⇒



2 Points

Q	Response
2,3	No
2,4	No
2,5	Yes

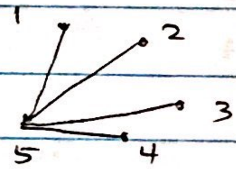
⇒



1 Point

Q	Response
3,4	No
3,5	Yes

⇒



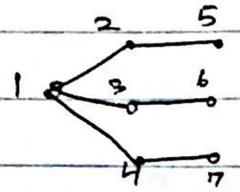
1 Point

Q	Response
4,5	Yes ⇒ connected
	No ⇒ Disconnected

1 Point

minimum # of questions = 10

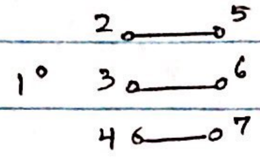
③ Vertex Cover Problem



} 2 Point

greedy: vertex with degree 3 = 1 ⇒ vertex cover {1}

Remove all incident edges



choose vertices 2, 3, 4 ⇒ vertex cover {1, 2, 3, 4}

\# of vertices = 4

} 2 Point

optimal: vertices {2, 3, 4}

\# of vertices = 3

} 1 Point

other solutions exist

no credit if counterexample has <5 or >7 vertices

if theoretical arguments are presented but no counterexample

(4) Peeking a bit string

$x_1 \quad x_2 \quad x_3 \quad x_4$

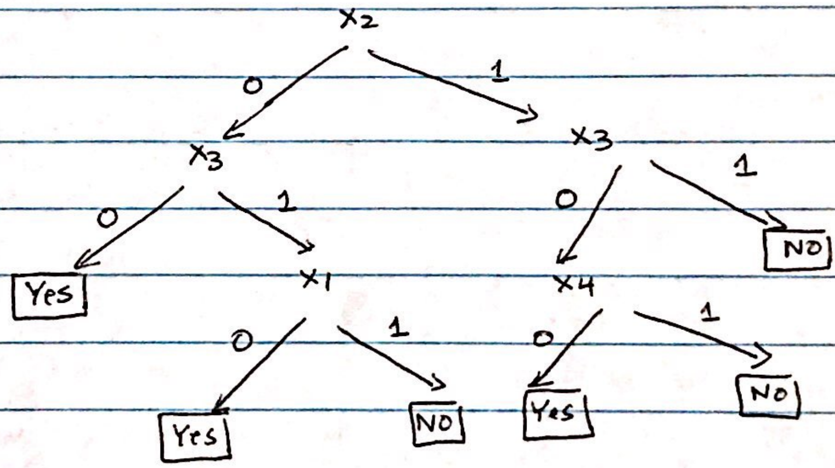
observation/scratch work:

~~If player peeks at x_1 or x_4,~~
adversary answers 1.
This reduces the problem to 3 bit string, without loss of generality $x_1 x_2 x_3$
Now the player will be forced to peek at all 3 bits because
player seeks x_1 or x_3 ⇒ Adversary answer 1 } and so on
player seeks x_2 ⇒ Adversary answers 0 }

(b) The first bit player peeks at is $\boxed{x_2}$ 1 Point

(x_3 is also a correct answer by symmetry)

(c)



3 Points

(d) 3 peeks 1 Point

(5)

(a) ∞

2 Points

(b) $f(x) = 256 \cdot \frac{1}{x}$ } 3 Points

{ any function $f(x)$ that satisfies following properties will work

(i) $f(x)$ monotonically decreasing

(ii) $f(x) > 0$ for $x > 0$

(iii) $f(1) = 256$ }

If ~~adversary~~ player stops after finite # of steps and

(i) guesses any integer for which $f(x) < 0$, then produce above fn

(ii) guesses never crosses below 0, then make the fn dip below 0 after the maximum N for which ~~adv~~ player peeked.