CSE 102: Spring 2021

Quiz # 2: April 14 (15 points)

30 minutes (Quiz) + 10 minutes (uploading) = 40 minutes

You may use the following fact:

- $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.

Assume, n is a positive integer, unless specified otherwise.

Problems

1. (2 points)

   $$T(n) = \begin{cases} 0 & \text{for } n = 0 \\ T(n-1) + n & \text{for } n > 0 \end{cases}$$

   Using induction, prove that $T(n) \leq n^2 \ \forall n \geq 0$.

2. (2 points)

   $$T(n) = \begin{cases} 2 & \text{for } n = 0 \\ T(n-1) + 1 & \text{for } n > 0 \end{cases}$$

   Using iteration, find an *exact* solution for $T(n)$.

3. (2 points)

   Let $a > 1$. Consider the following recurrence:

   $T(n) = \sqrt{a}T(\frac{n}{a}) + \sqrt{n}$

   Can we apply Master's Theorem to this recurrence? If not, explain why. If yes, which case and derive the asymptotic growth of $T(n)$.

4. (2 points)

   Recall that, $f(n) = \Omega(g(n))$ means that
   $\exists c > 0, \exists n_0 > 0, \forall n \geq n_0 : 0 \leq cg(n) \leq f(n)$.

   Prove that $\sum_{i=1}^{n}(ia^i) = \Omega(n^2 a)$ for $a > 1$ by specifying a c for which the inequality holds. [Do **NOT** use limits]

5. (2 points)

   Assume $a > 1$. Recall that $f(n) = \theta(g(n))$ means that
   $\exists c_1 > 0, \exists c_2 > 0, \exists n_0 > 0, \forall n \geq n_0 : c_1 g(n) \leq f(n) \leq c_2 g(n)$.

   It can be easily proved that $\sum_{j=1}^{n}(ja^n) = \theta(n^k a^n)$ for *some* k.

   Your task is to find a k for which the above statement holds.

6. (5 points)

   $$T(n) = \begin{cases} 0 & \text{for } n = 0 \\ 1 & \text{for } n = 1 \\ T(n-2) + (n-2) & \text{for } n \geq 2 \end{cases}$$

   (a) (2 points) Derive an expression for $T(n)$ after k iterations, and

   (b) (3 points) Assume n is even. Use above result to solve for $T(n)$, that is, find an *exact* expression for $T(n)$ in terms of n.