

Making change proof two ways

Proof of \star by mathematical induction ($b = 8$)

Basis step: WTS property is true about 8

Recursive step: Consider an arbitrary $n \geq 8$. Assume (as the IH) that there are nonnegative integers x, y such that $n = 5x + 3y$. WTS that there are nonnegative integers x', y' such that $n + 1 = 5x' + 3y'$. We consider two cases, depending on whether any 5 cent coins are used for n .

Case 1: Assume $x \geq 1$. Define $x' = x - 1$ and $y' = y + 2$ (both in \mathbb{N} by case assumption). Calculating:

$$\begin{aligned} 5x' + 3y' &\stackrel{\text{by def}}{=} 5(x - 1) + 3(y + 2) = 5x - 5 + 3y + 6 \\ &\stackrel{\text{rearranging}}{=} (5x + 3y) - 5 + 6 \\ &\stackrel{\text{IH}}{=} n - 5 + 6 = n + 1 \end{aligned}$$

Case 2: Assume $x = 0$. Therefore $n = 3y$, so since $n \geq 8$, $y \geq 3$. Define $x' = 2$ and $y' = y - 3$ (both in \mathbb{N} by case assumption). Calculating:

$$\begin{aligned} 5x' + 3y' &\stackrel{\text{by def}}{=} 5(2) + 3(y - 3) = 10 + 3y - 9 \\ &\stackrel{\text{rearranging}}{=} 3y + 10 - 9 \\ &\stackrel{\text{IH and case}}{=} n + 10 - 9 = n + 1 \end{aligned}$$

Proof of \star by strong induction ($b = 8$ and $j = 2$)

Basis step: WTS property is true about 8, 9, 10

Recursive step: Consider an arbitrary $n \geq 10$. Assume (as the IH) that the property is true about each of 8, 9, 10, \dots , n . WTS that there are nonnegative integers x', y' such that $n + 1 = 5x' + 3y'$.