Definitions

Term	Notation Example(s)	We say in English
n-tuple	(x_1, x_2, x_3)	The 3-tuple of x_1, x_2, and x_3
	$(3, 4)$	The 2-tuple or **ordered pair** of 3 and 4
sequence	$x_1, \ldots, x_n$	A sequence x_1 to x_n
	$x_1, \ldots, x_n$ where $n = 0$	An empty sequence
	$x_1, \ldots, x_n$ where $n = 1$	A sequence containing just x_1
	$x_1, \ldots, x_n$ where $n = 2$	A sequence containing just x_1 and x_2 in order
	x_1, x_2	A sequence containing just x_1 and x_2 in order
set		Unordered collection of objects. The set of ...
all integers	$\mathbb{Z}$	The (set of all) integers (whole numbers including negatives, zero, and positives)
all positive integers	$\mathbb{Z}^+$	The (set of all) strictly positive integers
all natural numbers	$\mathbb{N}$	The (set of all) natural numbers. **Note**: we use the convention that 0 is a natural number.
roster method	$\{43, 7, 9\}$	The set whose elements are 43, 7, and 9
	$\{9, \mathbb{N}\}$	The set whose elements are 9 and $\mathbb{N}$
set builder notation	$\{x \in \mathbb{Z} \mid x > 0\}$	The set of all x from the integers such that x is greater than 0
	$\{3x \mid x \in \mathbb{Z}\}$	The set of all integer multiples of 3. **Note**: we use the convention that writing two numbers next to each other means multiplication.
function definition	$f(x) = x + 4$	Define f of x to be $x + 4$
function application	$f(7)$	f of 7 **or** f applied to 7 **or** the image of 7 under f
	$f(z)$	f of z **or** f applied to z **or** the image of z under f
	$f(g(z))$	f of g of z **or** f applied to the result of g applied to z
absolute value	$\lvert -3 \rvert$	The absolute value of -3
square root	$\sqrt{9}$	The non-negative square root of 9
summation notation	$\sum_{i=1}^{n} i$	The sum of the integers from 1 to n, inclusive
	$\sum_{i=1}^{n} i^2 - 1$	The sum of $i^2 - 1$ (i squared minus 1) for each i from 1 to n, inclusive
quotient, integer division	n **div** m	The (integer) quotient upon dividing n by m; informally: divide and then drop the fractional part
modulo, remainder	n **mod** m	The remainder upon dividing n by m

Defining sets

To define a set using **roster method**, explicitly list its elements. That is, start with { then list elements of the set separated by commas and close with }.

To define a set using **set builder definition**, either form “The set of all x from the universe U such that x is ...” by writing

$$\{x \in U \mid ...x...\}$$

or form “the collection of all outputs of some operation when the input ranges over the universe U” by writing

$$\{...x... \mid x \in U\}$$

We use the symbol $\in$ as “is an element of” to indicate membership in a set.

Example sets: For each of the following, identify whether it's defined using the roster method or set builder notation.

$\{-1, 1\}$

$\{0, 0\}$

$\{-1, 0, 1\}$

$\{(x, x, x) \mid x \in \{-1, 0, 1\}\}$

$\emptyset$

$\mathbb{N} = \{x \in \mathbb{Z} \mid x \geq 0\}$

$\mathbb{Z}^+ = \{x \in \mathbb{Z} \mid x > 0\}$

$\{\mathtt{A}, \mathtt{C}, \mathtt{U}, \mathtt{G}\}$

$\{\mathtt{AUG}, \mathtt{UAG}, \mathtt{UGA}, \mathtt{UAA}\}$

Least greatest proofs

Prove or **disprove**: There is a least prime number.

Prove or **disprove**: There is a greatest integer.

Approach 1, De Morgan's and universal generalization:

Approach 2, proof by contradiction:

Extra examples: Prove or disprove that $\mathbb{N}$, $\mathbb{Q}$ each have a least and a greatest element. Prove that there is no greatest prime number.

Gcd def

Greatest common divisor Let a and b be integers, not both zero. The largest integer d such that d is a factor of a and d is a factor of b is called the greatest common divisor of a and b and is denoted by $gcd(a, b)$.