2.4 Numerical Integration

Consider a function $f(x)$ where f is continuous.

2.4.A

We will use the **Trapezoidal Method** to integrate the following series of points $(x_i, f(x)_i))$ from $x = 0$ to $x = 0.2$:

$$(0,0),(0.05,0.15),(0.1,0.3),(0.15,0.45),(0.2,0.6) \tag{15}$$

A plot of the series:

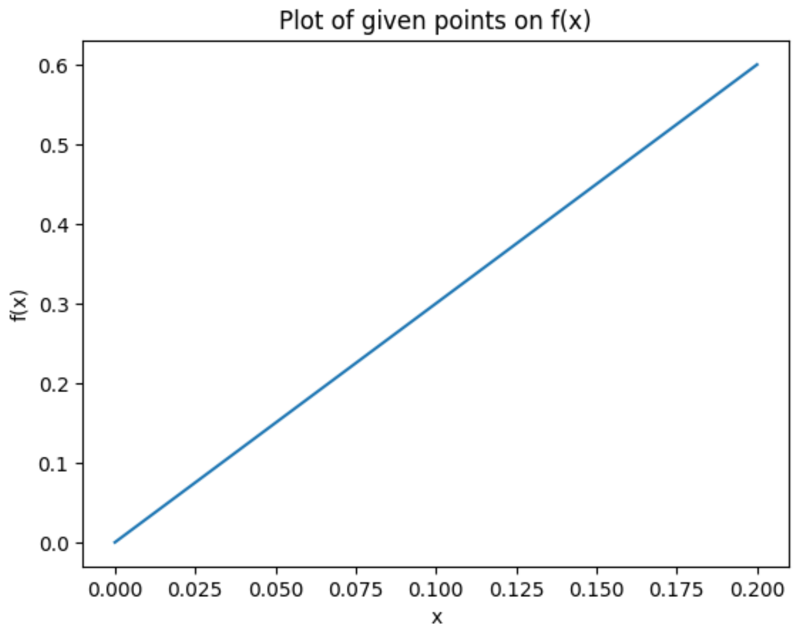

Figure 1:

The following is a Python implementation of the Trapezoidal Method:

```
# define points
x = np.array([0, 0.05, 0.1, 0.15, 0.2])
y = np.array([0, 0.15, 0.3, 0.45, 0.6])
```

```
# define function
def my_trapezoidal(x, y):
    sum = 0
    for i in range(1, len(x)):
        sum = sum + (x[i] - x[i - 1]) * (y[i] + y[i - 1]) /2
    return sum
```

Calling `my_trapezoidal()` on the above points will produce an output of 0.060 (within the margin of error of floating point calculations). This is equal to the analytical solution determined by treating the set of points as a triangle and computing its area.

2.4.B

Given the linearity of the set of points in Part 2.3.A, it may have been reasonable to use an integral approximation algorithm which simply treated the integral as a triangle with:

$$base = x[-1] - x[0], \qquad height = y[-1] - y[0] \tag{16}$$

However, the Trapezoidal Method would, in theory, be more applicable to a variety of series of points (such as the series in Part 2.3.B), so I decided to use the Trapezoidal Method to approximate the integral.

Aside from the given information that f is continuous, we have no information describing the behaviour of f between the provided discrete points, so any attempts to predict what those may be would further reduce the accuracy of our approximation; this is why I thought it best to simply connect the dots and compute the area underneath using the Trapezoidal Method.

2.4.C

Consider the following series of points $(x_i, f(x_i))$:

(0, 1), (0.13, 0.5198), (0.37, 0.6207), (0.49, 0.1728), (0.81, 1.259),
(1.06, 0.121), (1.19, 0.6467), (1.61, 0.6537), (1.94, 1.113), (2.06, 1.835)

A plot of the series:

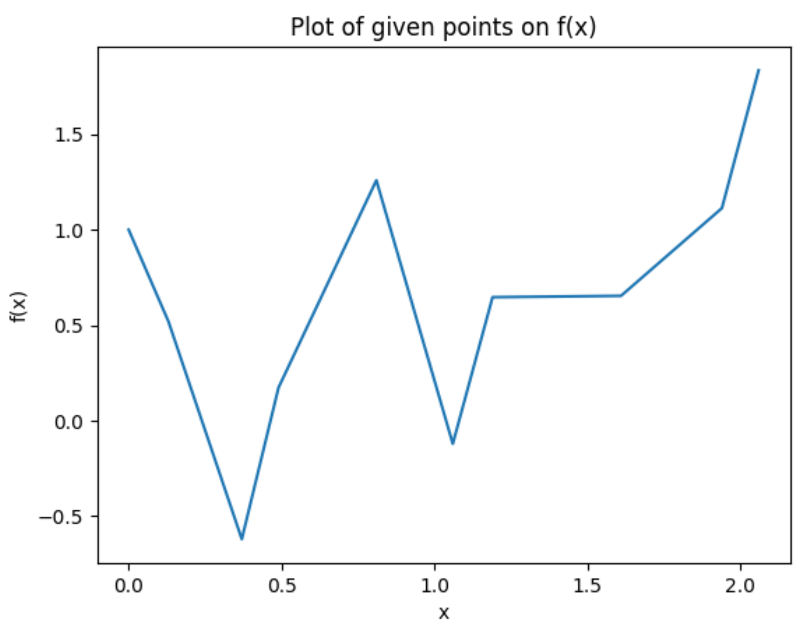

Figure 2:

Calling `my_trapezoidal()` on the above points (to integrate the series from $x = 0$ to $x = 2.06$) will produce a result of approximately 1.20678.

2.4.D

In terms of integrating the given discrete series', I am 100% confident that we have accurately determined the area under each set of points. In terms of approximating the integral of $f(x)$ using the above series', I am not very confident. The Trapezoidal Method's job is to connect the points and compute the area underneath the set of discrete points. As such, the Trapezoidal Method does include any estimation of the behaviour of $f(x)$ between the given points. This is good for us, however, since we have no information regarding the behaviour of $f(x)$ between the given points and any guesses would likely impact our approximated integral of $f(x)$ negatively. For this reason, the Trapezoidal Method will give the best approximation given the information in this scenario, however, I am not confident in even our best approximation.

In order to be more confident, it would be helpful to have more points of $f(x)$ on the given interval. This would disambiguate the behaviour of $f(x)$ and allow us to make a more informed (and likely more accurate) approximation. In theory, the more we know about $f(x)$, the more accurately (and confidently) we can approximate its integral, so the accuracy (and confidence) of our approximation should be proportional to the number of points of $f(x)$ we are given (assuming the given points are relatively evenly distributed across the given interval).