Version 2024.1

Assignment 1

Mathematics for Robotics

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This assignment is part of the *Fall 2024* offering of *ROB310: Mathematics for Robotics* at the University of Toronto. Please review the information below carefully. Submissions that do not adhere to the guidelines below may lose marks.

Release Date: .... September 16, 2024 - 00:00
Due Date: .... September 22, 2024 - 23:59
Weight: .... 3.75% or 0%
Late Penalties: .... -10% per day and -100% after 3 days

Submission Information:

- Download this document as a PDF and fill in your information below.
- Answer the questions to the best of your ability.
- Typeset your answers, labelling the questions identically to this document. We recommend using LaTeX, but will accept the use of other typesetting software (such as Microsoft Word or Google Docs).
- Save your typeset answers as a PDF and combine it with the cover-sheet of this document. Do NOT include the question descriptions.
- Submit your PDF on Crowdmark titled `lastname-firstname-rob310f24-a1.pdf`.

Name (First, Last): ....

UTORId (e.g., gummalur): ....

Acknowledgement:

I hereby affirm that the work I am submitting for this assignment is entirely my own. I have not received any unauthorized assistance, and I understand that plagiarism or any other form of academic dishonesty is a violation of the University's academic integrity policy.

I agree to the above statement....................

Date (dd/mm/yyyy): ....

Problem 1.1

A car moves on a straight road with an unknown but constant slope. It is assumed that the inertia of the wheels is negligible, that the friction force is proportional to the speed of the car, and that the engine imparts a force u.

Applying Newton's law, we obtain the following mathematical model of the system:

$$m\ddot{x}(t) = -b\dot{x}(t) + u(t) + mg\sin\theta,$$

where m, b, g, $\theta \in \mathbb{R}$ are the mass, the coefficient of friction, the gravitational constant, and the angle, respectively. The input to the system is $u(t)$. Since we are interested in controlling the speed $\dot{x}(t)$ of the car, we rewrite the model using $v(t) = \dot{x}(t)$,

$$m\dot{v}(t) = -bv(t) + u(t) + mg\sin\theta.$$

The output we can measure is:

$$y(t) = v(t).$$

Part 1.1.A

Assume that the road is flat, that is, $\theta = 0°$. If the input and output is sampled with a sampling time of T, derive a discrete-time representation of the system:

$$v_{k+1} = A_D v_k + B_D u_k, \quad y_k = v_k$$

with $v_k = v(kT)$ and the other variables being defined accordingly.

Provide a **closed-form** analytic expression for A_D and B_D.

Part 1.1.B

For $m = 1$, $b = 0.1$, $T = 0.01$, $\theta = 0°$ and $g = 10$, calculate the numeric values for A_D and B_D based on the result from Part 1.1.A.

Part 1.1.C

Compute A_D and B_D using `expm` within `scipy`. Compare the result with Part 1.1.B.

Part 1.1.D

Compute A_D and B_D using `cont2discrete` within `scipy`. Use a zero-order hold for the discretization method. Compare the results with Part 1.1.B and 1.1.C.

Part 1.1.E

Simulate the system obtained in Part 1.1.A with the parameter values from Part 1.1.B for a step input $u_k = 1$ for $k = 1, \ldots, 10000$ with $v_0 = 0$. Generate a plot showing the sequence $\{y_k\}$. The horizontal axis should represent the time in seconds. Provide a brief explanation of the result. Explain the role of b.

Part 1.1.F

Your system model above is unfortunately not quite correct. In the real-world system, the coefficient of friction is $\tilde{b} = 0.4$. Generate a plot of the error between the model output and the real-world output for the input and initial condition given in Part 1.1.E.

Part 1.1.G

Assume that $m = 1$, $b = 0.1$, $T = 0.01$, $\theta = 30°$, and $g = 10$. Simulate the resulting system with the parameter for a step input $u_k = 1$ for $k = 1, \ldots, 10000$ with $v_0 = 0$. Generate a plot showing the sequence $\{y_k\}$. The horizontal axis should represent the time in seconds. Provide a brief explanation of the result comparing it with that of Part 1.1.E.

Problem 1.2

Part 1.2.A

Let $f(x) = x + a\cos x$, where $a \in \mathbb{R}$ is some constant. Show that the equation $f(x) = 0$ has at least one solution.

Part 1.2.B

Let $a = 1$. Using the bisection method with an initial interval of $[-5, 5]$, find the root with an accuracy of 10^{-4}. How many iterations are required?