

## Before we start

If you or someone you know is suffering from food and/or housing insecurities there are UCSD resources here to help:

Basic Needs Office: <https://basicneeds.ucsd.edu/>

Triton Food Pantry (in the old Student Center) is free and anonymous, and includes produce:

<https://www.facebook.com/tritonfoodpantry/>

Mutual Aid UCSD: <https://mutualaiducsd.wordpress.com/>

If you find yourself in an uncomfortable situation, ask for help. We are committed to upholding University policies regarding nondiscrimination, sexual violence and sexual harassment.

Counseling and Psychological Services (CAPS) at 858 5343755 or <http://caps.ucsd.edu>

OPHD at (858) 534-8298, [ophd@ucsd.edu](mailto:ophd@ucsd.edu) , <http://ophd.ucsd.edu>. CARE at Sexual Assault Resource Center at 858 5345793 [sarc@ucsd.edu](mailto:sarc@ucsd.edu) <http://care.ucsd.edu>

Welcome to CSE 20: Discrete Math for Computer Science in Fall 2021!

## Themes and applications for CSE 20

- **Technical skepticism:** Know, select and apply appropriate computing knowledge and problem-solving techniques. Reason about computation and systems. Use mathematical techniques to solve problems. Determine appropriate conceptual tools to apply to new situations. Know when tools do not apply and try different approaches. Critically analyze and evaluate candidate solutions.
- **Multiple representations:** Understand, guide, shape impact of computing on society/the world. Connect the role of Theory CS classes to other applications (in undergraduate CS curriculum and beyond). Model problems using appropriate mathematical concepts. Clearly and unambiguously communicate computational ideas using appropriate formalism. Translate across levels of abstraction.

**Applications:** Numbers (how to represent them and use them in Computer Science), Recommendation systems and their roots in machine learning (with applications like Netflix), “Under the hood” of computers (circuits, pixel color representation, data structures), Codes and information (secret message sharing and error correction), Bioinformatics algorithms and genomics (DNA and RNA).

## Introductions

Class website: <http://cseweb.ucsd.edu/classes/fa21/cse20-a>

**Pro-tip:** the URL structure is your map to finding your course website for other CSE classes.

**Pro-tip:** you can use MATH109 to replace CSE20 for prerequisites and other requirements.

Instructor: Prof. Mia Minnes “Minnes” rhymes with Guinness, [minnes@eng.ucsd.edu](mailto:minnes@eng.ucsd.edu), <http://cseweb.ucsd.edu/minnes>

Our team: Four TAs and 10 tutors + all of you

Fill in contact info for students around you, if you’d like:

On an average week, MWF Lectures + review quizzes, W Discussion, T HW due, Office hours, Piazza. Project parts will be due some weeks.

Due dates: Review quizzes, HW, and Project, see course calendar and Canvas.

Education research: CSE 20 is participating in a project on retention and sense of community in UCSD majors; see research plan. If you consent to participate in this study, no action is needed. If you DO NOT consent to participate in this study, or you choose to opt-out at any time during the academic year, sign and submit this form to the research contact at [retentionstudy@cs.ucsd.edu](mailto:retentionstudy@cs.ucsd.edu).

## Friday September 24

What data should we encode about each Netflix account holder to help us make effective recommendations?

In machine learning, clustering can be used to group similar data for prediction and recommendation. For example, each Netflix user's viewing history can be represented as a  $n$ -tuple indicating their preferences about movies in the database, where  $n$  is the number of movies in the database. People with similar tastes in movies can then be clustered to provide recommendations of movies for one another. Mathematically, clustering is based on a notion of distance between pairs of  $n$ -tuples.

In the table below, each row represents a user's ratings of movies: ✓ (check) indicates the person liked the movie, ✗ (x) that they didn't, and • (dot) that they didn't rate it one way or another (neutral rating or didn't watch).

Person	Fyre	Frozen II	Picard	Ratings written as a 3-tuple
$P_1$	✗	•	✓	
$P_2$	✓	✓	✗	
$P_3$	✓	✓	✓	
$P_4$	•	✗	✓	

**Conclusion:** Modeling involves choosing data types to represent and organize data

## Review

1. Please complete the beginning of the quarter survey <https://forms.gle/gvibFnNixxqcWbaU8>
2. We want you to be familiar with class policies and procedures so you are ready to have a successful quarter. Please take a look at the class website <http://cseweb.ucsd.edu/classes/fa21/cse20-a> and answer the questions about it on Gradescope.
3. Modeling:
  - (a) Using the example movie database from class with the 3 movies Fyre, Frozen II, Picard, which of the following is a 3-tuple that represents the ratings of a user who liked Frozen II? (Select all and only that apply.)
    - i. 1
    - ii.  $(0, 0, 0)$
    - iii.  $[1, 1, 1]$
    - iv.  $\{-1, 0, 1\}$
    - v.  $(1, -1, 0)$
    - vi.  $(0, 1, 1)$
    - vii.  $(1, 1, 1, 1)$
  - (b) Using the example movie database from class with the 3 movies Fyre, Frozen II, Picard, how many distinct (different) 3-tuples of ratings are there?

# Monday September 27

Term	Notation Example(s)	We say in English
$n$ -tuple	$(x_1, x_2, x_3)$ $(3, 4)$	The 3-tuple of $x_1$ , $x_2$ , and $x_3$ The 2-tuple or <b>ordered pair</b> of 3 and 4
sequence	$x_1, \dots, x_n$ $x_1, \dots, x_n$ where $n = 0$ $x_1, \dots, x_n$ where $n = 1$ $x_1, \dots, x_n$ where $n = 2$ $x_1, x_2$	A sequence $x_1$ to $x_n$ An empty sequence A sequence containing just $x_1$ A sequence containing just $x_1$ and $x_2$ in order A sequence containing just $x_1$ and $x_2$ in order
set		Unordered collection of objects. The set of ...
all integers	$\mathbb{Z}$	The (set of all) integers (whole numbers including negatives, zero, and positives)
all positive integers	$\mathbb{Z}^+$	The (set of all) strictly positive integers
all natural numbers	$\mathbb{N}$	The (set of all) natural numbers. <b>Note:</b> we use the convention that 0 is a natural number.
roster method	$\{43, 7, 9\}$ $\{9, \mathbb{N}\}$	The set whose elements are 43, 7, and 9 The set whose elements are 9 and $\mathbb{N}$
set builder notation	$\{x \in \mathbb{Z} \mid x > 0\}$ $\{3x \mid x \in \mathbb{Z}\}$	The set of all $x$ from the integers such that $x$ is greater than 0 The set of all integer multiples of 3. <b>Note:</b> we use the convention that writing two numbers next to each other means multiplication.
function definition	$f(x) = x + 4$	Define $f$ of $x$ to be $x + 4$
function application	$f(7)$ $f(z)$ $f(g(z))$	$f$ of 7 <b>or</b> $f$ applied to 7 <b>or</b> the image of 7 under $f$ $f$ of $z$ <b>or</b> $f$ applied to $z$ <b>or</b> the image of $z$ under $f$ $f$ of $g$ of $z$ <b>or</b> $f$ applied to the result of $g$ applied to $z$
absolute value	$ -3 $	The absolute value of $-3$
square root	$\sqrt{9}$	The non-negative square root of 9
summation notation	$\sum_{i=1}^n i$ $\sum_{i=1}^n i^2 - 1$	The sum of the integers from 1 to $n$ , inclusive The sum of $i^2 - 1$ ( $i$ squared minus 1) for each $i$ from 1 to $n$ , inclusive
quotient, integer division	$n \text{ div } m$	The (integer) quotient upon dividing $n$ by $m$ ; informally: divide and then drop the fractional part
modulo, remainder	$n \text{ mod } m$	The remainder upon dividing $n$ by $m$

## Data Types: sets, $n$ -tuples, and strings

To define a set using **roster method**, explicitly list its elements. That is, start with  $\{$  then list elements of the set separated by commas and close with  $\}$ .

To define a set using **set builder definition**, either form “The set of all  $x$  from the universe  $U$  such that  $x$  is ...” by writing

$$\{x \in U \mid \dots x \dots\}$$

or form “the collection of all outputs of some operation when the input ranges over the universe  $U$ ” by writing

$$\{\dots x \dots \mid x \in U\}$$

We use the symbol  $\in$  as “is an element of” to indicate membership in a set.

**Example sets:** For each of the following, identify whether it’s defined using the roster method or set builder notation.

$$\{-1, 1\}$$

$$\{0, 0\}$$

$$\{-1, 0, 1\}$$

$$\{(x, x, x) \mid x \in \{-1, 0, 1\}\}$$

$$\emptyset$$

$$\mathbb{N} = \{x \in \mathbb{Z} \mid x \geq 0\}$$

$$\mathbb{Z}^+ = \{x \in \mathbb{Z} \mid x > 0\}$$

$$\{\text{A}, \text{C}, \text{U}, \text{G}\}$$

$$\{\text{AUG}, \text{UAG}, \text{UGA}, \text{UAA}\}$$

Term	Examples: (add additional examples from class)
<b>set</b> unordered collection of elements <i>repetition doesn't matter</i> <i>Equal means agree on membership of all elements</i>	$7 \in \{43, 7, 9\}$ $2 \notin \{43, 7, 9\}$
<b><math>n</math>-tuple</b> ordered sequence of elements with $n$ “slots” <i>repetition matters, fixed length</i> <i>Equal means corresponding components equal</i>	
<b>string</b> ordered finite sequence of elements each from specified set <i>repetition matters, arbitrary finite length</i> <i>Equal means same length and corresponding characters equal</i>	

RNA is made up of strands of four different bases that match up in specific ways. The bases are elements of the set  $B = \{\mathbf{A}, \mathbf{C}, \mathbf{G}, \mathbf{U}\}$ .

Formally, to define the set of all RNA strands, we need more than roster method or set builder descriptions.

**New! Recursive Definitions of Sets:** The set  $S$  (pick a name) is defined by:

Basis Step:	Specify finitely many elements of $S$
Recursive Step:	Give rule(s) for creating a new element of $S$ from known values existing in $S$ , and potentially other values.

The set  $S$  then consists of all and only elements that are put in  $S$  by finitely many (a nonnegative integer number) of applications of the recursive step after the basis step.

**Definition** The set of nonnegative integers  $\mathbb{N}$  is defined (recursively) by:

Basis Step:
Recursive Step:

Examples:

**Definition** The set of all integers  $\mathbb{Z}$  is defined (recursively) by:

Basis Step:
Recursive Step:

Examples:

**Definition** The set of RNA strands  $S$  is defined (recursively) by:

Basis Step:	$\mathbf{A} \in S, \mathbf{C} \in S, \mathbf{U} \in S, \mathbf{G} \in S$
Recursive Step:	If $s \in S$ and $b \in B$ , then $sb \in S$

where  $sb$  is string concatenation.

Examples:

**Definition** The set of bitstrings (strings of 0s and 1s) is defined (recursively) by:

Basis Step:
Recursive Step:

Examples:

## Review

1. Colors can be described as amounts of red, green, and blue mixed together<sup>1</sup> Mathematically, a color can be represented as a 3-tuple  $(r, g, b)$  where  $r$  represents the red component,  $g$  the green component,  $b$  the blue component and where each of  $r, g, b$  must be a value from this collection of numbers:

{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255}

- (a) **True** or **False**:  $(1, 3, 4)$  fits the definition of a color above.
- (b) **True** or **False**:  $(1, 100, 200, 0)$  fits the definition of a color above.
- (c) **True** or **False**:  $(510, 255)$  fits the definition of a color above.
- (d) **True** or **False**: There is a color  $(r_1, g_1, b_1)$  where  $r_1 + g_1 + b_1$  is greater than 765.
- (e) **True** or **False**: There is a color  $(r_2, g_2, b_2)$  where  $r_2 + g_2 + b_2$  is equal to 1.
- (f) **True** or **False**: Another way to write the collection of allowed values for red, green, and blue components is

$$\{x \in \mathbb{N} \mid 0 \leq x \leq 255\}$$

.

- (g) **True** or **False**: Another way to write the collection of allowed values for red, green, and blue components is

$$\{n \in \mathbb{Z} \mid 0 \leq n \leq 255\}$$

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- (h) **True** or **False**: Another way to write the collection of allowed values for red, green, and blue components is

$$\{y \in \mathbb{Z} \mid -1 < y \leq 255\}$$

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## 2. Recursive definition questions

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<sup>1</sup>This RGB representation is common in web applications. Many online tools are available to play around with mixing these colors, e.g. [https://www.w3schools.com/colors/colors\\_rgb.asp](https://www.w3schools.com/colors/colors_rgb.asp).



# Wednesday September 29

Sets from other sets Functions

# Review

# Friday October 1

Number representations

# Review