Definitions

Term	Notation Example(s)	We say in English ...
sequence	$x_1, \ldots, x_n$	A sequence x_1 to x_n
summation	$\sum_{i=1}^{n} x_i$ or $\displaystyle\sum_{i=1}^{n} x_i$	The sum of the terms of the sequence x_1 to x_n
all reals	$\mathbb{R}$	The (set of all) real numbers (numbers on the number line)
all integers	$\mathbb{Z}$	The (set of all) integers (whole numbers including negatives, zero, and positives)
all positive integers	$\mathbb{Z}^+$	The (set of all) strictly positive integers
all natural numbers	$\mathbb{N}$	The (set of all) natural numbers. **Note**: we use the convention that 0 is a natural number.
piecewise rule definition	$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$	Define f of x to be x when x is nonnegative and to be $-x$ when x is negative
function application	$f(7)$	f of 7 **or** f applied to 7 **or** the image of 7 under f
	$f(z)$	f of z **or** f applied to z **or** the image of z under f
	$f(g(z))$	f of g of z **or** f applied to the result of g applied to z
absolute value	$\|-3\|$	The absolute value of -3
square root	$\sqrt{9}$	The non-negative square root of 9

Data types

Term	**Examples**: (add additional examples from class)
set unordered collection of elements *repetition doesn't matter* *Equal sets agree on membership of all elements*	$7 \in \{43, 7, 9\}$ $\quad$ $2 \notin \{43, 7, 9\}$
n**-tuple** ordered sequence of elements with n "slots" ($n > 0$) *repetition matters, fixed length* *Equal n-tuples have corresponding components equal*	
string ordered finite sequence of elements each from specified set *repetition matters, arbitrary finite length* *Equal strings have same length and corresponding characters equal*	

Special cases:

When $n = 2$, the 2-tuple is called an **ordered pair**.

A string of length 0 is called the **empty string** and is denoted λ.

A set with no elements is called the **empty set** and is denoted $\{\}$ or $\emptyset$.

Defining sets

To define a set using **roster method**, explicitly list its elements. That is, start with { then list elements of the set separated by commas and close with }.

To define a set using **set builder definition**, either form "The set of all x from the universe U such that x is ..." by writing

$$\{x \in U \mid ...x...\}$$

or form "the collection of all outputs of some operation when the input ranges over the universe U" by writing

$$\{...x... \mid x \in U\}$$

We use the symbol $\in$ as "is an element of" to indicate membership in a set.

Example sets: For each of the following, identify whether it's defined using the roster method or set builder notation and give an example element.

$\{-1, 1\}$

$\{0, 0\}$

$\{-1, 0, 1\}$

$\{(x, x, x) \mid x \in \{-1, 0, 1\}\}$

$\{\}$

$\{x \in \mathbb{Z} \mid x \geq 0\}$

$\{x \in \mathbb{Z} \mid x > 0\}$

$\{\mathtt{A}, \mathtt{C}, \mathtt{U}, \mathtt{G}\}$

$\{\mathtt{AUG}, \mathtt{UAG}, \mathtt{UGA}, \mathtt{UAA}\}$

Set operations

To define a set we can use the roster method, set builder notation, a recursive definition, and also we can apply a set operation to other sets.

New! Cartesian product of sets and **set-wise concatenation of sets of strings**

Definition: Let X and Y be sets. The **Cartesian product** of X and Y, denoted $X \times Y$, is the set of all ordered pairs (x, y) where $x \in X$ and $y \in Y$

$$X \times Y = \{(x, y) \mid x \in X \text{ and } y \in Y\}$$

Definition: Let X and Y be sets of strings over the same alphabet. The **set-wise concatenation** of X and Y, denoted $X \circ Y$, is the set of all results of string concatenation xy where $x \in X$ and $y \in Y$

$$X \circ Y = \{xy \mid x \in X \text{ and } y \in Y\}$$

Pro-tip: the meaning of writing one element next to another like xy depends on the data-types of x and y. When x and y are strings, the convention is that xy is the result of string concatenation. When x and y are numbers, the convention is that xy is the result of multiplication. This is (one of the many reasons) why is it very important to declare the data-type of variables before we use them.

Fill in the missing entries in the table:

Set	**Example elements in this set**:
B	`A` `C` `G` `U`
	$(\texttt{A}, \texttt{C})$ $(\texttt{U}, \texttt{U})$
$B \times \{-1, 0, 1\}$	
$\{-1, 0, 1\} \times B$	
	$(0, 0, 0)$
$\{\texttt{A}, \texttt{C}, \texttt{G}, \texttt{U}\} \circ \{\texttt{A}, \texttt{C}, \texttt{G}, \texttt{U}\}$	
	`GGGG`