Practice problems for the Normal Distribution and the Central Limit Theorem

Problem 1. Given a standard normal distribution, find the area under the curve which lies

(a) to the left of $z = 1.43$;

(b) to the right of $z = -0.89$;

(c) between $z = -2.16$ and $z = -0.65$;

(d) to the left of $z = -1.39$;

(e) to the right of $z = 1.96$;

(f) between $z = -0.48$ and $z = 1.74$;

Problem 2. Given a standard normal distribution, find the value of k such that

(a) $P(Z < k) = 0.0427$;

(b) $P(Z > k) = 0.2946$;

(c) $P(-0.93 < Z < k) = 0.2946$.

Problem 3. Given the normally distributed variable X with mean 18 and standard deviation 2.5, find

(a) $P(X < 15)$;

(b) the value of k such that $P(X < k) = 0.2236$;

(c) the value of k such that $P(X > k) = 0.1814$;

(d) $P(17 < X < 21)$.

Problem 4. A soft-drink machine is regulated so that it discharges an average of 200 milliliters per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 milliliters,

(a) What fraction of the cups will contain more than 224 milliliters?

(b) What is the probability that a cup contains between 191 and 209 milliliters?

(c) How many cups are expected to overflow if 230 milliliter cups are used for the next 1000 drinks?

(d) Below what value do we get the smallest 25% of the drinks?

Problem 5. The random variable X, representing the number of cherries in a cherry puff, has the following probability distribution

x	4	5	6	7
P(X=x)	0.2	0.4	0.3	0.1

(a) Find the mean μ and the variance σ^2 of X.

(b) Find the mean $\mu_{\bar{x}}$ and the variance $\sigma_{\bar{x}}^2$ of the mean $\overline{X}$ for random samples of 36 cherry puffs.

(c) Approximate the probability that the average number of cherries in 36 cherry puffs will be less than 5.5.

Problem 6. The average life of a bread-making machine is 7 years, with a standard deviation of 1 year. Assuming that the lives of these machines follow approximately a normal distribution, find:

(a) The probability that the mean life of a random sample of 9 such machines falls between 6.4 and 7.2 years;

(b) The value x that is such that 15% of the means computed from random samples of size 9 would fall above x.

Problem 7. An astronomer is interested in measuring, in light years the distance to a distant star. The values of the measurements are i.i.d. with a common mean d, and a common variance of 4. How many measurements does the astronomer need to make in order to estimate the distance to within ± 0.5 light years of d with probability 95%?