Solutions to practice problems for Discrete Probability Distributions

Problem 1.

(a) $P(X \leq 2, Y = 1) = P(X = 0, Y = 1) + P(X = 1, Y = 1) + P(X = 2, Y = 1) = \frac{1}{30} + \frac{2}{30} + \frac{3}{30} = \frac{1}{5}$.

(b) $P(X > 2, Y \leq 1) = P(X = 3, Y = 0) + P(X = 3, Y = 1) = \frac{3}{30} + \frac{4}{30} = \frac{7}{30}$.

(c) $P(X > Y) = P(X > 0, Y = 0) + P(X > 1, Y = 1) + P(X > 2, Y = 2) = \frac{3}{5}$.

(d) $P(X + Y = 4) = P(X = 3, Y = 1) + P(X = 2, Y = 2) = \frac{4}{30} + \frac{4}{30} = \frac{4}{15}$.

Problem 2.

(a)

$$f(x) = \int_{-\infty}^{+\infty} f(x, y)\ dy = \int_0^1 \frac{2}{3}(x + 2y)\ dy = \frac{2}{3}(x + 1), \quad 0 \leq x \leq 1.$$

(b)

$$f(y) = \int_{-\infty}^{+\infty} f(x, y)\ dx = \int_0^1 \frac{2}{3}(x + 2y)\ dy = \frac{2}{3}(2y + \frac{1}{2}), \quad 0 \leq y \leq 1.$$

(c) $P(X < 0.5) = \int_0^{0.5} \frac{2}{3}(x + 1)\ dx = \frac{5}{12}$

Problem 3.

(a) $P(X + Y < 0.5) = \int_0^{0.5} \int_0^{0.5-x} 24\ xy\ dx\ dy = 12[\frac{1}{8}x^2 - \frac{1}{3}x^3 + \frac{1}{4}x^4]_0^{0.5} = \frac{1}{16}$

(b) $g(x) = \int_{-\infty}^{+\infty} f(x, y)\ dy = \int_0^{1-x} 24\ xy\ dy = 12\ x(1 - x)^2,\ 0 \leq x \leq 1$

(c) First we need to calculate $f(y|x) = \frac{f(x,y)}{g(x)} = \frac{24xy}{12x(1-x)^2} = \frac{2y}{(1-x)^2}$. The probability asked is $P\left(Y < \frac{1}{8} | X = \frac{3}{4}\right) = \int_0^{\frac{1}{8}} f(y|x = \frac{3}{4})\ dy = \int_0^{\frac{1}{8}} \frac{2y}{(1-\frac{3}{4})^2}\ dy = \frac{1}{4}$.

Problem 4.

(a)

x	1	2	3
g(x)	0.1	0.35	0.55

(b)

y	1	2	3
h(y)	0.2	0.5	0.3

(c) $P\left(Y=3|X=2\right)=\frac{f(y=3,x=2)}{g(x=2)}=\frac{0.2}{0.05+0.1+0.2}=0.571$

(d) Since $f(y=3,x=2)=0.2\neq h(y=3)g(x=2)=0.35\cdot 0.3=0.105$, the two variables are dependent of each other.