CSE 102: Spring 2021
Extra Credit Problems

No human assistance, real or virtual, allowed.
Internet sources are allowed and must be cited.
Each of the following problem is worth 1 point.

1. **Greedy-Vertex-Cover:** Consider the following greedy strategy: Select a vertex of highest degree, and remove all of its incident edges. Give an example to show that this algorithm does not have an approximation ratio of 2.

2. **Traveling Salesman Problem on a plane:** Suppose that the vertices for an instance of the traveling salesman problems are points in the plane and that the cost $c(u, v)$ is the Euclidean distance between points u and v. Show that an optimal tour never crosses itself.

3. **Traveling Salesman Problem: Nearest Link Strategy:** Suppose you use Kruskal's algorithm to build a minimum spanning tree, which is then used to create a solution to the traveling salesman problem. This strategy is called Nearest Link strategy. Prove that this strategy does not produce an optimal tour.

4. **Traveling Salesman Problem: Nearest Link vs Nearest Neighbor Strategy:** Create an example to establish that nearest link strategy can do better than nearest neighbor strategy. Create another example to establish that nearest neighbor strategy can do better than nearest link strategy.

5. **SAT to 3SAT:** Describe a conversion of a SAT problem to an equivalent 3SAT problem so that the original SAT problem is satisfiable iff the 3SAT problem is satisfiable. The conversion should be possible in polynomial time.

6. **3SAT to Constrained-3SAT:** Describe a conversion of a 3SAT problem to a equivalent constrained-3SAT problem where no variable appears in > 3 clauses so that the original SAT problem is satisfiable iff the 3SAT problem is satisfiable. The conversion should be possible in polynomial time.

Remark: Observe that after this transformation, each literal appears at most twice in the entire set of clauses but 3SAT still allows clauses with 3 literals. Therefore, the above constrained-3SAT problem is not a 2SAT problem, which restricts the number of literals to less than or equal to 2 in each clause.

P Versions of Simplified NP-Complete problems

Following problems are worth 2.5 to 5 points, depending upon the level of difficulty (you will know when you attempt them).

1. **Vertex Cover for a Tree:** Give an efficient greedy algorithm that finds an optimal vertex cover for a tree (acyclic, connected, undirected graph) in polynomial time. With careful implementation, devise a linear algorithm.

2. **3SAT with one literal:** Prove that a 3SAT problem can be solved in polynomial time if each literal appears at most once in the entire set of clauses.

3. **2-chromatic Graphs:** Devise an algorithm to determine the chromatic number of graphs with the property that each vertex has degree at most 2. The running time of your algorithm should be $O(n)$, where n is the number of vertices.

4. **Clique-3:** Describe an $O(n)$ algorithm for Clique-3, where n is the number of vertices in a graph.

5. **Clique-4:** Describe a polynomially-bounded algorithm to determine if a graph has 4-clique. What is the worst-case complexity of your algorithm?

Proof Problems

1. **Traveling Salesman Problem on the earth:** Suppose that the vertices for an instance of the traveling salesman problems are cities on the earth and that the cost $c(u, v)$ is the spherical distance between cities u and v. Prove that an optimal tour never crosses itself or provide a counterexample.

2. **Hamiltonian Cycle:** Give necessary and sufficient conditions for an undirected graph with maximum degree 2 to have a Hamiltonian Cycle. Outline an efficient algorithm to test the conditions.

3. **Planar Graph Node Coloring:** Prove that any planar graph can be colored using at most 6 colors. No two adjacent nodes can have the same color. You may use Euler's formula that $v - e + f = 2$, where v is number of vertices, e is number of edges, and f is number of faces. [Hint: First, prove that any planar graph with $v \geq 3$ must satisfy $e \leq 3n - 6$. Then, establish that a planar graph must have at least one vertex of degree at most 5. Then use induction to prove the above result.]

   [Remark: Planar Graph Region coloring is the problem of coloring a map so that no contiguous region has the same color, as in a map. Four color conjecture stated that every planar graph is 4-colorable. This conjecture was proved in 1976 for the first time using a computer.]

4. **Euler Path:** Show that an undirected connected graph with possibly multiple edges between certain pairs of nodes has an Eulerian tour iff all its vertices have even degree.