CSE 102: SPRING 2021

QUIZ 1 Solutions

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(1) $2c\sqrt{n} < n \Rightarrow \underset{\text{since } c=1}{2\sqrt{n} < n} \Rightarrow 2 < \sqrt{n} \Rightarrow 4 < n$ 1 Point

$\Rightarrow \boxed{n_0 = 5}$ ▨ 1 Point

(2) $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{\lim_{n\to\infty} f(n)}{\lim_{n\to\infty} g(n)} = \frac{L_1}{L_2} = L$ where $0 < L < \infty$ 1 Point

Therefore, $f(n) = \Theta(g(n))$ ▨ 1 Point

(3) $f(n) = o(g(n)) \not\Rightarrow \ln f(n) = o(\ln(g(n)))$

Counter-example: $f(n) = n$; $g(n) = n^2$ Example 1 Point

$n = o(n^2)$

$\left.\begin{array}{l} \ln f(n) = \ln n \\ \ln g(n) = \ln n^2 = 2 \ln n \end{array}\right\} \Rightarrow \begin{array}{l} \ln f(n) \neq o(\ln(g(n))) \\ \text{In fact } \ln f(n) = \Theta(\ln(g(n))) \end{array}$ 1 Point ~~1 Point~~

▨

other counter-examples exist

(4) $f(n) = (2n)^n$

Step 1: $\ln f(n) = n \ln(2n)$

Step 2: $\frac{d}{dn} \ln f(n) = \frac{d}{dn} n(\ln(2n))$

$\Rightarrow \underbrace{\frac{f'(n)}{f(n)}}_{\text{composition rule}} = \underbrace{1 \cdot \ln(2n) + n \cdot \cancel{\ ?\ } \cdot \frac{d(\ln(2n))}{dn}}_{\text{multiplication rule}}$ $\boxed{1 \text{ Point}}$

$\boxed{1 \text{ Point}}$

$\Rightarrow \frac{f'(n)}{f(n)} = \ln(2n) + \cancel{n} \cdot \underbrace{\frac{1}{2\cancel{n}} \cdot \frac{d(2n)}{dn}}_{\text{composition rule}}$ $\boxed{1 \text{ Point}}$

$\Rightarrow \frac{f'(n)}{f(n)} = \ln(2n) + \frac{1}{\cancel{2}} \cdot \cancel{2} = \ln(2n) + 1$

$\Rightarrow f'(n) = f(n)[1 + \ln(2n)]$
$= (2n)^n [1 + \ln(2n)]$ $\boxed{1 \text{ Point}}$

No credit if step 1 is missing

(5) $f(n) = n^n \Rightarrow \ln f(n) = n \ln n \Rightarrow f(n) = e^{n \ln n}$

$g(n) = e^{n \ln n}$

$\therefore f(n) = g(n)$ 1 Point

$\therefore f(n) = \Theta(g(n))$ 1 Point

(6) $f(n) = a_0 + a_1 n + \cdots + a_k n^k$

$g(n) = n^{k-1}$

Let $h(n) = n$ 1 Point

then $$\lim_{n\to\infty} \frac{f(n)/g(n)}{h(n)} = \lim_{n\to\infty} \frac{a_0 + a_1 n + \cdots + a_k n^k}{n^{k-1} \cdot n}$$

$$= \lim_{n\to\infty} \underbrace{\frac{a_0}{n^k} + \cdots + \frac{a_{k-1}}{n}}_{0} + a_k$$

$$= a_k.$$

1.5 Point

Since $a_k > 0 \Rightarrow \frac{f(n)}{g(n)} = \Theta(h(n))$ .5 Point

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