# CSE 102 Spring 2021
# Advanced Homework Assignment 1

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## 1 AdvHW1

**1. Prove that $\binom{2n}{n} = \Theta(\frac{4^n}{\sqrt{n}})$, where $\binom{m}{k}$ denotes the binomial coefficient $\binom{m}{k} = \frac{m!}{k!(m-k)!}$, for $(0 \leq k \leq m)$.**

*Proof.* By using $Stirling's\ Formula$, we can get:[1]

$$\begin{aligned}
\binom{2n}{n} &= \frac{(2n)!}{n! \cdot n!} \\
&= \frac{\sqrt{2\pi 2n} \cdot (\frac{2n}{e})^{2n}}{(\sqrt{2\pi n} \cdot (\frac{n}{e})^n)^2} \\
&= \frac{\sqrt{4\pi n} \cdot \frac{4^n \cdot n^{2n}}{e^{2n}}}{(\sqrt{2\pi n} \cdot \frac{n^n}{e^n})^2} \\
&= \frac{4^n}{\sqrt{\pi n}}
\end{aligned}$$

Therefore, we can observe that:

$$\begin{aligned}
\frac{\sqrt{\pi}}{100} \cdot \frac{4^n}{\sqrt{\pi n}} \leq & \frac{4^n}{\sqrt{\pi n}} \leq \sqrt{\pi} \cdot \frac{4^n}{\sqrt{\pi n}} \\
\frac{1}{100} \cdot \frac{4^n}{\sqrt{n}} \leq & \frac{4^n}{\sqrt{\pi n}} \leq \frac{4^n}{\sqrt{n}} \\
\frac{1}{100} \cdot \frac{4^n}{\sqrt{n}} \leq & \binom{2n}{n} \leq \frac{4^n}{\sqrt{n}}
\end{aligned}$$

Hence, we have $\binom{2n}{n} = \Theta(\frac{4^n}{\sqrt{n}})$. □

# References

[1] Wikipidea, https://en.wikipedia.org/wiki/Stirling