## Making change proof two ways

Proof of  $\star$  by mathematical induction (b = 8)

Basis step: WTS property is true about 8

**Recursive step**: Consider an arbitrary  $n \ge 8$ . Assume (as the IH) that there are nonnegative integers x, y such that n = 5x + 3y. WTS that there are nonnegative integers x', y' such that n + 1 = 5x' + 3y'. We consider two cases, depending on whether any 5 cent coins are used for n.

Case 1: Assume  $x \ge 1$ . Define x' = x - 1 and y' = y + 2 (both in  $\mathbb N$  by case assumption). Calculating:

$$5x' + 3y' \stackrel{\text{by def}}{=} 5(x - 1) + 3(y + 2) = 5x - 5 + 3y + 6$$

$$\stackrel{\text{rearranging}}{=} (5x + 3y) - 5 + 6$$

$$\stackrel{\text{IH}}{=} n - 5 + 6 = n + 1$$

Case 2: Assume x=0. Therefore n=3y, so since  $n\geq 8,\ y\geq 3$ . Define x'=2 and y'=y-3 (both in  $\mathbb N$  by case assumption). Calculating:

$$5x' + 3y' \stackrel{\text{by def}}{=} 5(2) + 3(y - 3) = 10 + 3y - 9$$

$$\stackrel{\text{rearranging}}{=} 3y + 10 - 9$$

$$\stackrel{\text{IH and case}}{=} n + 10 - 9 = n + 1$$

Proof of  $\star$  by strong induction (b = 8 and j = 2)

Basis step: WTS property is true about 8, 9, 10

**Recursive step**: Consider an arbitrary  $n \ge 10$ . Assume (as the IH) that the property is true about each of  $8, 9, 10, \ldots, n$ . WTS that there are nonnegative integers x', y' such that n + 1 = 5x' + 3y'.