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Definitions

Term	Notation Example(s)	We say in English
n-tuple	(x_1, x_2, x_3)	The 3-tuple of x_1 , x_2 , and x_3
	(3,4)	The 2-tuple or ordered pair of 3 and 4
sequence	x_1, \ldots, x_n	A sequence x_1 to x_n
	x_1, \ldots, x_n where $n = 0$	An empty sequence
	x_1, \ldots, x_n where $n = 1$	A sequence containing just x_1
	x_1, \ldots, x_n where $n = 2$	A sequence containing just x_1 and x_2 in order
	x_1, x_2	A sequence containing just x_1 and x_2 in order
set		Unordered collection of objects. The set of \dots
all integers	\mathbb{Z}	The (set of all) integers (whole numbers including
11	777.+	negatives, zero, and positives)
all positive integers	\mathbb{Z}_{+}	The (set of all) strictly positive integers
all natural numbers	\mathbb{N}	The (set of all) natural numbers. Note : we use
	(42.7.0)	the convention that 0 is a natural number.
roster method	$\{43, 7, 9\}$	The set whose elements are 43, 7, and 9
	$\{9,\mathbb{N}\}$	The set whose elements are 9 and \mathbb{N}
set builder notation	$\{x \in \mathbb{Z} \mid x > 0\}$	The set of all x from the integers such that x is
		greater than 0
	$\{3x \mid x \in \mathbb{Z}\}$	The set of all integer multiples of 3. Note : we use
		the convention that writing two numbers next to
		each other means multiplication.
function definition	f(x) = x + 4	Define f of x to be $x + 4$
function application	f(7) = x + 1	f of 7 or f applied to 7 or the image of 7 under f
ranction application	f(z)	f of z or f applied to z or the image of z under f
	f(g(z))	f of g of z or f applied to the result of g applied
	$J(g(\sim))$	to z
absolute value	$\begin{vmatrix} -3 \\ \sqrt{9} \end{vmatrix}$	The absolute value of -3
square root	$\sqrt{9}$	The non-negative square root of 9
	$\sum_{i=1}^{n}$.	
summation notation	$\sum_{i=1}^{n} i$ $\sum_{i=1}^{n} i^2 - 1$	The sum of the integers from 1 to n , inclusive
	$\sum_{i=1}^{n} i^2$	The sum of $i^2 = 1$ (i covered minus 1) for soil :
	$\sum_{i=1}^{n} i^i - 1$	The sum of $i^2 - 1$ (<i>i</i> squared minus 1) for each <i>i</i> from 1 to <i>m</i> inclusive
	<i>i</i> =1	from 1 to n , inclusive
quotient, integer division	$n \operatorname{\mathbf{div}} m$	The (integer) quotient upon dividing n by m ; in-
· · · · · · · · · · · · · · · · · · ·		formally: divide and then drop the fractional part
modulo, remainder	$n \mod m$	The remainder upon dividing n by m

Defining sets

To define a set using **roster method**, explicitly list its elements. That is, start with { then list elements of the set separated by commas and close with }.

To define a set using **set builder definition**, either form "The set of all x from the universe U such that x is ..." by writing

$$\{x \in U \mid ...x...\}$$

or form "the collection of all outputs of some operation when the input ranges over the universe U" by writing

$$\{...x... \mid x \in U\}$$

We use the symbol \in as "is an element of" to indicate membership in a set.

Example sets: For each of the following, identify whether it's defined using the roster method or set builder notation.

Least greatest proofs

Prove or **disprove**: There is a least prime number.

Prove or **disprove**: There is a greatest integer.

Approach 1, De Morgan's and universal generalization:

Approach 2, proof by contradiction:

Extra examples: Prove or disprove that \mathbb{N} , \mathbb{Q} each have a least and a greatest element. Prove that there is no greatest prime number.

Gcd def

Greatest common divisor Let a and b be integers, not both zero. The largest integer d such that d is a factor of a and d is a factor of b is called the greatest common divisor of a and b and is denoted by gcd(a, b).