This week's highlights

- Evaluate the truth value of a compound proposition given truth values of its constituent variables.
- Prove propositional equivalences using truth tables
- Prove propositional equivalences using other known equivalences, e.g.
  - DeMorgan's laws
  - Double negation laws
  - Distributive laws, etc.
- Compute the CNF and DNF of a given compound proposition.
- Translate sentences from English to propositional logic using appropriate propositional variables and boolean operators.
- Form the converse, contrapositive, and inverse of a given conditional statement.
- Decide and justify whether or not a collection of propositions is consistent.

Lecture videos

Monday: No class in observance of Martin Luther King day.

A video to reflect on the role of algorithms in systemic racism. Credit: Safiya Umoja Noble.

Week 3 Day 1 YouTube playlist

Week 3 Day 2 YouTube playlist

Wednesday January 20

Proposition	Declarative sentence that is true or false (not both).
Propositional variable	Variable that represents a proposition.
Compound proposition	New propositions formed from existing propositions (potentially) usi logical operators.
Truth table	Table with 1 row for each of the possible combinations of truth valu of the input and an additional column that shows the truth value the result of the operation corresponding to a particular row.

Note: A propositional variable is one example of a compound proposition.

Friday January 22

The only way to make the conditional statement $p \to q$ false is to ______________________________

The **hypothesis** of $p \to q$ is ____________ The **antecedent** of $p \to q$ is ____________

The **conclusion** of $p \to q$ is ____________ The **consequent** of $p \to q$ is ____________

Input		Output				
		Conjunction	Exclusive or	Disjunction	Conditional	Biconditional
p	q	$p \wedge q$	$p \oplus q$	$p \vee q$	$p \to q$	$p \leftrightarrow q$
T	T	T	F	T	T	T
T	F	F	T	T	F	F
F	T	F	T	T	T	F
F	F	F	F	F	T	T

Examples

$p \to q \equiv \neg p \vee q$ because ______________________________

$p \leftrightarrow q$ is not logically equivalent to $p \wedge q$ because ______________________________

$\neg(p \leftrightarrow q) \equiv p \oplus q$ because ______________________________

$p \to q$ is not logically equivalent to $q \to p$ because ______________________________

$p \leftrightarrow q \equiv q \leftrightarrow p$ because ______________________________

The **converse** of $p \to q$ is ____________________

The **inverse** of $p \to q$ is ____________________ Which of these

The **contrapositive** of $p \to q$ is ____________________
is logically equivalent to $p \to q$?

Translation: Express each of the following sentences as compound propositions, using the given propositions.

"A sufficient condition for the warranty to be good is that you bought the computer less than a year ago"

w is "the warranty is good"

b is "you bought the computer less than a year ago"

"Whenever the message was sent from an unknown system, it is scanned for viruses."

s is "The message is scanned for viruses"

u is "The message was sent from an unknown system"

"I will complete my to-do list only if I put a reminder in my calendar"

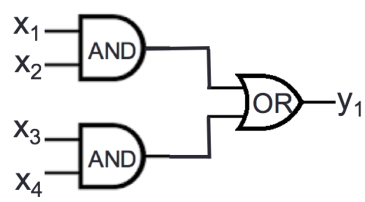
r is "I will complete my to-do list"

c is "I put a reminder in my calendar"

Review quiz questions

1. **Wednesday**

   (a) Consider the logic circuit



   For which of the following settings(s) of input values is the output $y_1 = 0$? (Select all and only those that apply.)

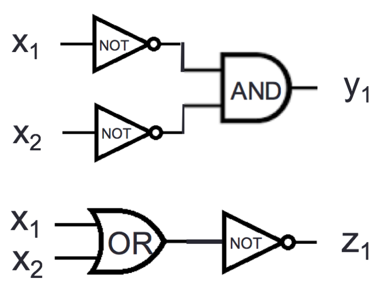
   i. $x_1 = 0$, $x_2 = 0$, $x_3 = 0$, and $x_4 = 0$

   ii. $x_1 = 1$, $x_2 = 1$, $x_3 = 1$, and $x_4 = 1$

   iii. $x_1 = 1$, $x_2 = 0$, $x_3 = 0$, and $x_4 = 1$

   iv. $x_1 = 0$, $x_2 = 0$, $x_3 = 1$, and $x_4 = 1$

   (b) Consider the logic circuits



   For which of the following settings(s) of input values do the outputs of these circuits have the same value, i.e. $y_1 = z_1$? (Select all and only those that apply.)

   i. $x_1 = 1$, $x_2 = 1$

   ii. $x_1 = 1$, $x_2 = 0$

   iii. $x_1 = 0$, $x_2 = 1$

   iv. $x_1 = 0$, $x_2 = 0$

2. **Wednesday** For each of the following propositions, indicate exactly one of:

   - There is no assignment of truth values to its variables that makes it true,
   - There is exactly one assignment of truth values to its variables that makes it true, or
   - There are exactly two assignments of truth values to its variables that make it true, or
   - There are exactly three assignments of truth values to its variables that make it true, or
   - *All* assignments of truth values to its variables make it true.

   (a) $x \wedge y \wedge (x \vee y)$

   (b) $\neg x \wedge y \wedge (x \vee y)$

   (c) $x \wedge \neg y \wedge (x \wedge y)$

   (d) $\neg x \wedge (y \vee \neg y)$

   (e) $x \wedge (y \vee \neg x)$

3. **Friday** For each of the following propositions, indicate exactly one of:

   - There is no assignment of truth values to its variables that makes it true,
   - There is exactly one assignment of truth values to its variables that makes it true, or
   - There are exactly two assignments of truth values to its variables that make it true, or
   - There are exactly three assignments of truth values to its variables that make it true, or
   - *All* assignments of truth values to its variables make it true.

   (a) $(p \leftrightarrow q) \oplus (p \wedge q)$

   (b) $(p \to q) \vee (q \to p)$

   (c) $(p \to q) \wedge (q \to p)$

   (d) $\neg(p \to q)$

4. **Friday Definition**: A collection of compound propositions is called **consistent** if there is an assignment of truth values to the propositional variables that makes each of the compound propositions true.

   For each of the following system specifications, identify the compound propositions that give their translations to logic and then determine if the translated collection of compound propositions is consistent.

   (a) Specification: If the computer is out of memory, then network connectivity is unreliable. No disk errors can occur when the computer is out of memory. Disk errors only occur when network connectivity is unreliable.

   Translation: M = "the computer is out of memory"; N = "network connectivity is unreliable"; D = "disk errors can occur".

   i.

$$\neg M \to N$$
$$\neg D \to M$$
$$D \to N$$

ii.

$$M \to \neg N$$
$$\neg D \land M$$
$$N \to D$$

iii.

$$M \to N$$
$$M \to \neg D$$
$$\neg N \to \neg D$$

(b) Specification: Whether you think you can, or you think you can't - you're right. [1]

Translation: T = "you think you can"; C = "you can".

i.

$$\begin{aligned} &T \to C \\ &\neg T \to \neg C \end{aligned}$$

ii.

$$\begin{aligned} &T \wedge C \\ &\neg T \wedge \neg C \end{aligned}$$

iii.

$$\begin{aligned} &T \to \neg T \\ &C \to \neg C \end{aligned}$$

(c) Specification: A secure password must be private and complicated. If a password is complicated then it will be hard to remember. People write down hard-to-remember passwords. If a password is written down, it's not private. The password is secure.

Translation: S = "the password is secure"; P = "the password is private"; C = "the password is complicated"; H = "the password is hard to remember"; W = "the password is written down".

i.

$$\begin{aligned} &\neg(P \wedge C) \to \neg S \\ &C \to H \\ &W \wedge H \\ &W \to \neg P \\ &S \end{aligned}$$

[1] Henry Ford

ii.

$$
\begin{aligned}
&(P \wedge C) \rightarrow S \\
&C \rightarrow H \\
&W \rightarrow H \\
&W \rightarrow P \\
&S
\end{aligned}
$$

iii.

$$
\begin{aligned}
&S \rightarrow (P \wedge C) \\
&C \rightarrow H \\
&H \rightarrow W \\
&W \rightarrow \neg P \\
&S
\end{aligned}
$$