Assignment 1 - Martingale
CS 7646 Machine Learning for Trading
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Martingale Project - Assignment 1

1. **In Experiment 1, estimate the probability of winning $80 within 1000 sequential bets. Explain your reasoning thoroughly.**

- After 1000 sequential bets, it is estimated that the probability of winning $80 is almost equal to 1. This can be seen both mathematically via the Binomial Distribution and graphically, according to Figure 2. If the probability of winning is 18/38 (the number of black spots on the American Roulette wheel), and the least number of bets it takes to reach $80 is 80 bets, then we can substitute into the following equation N= 10000, x = 80, π^x = (18/38) and find the cumulative probability for winning at least 80 times out of 1000. Accordingly, when looking at Figure 2, we can see that over the course of 1000 trials (bets), the mean value across all trials eventually converge to a value of $80 in winnings, meaning that eventually in this scenario, if we keep betting, we are almost guaranteed to eventually reach $80 in winnings.

$$P(x)=\frac{N!}{x!(N-x)!}\pi^x(1-\pi)^{N-x}$$

$$\sum_{i=80}^{1000} {}_{1000}C_i \cdot p^i \cdot (1-p)^{1000-i} = \mathbf{0.999999}$$

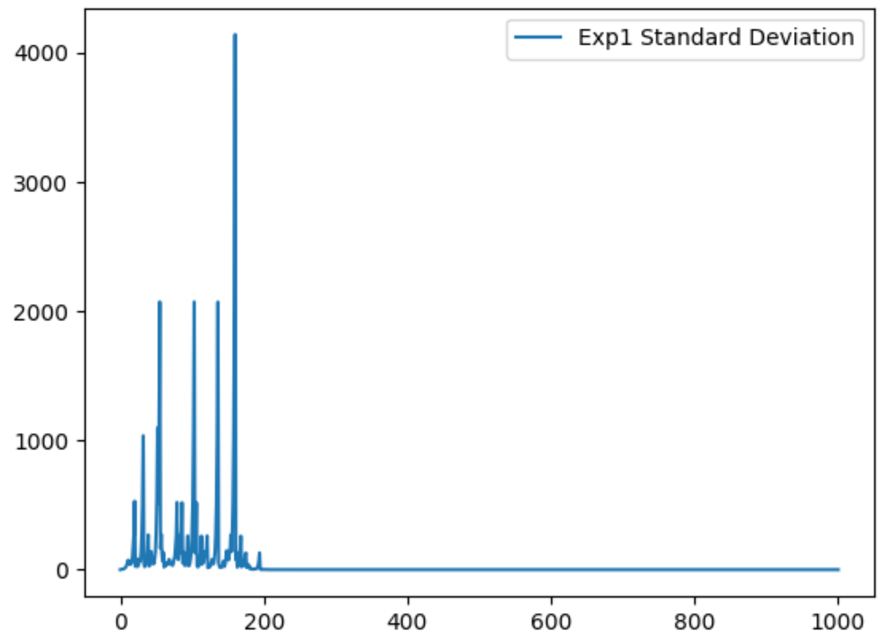
2. **In Experiment 1, what is the estimated expected value of our winnings after 1000 sequential bets? Explain your reasoning thoroughly.**

- The estimated expected value of our winnings in Experiment 1 after 1000 sequential bets is $80. This can be seen both mathematically via the Expected Value equation and visually through Figure 2 (seen in the appendix). Given that this game only has two possible outcomes (win or loss), we can represent the probabilities of said outcomes as a binomial distribution. As a result, we can manipulate the equation for expected value to be of the following form. If we substitute the values n=1000 and p= (18/38) into the equation, we can see that the expected value of our winnings after 1000 sequential bets is about $474. However, because we default any value over $80 to be $80 in our game, we can conclude that the estimated expected value of our winnings is $80 after 1000 sequential bets. Accordingly, when looking at Figure 2, we can see that over the course of 1000 trials (bets), the mean value across all trials eventually converge to a value of $80 in winnings, which is why it is safe to expect a winnings value of $80 after 1000 bets.

$$\begin{aligned} E[X] &= n\,p \\ &= 1000 * (\tfrac{18}{38}) \\ &= 474 \end{aligned}$$

$$474 > 80, \text{ therefore } E[X] = 80$$

3. **In Experiment 1, does the standard deviation reach a maximum value then stabilize or converge as the number of sequential bets increases? Explain why it does (or does not) thoroughly.**



- Yes, in Experiment 1, there exists a point where the Standard Deviation reaches a maximum value (as can be seen in the figure above), and then eventually there also exists a point where the standard deviation stabilizes at a single value (0) as the number of sequential bets increase. The standard deviation stabilizes at a value of 0 because within a given game, once the player has reached a winnings value of $80, the player has "won" the game and the remaining winnings values of that game are recorded as $80 as well.

4. **In Experiment 2, estimate the probability of winning $80 within 1000 sequential bets. Explain your reasoning using the experiment thoroughly.**

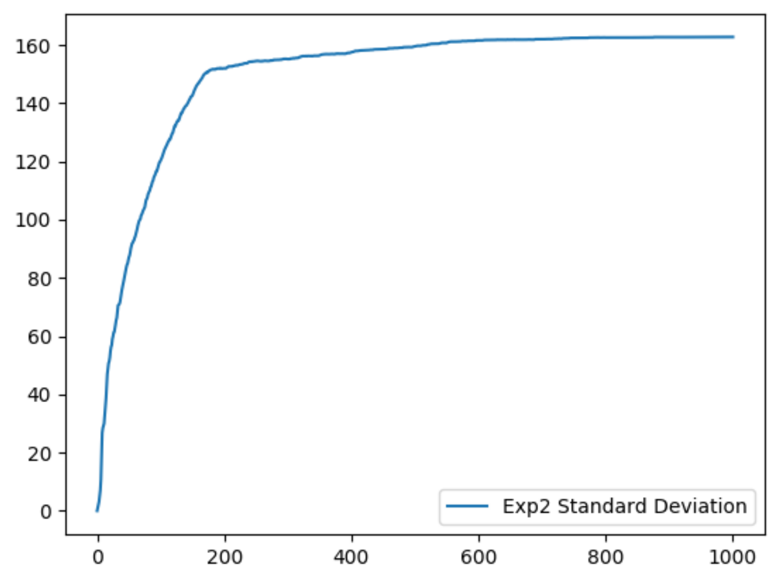
- Out of 1000 trials, 622 trials were recorded as "wins" (hitting the $80 winnings mark). As a result we can estimate the probability of winning $80 in experiment 2 to be 622/1000 = 62.2%. While Figure 4 shows mean value that are closer to $0 - $-50, this can be explained because of the difference in the lower and upper bound (upper bound defaulting at $80 and lower bound defaulting at -$256) that brought the mean values across each bet much lower. For example, if the probability of winning $80 was 0.5, the mean value of a given bet across 1000 trials would be (80 -256)/2 = -$88, which may not at first seem to represent the 50:50 odds of winning.

5. **In Experiment 2, what is the estimated expected value of our winnings after 1000 sequential bets? Explain your reasoning thoroughly.**

- By taking the number of wins and the number of losses and multiplying them by the upper bounds and lower bounds of the winnings values ($80 and -$256 respectively), summing them, and then dividing the sum by 1000, we can come up with an estimated expected value of our winnings in experiment 2 after 1000 sequential bets. As a result, we can say that on average, over the course of 1000 games, our estimated value of our winnings at the end of the 1000 bets will be -$47.01.

$$\frac{(\$80 * 622) + [(1000-622)*(-\$256)]}{1000} = -47.008$$

6. **In Experiment 2, does the standard deviation reach a maximum value then stabilize or converge as the number of sequential bets increases? Explain why it does (or does not) thoroughly.**



- Yes, in Experiment 2, there exists a point where the Standard Deviation reaches a maximum value (as can be seen in the figure above), and then eventually there also exists a point where the standard deviation begins to stabilize as the number of sequential bets increase. However, unlike Experiment 1, the standard deviation doesn't stabilize around 0. Instead this graph above convergence around a standard deviation of 160. Accordingly, this high standard deviation could be due to the range between the maximum and minimum defaults of the upper and lower bounds of winnings ($80 and -$256). Also, because the probability of winning is 0.62 and the probability of losing is 0.38 (neither being as definitive as 0.99999 in Experiment1), it is more likely to have a wider spread of varying winnings values.

Appendix

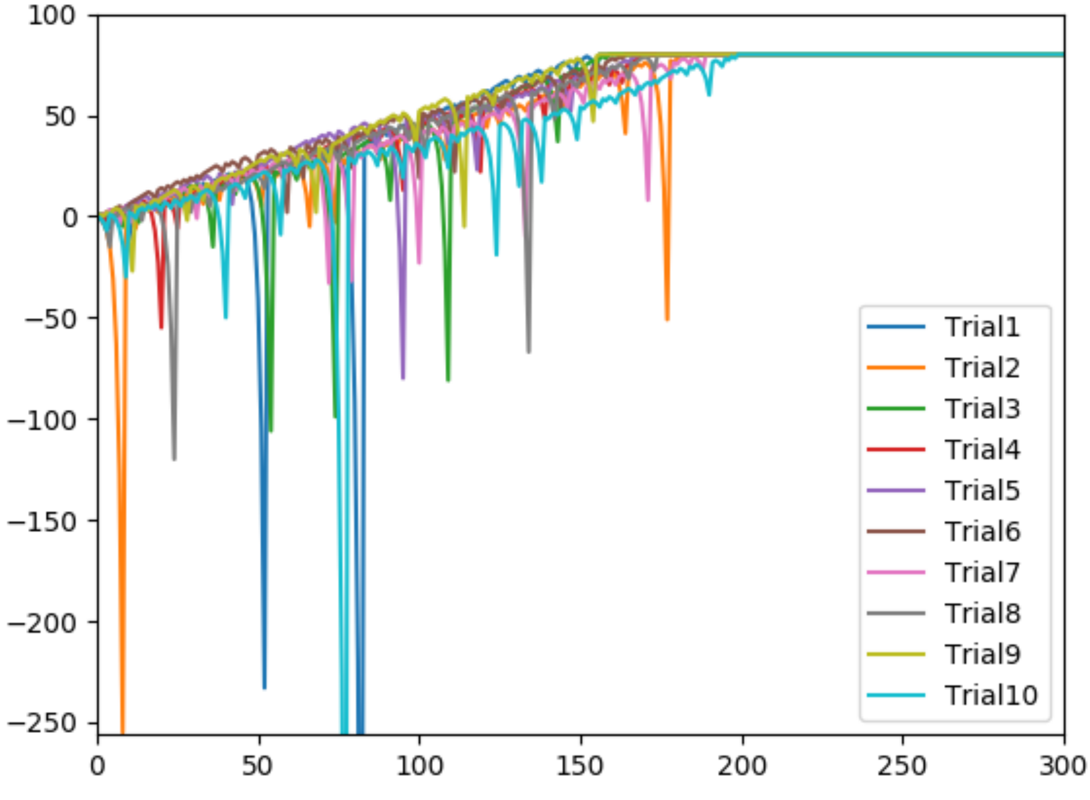


Figure 1: Simple Simulator 10 Trials Winnings

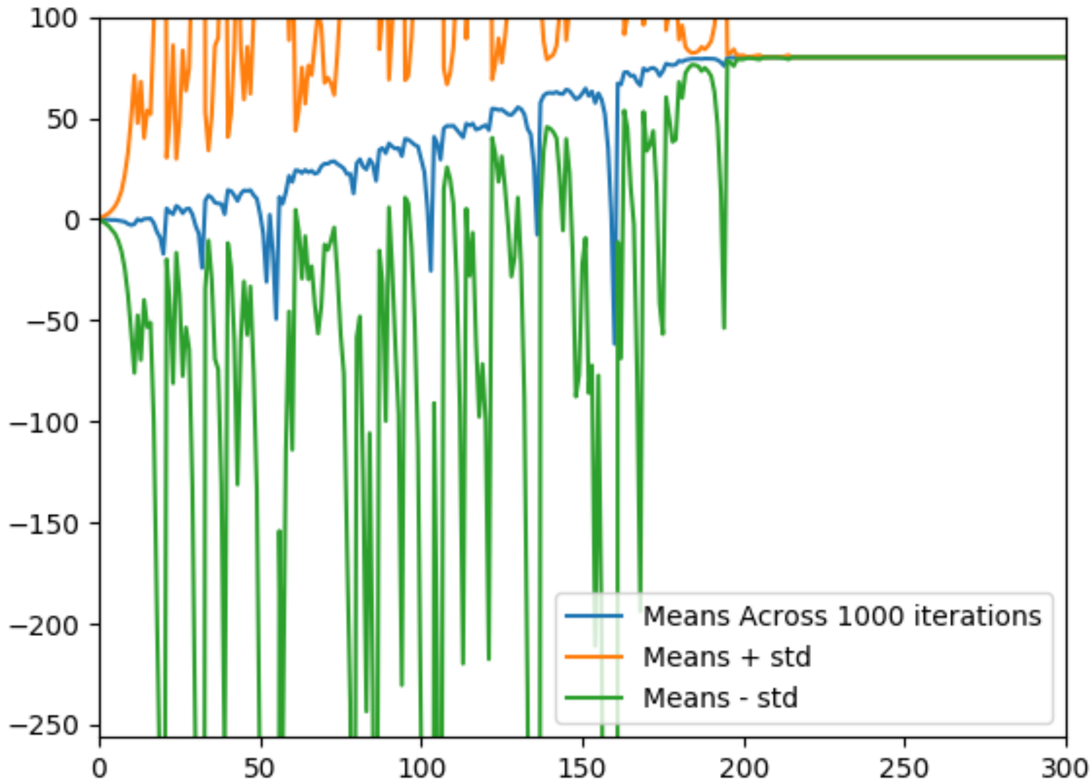


Figure 2: Simple Simulator 1000 Trials Mean Values with Standard Deviation

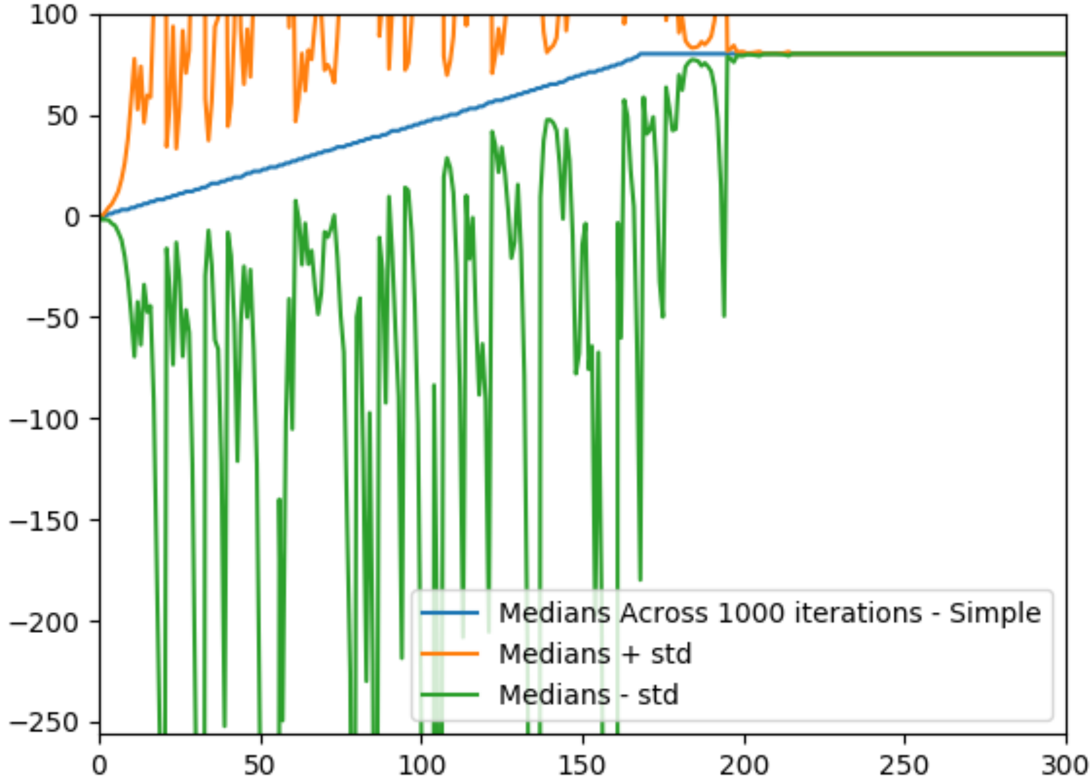


Figure 3: Simple Simulator 1000 Trials Median Values with Standard Deviation

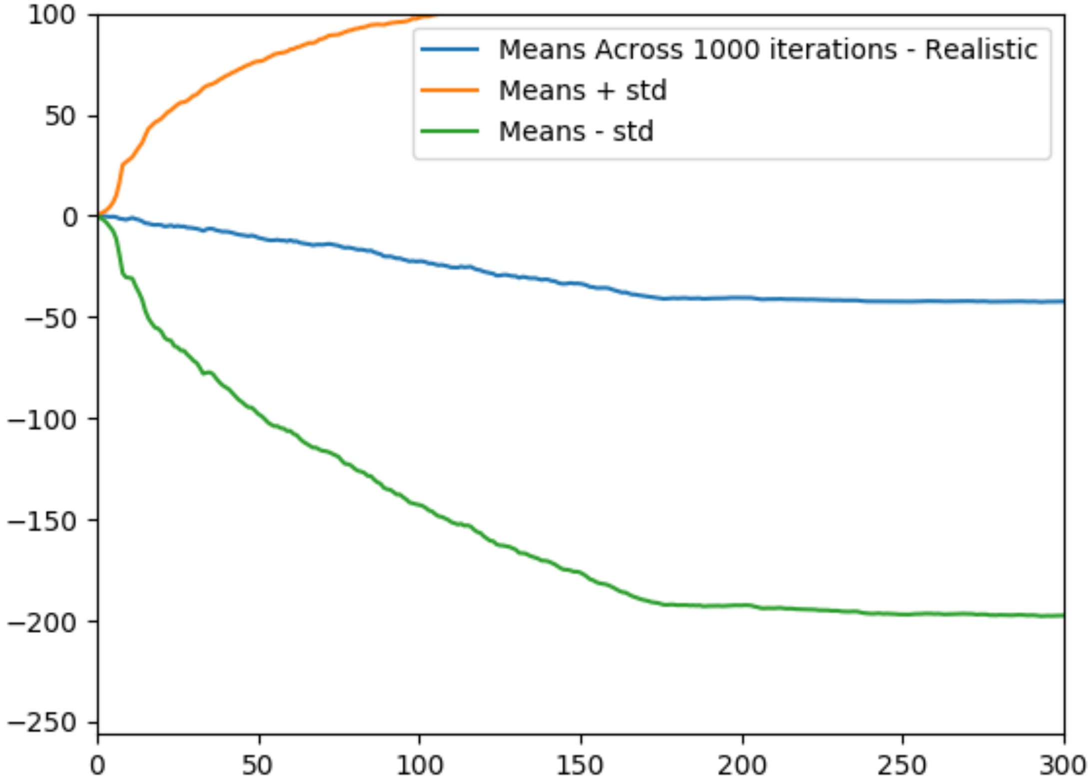


Figure 4: Realistic Simulator 1000 Trials Mean Values with Standard Deviation

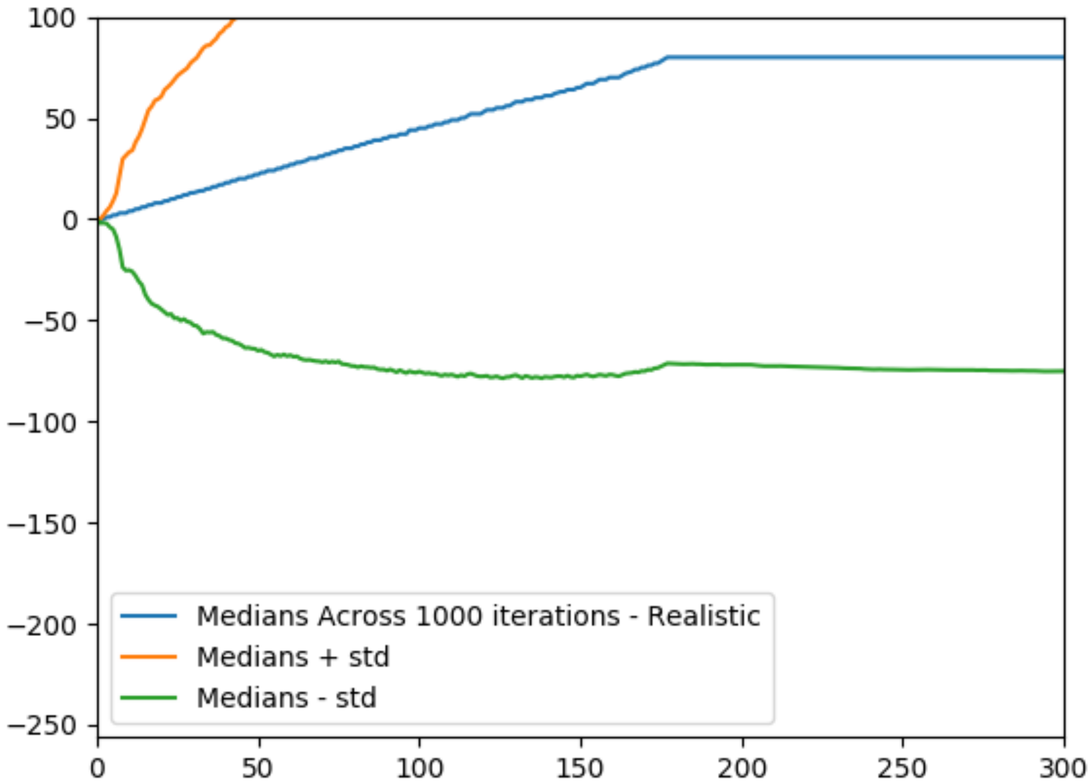


Figure 5: Realistic Simulator 1000 Trials Median Values with Standard Deviation