1. Propositions and Formulas

a)

|  |  |  |  |
| --- | --- | --- | --- |
| p | q | q V p | p ^ (q V p) |
| T | T | T | T |
| T | F | T | T |
| F | T | T | F |
| F | F | F | F |

p ^ (q V p) is contingent formula and this formula is satisfiable

b)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| p | q | p → q | ¬q | (p→q)∧¬q | ((p→q)∧¬q)→p |
| T | T | T | F | F | T |
| T | F | F | T | F | T |
| F | T | T | F | F | T |
| F | F | T | T | T | F |

((p→q)∧¬q)→p is a contingent formula and this formula is satisfiable

c)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | ¬p | ¬q | p∨q | ¬p∧¬q | ¬(¬p∧¬q) | (p∨q)↔¬(¬p∧¬q) |
| T | T | F | F | T | F | T | T |
| T | F | F | T | T | F | T | T |
| F | T | T | F | T | F | T | T |
| F | F | T | T | F | T | F | T |

(p∨q)↔¬(¬p∧¬q) is tautology formula and this formula is satisfiable

2. Logical Equivelance

a)

p∧(q∨r)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p | q | r | q∨r | p∧(q∨r) |
| T | T | T | T | T |
| T | F | T | T | T |
| T | T | F | T | T |
| T | F | F | F | F |
| F | T | T | T | F |
| F | F | T | T | F |
| F | T | F | T | F |
| F | F | F | F | F |

(p∧q)∨(p∧r)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| p | q | r | p∧q | p∧r | (p∧q)∨(p∧r) |
| T | T | T | T | T | T |
| T | F | T | F | T | T |
| T | T | F | T | F | T |
| T | F | F | F | F | F |
| F | T | T | F | F | F |
| F | F | T | F | F | F |
| F | T | F | F | F | F |
| F | F | F | F | F | F |

Based on 2 truth tables shown above, we can see that p∧(q∨r)≡(p∧q)∨(p∧r)

b)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| p | q | ¬p | ¬q | ¬p∨¬q | p∧q | ¬(¬p∨¬q) |
| T | T | F | F | F | T | T |
| T | F | F | T | T | F | F |
| F | T | T | F | T | F | F |
| F | F | T | T | T | F | F |

Based on a truth shown above, we can see that p∧q≡¬(¬p∨¬q)

c)

+ First, using the logical equivalent formula A → B ≡ ¬A ∨ B. When A is p and B is q.

We have, p → q ≡ ¬p ∨ q.

+ While, A∨B≡¬(¬A∧¬B). When A is ¬p and B is q.

So, we have ¬p ∨ q ≡¬(¬¬p∧¬q).

+ Moreover, A ≡ ¬¬A. When A is p.

We have p ≡ ¬¬p. So ¬(¬¬p∧¬q) ≡ ¬(p∧¬q).

With all three statements above and using substitution,

We can conclude that p → q ≡ ¬(p∧¬q).

d)

+ First, using the logical equivalent formula A↔B ≡( A→B) ∧ (B→A). When A is p and B is q,

We have p↔q ≡ (p→q)∧(q→p).

+ Moreover, A → B ≡ ¬A ∨ B. When A is p and B is q

We have p → q ≡ ¬p ∨ q.

+ In addition, using the logical equivalent formula A → B ≡ ¬A ∨ B again but when A is q and B is p

We have q → p ≡ ¬q ∨ p

With all 3 statements above and using substitution.

We can conclude that p↔q≡(¬p∨q)∧(¬q∨p).

3. Logical Implications and Proofs

a)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p | q | ¬q | p∨q | (p∨q)∧¬q |
| T | T | F | T | F |
| T | F | T | T | T |
| F | T | F | T | F |
| F | F | T | F | F |

Based on the truth table above, whenever (p∨q)∧¬q is true then p is true.

We can conclude that: (p∨q)∧¬q⊨p

b)

(p↔q)∧q⊨p

1 p↔q Premise

2 q Premise

3 (p→q)∧(q→p) Equivalent to line 1 using A↔B ≡( A→B) ∧ (B→A)

4 (q→p)∧(p→q) Equivalent to line 3 using A ∧ B ≡ B ∧ A

5 q→p Using logical implication of line 3 using A∧B⊨A

6 p Using logical implication of line 5 AND line 2 using

(A→B)∧A⊨B

4. Truth values of fully quantified predicates

a)

|  |  |  |  |
| --- | --- | --- | --- |
| x | y | xy | xy >= 0 |
| 0 | 0 | 0 | T |
| 0 | 1 | 0 | T |
| 0 | -1 | 0 | T |
| 1 | 0 | 0 | T |
| 1 | 1 | 1 | T |
| -1 | -1 | 1 | T |
| -1 | 0 | 0 | T |
| -1 | 1 | -1 | F |
| 1 | -1 | -1 | F |

b)

∀x∃y p(x, y) is true because we can choose y=0 for every x

c)

∃x∀y p(x, y) is true because we can choose x =0 for every y

5. Proofs with Predicates

Let x is “my teapot”

A is “ purple”

B is “holds water”

a) Translate syllogism to set – theoretic notation

My teapot is purple x ∈ A

My teapot holds water x ∈ B

There exist purple things that hold water (A ∩ B ≠ ⊘)

b) Translate set -theoretic notation to the notation of predicate logic

Let p(x) is “ My teapot is purple”

h(x) is “My teapot holds water”

x ∈ A: p(x)

x ∈ B: h(x)

A ∩ B ≠ ⊘: ∃y ∈ (p(y) ∧ h(y))

c)

p(x) ∧ (x ∈ S) ⊨ ∃y ∈ S p(y)

“My teapot is purple” and “ My teapot holds water” implies “ There exist purple things that hold water”

Proof:

1 p(x) Premise

2 h(x) Premise

3 p(x) ∧ h(x) Using logical implication from line 1 and line 2

using A,B ⊨A ∧ B

4 ∃y ∈ (p(y) ∧ h(y)) Using logical implication from line 3 using existential

Generalization Q(a) → ∃xQ(x)

6.Proofs

a)

Contrapositive: Suppose x, y ∈ Z, if not 5 ∤ x and 5 ∤ y then not 5 ∤ xy. In other words, suppose x, y ∈ Z, if 5|x or 5|y then 5|xy

Proof:

Case 1: Let 5|x is true then x = 5a (a ∈ Z).

From this we get xy = 5(ay), which means 5 | xy.

Case 2: Let 5|y is true then y = 5a (a ∈ Z)

From this we get xy = 5(ax), which means 5 | xy.

All two cases above prove that suppose x, y ∈ Z, if 5|x or 5|y then 5|xy.

In other words, Suppose x, y ∈ Z, if not 5 ∤ x and 5 ∤ y then not 5 ∤ xy

b)

Proof by contradiction:

Assume, a and b are two odd perfect squares, then a + b is a perfect square.

+ When a number is a perfect square then that number is always congruent to 0 or 1 mod 4.

Proof:

case 1: square number is even (2a) ^ 2 = 4a^2 ≡ 0 (mod 4)

case 2: square number is odd (2a +1) ^2 = 4a^2 + 4a + 1 ≡ 1 (mod 4)

+ Let a and b are two odd perfect squares, then a + b is a perfect square

So, a = (2x + 1)^2 and b = (2y +1) ^ 2 for some x, y ∈ N

a + b = (2x + 1)^2 + (2y+1)^2

= 4x^2 + 4y^2 + 4x + 4y + 2

We can see that a +b ≡ 2 (mod 4) which contradicts with a theory proven above so a + b can not be a perfect square

Conclusion: if a and b are two odd perfect squares, then a + b is not a perfect square.

7.Induction

a)

Base case : the first number is 1 as n = 1, we have

Inductive step:

Assume the first n number we have

We have to prove that for the case first n + 1 numbers:

Proof:

This proves that:

8. Program Correctness

a) p(x,y) = (x≥0∧y = 1) ⋁ (x<0∧y = 0)

b)

if (x >= 0) :

{x >= 0} Choice Rule

y = 1

{y = 1} Assignment rule

{ x >= 0) } x not changed

{ x≥0∧y = 1} implication rule

{(x≥0∧y = 1) ⋁ (x<0∧y = 0)} implication rule

else:

{x<0} Choice rule

y = 0

{y = 0} Assignment rule

{x<0} x not changed

{ x<0∧y = 0} implication rule

{(x≥0∧y = 1) ⋁ (x<0∧y = 0)} implication rule

{(x≥0∧y = 1) ⋁ (x<0∧y = 0)} choice rule

9. Relations

a) R is irreflexive because fRf can not relate to itself

b) R is anti-symmetric because f must be compiled before g

c)R is transitive because there might be not directive dependency (g might depend on f through some intermediary files)

d)R is strict partial ordering because R is irreflexive, anti-symmetric and transitive

e)R is not an equivalence or total ordering relation because it is not reflexive and symmetric

10.Functions

a)

i.

f:Z→Z given by f(x) = 3x+ 7

By the definition: f(x) = f(y)

So: 3x + 7 = 3y+7

3x = 3y

X= y

In conclusion: f is injective

ii.

f is not injective because 2 QUT students could have same last name but have different QUT student numbers

b)

i.

Let x, y ∈ S

By definition: f(x) = f(y)

Suppose that f(x) = j and x = sj

So f(x) = f(y) = j

f(y) = j

So y =sj = x

So f is injective

ii)

R is reflexive because f(a) = f(a).

Suppose aRb and bRa:

Because f is injective so f(a) = f(b). So a = b and vice versa.

R is anti-symmetric.

Suppose aRb and bRc

So f(a) f(b) and f(b) f(c)

So f(a) f(c)

So aRc

R is transitive

Let x, y ∈ S

So x=si and y = sj (i,j ∈ Z)

If xRy, x y so si sj, I  j

If yRx, y x so sj si, j i

If xRy V yRx , y = x or i= j since R is reflexive, transitive and anti-symmetric

So, xRy V yRx V x = y

R is total ordering on R