Recurrence Equations

Master Theorem Analysis

$a \geq 1$, $b > 1$, $f(n)$ asymptotically positive

$$T(n) = a\,T\left(\frac{n}{b}\right) + f(n)$$

a = # of subproblems

b = $\frac{\text{size of original problem}}{\text{size of subproblem}}$ } "Division" Work

Example: $a = 2$, $b = 2$ — 2 problems of $\frac{1}{2}$ size

$f(n)$ = } Extra "Combine" Work

special case $a = b \Rightarrow \log_b(a) = \log_b(b) = 1$

Case 1: $f(n) = O(n^{1-\epsilon}) \Rightarrow T(n) = \Theta(n)$ "Division" Dominates

Case 2: $f(n) = \Theta(n) \Rightarrow T(n) = \Theta(n \log n)$ Division + Combine Partnership

Case 3: $f(n) = \Omega(n^{1+\epsilon}) \Rightarrow T(n) = \Theta(f(n))$ "Combine" Dominates
+ regularity

General case: $\log_b(a) = c$

Case 1 $f(n) = O(n^{c-\epsilon}) \Rightarrow T(n) = \Theta(n^c)$ "Division" Dominates

2: $f(n) = \Theta(n^c) \Rightarrow T(n) = \Theta(n^c \log n)$ Division + Combine Partnership

3: $f(n) = \Omega(n^{c+\epsilon}) \Rightarrow T(n) = \Theta(f(n))$
+ regularity

Need to know how to compute $\log_b(a)$

a	1	....	$\sqrt[k]{b}$		$\sqrt[3]{b}$	$\sqrt[2]{b}$	b	b^2	b^3	...	b^k	...
$\log_b(a)$	0		$1/k$		$1/3$	$1/2$	1	2	3	...	k	...