DSCI 122 Fall 2025 HW5

Show the details of your answer and write down the row operations you are using.

Problem 1

Compute the product AB in the following cases:

a) $A = \begin{bmatrix} 1 & -1 & 0 & 1 \\ 2 & 1 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 1 & 3 \\ -1 & 0 \\ 0 & 2 \end{bmatrix}$.

b) $A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 3 & 2 \\ 2 & 1 & 1 \end{bmatrix}$

Problem 2

Consider the matrices $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 1 \end{bmatrix}$

Find the elimination matrices E_1 and E_2 such that $E_2E_1A = B$.

Problem 3

Say if the matrix is invertible or not then compute its inverse:

a) $A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$.

b) $A = \begin{bmatrix} 2 & 3 & 4 \\ 2 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix}$.

Problem 4

a) Let A be an $n \times n$ matrix and $x \in \mathbb{R}^n$ such that $Ax = 7x$. Express $A^{-1}x$ in terms of x.

b)Let A, B and C be three invertible matrices. Does the equation $C^{-1}(A+X)B^{-1} = I_n$ have a solution X? If yes express the solution in terms of A, B and C.

Problem 5

Suppose that A is an $m \times n$ matrix and there exist $n \times m$ matrices C and D such that $CA = I_n$ and $AD = I_m$. Show that $D = C$ and $m = n$.