CSCE 580 – Artificial Intelligence

Project A- Report

Shruti Jadhav

Introduction:

The concept of Entropy and missing information was explored through this activity using the Mastermind game. The data was collected and analyzed through the questions presented in this report. An important rule of the game that is different from original mastermind game is enforced in this activity. The changed rule is that feedback corresponds to a specific position of a guess. This is significant as it changes the results and analysis compared to different versions of this game. For more information on the activity please visit https://modelai.gettysburg.edu/2025/mastermind/

Data:

The mastermind game was played for a total of 6 times. The data extracted from all the trials are presented in the table below.

Table 1: Data Collected from Game Trials

Trial	Round	Guess	Feedback	Possible codes Remaining (N)	Entropy H = Log2(N)
1	1	Navy, Yellow, Brown, Red	white, -,-,black	48	5.585
	2	Blue, Navy, Green, Red	Black, Black,-,Black	3	1.585
	3	Blue, Navy, Navy, Red	Black, Black,Black,Black	1	0
2	1	Red, Yellow, Green, Navy	Black, Black,-,-	16	4
	2	Red, Yellow, Brown, Blue	Black, Black, -, Black	3	1.585
	3	Red, Yellow, Blue, Blue	Black, Black, -, Black	2	1
	4	Red, Yellow, red, Blue	Black, Black, -, Black	1	0
	5	Red, Yellow,Yellow, Blue	Black, Black,Black,Black	1	0
3	1	Yellow, Red, Navy, Brown	-,-,-,white	54	5.755
	2	Brown, Brown, Green, Blue	white,-,white,-	16	4
	3	Green, Brown, Brown,	black,-,black,black	1	0
	4	Green, Green, Brown, Green	Black, Black,Black,Black	1	0
4	1	Red, Blue, Yellow, Brown	white,black,white,-	80	6.322
	2	Green, Blue, Red, Yellow	-,black,black,black	3	1.585
	3	Navy, Blue, Red, Yellow	-,black,black,black	2	1
	4	Yellow, Blue, Red, Yellow	Black, Black,Black,Black	1	0
5	1	Yellow, Red, Green, Blue	-,-,white,white	144	7.170
	2	Navy, Brown, Blue, Green	white, white, white, white	36	5.170
	3	Green, Blue, Brown, Navy	black,black	1	0
6	1	Red, Blue, Green, Brown	white,-,white,-	144	7.170
	2	Navy, Red, Yellow, Green	-,white,black,white	8	3
	3	Yellow, Green, Yellow, Red	white,white,black,black	1	0
	4	Green, Yellow, Yellow, Red	black,black,black,black	1	0

Questions and Analysis:

1. Describe the state space for the codes. How many codes are possible?

There are 4 guessing spaces, with 6 colors. Each guessing spot can take on any of the 6 colors, therefore there are 6^4 codes possible i.e. **N = 6^4 = 1296.**

2. Describe the state space for the feedback. How many feedbacks are possible?

The feedbacks that we can obtain are either black pegs or white pegs. There are 4 feedback spaces. After calculating all permutations of black, white or empty feedback spaces, it is found that **14 feedbacks** are possible.

3. Calculate the entropy for the full state space of codes at the start. How many bits of missing information are there?

Using the Entropy formula, H= Log2(N), we find out that using N=1296 at the beginning there are **H = Log2(1296) = 10.34 bits** of missing information.

4. Make a chart showing log2(x) from 1 to the maximum number of codes.

The following graph shows the progression of the log2(x) function from N=1 to N=1296.

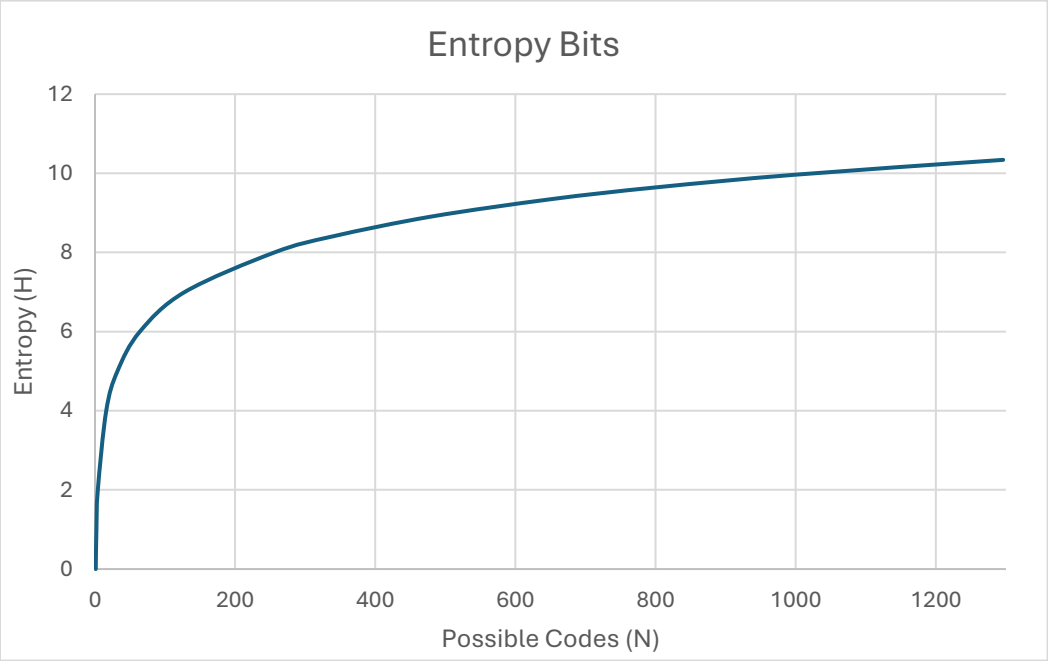


Figure 1: Entropy-No. of Codes Relation

5. Explore value of a black feedback peg:

a. Compute the number of codes if you have zero, one, two, three, or four black feedback pegs.

Due to the nature of the rule of this game that corresponds feedback to a specific placement of the guess, the number of codes if we have a black feedback peg is calculated by simply finding the combination of values for remaining spaces without feedback. This is found by using formula: $N=C^{G-B}$ where N is number of codes, C is number of colors (6), G is guess pegs (4), and B is number of black pegs. Therefore the number of codes is in Table 2 below.

Table 2: Black Feedback Peg Value Exploration

Black Feedback Peg	Number of Codes	Entropy
0	1296	10.340
1	216	7.755
2	36	5.170
3	6	2.585
4	1	0

b. Compute the entropy if you have zero, one, two, three, or four black feedback pegs.

Entropy for the number of black feedback pegs is also provided in Table 2 above.

c. Compute the information gain for gaining a black peg.

The information gain can be found by finding the difference between the entropy after gaining one black peg. Gain = $H_{old} - H_{new}$. Gain = 10.340 – 7.755 = 2.585. Therefore, information gain is **2.585 bits.**

6. Explore value of a white feedback peg:

a. Compute the number of codes if you have zero or one white feedback pegs.

Given a white peg indicates nature of placement of a color in other spots, the relationship between all possible outcomes for other open positions must be coupled with all color possibilities for a current position. The formula to find number of codes should first include, open positions for all color combinations, open position for color which got white feedback as well as all colors except current for the current position. This can be visualized by the formula: $N = C^{G-W} * W! * \binom{G}{W}$ where G is the number of Guess pegs (4), C is number of colors (6), and W is number of white feedback pegs. Therefore, the number of codes is in the Table 3 below.

Table 3: White Feedback Peg Value Exploration

White Feedback Peg	Number of Codes	Entropy
0	1296	10.340
1	864	9.755

b. Compute the entropy if you have zero or one white feedback pegs.

Entropy for white feedback pegs is also provided in Table 3 above.

c. Compute the information gain for gaining your first white peg.

The information gain can be found by finding the difference between the entropy after gaining one black peg. Gain = $H_{old} - H_{new}$. Gain = 10.340 – 9.755= 0.585. Therefore, information gain is **0.585 bits.**

7. Which was it easier for you to calculate the black peg value or the white peg value? Why?

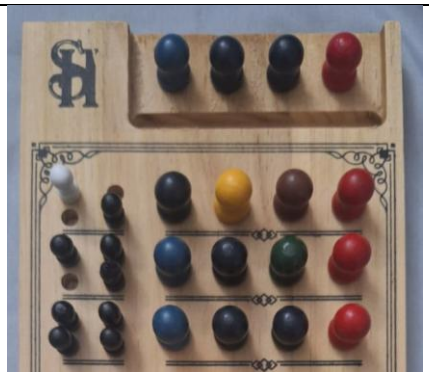
It was easier to calculate the black peg value. A position-specific black peg fixed one color in the correct place, which reduced the search space to only the remaining positions. That gave me a simple formula and made the entropy calculation straightforward. White peg values on the other hand required complex thinking of the problem, along with considering values for the spots with the white feedback peg. This created dependencies and made the reasoning to find a formula more complicated.

If mastermind were to be played with original rules (i.e. blind feedback) then the black peg calculation would have been a bit more intensive than what is currently presented with the modified rule. However, even in that case calculation for the white pegs would have remained more difficult overall.

8. Assuming it was the first turn of the game: Would you rather your feedback be a black peg or two white pegs?

If it was the first turn of the game, I would rather my feedback be a black peg. Not only does it visually confirm placements of a color, but also as seen from these calculations it reduces entropy for future guesses, getting me closer to the real solution faster, as seen from the graph and data. Furthermore, as supported by the calculations it provides more information gain of 2.585 bits compared to 1 white peg of 0.585 bits.

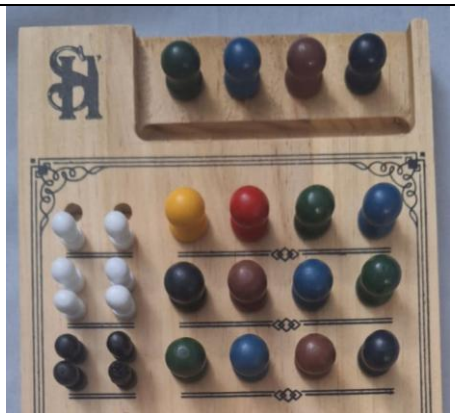
Appendix 1: Images from Mastermind Trials


Trial 1


Trial 2


Trial 3


Trial 4


Trial 5


Trial 6