Solutions to practice problems for Expectation Value, Variance and Covariance.

Problem 1. $E[X] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xf(x,y)\,dx\,dy = \int_{-a}^{a}\int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} \frac{1}{\pi a^2}\,dx\,dy = 0$

Problem 2. $E[X] = 0\times0.41+1\times0.37+2\times0.16+3\times0.05+4\times0.01 = 0.88$

Problem 3. $E[X] = \$4000 \times 0.3 + \$1000 \times 0.7 = \$500$.
$Var[X] = \sum [x - E[X]]^2 \cdot f(x) = (4000-500)^2 \cdot 0.3 + (-1000-500)^2 \cdot 0.7 = \$5250000$.

Problem 4. Premium $- 0.002 \times 200000 + 100000 \times 0.01 + 50000 \times 0.1 = 500 \Rightarrow$ Premium $= 6900$

Problem 5.

(a) $E[g(X)] = [2(-3)+1]^2\,\frac{1}{6} + [2(6)+1]^2\,\frac{1}{2} + [2(9)+1]^2\,\frac{1}{3} = 209$.

(b) $Var[g(X)] = E\left[\left\{g(X) - \mu_{g(X)}\right\}^2\right] = \left\{[2(-3)+1]^2 - 209\right\}^2\frac{1}{6} + \left\{[2(6)+1]^2 - 209\right\}^2\frac{1}{2} + \left\{[2(9)+1]^2 - 209\right\}^2\frac{1}{3} = 14144$

Problem 6.

(a) $E[g(X,Y)] = 2\cdot1(0.1) + 2\times9(0.2) + 2\times25(0.1) + 4\times1(0.15) + 4\times9(0.3) + 4\times25(0.15) = 35.2$.

(b) $\mu_X = 2\times(0.1+0.2+0.1) + 4\times(0.15+0.3+0.15) = 3.2$
$\mu_Y = 1\times0.25 + 3\times0.5 + 5\times0.25 = 3$

Problem 7.

(a) $Var[X] = E(X^2) - [E(X)]^2 = 4\times0.01 + 9\times0.25 + 16\times0.4 + 25\times0.3 + 36\times0.04 - [2\times0.01 + 3\times0.25 + 4\times0.4 + 5\times0.3 + 6\times0.04]^2 = 17.63 - (4.11)^2 = 0.738$

(b) $E[Z] = E[3X-2] = 3E[X] - 2 = 3\times4.11 - 2 = 10.33$
$Var[Z] = Var[3X-2] = 9Var[X] = 9\times0.738 = 6.64$.

Problem 8. $Cov[X,Y] = E[XY] - \mu_X\mu_Y$
$E[XY] = \int_0^1\int_0^1 x\ y\ \frac{2}{3}\ (x+2y)\ dx\ dy = \frac{1}{3}$
$f(x) = \int_0^1 \frac{2}{3}\ (x+2y)\ dy = \frac{2}{3}\ (x+1)$
$\mu_X = \int_0^1 x\ f(x)dx = \int_0^1 x\frac{2}{3}\ (x+1)\ dx = \frac{5}{9}$

$g(y) = \int_0^1 \frac{2}{3}\ (x + 2y)\ dx = \frac{1}{3}\ (4y + 1)$
$\mu_Y = \int_0^1 y\ \frac{1}{3}\ (4y + 1)dy = \frac{33}{54}$
$Cov[X, Y] = \frac{1}{3} - \frac{5}{9}\ \frac{33}{54} = -0.00617$

Problem 9. $Var[Z] = Var[-2X + 4Y - 3] = Var[-2X] + Var[4Y] = 4Var[X] + 16Var[Y] = 68$

Problem 10. $E[Z] = E[XY] = E[X] \cdot E[Y] = \int_2^\infty x\frac{8}{x^3}\ dx \cdot \int_0^1 y\frac{2}{y}\ dy = 8$
Note: Pay attention to the indefinite integral.