# Solutions to practice problems for Discrete Probability Distributions

**Problem 1**. Let $X = x$ denote number of theft cases where the need to get drugs is stated as the main reason for perpetrating the crime. $p = 0.75$ and $n = 5$.

(a) $P\left(X = 2\right) = \text{bin}(x; n, p) = \text{bin}(2; 5, 0.75) = C_{5,2}\ p^2\ (1-p)^{n-x} = C_{5,2}\ 0.75^2\ 0.25^3 = 0.0879.$

(b) $P\left(X \leq 3\right) = C_{5,0}\ 0.75^0\ 0.25^5 + C_{5,1}\ 0.75\ 0.25^4 + C_{5,2}\ 0.75^2\ 0.25^3 + C_{5,3}\ 0.75^3\ 0.25^2 = 0.00098 + 0.01465 + 0.08789 + 0.26367 = 0.36719$

**Problem 2**.

(a) This deals with having 4 victories in the first 4 games. $P\left(X = 4\right) = C_{4,4}\ 0.9^4\ 0.1^0 = 0.6561.$

(b) For the series to go to seven games both teacms have to win three games in the first six. So the answer is $C_{6,3} 0.9^3 0.1^3 = 0.1458$

**Problem 3**.

(a) There are 12 face cards in a deck of 52 so $n = 7, A = 12, B = 40$. $X$ : number of cards that are face cards if 7 are selected. $P\left(X = 2\right) = \text{hyp}(x; A, B, n) = \frac{C_{A,x}\ C_{B,n-x}}{C_{A+B,n}} = \frac{C_{12,2}\ C_{40,5}}{C_{52,7}}.$

(b) There are 4 queens so $n = 7, A = 4, B = 48$. $X$ : number of queens in 7 cards. $P\left(X = 1, 2, 3, 4\right) = \text{hyp}(x; A, B, n) = \frac{C_{4,1}\ C_{48,6}}{C_{52,7}} + \frac{C_{4,2}\ C_{48,5}}{C_{52,7}} + \frac{C_{4,3}\ C_{42,4}}{C_{52,7}} + \frac{C_{4,4}\ C_{48,3}}{C_{52,7}}.$

**Problem 4**. $p = 0.7$, and $n = 18$.

$P\left(X = 10\right) + P\left(X = 11\right) + P\left(X = 12\right) + P\left(X = 13\right) \simeq \text{bin}(10; 18, 0.7) + \text{bin}(11; 18, 0.7) + \text{bin}(12; 18, 0.7) + \text{bin}(13; 18, 0.7)$, since for values of $A \gg B$ we can approximate the hypergeometric distribution by a binomial. The final result will be,

$C_{18,10}\ 0.7^10\ 0.3^8 + C_{18,11}\ 0.7^11\ 0.3^7 + C_{18,12}\ 0.7^12\ 0.3^6 + C_{18,13}\ 0.7^13\ 0.3^5 = 0.0811 + 0.1376 + 0.1873 + 0.2017 = 0.6078.$

**Problem 5**. $A = 3$, $B = 17$, $n = 5$

(a) $P(X=0) = \text{hyp}(x; A, B, n) = \frac{C_{3,0}\ C_{17,5-0}}{C_{20,5}} = 0.3991.$

(b) $P(X=2) = \text{hyp}(x; A, B, n) = \frac{C_{3,2}\ C_{17,5-2}}{C_{20,5}} = 0.1316.$

**Problem 6**. $p = 0.8$, let $X = x$ denote number of people that believe in the story.

(a) $C_{5,2}\ 0.8^4\ 0.2^2 = C_{5,3}\ 0.8^4\ 0.2^2$.

(b) $0.8^1\ 0.2^2$.

**Problem 7**. The probability asked is described by a Binomial distribution with parameters $p = 0.001$ and $n = 10000$. For these extreme parameters its calculation is difficult, but we can approximate it by a Poisson distribution with parameters $\lambda = n \times p = 10$. So the probability will be calculated by $P(X = x) = \text{Poisson}(x; \lambda) = \frac{e^{-\lambda}\ \lambda^x}{x!}$.

The probability asked is that $P(X = 6, 7, 8) = \text{Poisson}(6; 10) + \text{Poisson}(7; 10) + \text{Poisson}(8; 10) = \frac{e^{-10}\ 10^6}{6!} + \frac{e^{-10}\ 10^7}{7!} + \frac{e^{-10}\ 10^8}{8!} = 0.0631 + 0.0901 + 0.1126 = 0.2657$

**Problem 8**. $\lambda = 3$.

(a) $P(X = 5) = \frac{e^{-3}\ 3^5}{5!} = 0.1008$

(b) $P(X = 0, 1, 2) = e^{-3} + \frac{e^{-3}\ 3^1}{1!} + \frac{e^{-3}\ 3^2}{2!} = 0.0498 + 0.1494 + 0.224 = 0.4232$

(b) $P(X \geq 2) = 1 - P(X = 0, 1) = 1 - 0.0498 - 0.1494 = 0.8008$