Quiz # 1: April 7, 2021 (15 points)

30 minutes (Quiz) + 10 minutes (uploading) = 40 minutes

You may use use any definitions. You may use the limit statements or any facts in the AsymptoticSummarySKL document (enclosed with Lecture 1 module), *unless stated otherwise*. Reference to any other theorems, lemmas, or exercises will earn no credit.

In the following, assume that $f(n)$ and $g(n)$ are asymptotically positive functions.

1. (2 points) Since $n = \omega(2\sqrt{n})$, we know that,

$$\forall c > 0, \exists n_0 > 0, \forall n \geq n_0 : 0 \leq 2c\sqrt{n} < n.$$

   Assume, $c = 1$. Provide the smallest integer n_0 for which the above inequality is satisfied. Derive the result.

2. (2 points) Let $\lim_{n \to \infty} f(n) = L_1$, where $0 < L_1 < \infty$.

   Let $\lim_{n \to \infty} g(n) = L_2$, where $0 < L_2 < \infty$.

   Prove or disprove that $f(n) = \theta(g(n))$.

3. (2 points) Prove or disprove: If $f(n) = o(g(n))$, then $\ln f(n) = o(\ln g(n))$.

4. (4 points) Compute the derivative of $f(n) = (2n)^n$ using the following methodology (do not use internet derivative calculator).

   Step 1. Take ln on both sides to derive $\ln f(n) = $ ......

   Step 2. Now, take derivative on both sides.

   Now, complete the remaining steps.

5. (2 points) Let $f(n) = n^n$ and $g(n) = e^{(n \ln n)}$. List exactly ONE of the following choices that most accurately reflects the relationship between $f(n)$ and $g(n)$. If none is correct, write NONE.

(a) $f(n) = \Omega(g(n))$

(b) $f(n) = \omega(g(n))$

(c) $f(n) = \theta(g(n)$

(d) $f(n) = o(g(n))$

(e) $f(n) = O(g(n))$

You must show all your work to demonstrate how you arrived at the conclusion. [No points for correct answer without proof].

6. (3 points) Let $f(n) = a_0 + a_1 n + \ldots + a_k n^k$, where $k > 0$ and for $0 \le i \le k, a_i > 0$. Let $g(n) = n^{k-1}$.

Find a simple $h(n)$ such that $f(n)/g(n) = \theta(h(n))$. Prove your assertion.