Solutions to practice problems for Conditional Probability

Problem 1.

(a) Probability that the convict committed armed robbery given that the convict pushed dope.

(b) Probability that the convict did not push dope given that the convict committed armed robbery.

(c) Probability that the convict did not commit armed robbery given that the convict did not push dope.

Problem 2. Let F_1 and F_2 denote the events that fire engine 1 and fire engine 2 are available, respectively. We have that $P(F_1) = 0.96$ and $P(F_2) = 0.96$.

- $P(F_1' \text{ and } F_2') = P(F_1')^2 = [1 - P(F_1)]^2 = (1 - 0.96)^2 = 0.0016$.
- 0.9984.

Problem 3. Let XR denote the event that patient gets an X-ray, CF the event that the patient gets a cavity filled and TE the event that the patient gets a tooth extracted. Given this, the probabilities given can be written in the following way: $P(\text{XR}) = 0.6$, $P(\text{CF}|\text{XR}) = 0.3$ and $P(\text{TE}|\text{XR, CF}) = 0.1$.

What is asked is $P(\text{TE and XR and CF})$ which can be calculated by using the multiplicative rule:

$$
\begin{aligned}
P(\text{TE and XR and CF}) = P(\text{XR}) \cdot P(\text{CF}|\text{XR}) \cdot P(\text{TE}|\text{XR, CF}) = \\
= 0.6 \times 0.3 \times 0.1 = 0.018
\end{aligned}
$$

Solutions to practice problems for Bayes Rule

Problem 4. Let PD denote the event that a diagnose is positive and C the event the patient has cancer. The probabilities given are: $P(\text{C}) = 0.05$, $P(\text{PD}|\text{C}) = 0.78$ and $P(\text{PD}|\text{C'}) = 0.06$.

(a) From the theorem of total probability, $P\,(\text{D}) = P\,(\text{C}) \cdot P\,(\text{D}|\text{C}) + P\,(\text{C'}) \cdot P\,(\text{D}|\text{C'}) = 0.05 \times 0.78 + 0.06 \times 0.95 = 0.096$.

(b) From Bayes Rule, $P\,(\text{C}|\text{D}) = \frac{\text{P(D|C)} \cdot \text{P(C)}}{\text{P(D|C)} \cdot \text{P(C)} + \text{P(D|C')} \cdot \text{P(C')}} = \frac{0.78 \times 0.05}{0.096} = 0.406$.

Problem 5.

(a) From the multiplicative rule, $P\,(\text{A and B and C}) = P\,(\text{A}) \cdot P\,(\text{B}|\text{A}) \cdot P\,(\text{C}|\text{A and B}) = 0.3 \times 0.75 \times 0.2 = 0.045$.

(b)

$$
\begin{aligned}
P\,(\text{B' and C}) &= P\,((B' \cap C \cap A) \cup (B' \cap C \cap A')) \\
&= P\,(B' \cap C|A) \cdot P(B'|A) \cdot P(A) + P(B' \cap C|A') \cdot P(B'|A') \cdot P(A') \\
&= 0.8 \times (1 - 0.75) \times 0.3 + 0.9 \times (1 - 0.2) \times 0.7 \\
&= 0.564.
\end{aligned}
$$

(c)

$$
\begin{aligned}
P(C) = &\; P(C|A \cap B) \cdot P(B|A) \cdot P(A) + P(C|A' \cap B) \cdot P(B|A') \cdot P(A) \\
&+ P(C|A \cap B') \cdot P(B'|A) \cdot P(A) + P(C|A' \cap B') \cdot P(B'|A') \cdot P(A') \\
= &\; 0.63
\end{aligned}
$$

(d) 0.1064.