

**Monday January 25**

**Definition** A compound proposition is in **disjunctive normal form** (DNF) means that it is an OR of ANDs of variables and their negations.

**Definition** A compound proposition is in **conjunctive normal form** (CNF) means that it is an AND of ORs of variables and their negations.

*Extra example:* A compound proposition that gives output ? is:

# Review

# Friday January 22

The only way to make the conditional statement  $p \rightarrow q$  false is to \_\_\_\_\_

The **hypothesis** of  $p \rightarrow q$  is \_\_\_\_\_ The **antecedent** of  $p \rightarrow q$  is \_\_\_\_\_

The **conclusion** of  $p \rightarrow q$  is \_\_\_\_\_ The **consequent** of  $p \rightarrow q$  is \_\_\_\_\_

Input		Output				
$p$	$q$	Conjunction $p \wedge q$	Exclusive or $p \oplus q$	Disjunction $p \vee q$	Conditional $p \rightarrow q$	Biconditional $p \leftrightarrow q$
$T$	$T$	$T$	$F$	$T$	$T$	$T$
$T$	$F$	$F$	$T$	$T$	$F$	$F$
$F$	$T$	$F$	$T$	$T$	$T$	$F$
$F$	$F$	$F$	$F$	$F$	$T$	$T$

## Examples

$p \rightarrow q \equiv \neg p \vee q$  because \_\_\_\_\_

$p \leftrightarrow q$  is not logically equivalent to  $p \wedge q$  because \_\_\_\_\_

$\neg(p \leftrightarrow q) \equiv p \oplus q$  because \_\_\_\_\_

$p \rightarrow q$  is not logically equivalent to  $q \rightarrow p$  because \_\_\_\_\_

$p \leftrightarrow q \equiv q \leftrightarrow p$  because \_\_\_\_\_

The **converse** of  $p \rightarrow q$  is \_\_\_\_\_

The **inverse** of  $p \rightarrow q$  is \_\_\_\_\_ Which of these is logically equivalent to  $p \rightarrow q$ ?

The **contrapositive** of  $p \rightarrow q$  is \_\_\_\_\_

**Translation:** Express each of the following sentences as compound propositions, using the given propositions.

“A sufficient condition for the warranty to be good is	$w$ is “the warranty is good”
that you bought the computer less than a year ago”	$b$ is “you bought the computer less than a year ago”

“Whenever the message was sent from an unknown system, it is scanned for viruses.”	$s$ is “The message is scanned for viruses”
	$u$ is “The message was sent from an unknown system”

<p>“I will complete my to-do list only if I put a reminder in my calendar”</p>	<p><math>r</math> is “I will complete my to-do list”  <math>c</math> is “I put a reminder in my calendar”</p>
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## Review

1. For each of the following propositions, indicate exactly one of:

- There is no assignment of truth values to its variables that makes it true,
- There is exactly one assignment of truth values to its variables that makes it true, or
- There are exactly two assignments of truth values to its variables that make it true, or
- There are exactly three assignments of truth values to its variables that make it true, or
- *All* assignments of truth values to its variables make it true.

(a)  $(p \leftrightarrow q) \oplus (p \wedge q)$

(b)  $(p \rightarrow q) \vee (q \rightarrow p)$

(c)  $(p \rightarrow q) \wedge (q \rightarrow p)$

(d)  $\neg(p \rightarrow q)$

2. **Definition:** A collection of compound propositions is called **consistent** if there is an assignment of truth values to the propositional variables that makes each of the compound propositions true.

For each of the following system specifications, identify the compound propositions that give their translations to logic and then determine if the translated collection of compound propositions is consistent.

- (a) Specification: If the computer is out of memory, then network connectivity is unreliable. No disk errors can occur when the computer is out of memory. Disk errors only occur when network connectivity is unreliable.

Translation:  $M$  = “the computer is out of memory”;  $N$  = “network connectivity is unreliable”;  $D$  = “disk errors can occur”.

i.

$$\neg M \rightarrow N$$

$$\neg D \rightarrow M$$

$$D \rightarrow N$$

ii.

$$M \rightarrow \neg N$$

$$\neg D \wedge M$$

$$N \rightarrow D$$

iii.

$$M \rightarrow N$$

$$M \rightarrow \neg D$$

$$\neg N \rightarrow \neg D$$

(b) Specification: Whether you think you can, or you think you can't - you're right. <sup>1</sup>

Translation:  $T$  = "you think you can";  $C$  = "you can".

i.

$$\begin{aligned}T &\rightarrow C \\ \neg T &\rightarrow \neg C\end{aligned}$$

ii.

$$\begin{aligned}T &\wedge C \\ \neg T &\wedge \neg C\end{aligned}$$

iii.

$$\begin{aligned}T &\rightarrow \neg T \\ C &\rightarrow \neg C\end{aligned}$$

(c) Specification: A secure password must be private and complicated. If a password is complicated then it will be hard to remember. People write down hard-to-remember passwords. If a password is written down, it's not private. The password is secure.

Translation:  $S$  = "the password is secure";  $P$  = "the password is private";  $C$  = "the password is complicated";  $H$  = "the password is hard to remember";  $W$  = "the password is written down".

i.

$$\begin{aligned}\neg(P \wedge C) &\rightarrow \neg S \\ C &\rightarrow H \\ W &\wedge H \\ W &\rightarrow \neg P \\ S\end{aligned}$$

ii.

$$\begin{aligned}(P \wedge C) &\rightarrow S \\ C &\rightarrow H \\ W &\rightarrow H \\ W &\rightarrow P \\ S\end{aligned}$$

iii.

$$\begin{aligned}S &\rightarrow (P \wedge C) \\ C &\rightarrow H \\ H &\rightarrow W \\ W &\rightarrow \neg P \\ S\end{aligned}$$

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<sup>1</sup>Henry Ford