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Assignment 2

Mathematics for Robotics

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This assignment is part of the *Fall 2024* offering of *ROB310: Mathematics for Robotics* at the University of Toronto. Please review the information below carefully. Submissions that do not adhere to the guidelines below may lose marks.

Release Date: .... September 23, 2024 - 00:00
Due Date: .... September 29, 2024 - 23:59
Weight: .... 3.75% or 0%
Late Penalties: .... -10% per day and -100% after 3 days

Submission Information:

- Download this document as a PDF and fill in your information below.
- Answer the questions to the best of your ability.
- Typeset or **neatly hand-write** your answers, labelling the questions identically to this document. We recommend using LaTeX, but will accept the use of other typesetting software (such as Microsoft Word or Google Docs).
- Save your answers as a PDF and combine it with the cover-sheet of this document. Do NOT include the question descriptions.
- Submit your PDF on Crowdmark titled `lastname-firstname-rob310f24-a2.pdf`.

Name (First, Last): .... Jaden Stanshall

UTORId (e.g., gummalur): .... stanshal

Acknowledgement:

I hereby affirm that the work I am submitting for this assignment is entirely my own. I have not received any unauthorized assistance, and I understand that plagiarism or any other form of academic dishonesty is a violation of the University's academic integrity policy.

I agree to the above statement.. ✔

Date (dd/mm/yyyy): 29/11/2024

Problem 2.1

Consider the centered difference approach for numerically approximating the derivative of $f(x)$:

$$f'(x) \approx \frac{f(x+\Delta) - f(x-\Delta)}{2\Delta},$$

where Δ is the numerical step-size with $|\Delta| \ll 1$.

Part 2.1.A

Determine whether the algorithm is consistent or not. Show your work.

Part 2.1.B

Show that the order of accuracy of the algorithm is 2.

Part 2.1.C

State the conditions under which the algorithm numerically stable, or indicate that no such conditions exist. Show your work.

Problem 2.2

State the method you would use to find the root of $f : \mathbb{R} \to \mathbb{R}$ in each of the following cases.

Part 2.2.A

f is once continuously differentiable and f' is inexpensive to evaluate

Part 2.2.B

f is Lipschitz continuous with constant $0 \leq L < 1$

Part 2.2.C

f is once continuously differentiable and f' is extensive to evaluate

Part 2.2.D

f is continuous but not differentiable

Part 2.3

You are given a series of points $(x_i, f(x_i))$ that are obtained from a continuous function, f.

Part 2.3.A

Write your own numerical integration algorithm for integrating the following series from $x = 0$ to $x = 0.2$:

$$\{(0, 0), (0.05, 0.15), (0.1, 0.3), (0.15, 0.45), (0.2, 0.6)\}$$

Part 2.3.B

Explain why you chose the algorithm you chose in Part 2.3.A.

Part 2.3.C

Integrate the following series from $x = 0$ to $x = 2.06$:

$$\{(0, 1), (0.13, 0.5198), (0.37, -0.6207), (0.49, 0.1728), (0.81, 1.259), \ldots,$$
$$(1.06, -0.121), (1.19, 0.6467), (1.61, 0.6537), (1.94, 1.113), (2.06, 1.835)$$

Part 2.3.D

How confident are you in your results from Part 2.3.A and 2.3.C? Explain your answer. Which information would you need to be more confident?