This week's highlights

- Evaluate the truth value of a compound proposition given truth values of its constituent variables.
- Prove propositional equivalences using truth tables
- Prove propositional equivalences using other known equivalences, e.g.
  - DeMorgan's laws
  - Double negation laws
  - Distributive laws, etc.
- Compute the CNF and DNF of a given compound proposition.
- Translate sentences from English to propositional logic using appropriate propositional variables and boolean operators.
- Form the converse, contrapositive, and inverse of a given conditional statement.
- Decide and justify whether or not a collection of propositions is consistent.

Lecture videos

Monday: No class in observance of Martin Luther King day.

A video to reflect on the role of algorithms in systemic racism. Credit: Safiya Umoja Noble.

Week 3 Day 1 YouTube playlist

Week 3 Day 2 YouTube playlist

Wednesday January 20

Proposition	Declarative sentence that is true or false (not both).
Propositional variable	Variable that represents a proposition.
Compound proposition	New propositions formed from existing propositions (potentially) usi logical operators.
Truth table	Table with 1 row for each of the possible combinations of truth valu of the input and an additional column that shows the truth value the result of the operation corresponding to a particular row.

Note: A propositional variable is one example of a compound proposition.

Logical operators aka propositional connectives

Conjunction	AND	$\wedge$	`\land`	2 inputs	Evaluates to T when **both** inputs are T
Exclusive or	XOR	$\oplus$	`\oplus`	2 inputs	Evaluates to T when **exactly one** of inputs i
Disjunction	OR	$\vee$	`\lor`	2 inputs	Evaluates to T when **at least one** of inputs
Negation	NOT	$\neg$	`\lnot`	1 input	Evaluates to T when its input is F

Input		Output		
		Conjunction	**Exclusive or**	**Disjunction**
p	q	$p \wedge q$	$p \oplus q$	$p \vee q$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	F

Input	Output
	Negation
p	$\neg p$
T	F
F	T

Input			Output		
p	q	r		$(p \wedge q) \oplus (\ (p \oplus q) \wedge r \ )$	$(p \wedge q) \vee (\ (p$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

Logical equivalence	Two compound propositions are **logically equivalent** means that they have the same truth values for all settings of truth values to their propositional variables.
Tautology	A compound proposition that evaluates to true for all settings of truth values to its propositional variables; it is abbreviated T.
Contradiction	A compound proposition that evaluates to false for all settings of truth values to its propositional variables; it is abbreviated F.
Contingency	A compound proposition that is neither a tautology nor a contradiction.

Extra Example: Which of the compound propositions in the table below are logically equivalent?

Input		Output				
p	q	$\neg(p \wedge \neg q)$	$\neg(\neg p \vee \neg q)$	$(\neg p \vee q)$	$(\neg q \vee \neg p)$	$(p \wedge q)$
T	T					
T	F					
F	T					
F	F					

(Some) logical equivalences) cf. Rosen pp. 26-28

$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$	**Commutativity** Ordering of terms
$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	**Associativity** Grouping of terms
$p \wedge F \equiv F$ $\quad p \vee T \equiv T$	$p \wedge T \equiv p$ $\quad p \vee F \equiv p$	**Absorption** aka short circuit evaluat
$\neg(p \wedge q) \equiv \neg p \vee \neg q$	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	**DeMorgan's Laws**

Can replace p and q with any compound proposition

Given an compound proposition, we can use

- Truth tables
- Logical equivalences

to compute its truth value for specific input values.

Now, given a truth table, how do we find a compound proposition that has the specified output values?

Application: design a circuit given a desired input-output relationship.

Input		Output	
p	q	$mystery_1$	$mystery_2$
T	T	T	F
T	F	T	F
F	T	F	F
F	F	T	T

Input			Output
p	q	r	?
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

A compound proposition that gives output $mystery_1$ is: ______________________

A compound proposition that gives output $mystery_2$ is: ______________________

Definition A compound proposition is in **disjunctive normal form** (DNF) means that it is an OR of ANDs of variables and their negations.

Definition A compound proposition is in **conjunctive normal form** (CNF) means that it is an AND of ORs of variables and their negations.

Extra example: A compound proposition that gives output ? is:

Friday January 22

The only way to make the conditional statement $p \to q$ false is to ______________________

The **hypothesis** of $p \to q$ is ____________ The **antecedent** of $p \to q$ is ____________

The **conclusion** of $p \to q$ is ____________ The **consequent** of $p \to q$ is ____________

Input		Output				
		Conjunction	Exclusive or	Disjunction	Conditional	Biconditional
p	q	$p \wedge q$	$p \oplus q$	$p \vee q$	$p \to q$	$p \leftrightarrow q$
T	T	T	F	T	T	T
T	F	F	T	T	F	F
F	T	F	T	T	T	F
F	F	F	F	F	T	T

Examples

$p \to q \equiv \neg p \vee q$ because ______________________

$p \leftrightarrow q$ is not logically equivalent to $p \wedge q$ because ______________________

$\neg(p \leftrightarrow q) \equiv p \oplus q$ because ______________________

$p \to q$ is not logically equivalent to $q \to p$ because ______________________

$p \leftrightarrow q \equiv q \leftrightarrow p$ because ______________________

The **converse** of $p \to q$ is ______________

The **inverse** of $p \to q$ is ______________ Which of these

The **contrapositive** of $p \to q$ is ______________
is logically equivalent to $p \to q$?

Translation: Express each of the following sentences as compound propositions, using the given propositions.

"A sufficient condition for the warranty to be good is that you bought the computer less than a year ago"

w is "the warranty is good"
b is "you bought the computer less than a year ago"

"Whenever the message was sent from an unknown system, it is scanned for viruses."

s is "The message is scanned for viruses"
u is "The message was sent from an unknown system"

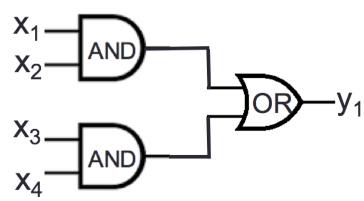
"I will complete my to-do list only if I put a reminder in my calendar"

r is "I will complete my to-do list"
c is "I put a reminder in my calendar"

Review quiz questions

1. **Wednesday**

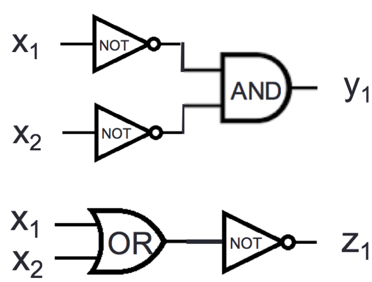
   (a) Consider the logic circuit

   For which of the following settings(s) of input values is the output $y_1 = 0$? (Select all and only those that apply.)

   i. $x_1 = 0$, $x_2 = 0$, $x_3 = 0$, and $x_4 = 0$
   ii. $x_1 = 1$, $x_2 = 1$, $x_3 = 1$, and $x_4 = 1$
   iii. $x_1 = 1$, $x_2 = 0$, $x_3 = 0$, and $x_4 = 1$
   iv. $x_1 = 0$, $x_2 = 0$, $x_3 = 1$, and $x_4 = 1$

   (b) Consider the logic circuits

   For which of the following settings(s) of input values do the outputs of these circuits have the same value, i.e. $y_1 = z_1$? (Select all and only those that apply.)

   i. $x_1 = 1$, $x_2 = 1$
   ii. $x_1 = 1$, $x_2 = 0$
   iii. $x_1 = 0$, $x_2 = 1$
   iv. $x_1 = 0$, $x_2 = 0$

2. **Wednesday** For each of the following propositions, indicate exactly one of:

   - There is no assignment of truth values to its variables that makes it true,
   - There is exactly one assignment of truth values to its variables that makes it true, or
   - There are exactly two assignments of truth values to its variables that make it true, or
   - There are exactly three assignments of truth values to its variables that make it true, or
   - *All* assignments of truth values to its variables make it true.

   (a) $x \wedge y \wedge (x \vee y)$

   (b) $\neg x \wedge y \wedge (x \vee y)$

   (c) $x \wedge \neg y \wedge (x \wedge y)$

   (d) $\neg x \wedge (y \vee \neg y)$

   (e) $x \wedge (y \vee \neg x)$

3. **Friday** For each of the following propositions, indicate exactly one of:

   - There is no assignment of truth values to its variables that makes it true,
   - There is exactly one assignment of truth values to its variables that makes it true, or
   - There are exactly two assignments of truth values to its variables that make it true, or
   - There are exactly three assignments of truth values to its variables that make it true, or
   - *All* assignments of truth values to its variables make it true.

   (a) $(p \leftrightarrow q) \oplus (p \wedge q)$
   (b) $(p \to q) \vee (q \to p)$
   (c) $(p \to q) \wedge (q \to p)$
   (d) $\neg(p \to q)$

4. **Friday Definition**: A collection of compound propositions is called **consistent** if there is an assignment of truth values to the propositional variables that makes each of the compound propositions true.

   For each of the following system specifications, identify the compound propositions that give their translations to logic and then determine if the translated collection of compound propositions is consistent.

   (a) Specification: If the computer is out of memory, then network connectivity is unreliable. No disk errors can occur when the computer is out of memory. Disk errors only occur when network connectivity is unreliable.

   Translation: M = "the computer is out of memory"; N = "network connectivity is unreliable"; D = "disk errors can occur".

   i.

$$\neg M \to N$$
$$\neg D \to M$$
$$D \to N$$

ii.

$$M \to \neg N$$
$$\neg D \wedge M$$
$$N \to D$$

iii.

$$M \to N$$
$$M \to \neg D$$
$$\neg N \to \neg D$$

(b) Specification: Whether you think you can, or you think you can't - you're right. [1]

Translation: T = "you think you can"; C = "you can".

i.

$$T \to C$$
$$\neg T \to \neg C$$

ii.

$$T \wedge C$$
$$\neg T \wedge \neg C$$

iii.

$$T \to \neg T$$
$$C \to \neg C$$

(c) Specification: A secure password must be private and complicated. If a password is complicated then it will be hard to remember. People write down hard-to-remember passwords. If a password is written down, it's not private. The password is secure.

Translation: S = "the password is secure"; P = "the password is private"; C = "the password is complicated"; H = "the password is hard to remember"; W = "the password is written down".

i.

$$\neg(P \wedge C) \to \neg S$$
$$C \to H$$
$$W \wedge H$$
$$W \to \neg P$$
$$S$$

[1] Henry Ford

ii.

$$(P \wedge C) \to S$$
$$C \to H$$
$$W \to H$$
$$W \to P$$
$$S$$

iii.

$$S \to (P \wedge C)$$
$$C \to H$$
$$H \to W$$
$$W \to \neg P$$
$$S$$