BERRY COLLEGE

Mount Berry, Georgia

Speeding Cars and Traffic Lights:

A Stochastic Model of the Time Benefit of Speeding on Roads with Traffic Lights

A Thesis submitted in partial satisfaction

of the requirements for the Honors Degree

in

Mathematics

by

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December 2025

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1 Abstract

In this paper we introduce a stochastic model T_k^α to represent the time saved by a speeding car on a road with traffic lights. We calculate the expected value for each distribution and provide bounds on the variance. Two special cases of the random variables are investigated and shown to be familiar distributions. Monte-Carlo simulations are used to investigate qualitative properties of the more complicated distributions.

2 Introduction

Imagine you are driving down the highway at a reasonable speed and then suddenly an aggravated driver speeds past you 20mph over the speed limit. You try to brush it off, but a minute later you hit a traffic light and realize that you have come to a stop right next to the same car that spent so much energy trying to pass you. Now this is just an anecdote but it does raise an interesting question. How much time does speeding really save? If a car is speeding by a constant amount without interruption, it can easily be shown that the time saved decreases with the speed limit [2]. This question is made more interesting when we consider the effects of traffic lights on time saved by speeding. The goals of this paper are threefold: the first to develop a model for this situation, the second to investigate the model with computational methods, and third to prove several general statements about the model.

The format of this paper is as follows; In Section 3 the model is introduced and T_k^α is defined. Section 4 investigates qualitative properties of T_k^α using Monte-Carlo simulations. Section 5 computes T_k^α for the edge cases of $\alpha = 1$ and $\alpha = 0$. Section 6 investigates the distributions $T_k^{.75}$ computationally since they are of particular interest for traffic. In section 7 we compute a general formula for the expected value of T_k^α and an expression for the variance of T_k^α from which we derive upper bounds. Section 8 concludes the paper with closing remarks and comments on future research.

3 Formulation of The Model

The Situation In general, the amount of time saved by speeding is a very complex quantity that is dependent such as traffic conditions, how much the car is speeding, how many traffic lights are present, and many other quantities. In this paper we look to see how a simplified model of traffic lights effects the time saved by speeding. The model is described as follows, imagine an arbitrarily long road with traffic lights evenly spaced every d units. On this road we have two cars with the same initial starting location traveling at constant speeds v_{sl} for the car traveling at the speed limit and $v_{sl} + \Delta v$ for the speeding car. We assume that each car stops instantly when they arrive at a red light and speeds back up to the corresponding speed instantly when the light turns green. Furthermore, we assume that each traffic light has the same cycle, but the relative phases for each light are uniformly chosen and independent from each other. For simplicity, we choose the units of time so that the period of the traffic light cycle is one unit. With this in place we can define the main variable of the model. We define the random variable T_k^α to be the time difference between when the cars pass the k^{th} traffic light if the traffic light cycles are such that the light is red α percent of the time.

This model can be visualized using time-space diagrams. In traffic modeling, time-space diagrams are used to visualize the relationships between the phases of traffic lights and how the relative phase differences affect drivers. This is done by visualizing traffic periods as periodic bands of colors representing traffic light colors and plotting the time-space path of a few test cars. The time-space diagram representing the situation we seek to model is shown in figure 2. One of the diagrams corresponds to an example of the speeding car saving a significant amount of time and the other corresponds to an example where the speeding car saves no time.

To actually formulate the model we need to do a bit more work. At each traffic light, we can decompose the the change in time saved as the sum of two terms

$$\Delta T_k^\alpha = \Delta t + \Delta W_\alpha(X) \tag{1}$$

where Δt is the time saved by traveling the road section between lights and $\Delta W_\alpha(X)$ is the amount of time the regular car waits at the light minus the amount of time the speeding car waits at the light, if the light has relative phase X and is red for α percent of the time. In the following paragraphs, we discuss these two terms, but this allows us to describe T_k^α more concretely. Define $T_0^\alpha = 0$ and let $X_1, X_2, ... \sim \text{Unif}(0, 1)$, then we define for each $k > 0$

$$T_k^\alpha = T_{k-1}^\alpha + \Delta t + \Delta W_\alpha(X_k). \tag{2}$$

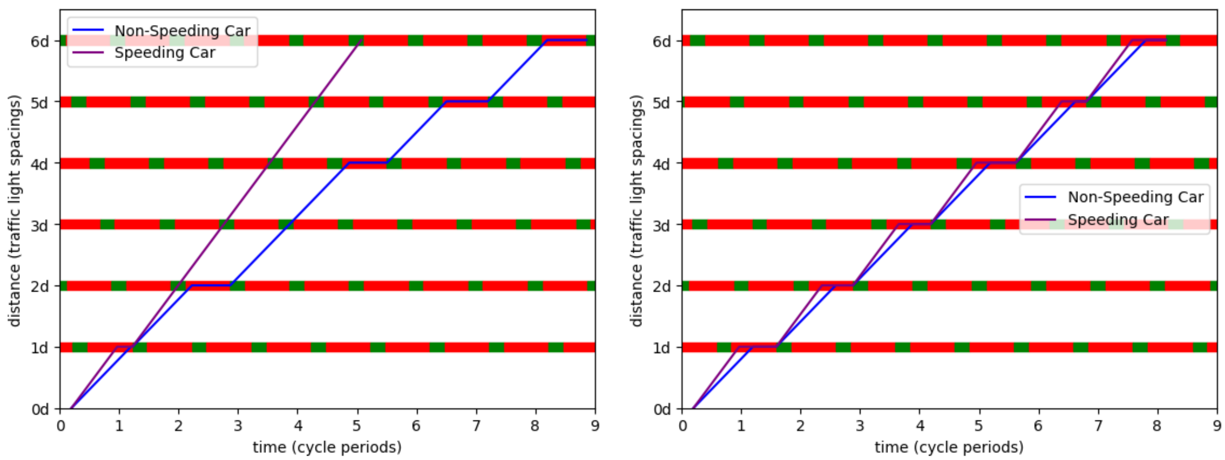

Figure 1:

Figure 2: Example time-space diagrams for a speeding and non-speeding car: one corresponding to a positive time benefit (left) and one corresponding to no net time benefit (right)

Δt The quantity Δt is the difference in times that it takes each car to travel a road segment of length d, so we can write

$$\Delta t = \frac{d}{v_{sl}} - \frac{d}{v_{sl} + \Delta v} = \frac{d\Delta v}{v_{sl}(v_{sl} + \Delta v)}. \tag{3}$$

From this expression we can see that as a function of Δv, we have $\Delta t = O(\frac{d}{v_{sl}^2})$. This justifies our claim from before that in the absence of stops, the time saved by speeding is a decreasing function in v_{sl}. Since Δt is the only place where the parameters d, v_{sl}, and Δv appear, we will combine these parameters into one, meaning T_k^α has Δt as a single parameter.

As previously described the term $\Delta W_\alpha(X)$ describes the difference in times that each car waits at a given stop light assuming the traffic cycle has relative phase X and is red for α percent of the time. To make this more concrete we define the concept of a wait function. A wait function $W : \mathbb{R} \to \mathbb{R}$ is a periodic function with period 1 such that $W(t)$ is the time a car waits at a traffic light if they arrive at the light at time t. In this model, we assume that the lights have cycles that are red for α units of time and then green for $1 - \alpha$ units of time. Under these conditions, we define for each $\alpha \in [0, 1]$ the wait function

$$W_\alpha(t) = (\alpha - \{t\})\chi_{[0,\alpha]}(t) \tag{4}$$

where $\{t\}$ indicate the fractional part of t and $\chi_{[0,\alpha]}$ is the characteristic function of the set $[0, \alpha]$. A few examples of wait functions are shown in Figure [3]. With these wait functions, we can now write

$$\Delta W_\alpha(X_k) = W_\alpha(T_{k-1}^\alpha + \Delta t + X_k) - W_\alpha(X_k) \tag{5}$$

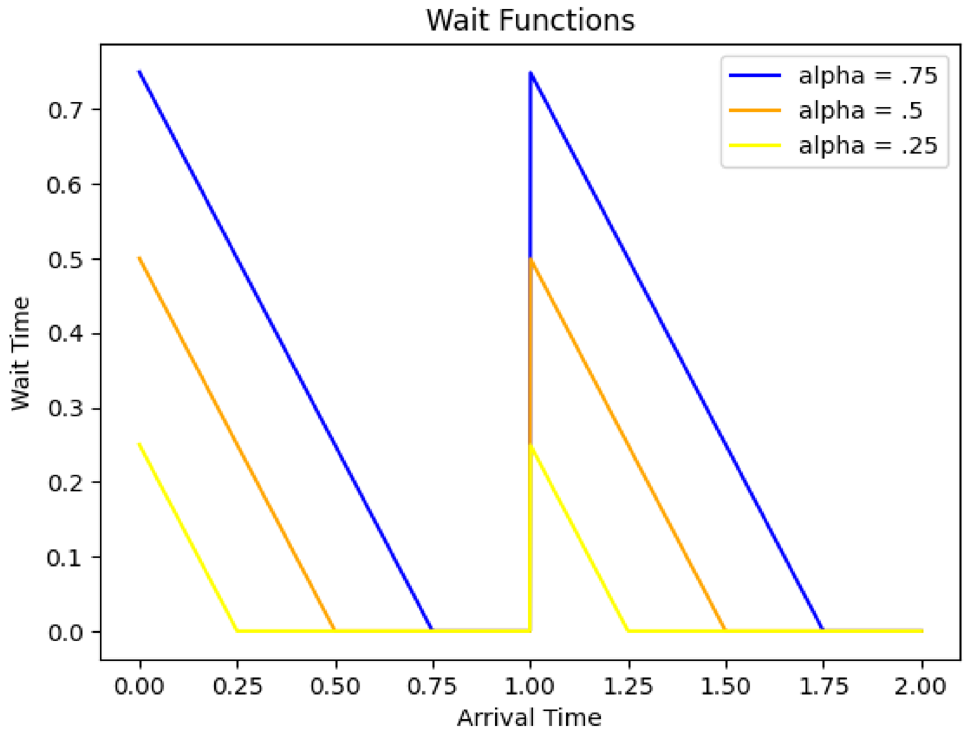

Figure 3: Plots of $W_\alpha(t)$ for $\alpha = .75, .5, .25$

since we know that if the speeding car arrives at the k^{th} traffic light at time X_k then the regular car will arrive at time $T^\alpha_{k-1} + \Delta t + X_k$.

We are now in a place to fully describe out model. For $\alpha \in [0, 1]$, positive Δt, and $X_1, X_2, ... \sim \text{Unif}([0, 1])$ we define the stochastic process $\{T^\alpha_k\}_{k=0}^\infty$ by $T^\alpha_0 = 0$ and for positive k

$$T^\alpha_k = T^\alpha_{k-1} + \Delta t + W_\alpha(T^\alpha_{k-1} + \Delta t + X_k) - W_\alpha(X_k). \tag{6}$$

This model is unique in a few different ways. First, the difference between consecutive terms are not independent nor identical, meaning that central limit theorem based arguments cannot be applied to this problem as one may apply them to more simple stochastic processes.

However, these sequences do appear to have apparently normal limiting distributions. Furthermore, because of the nature of the ΔW_α term, T_k^α is a mixed random variable for most choices of α and Δt. This means that common techniques for analyzing either continuous or discrete distributions cannot be applied to T_k^α unless you use more generalized measure theoretic versions of the same techniques. These features make this model quite difficult to deal with analytically, so we are forced to rely on computational help along with a few clever tricks to get a handle on the distributions.

4 Zoo of Distributions

Though this model is complicated to deal with analytically, the definition of T_k^α allows for the distribution to be approximated using a Monte-Carlo simulation by using the simulated distribution of T_{k-1}^α to simulate T_k^α. Through the use of simulation we can understand the qualitative behavior of the model. We simulated T_k^α for various values of α, Δt, and k using a 500,000 point Monte-Carlo simulation. The results are presented in figures 4 through 6.

From these figures we can better understand the qualitative properties of T_k^α. For example, these figures show how α affects the location of point probabilities in the distribution. These point probabilities are displayed by bins that have a much higher density than neighboring bins. From figures 4 to 6, we can see the general trend that for small α, the point probabilities are located very close to the mean of the distribution and for larger α the point probabilities have a larger contribution throughout the support of the distribution. We also can see that for $\alpha = .75$ the support of the distribution has regularly spaced gaps while the support of $T_k^{.25}$ does not appear to have any gaps and the support of $T_k^{.5}$ only has gaps for some values of Δt and k.

From these figures we also can gain some insight about the expected value and variance. For fixed k, we can see that the expected value of T_k^α increases with Δt but is constant with respect to α. We can also see that for a fixed value of k the variance of T_k^α the variance appears to increase with α. The relationship between variance of the distributions and Δt is harder to determine from figures 4 through 6. To understand that relationship, we performed Monte-Carlo simulations of T_1^α for various values of α and Δt. The resulting plot of the variance of each simulation as a function of Δt is given in figure 7. This figure suggests that the variance of T_1^α is symmetric about $\Delta t = .5$ and tends to decrease to zero as α approaches 0.

One last thing figures 4 through 6 demonstrate is that for large values of k, it appears that T_k^α approaches a normal distribution. To see this, we simulated T_{70}^α using a Monte-Carlo simulation with 100,000 points and plotted the cdf of a normal distribution sharing the simulated mean and variance of T_k^α for $\Delta t = .75$. The resulting plot is shown in figure 8. This figure shows that the distributions of T_{70}^α are fairly close to it's normal counterpart. This is an interesting property, because T_k^α can be written as a sum of non-independent and non-identical random variables meaning the central limit theorem does not apply to this sequence of random variables, yet T_k^α seems to approach a normal distribution regardless.

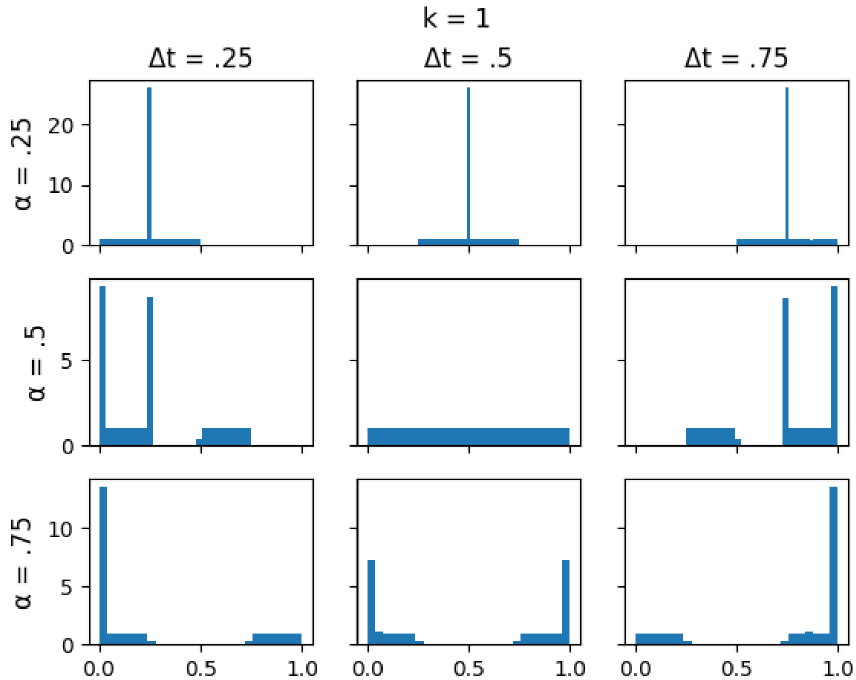

Figure 4: Monte-Carlo simulation of the distribution of T_1^α for various choices of α and Δt

5 Analysis of T_k^1 and T_k^0

While most values of α lead to complicated mixed distributions that do not have known closed form expressions, The edge cases of the family T_k^α turn out to be familiar distributions in disguise. Namely for each k, we have found that T_k^1 is a shifted binomial distribution and T_k^0 is a random variable that takes on a single value. These two cases, serve not only as endpoints of the family of distributions but also as extremal cases for certain properties of the distributions.

The Case of $\alpha = 1$ As previously mentioned, for each k, we know T_k^1 is a shifted binomial random variable. We will show this fact in two parts, first we will show that the support of

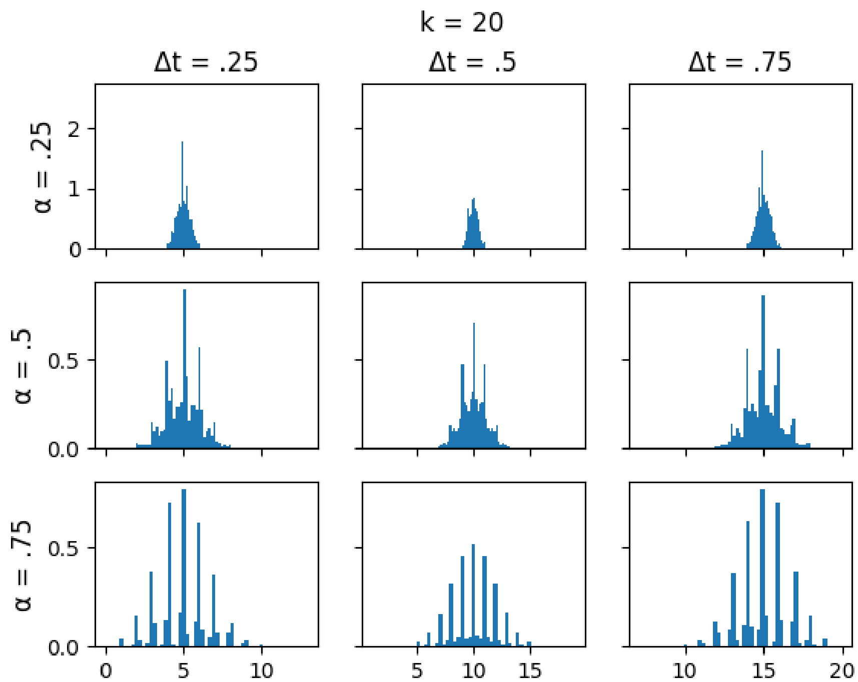

Figure 5: Monte-Carlo simulation of the distribution of T^{α}_{20} for various values of α and Δt

T^1_k is contained in the integers and second we will show that T^1_k is a constant plus the sum of identical and independent Bernoulli random variables, justifying our claim that T^1_k is a shifted binomial random variable.

We demonstrate the first fact through an inductive argument on k. First note that $T^1_0 = 0$ has support in the integers. Now for $k > 0$, supposing T^1_{k-1} has support in the integers, we can write

$$
\begin{aligned}
T^1_k &= T^1_{k-1} + \Delta t + W_1(T^1_{k-1} + \Delta t + X_k) - W_1(X_k) \\
&= T^1_{k-1} + \Delta t + (1 - \{T^1_{k-1} + \Delta t + X_k\}) - (1 - \{X_k\}) \\
&= T^1_{k-1} + \Delta t + \{X_k\} - \{T^1_{k-1} + \Delta t + X_k\}.
\end{aligned}
$$

By assumption T^1_{k-1} can only take on integer values, thus the T^1_{k-1} term inside the fractional

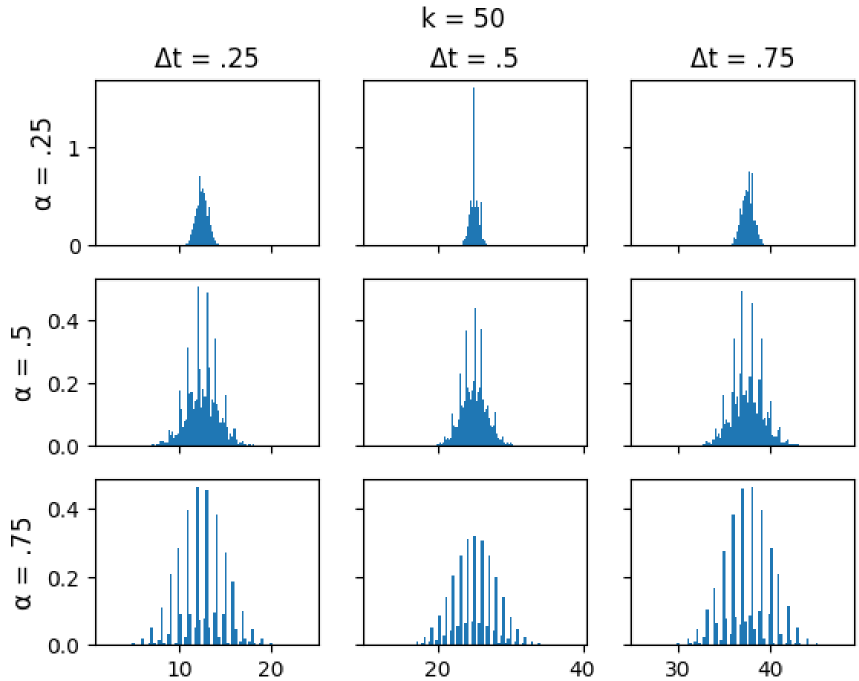

Figure 6: Monte-Carlo simulation of the distribution of T_{50}^{α} for various choices of α and Δt

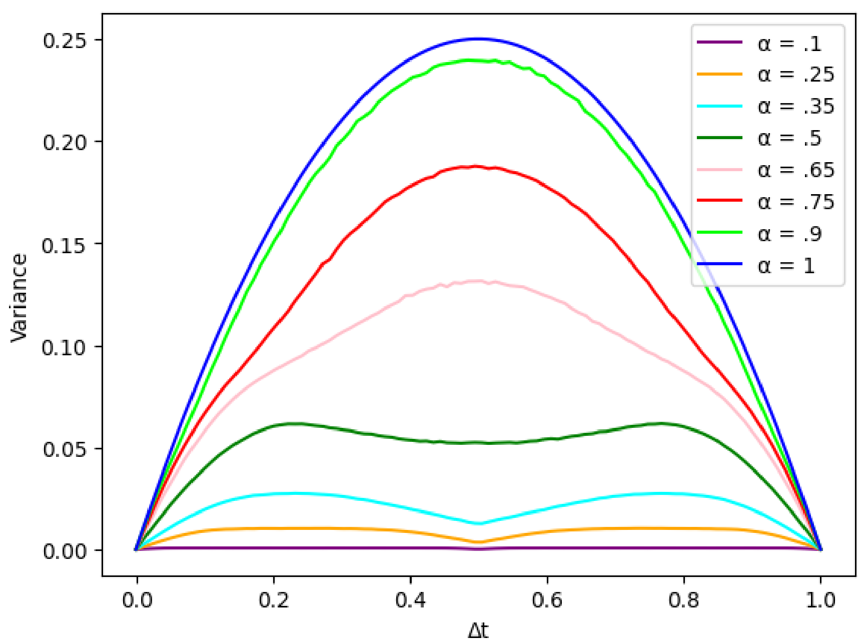

Figure 7: Simulated values of $\mathbb{V}(T_1^{\alpha})$ for selected values of α as a function of $\{\Delta t\}$

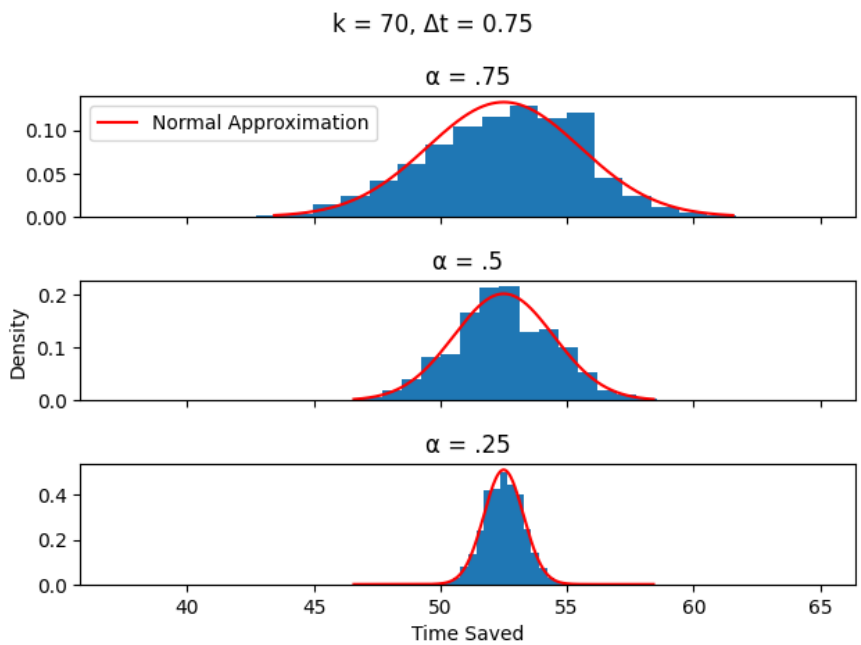

Figure 8: Simulated distributions of T_{70}^{α} for various choices of alpha along with pdf of normal distribution with corresponding mean and standard deviation.

part of the computation does not affect T_k^1. Using this fact gives

$$\begin{aligned}
T_k^1 &= T_{k-1}^1 + \Delta t + \{X_k\} - \{\Delta t + X_k\} \\
&= T_{k-1}^1 + \Delta t + (\{X_k\} - \{\Delta t\} - \{X_k\})\chi_{[0,1-\{\Delta t\})}(X_k) + (\{X_k\} - \{\Delta t\} - \{X_k\} - 1)\chi_{(1-\{\Delta t\},1]}(X_k) \\
&= T_{k-1}^1 + \Delta t - \{\Delta t\} + \chi_{(1-\{\Delta t\},1]}(X_k) \\
&= T_{k-1}^1 + \lfloor \Delta t \rfloor + \chi_{(1-\{\Delta t\},1]}(X_k).
\end{aligned}$$

But this shows that T_k^1 is sum of integers, we know that T_k^1 can only take on integral values, thus establishing our claim.

With this result, we can show the second fact. Notice that for each k, the quantity $\chi_{(1-\{\Delta t\},1]}(X_k)$ is a random variable with outcomes 0 and 1 with probabilities $\{1 - \Delta t\}$ and $\{\Delta t\}$ respectively. This means that

$$\chi_{(1-\{\Delta t\},1]}(X_k) := B_k \sim \mathrm{Bern}(\{\Delta t\}). \tag{7}$$

Thus, we can rewrite our expression for T_k^1 as

$$\begin{aligned}
T_k^1 &= T_{k-1}^1 + \lfloor \Delta t \rfloor + B_k \\
&= \sum_{n=1}^{k} \lfloor \Delta t \rfloor + B_n \\
&= k\lfloor \Delta t \rfloor + \sum_{n=1}^{k} B_n.
\end{aligned}$$

Since we can write T_k^1 as the sum of k independent and identical Bernoulli variables with parameter $\{\Delta t\}$ we know

$$[T_k^1 - k\lfloor \Delta t \rfloor] \sim \mathrm{Bin}(k, \{\Delta t\}). \tag{8}$$

From this we immediately know the mean and variance of T_k^1 to be

$$\mathbb{E}(T_k^1) = k\lfloor \Delta t \rfloor + k\{\Delta t\} = k\Delta t$$

$$\mathbb{V}(T_k^1) = k\{\Delta t\}(1 - \{\Delta t\})$$

The Case of $\alpha = 0$ This case is much easier to analyze than the previous case. This is because of the simple fact that $W_0(t) = 0$. Meaning that T_k^0 is not a function of any of the X_k's. Thus is not a very random variable at all. This makes sense because W_0 corresponds to a light cycle in which the light is always green. Meaning that this reduces to a model of the time saved without considering any traffic lights. We can see this fact since

$$\begin{aligned} T_k^0 &= T_{k-1}^0 + \Delta t + W_0(T_{k-1}^\alpha + \Delta t + X_k) - W_0(X_k) \\ &= T_{k-1}^0 + \Delta t. \end{aligned}$$

Thus by an inductive argument, we see

$$T_k^0 = k\Delta t. \tag{9}$$

From this we get the somewhat trivial result that

$$\mathbb{E}(T_k^0) = k\Delta t$$

and

$$\mathbb{V}(T_k^0) = 0.$$

6 Computational Results for $T_k^{0.75}$

The case for $\alpha = 0.25$ is of special interest because if a traffic light has four equally timed phases, two for through traffic and two for turning traffic, then the light will have wait function $W_{0.75}(t)$. This particular example, for Δt between 0.25 and 0.75 turns out to have easily separable continuous and discrete parts. We can break down these distributions into three main parts, the forbidden regions, the discrete part, and the continuous part In figure 9 the simulated distribution of $T_{18}^{0.75}$ for $\Delta t = .3$ provides an example of each of these parts of the distributions. To construct this figure, we first simulated $T_{18}^{0.75}$ using a Monte-Carlo simulation with 500,000 points and then filtered out any points that were within a small tolerance of the known locations of the point probabilities. The full distribution, and the two filtered parts of the data were plotted as histograms.

From the figure, we can clearly see the properties of the support of this distribution. One thing that this figure shows is that for $\alpha = 0.75$, we know that intervals of the form $(n + 0.25, n + 0.75)$ have probability 0 for any integer n. This is in contrast to several common continuous distributions such as a normal distribution in which every open interval has a non-zero probability. We also can see that the discrete part of this distribution is located at integral values. This discrete part appears to be close to symmetric about it's mean and appears to have the same general shape as the original distribution. Similarly, the continuous part appears to be roughly symmetric about it's mean and have a similar general shape to the discrete part. However, the distribution within a given connected component appears to have a jump at integral values. This suggests that a closed form expression for

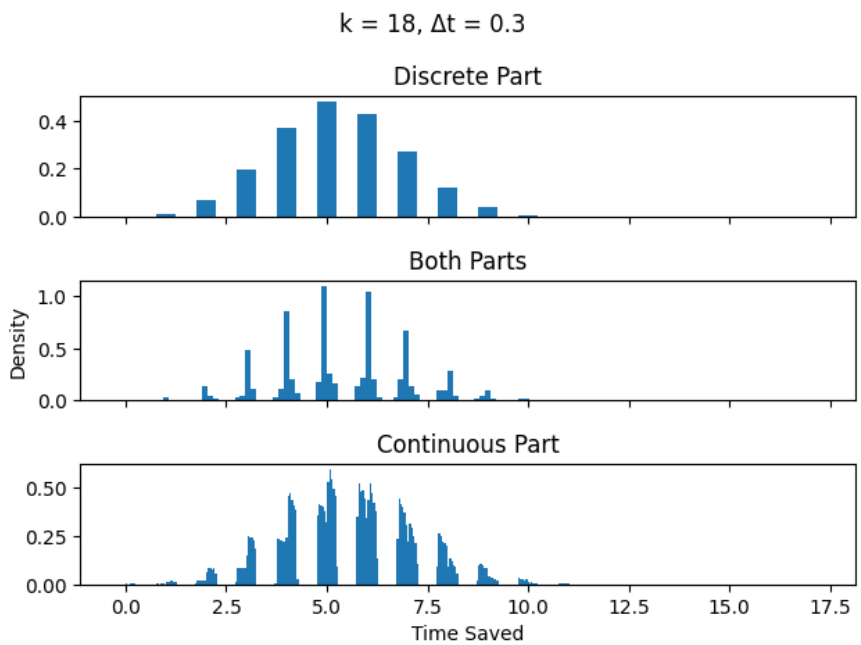

Figure 9: Simulation of distribution of $T_{18}^{0.75}$ for $\Delta t = .3$ broken into it's discrete and continuous parts

this distribution would likely greatly grow in complexity as k gets larger.

7 Analytic results for T_k^α

As mentioned previously, except for a few special values of α , the distribution of T_k^α becomes very hard to calculate, and any explicit form of the distribution would likely be too convoluted to easily extract general results on quantities such as expected value or variance. Fortunately, using the law of total expectation, we can extract some useful information about T_k^α without having an explicit form for it's distribution [1].

Expected Value of T_k^α One piece of information we can glean from Equation [6] is the expected value of T_k^α. We do this by considering the conditional random variable

$$[T_k^\alpha | T_{k-1}^\alpha = t] = t + \Delta t + W_\alpha(t + \Delta t + X_k) - W_\alpha(X_k). \tag{10}$$

This conditional random variable is solely a function of X_k, so we can calculate it's expected value as follows

$$\begin{aligned}
\mathbb{E}(T_k^\alpha | T_{k-1}^\alpha) &= \mathbb{E}(T_{k-1}^\alpha + \Delta t + W_\alpha(T_{k-1}^\alpha + \Delta t + X_k) - W_\alpha(X_k)) \\
&= T_{k-1}^\alpha + \Delta t + \mathbb{E}(W_\alpha(T_{k-1}^\alpha + \Delta t + X_k) - W_\alpha(X_k)) \\
&= T_{k-1}^\alpha + \Delta t + \int_0^1 W_\alpha(T_{k-1}^\alpha + \Delta t + x) - W(x)_\alpha dx \\
&= T_{k-1}^\alpha + \Delta t + \int_0^1 W_\alpha(T_{k-1}^\alpha + \Delta t + x) dx - \int_0^1 W_\alpha(x) dx \\
&= T_{k-1}^\alpha + \Delta t + \int_{T_{k-1}^\alpha + \Delta t}^{1 - T_{k-1}^\alpha + \Delta t} W_\alpha(x) dx - \int_0^1 W_\alpha(x) dx \\
&= T_{k-1}^\alpha + \Delta t
\end{aligned}$$

where the integral terms cancel out because integral of a periodic function over any period is the same. From this, the law of total expectation implies

$$\begin{aligned}\mathbb{E}(T_k^\alpha) &= \mathbb{E}(\mathbb{E}(T_k^\alpha | T_{k-1}^\alpha)) \\ &= \mathbb{E}(T_{k-1}^\alpha + \Delta t) \\ &= \mathbb{E}(T_{k-1}^\alpha) + \Delta t.\end{aligned}$$

However this and the fact that $\mathbb{E}(T_0^\alpha) = 0$ imply by induction that

$$\mathbb{E}(T_k^\alpha) = k\Delta t. \tag{11}$$

This shows that the expected value of T_k^α does not depend on the choice of wait function. In other words, the specifics of the traffic light cycles lead to the same average time saved when speeding. This means that the example given in the introduction is slightly misleading in the sense that it would've been just as likely for the reckless driver to get lucky and pass through the light while you are stopped at the red light.

Variance of T_k^α Though we know the expected value of T_k^α, this does not tell us anything about the shape of the distributions of T_k^α and how far from their mean they vary. To try to understand that, we wish to understand the variance of the random variables. To get a handle on the variance of T_k^α we can use a similar trick to what we did for the expected value, only applied to $(T_k^\alpha)^2$. As before consider the conditional random variable $(T_k^\alpha)^2 | T_{k-1}^\alpha$, then the expected value is given by

$$
\begin{aligned}
\mathbb{E}((T_k^\alpha)^2|T_{k-1}^\alpha) =&\mathbb{E}((T_{k-1}^\alpha + \Delta t + W_\alpha(T_{k-1}^\alpha + \Delta t + X_k) - W_\alpha(X_k))^2) \\
=&(T_{k-1}^\alpha)^2 + 2T_{k-1}^\alpha \Delta t + \Delta t^2 + (T_{k-1}^\alpha + \Delta t)\mathbb{E}(W_\alpha(T_{k-1}^\alpha + \Delta t + X_k) - W_\alpha(X_k)) \\
&+ \mathbb{E}(W_\alpha^2(T_{k-1}^\alpha + \Delta t + X_k)) - 2\mathbb{E}(W_\alpha(T_{k-1}^\alpha + \Delta t + X_k)W_\alpha(X_k)) + \mathbb{E}(W_\alpha^2(X_k)) \\
=&(T_{k-1}^\alpha)^2 + 2T_{k-1}^\alpha \Delta t + \Delta t^2 + 2\mathbb{E}(W_\alpha^2(X_k)) - 2\mathbb{E}(W_\alpha(T_{k-1}^\alpha + \Delta t + X_k)W_\alpha(X_k)) \\
=&(T_{k-1}^\alpha)^2 + 2T_{k-1}^\alpha \Delta t + \Delta t^2 + \frac{2}{3}\alpha^3 - 2\int_0^1 W_\alpha(T_{k-1}^\alpha + \Delta t + x)W_\alpha(x)dx.
\end{aligned}
$$

Writing the last term as $I(T_{k-1}^\alpha)$, using the law of total expectation, and subtracting $\mathbb{E}(T_k^\alpha)^2$ we get

$$
\begin{aligned}
\mathbb{V}(T_k^\alpha) &= \mathbb{E}((T_k^\alpha)^2) - \mathbb{E}(T_k^\alpha)^2 \\
&= \mathbb{E}((T_{k-1}^\alpha)^2) + 2\mathbb{E}(T_{k-1}^\alpha)\Delta t + \Delta t^2 + \frac{2}{3}\alpha^3 - 2\mathbb{E}(I(T_{k-1}^\alpha)) - \mathbb{E}(T_k^\alpha)^2 \\
&= \mathbb{E}((T_{k-1}^\alpha)^2) + 2(k-1)\Delta t^2 + \Delta t^2 - k^2\Delta t^2 + \frac{2}{3}\alpha^3 - 2\mathbb{E}(I(T_{k-1}^\alpha)) \\
&= \mathbb{E}((T_{k-1}^\alpha)^2) - ((k-1)\Delta t)^2 + \frac{2}{3}\alpha^3 - 2\mathbb{E}(I(T_{k-1}^\alpha)) \\
&= \mathbb{E}((T_{k-1}^\alpha)^2) - \mathbb{E}(T_{k-1}^\alpha)^2 + \frac{2}{3}\alpha^3 - 2\mathbb{E}(I(T_{k-1}^\alpha)) \\
&= \mathbb{V}(T_{k-1}^\alpha) + \frac{2}{3}\alpha^3 - 2\mathbb{E}(I(T_{k-1}^\alpha)).
\end{aligned}
$$

Since the integral expression is strictly greater than zero, using an inductive argument it can be shown that

$$
\mathbb{V}(T_k^\alpha) \leq \frac{2}{3}\alpha^3 k. \tag{12}
$$

Comparing this upper bound to figure 7 we see that this bound appears to be strict for some values of Δt when α is less than .5. Better bounds likely can be obtained by finding a larger lower bound for the expected value of the integral expression. This would likely require more information about T_k^α than it's expected value, meaning it would likely involve calculation of T_k^α or more sophisticated techniques.

8 Conclusion

In this paper we have developed and analyzed a simple model for the time saved by a car speeding subject to traffic lights. This model is novel both in the situation being modeled and in the distributions generated by the model. In terms of the phenomenon being modeled, we have found that in this simplified situation, the specific details of the individual traffic light cycles on average do not affect the time saved by speeding, however the cycle of the traffic lights and speeding amount can increase the maximum and decrease the minimum amount of time that could be saved while speeding. In terms of the mathematical model, we have found a family of mixed distributions that can be expanded as a sum of non-independent and non-identical random variables yet appears to have behavior similar to what the central limit theorem would imply.

Further work could either work on understanding this model better or attempt to use similar techniques on more accurate models. For example, while we were able to extract some information from T_k^α, we have not been able to calculate T_k^α exactly for k greater than 1. This information could be used to calculate the variance of the random variables exactly, leading to a better understanding of the limiting behavior of these distributions. From a model standpoint, further work could see what happens when some of the simplifying assumptions are dropped. For example, this model assumes that the traffic light cycles are identical and independent, however in most situations, these two assumptions are false. There typically are procedures in place for how traffic light cycles should be calibrated, these procedures can greatly affect how often speeding and normal cars hit red lights.

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