



# Definitions

Term	Notation Example(s)	We say in English
$n$ -tuple	$(x_1, x_2, x_3)$ $(3, 4)$	The 3-tuple of $x_1$ , $x_2$ , and $x_3$ The 2-tuple or <b>ordered pair</b> of 3 and 4
sequence	$x_1, \dots, x_n$ $x_1, \dots, x_n$ where $n = 0$ $x_1, \dots, x_n$ where $n = 1$ $x_1, \dots, x_n$ where $n = 2$ $x_1, x_2$	A sequence $x_1$ to $x_n$ An empty sequence A sequence containing just $x_1$ A sequence containing just $x_1$ and $x_2$ in order A sequence containing just $x_1$ and $x_2$ in order
set		Unordered collection of objects. The set of ...
all integers	$\mathbb{Z}$	The (set of all) integers (whole numbers including negatives, zero, and positives)
all positive integers	$\mathbb{Z}^+$	The (set of all) strictly positive integers
all natural numbers	$\mathbb{N}$	The (set of all) natural numbers. <b>Note:</b> we use the convention that 0 is a natural number.
roster method	$\{43, 7, 9\}$ $\{9, \mathbb{N}\}$	The set whose elements are 43, 7, and 9 The set whose elements are 9 and $\mathbb{N}$
set builder notation	$\{x \in \mathbb{Z} \mid x > 0\}$ $\{3x \mid x \in \mathbb{Z}\}$	The set of all $x$ from the integers such that $x$ is greater than 0 The set of all integer multiples of 3 <b>Note:</b> we use the convention that writing two numbers next to each other means multiplication.
function definition	$f(x) = x + 4$	Define $f$ of $x$ to be $x + 4$
function application	$f(7)$ $f(z)$ $f(g(z))$	$f$ of 7 <b>or</b> $f$ applied to 7 <b>or</b> the image of 7 under $f$ $f$ of $z$ <b>or</b> $f$ applied to $z$ <b>or</b> the image of $z$ under $f$ $f$ of $g$ of $z$ <b>or</b> $f$ applied to the result of $g$ applied to $z$
absolute value	$ -3 $	The absolute value of $-3$
square root	$\sqrt{9}$	The non-negative square root of 9
summation notation	$\sum_{i=1}^n i$ $\sum_{i=1}^n i^2 - 1$	The sum of the integers from 1 to $n$ , inclusive The sum of $i^2 - 1$ ( $i$ squared minus 1) for each $i$ from 1 to $n$ , inclusive
quotient, integer division	$n \text{ div } m$	The (integer) quotient upon dividing $n$ by $m$ ; informally: divide and then drop the fractional part
modulo, remainder	$n \text{ mod } m$	The remainder upon dividing $n$ by $m$

# Netflix intro

What data should we encode about each Netflix account holder to help us make effective recommendations?

In machine learning, clustering can be used to group similar data for prediction and recommendation. For example, each Netflix user's viewing history can be represented as a  $n$ -tuple indicating their preferences about movies in the database, where  $n$  is the number of movies in the database. People with similar tastes in movies can then be clustered to provide recommendations of movies for one another. Mathematically, clustering is based on a notion of distance between pairs of  $n$ -tuples.

## Data types

Term	Examples: (add additional examples from class)
<b>set</b> unordered collection of elements <i>Equal means agree on membership of all elements</i>	$7 \in \{43, 7, 9\}$ $2 \notin \{43, 7, 9\}$
<b><math>n</math>-tuple</b> ordered sequence of elements with $n$ "slots" <i>Equal means corresponding components equal</i>	
<b>string</b> ordered finite sequence of elements each from specified set <i>Equal means same length and corresponding characters equal</i>	

$$\{-1, 1\} \quad \{0, 0\} \quad \{-1, 0, 1\} \quad \mathbb{Z} \quad \mathbb{N} = \{x \in \mathbb{Z} \mid x \geq 0\} \quad \emptyset \quad \mathbb{Z}^+ = \{x \in \mathbb{Z} \mid x > 0\}$$

*Which of the sets above are defined using the roster method? Which are defined using set builder notation?*

*Which of the sets above have 0 as an element?*

*Can you write any of the sets above more simply?*

## Rna def

RNA is made up of strands of four different bases that match up in specific ways. The bases are elements of the set  $B = \{\mathbf{A}, \mathbf{C}, \mathbf{G}, \mathbf{U}\}$ .

**Definition** The set of RNA strands  $S$  is defined (recursively) by:

Basis Step:  $\mathbf{A} \in S, \mathbf{C} \in S, \mathbf{U} \in S, \mathbf{G} \in S$   
Recursive Step: If  $s \in S$  and  $b \in B$ , then  $sb \in S$

where  $sb$  is string concatenation.

Examples:

To define a set we can use the **roster method**, the **set builder notation**, and also . . .

**New! Recursive Definitions of Sets:** The set  $S$  (pick a name) is defined by:

Basis Step:	Specify finitely many elements of $S$
Recursive Step:	Give a rule for creating a new element of $S$ from known values existing in $S$ , and potentially other values.

The set  $S$  then consists of all and only elements that are put in  $S$  by finitely many (a nonnegative integer number) of applications of the recursive step after the basis step.