153_project

2022-04-05

```
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.1 --

## v ggplot2 3.3.5     v purrr   0.3.4
## v tibble  3.1.6     v dplyr   1.0.8
## v tidyr   1.2.0     v stringr 1.4.0
## v readr   2.1.2     v forcats 0.5.1

## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```
library(TSA)
```

```
##
## Attaching package: 'TSA'

## The following object is masked from 'package:readr':
##
##     spec

## The following objects are masked from 'package:stats':
##
##     acf, arima

## The following object is masked from 'package:utils':
##
##     tar
```

```
library(forecast)
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo

## Registered S3 methods overwritten by 'forecast':
##   method       from
##   fitted.Arima TSA
##   plot.Arima   TSA
```

```
library(astsa)
```

```
##
## Attaching package: 'astsa'

## The following object is masked from 'package:forecast':
##
##     gas
```

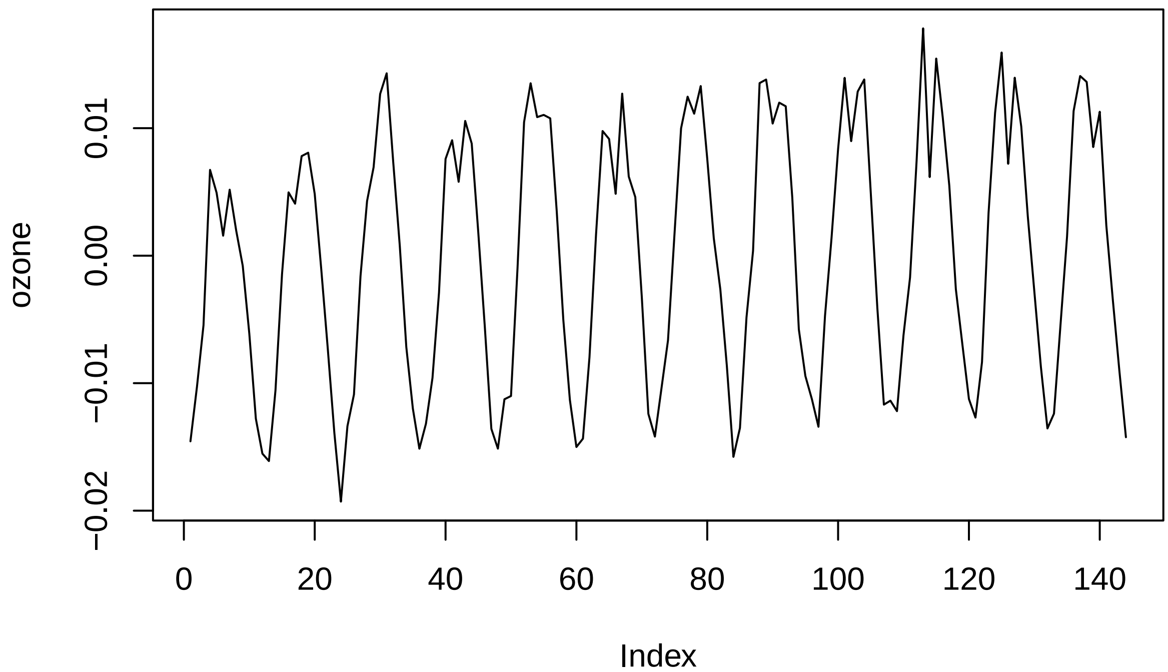
```
source('cleaning.R')
```

```
##
## Attaching package: 'lubridate'

## The following objects are masked from 'package:base':
##
##     date, intersect, setdiff, union

## New names:
## * `` -> ...1

## Rows: 1746661 Columns: 29
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## chr   (8): Address, State, County, City, NO2 Units, O3 Units, SO2 Units, CO ...
## dbl  (20): ...1, State Code, County Code, Site Num, NO2 Mean, NO2 1st Max Va...
## date  (1): Date Local
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
## `summarise()` has grouped output by 'year'. You can override using the `.groups` argument.
```

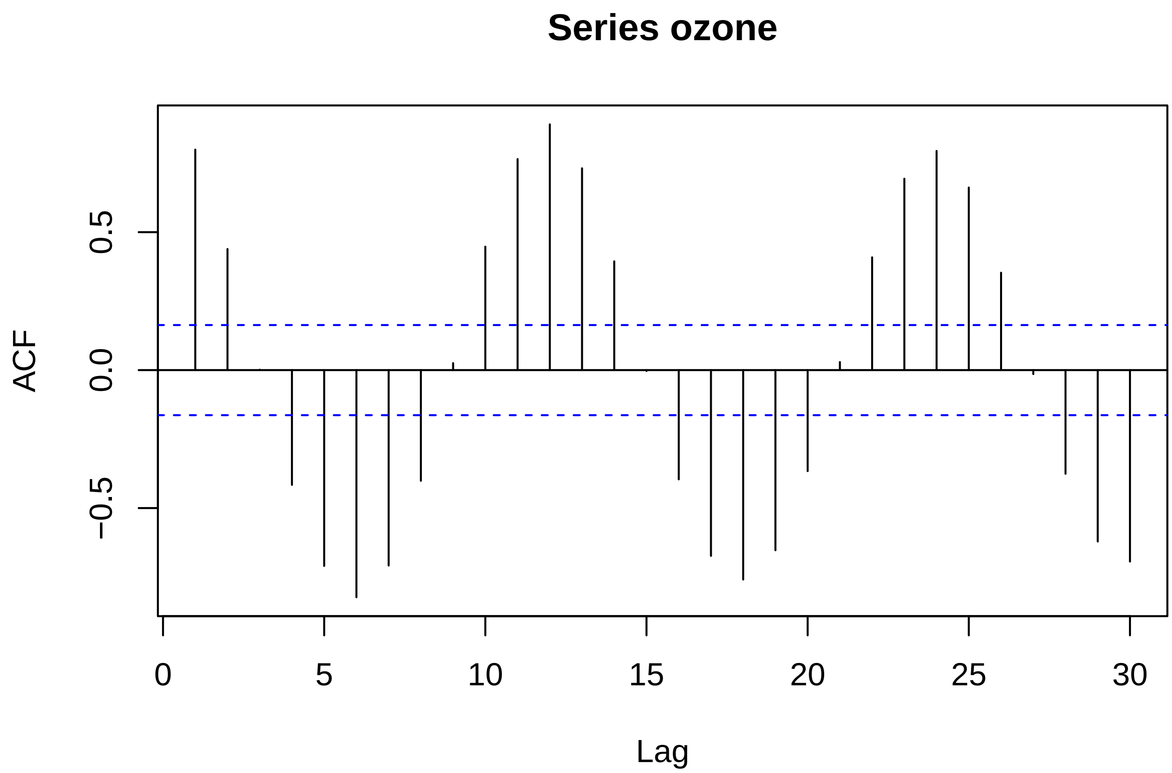
Original data

1. The original data have clear sign of seasonality, but there seems to be linear upward trend.
2. The ACF PACF plot shows the strong sign of seasonality

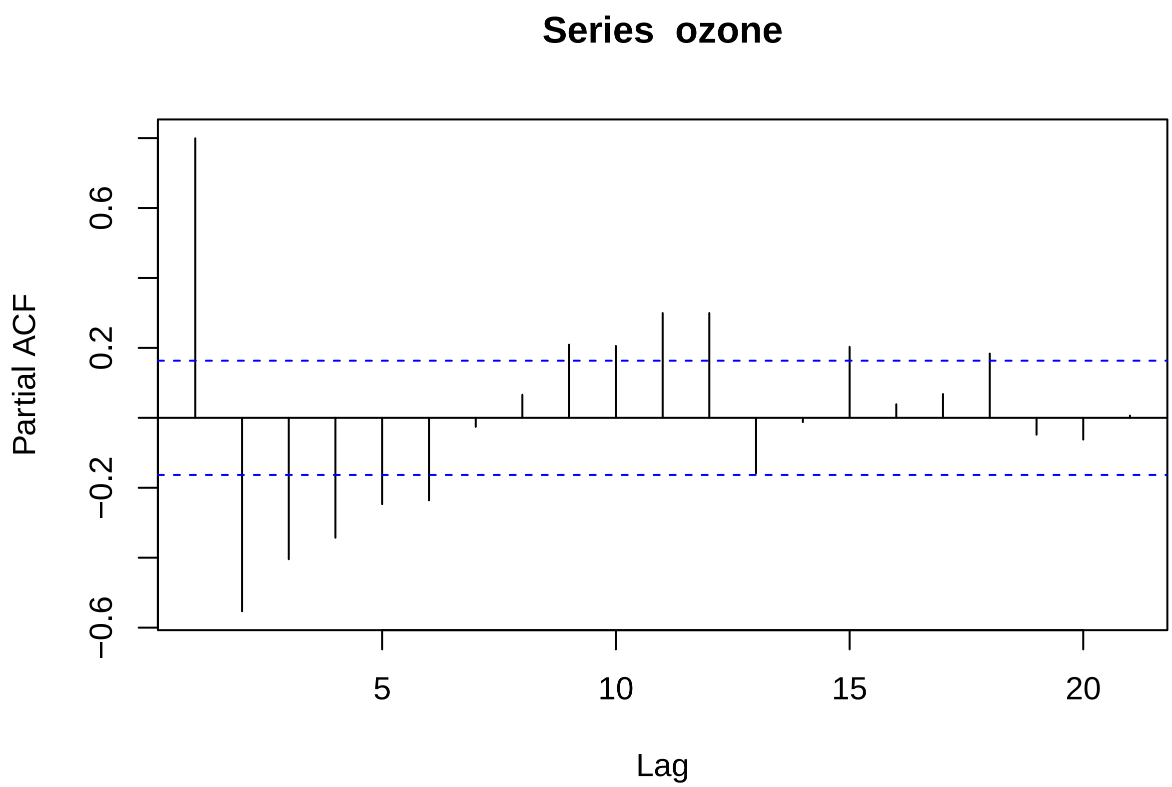
```
ozone <- phoenix$o3
ozone = ozone - mean(ozone) # mean centered
plot(ozone,type ="l")
```




```
acf(ozone,lag.max = 30)
```




```
pacf(ozone)
```



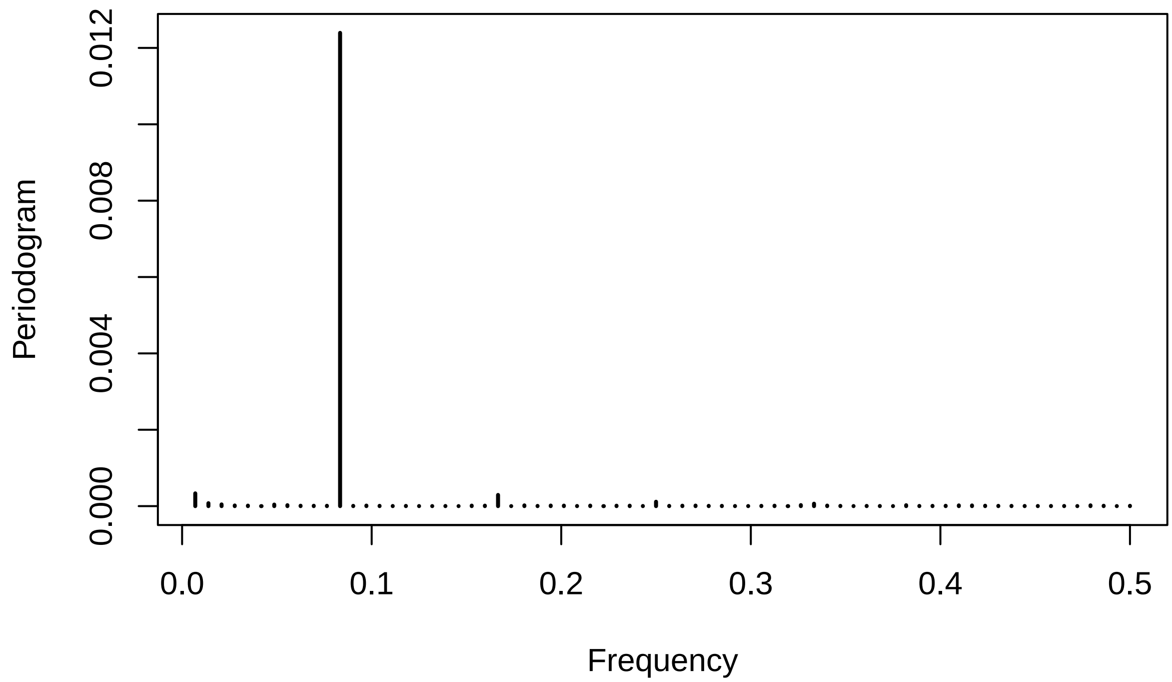
Sinusoidal fitting

Vt = ozone (1+B)Vt = Xt (Vt - f(t)) = Xt

$f(t) = -0.003 + 0.000042 * t - 0.0012 * sin(t) - 0.0123 * cos(t) + 0.000002 * t * sin(t) - 0.000011 * t * cos(t)$

1. There is one significant peak in the periodogram 2. The residual plot shows some seasonality but the plot seems to be AR process and possible seasonal ARMA

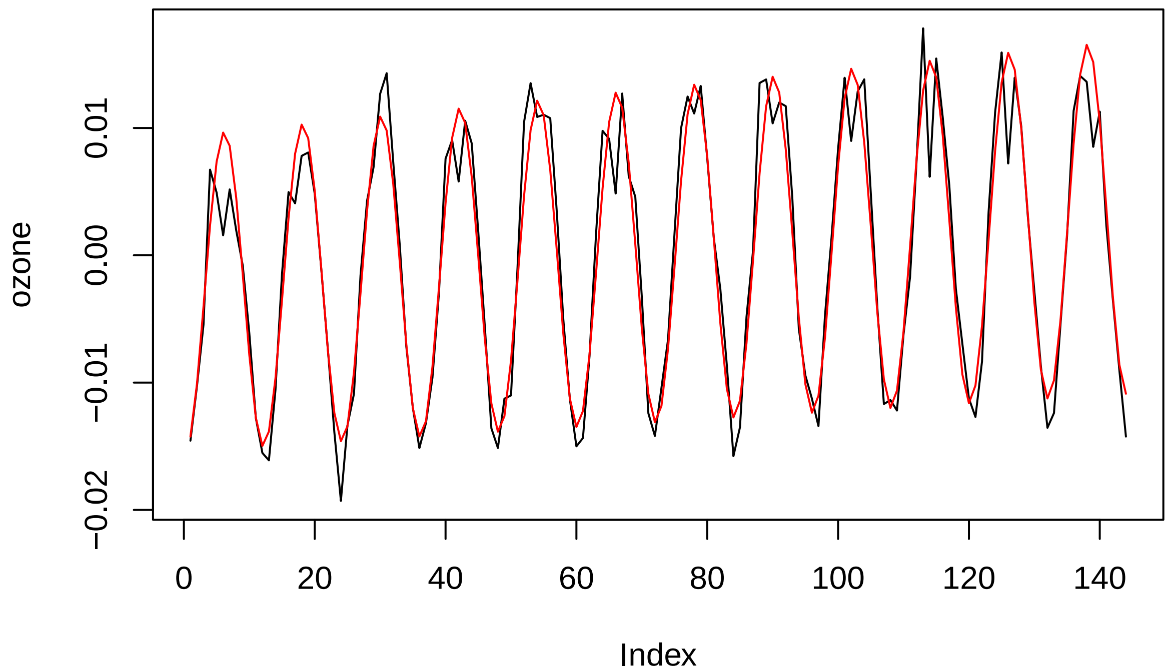
```
t = 1:length(ozone)
# Check the periodogram
periodo = periodogram(ozone,plot=TRUE,ylab="Periodogram", xlab="Frequency") # There is one significant f
```



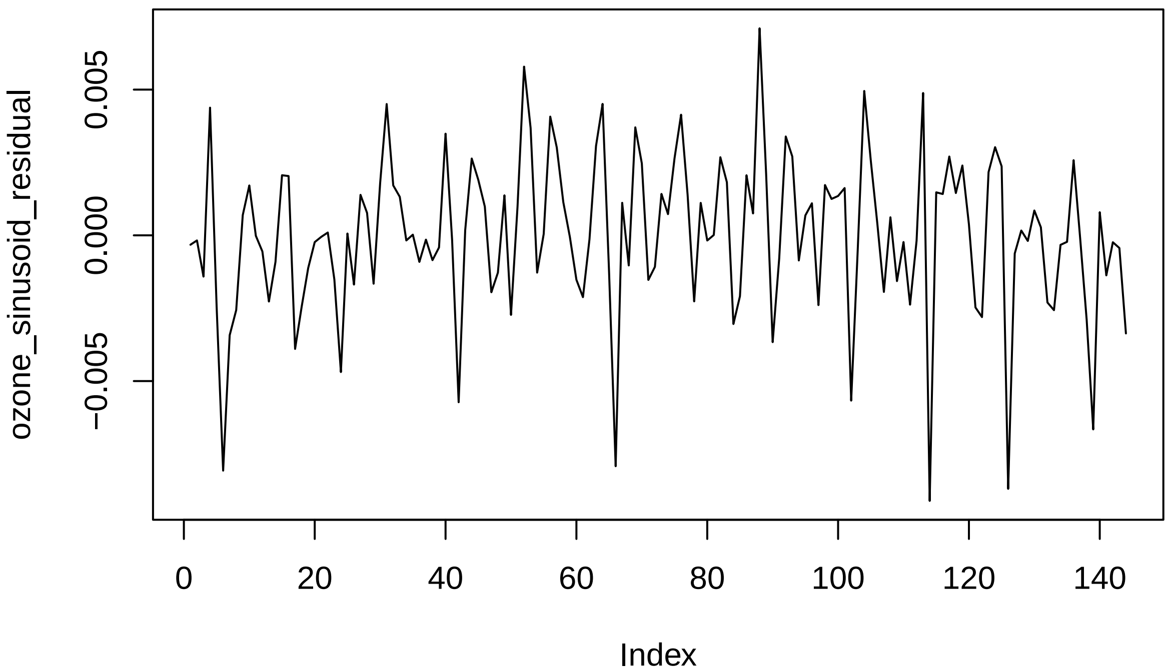

```r
# Get the high magnitudes in descending order
order_spec = sort(periodo$spec,decreasing = TRUE)

# Get the frequency that gives max magnitude
first_max = order_spec[1]
first_maximizing_freq = periodo$freq[periodo$spec==first_max]
first_sin_max = sin(2*pi*first_maximizing_freq*t)
first_cos_max = cos(2*pi*first_maximizing_freq*t)

# Max Sinusoidal fitting
ozone_sinusoid_model = lm(ozone ~ first_sin_max*(1+t)+first_cos_max*(1+t))
print(ozone_sinusoid_model$coefficients)
```

```
##     (Intercept)   first_sin_max               t   first_cos_max first_sin_max:t
##   -3.009489e-03   -1.160248e-03    4.163369e-05   -1.233604e-02    1.963185e-06
## t:first_cos_max
##   -1.058031e-05
```

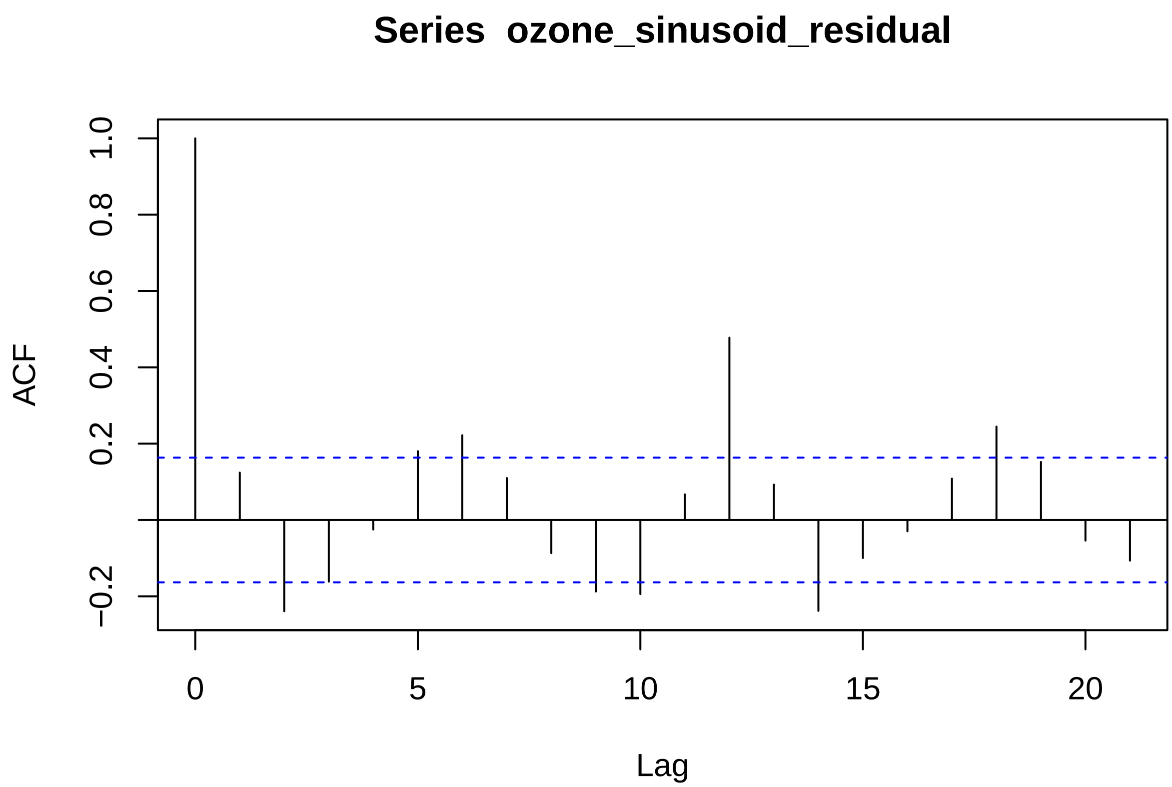
```r
# Overlay the sinusoidal fitting over the original plot
plot(ozone,type = "l")
lines(t,ozone_sinusoid_model$fitted.values,col = "red")
```



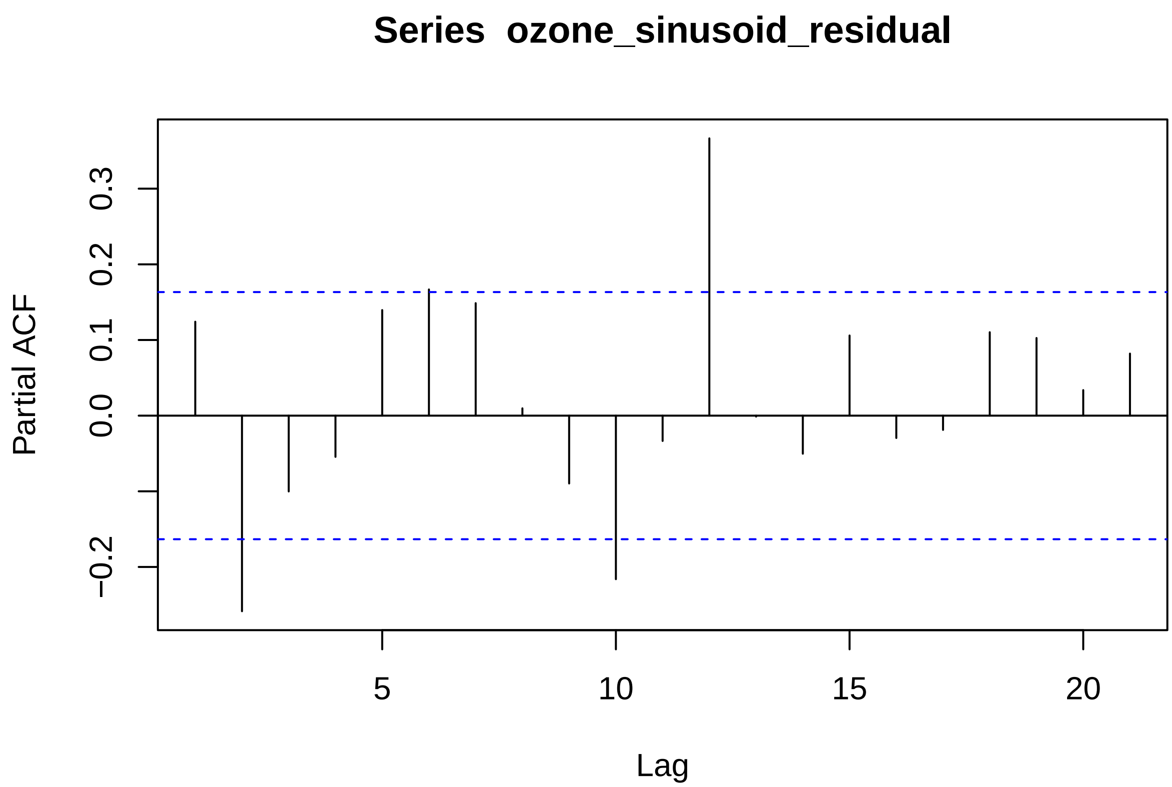

```
# Get the residual, hoping for removing seasonality
ozone_sinusoid_residual = ozone_sinusoid_model$residuals
plot(ozone_sinusoid_residual,type = "l") # residual seems to be stationary
```




```
stats::acf(ozone_sinusoid_residual)
```




```
stats::pacf(ozone_sinusoid_residual)
```

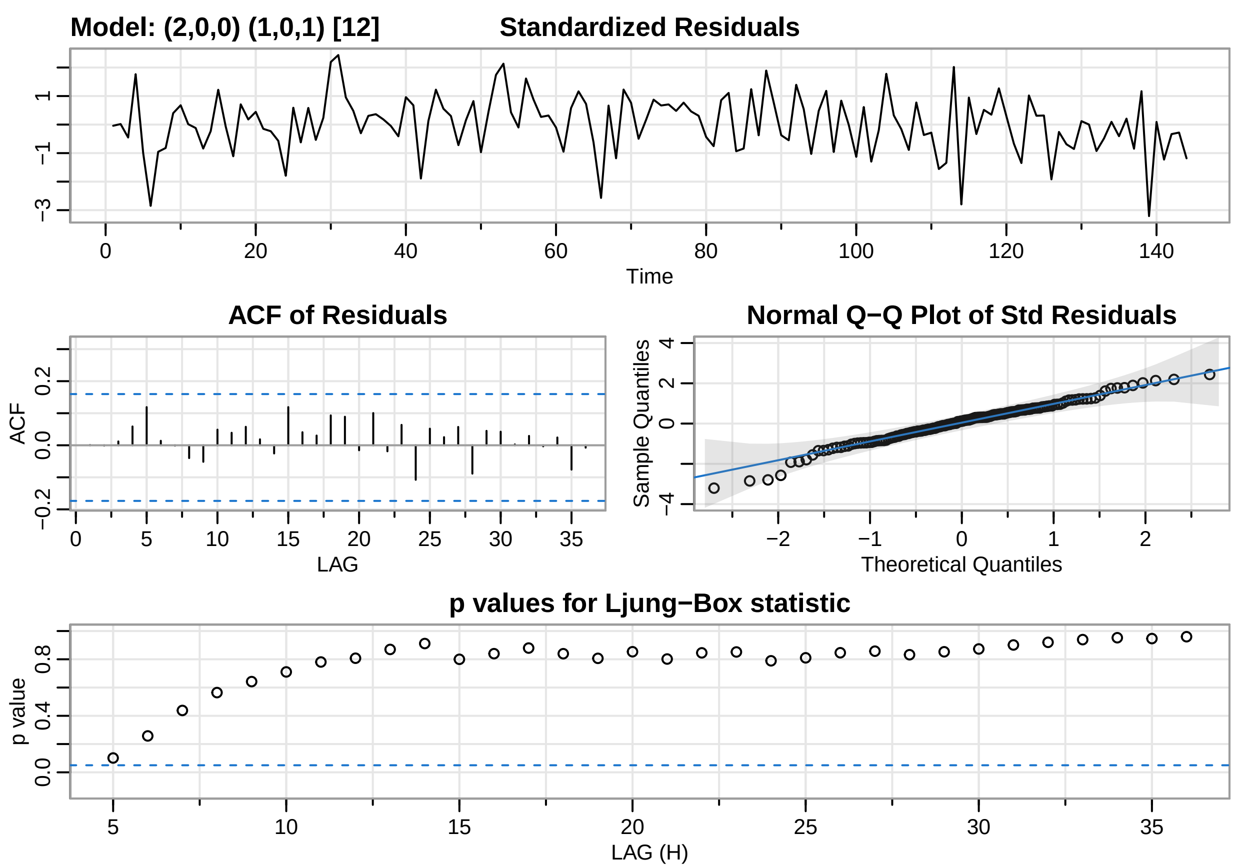


Model1 : SARIMA(2,0,0)(1,0,1)12

```
model1 <- sarima(ozone_sinusoid_residual, p=2, d=0, q=0, P=1, D=0, Q=1, S=12) # fit the model
```

```
## initial  value -5.921137
## iter   2 value -6.020457
## iter   3 value -6.079641
## iter   4 value -6.085871
## iter   5 value -6.091190
## iter   6 value -6.095582
## iter   7 value -6.095958
## iter   8 value -6.095969
## iter   9 value -6.095970
## iter   9 value -6.095970
## iter   9 value -6.095970
## final  value -6.095970
## converged
## initial  value -6.075290
## iter   2 value -6.078754
## iter   3 value -6.081474
## iter   4 value -6.085882
## iter   5 value -6.088012
## iter   6 value -6.092257
```

```
## iter   7 value -6.095296
## iter   8 value -6.097651
## iter   9 value -6.099009
## iter  10 value -6.100678
## iter  11 value -6.101042
## iter  12 value -6.101155
## iter  13 value -6.101161
## iter  14 value -6.101163
## iter  15 value -6.101163
## iter  16 value -6.101166
## iter  17 value -6.101167
## iter  18 value -6.101167
## iter  19 value -6.101168
## iter  20 value -6.101168
## iter  21 value -6.101169
## iter  22 value -6.101169
## iter  22 value -6.101169
## iter  22 value -6.101169
## final  value -6.101169
## converged
```




```
coeff_table <- as.data.frame(model1$ttable)
coeff_table <-coeff_table %>% mutate(ci_lower = Estimate-1.96*SE,ci_upper =  Estimate+1.96*SE)
coeff_table # show estimated coefficient and its ci
```

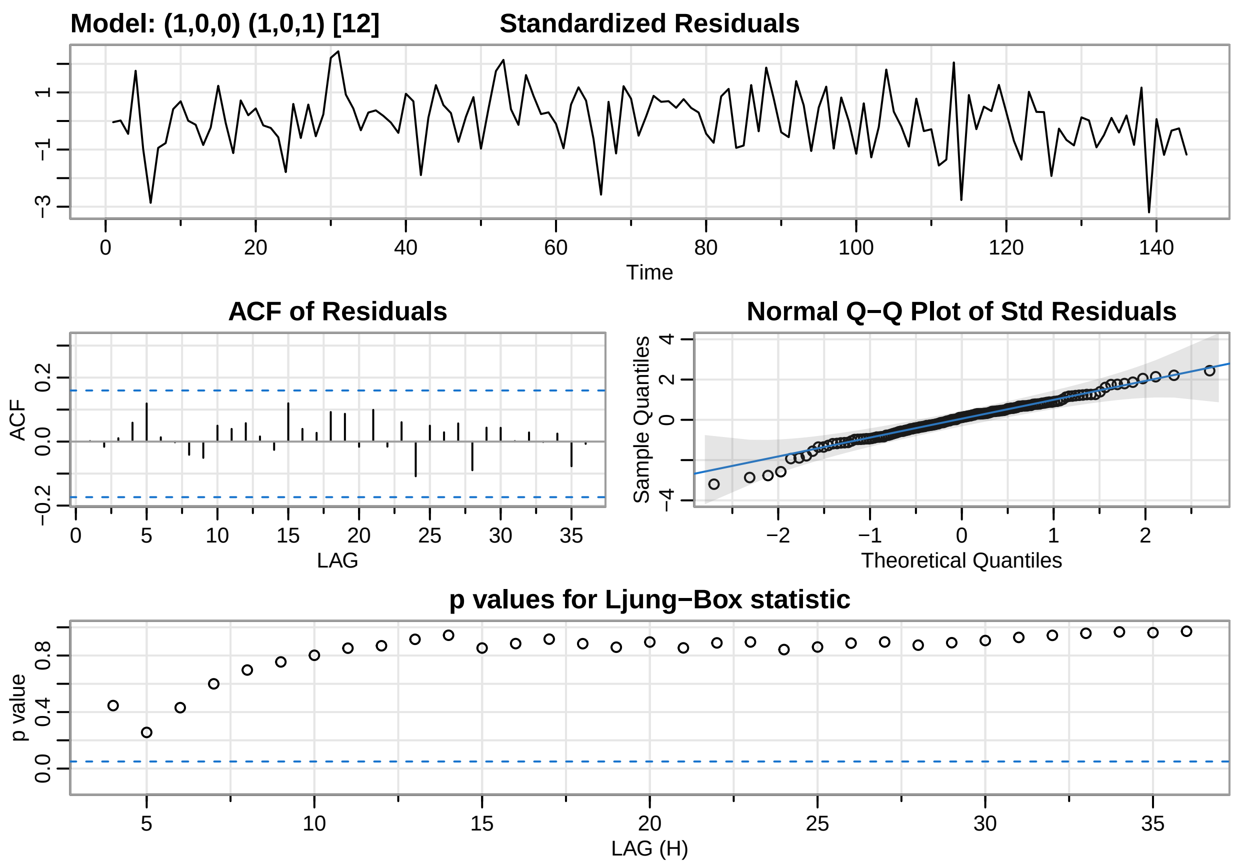
```
##        Estimate      SE t.value p.value   ci_lower   ci_upper
```

```
## ar1     0.0903 0.0837  1.0792  0.2824 -0.073752  0.254352
## ar2    -0.0160 0.0881 -0.1810  0.8566 -0.188676  0.156676
## sar1    0.9462 0.0545 17.3486  0.0000  0.839380  1.053020
## sma1   -0.7176 0.1439 -4.9858  0.0000 -0.999644 -0.435556
## xmean  -0.0002 0.0006 -0.3728  0.7099 -0.001376  0.000976
```

Model2: SARIMA(1,0,0)(1,0,1)12

```
model2 <- sarima(ozone_sinusoid_residual, p=1, d=0, q=0, P=1, D=0, Q=1, S=12) # fit the model
```

```
## initial  value -5.924525
## iter   2 value -6.014280
## iter   3 value -6.072474
## iter   4 value -6.080884
## iter   5 value -6.087853
## iter   6 value -6.094164
## iter   7 value -6.094618
## iter   8 value -6.094677
## iter   9 value -6.094683
## iter  10 value -6.094683
## iter  11 value -6.094684
## iter  12 value -6.094684
## iter  12 value -6.094684
## iter  12 value -6.094684
## final  value -6.094684
## converged
## initial  value -6.075303
## iter   2 value -6.078312
## iter   3 value -6.083138
## iter   4 value -6.089196
## iter   5 value -6.092910
## iter   6 value -6.095028
## iter   7 value -6.096554
## iter   8 value -6.098751
## iter   9 value -6.100853
## iter  10 value -6.101030
## iter  11 value -6.101038
## iter  12 value -6.101038
## iter  13 value -6.101041
## iter  14 value -6.101048
## iter  15 value -6.101050
## iter  16 value -6.101052
## iter  17 value -6.101053
## iter  18 value -6.101054
## iter  19 value -6.101055
## iter  20 value -6.101055
## iter  20 value -6.101055
## iter  20 value -6.101055
## final  value -6.101055
## converged
```




```r
coeff_table <- as.data.frame(model2$ttable)
coeff_table <-coeff_table %>% mutate(ci_lower = Estimate-1.96*SE,ci_upper =  Estimate+1.96*SE)
coeff_table # show estimated coefficient and its ci
```

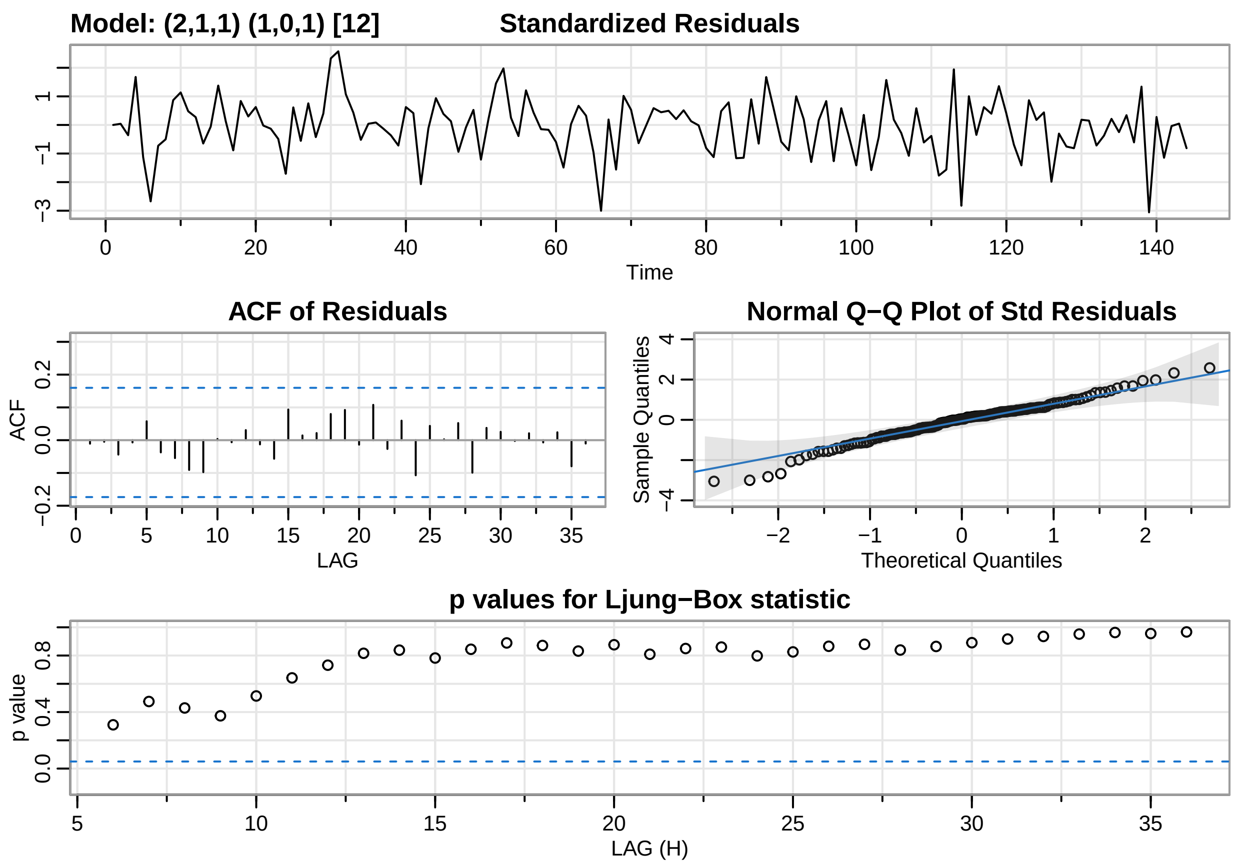
```
##       Estimate     SE t.value p.value  ci_lower  ci_upper
## ar1     0.0888 0.0832  1.0667  0.2879 -0.074272  0.251872
## sar1    0.9489 0.0498 19.0430  0.0000  0.851292  1.046508
## sma1   -0.7226 0.1379 -5.2393  0.0000 -0.992884 -0.452316
## xmean  -0.0002 0.0006 -0.3640  0.7164 -0.001376  0.000976
```

Model3: SARIMA(2,1,1)(1,0,1)12

```r
model3 <- sarima(ozone_sinusoid_residual, p=2, d=1, q=1, P=1, D=0, Q=1, S=12) # fit the model
```

```
## initial  value -5.626415
## iter   2 value -5.817904
## iter   3 value -5.967005
## iter   4 value -6.000235
## iter   5 value -6.022716
## iter   6 value -6.038025
## iter   7 value -6.061036
## iter   8 value -6.062258
```

```
## iter   9 value -6.063975
## iter  10 value -6.065519
## iter  11 value -6.066125
## iter  12 value -6.066443
## iter  13 value -6.067328
## iter  14 value -6.067686
## iter  15 value -6.067780
## iter  16 value -6.067831
## iter  17 value -6.067832
## iter  18 value -6.067883
## iter  19 value -6.067920
## iter  20 value -6.067946
## iter  21 value -6.067967
## iter  22 value -6.068029
## iter  23 value -6.068059
## iter  24 value -6.068062
## iter  25 value -6.068063
## iter  25 value -6.068063
## iter  25 value -6.068063
## final  value -6.068063
## converged
## initial  value -6.049027
## iter   2 value -6.063938
## iter   3 value -6.069719
## iter   4 value -6.077660
## iter   5 value -6.081289
## iter   6 value -6.083983
## iter   7 value -6.086264
## iter   8 value -6.089727
## iter   9 value -6.093924
## iter  10 value -6.097960
## iter  11 value -6.098795
## iter  12 value -6.099032
## iter  13 value -6.099192
## iter  14 value -6.099209
## iter  15 value -6.099215
## iter  16 value -6.099216
## iter  17 value -6.099217
## iter  18 value -6.099217
## iter  19 value -6.099217
## iter  20 value -6.099218
## iter  21 value -6.099219
## iter  22 value -6.099220
## iter  23 value -6.099220
## iter  23 value -6.099220
## iter  23 value -6.099220
## final  value -6.099220
## converged
```




```
coeff_table <- as.data.frame(model3$ttable)
coeff_table <-coeff_table %>% mutate(ci_lower = Estimate-1.96*SE,ci_upper =  Estimate+1.96*SE)
coeff_table # show estimated coefficient and its ci
```

```
##          Estimate     SE  t.value p.value  ci_lower  ci_upper
## ar1        0.0192 0.0892   0.2150  0.8301 -0.155632  0.194032
## ar2       -0.0791 0.0894  -0.8846  0.3779 -0.254324  0.096124
## ma1       -0.9352 0.0317 -29.5144  0.0000 -0.997332 -0.873068
## sar1       0.9602 0.0366  26.2225  0.0000  0.888464  1.031936
## sma1      -0.7544 0.1106  -6.8202  0.0000 -0.971176 -0.537624
## constant   0.0000 0.0001  -0.0145  0.9884 -0.000196  0.000196
```

Model4: SARIMA(2,0,1)(1,0,1)12

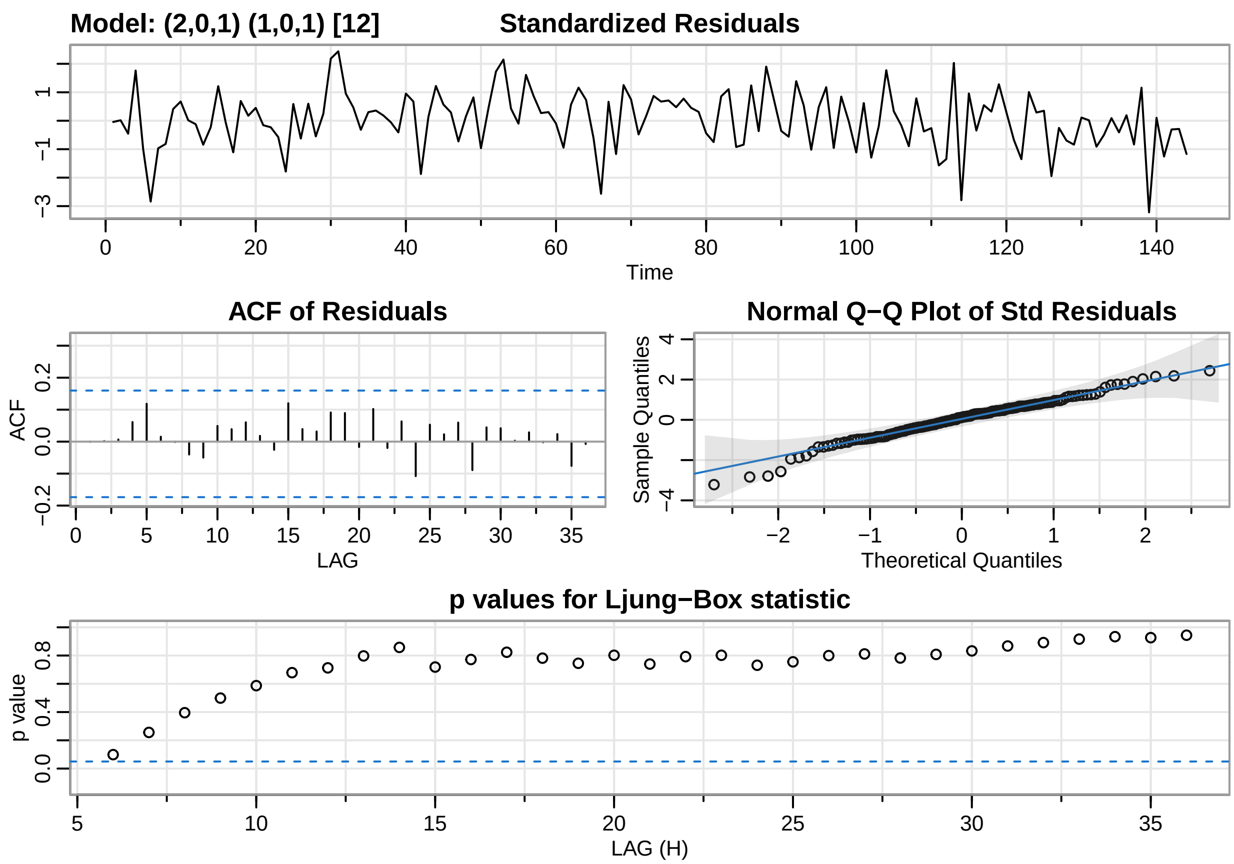
```
model4 <- sarima(ozone_sinusoid_residual, p=2, d=0, q=1, P=1, D=0, Q=1, S=12) # fit the model
```

```
## initial  value -5.921137
## iter   2 value -6.021739
## iter   3 value -6.079075
## iter   4 value -6.085438
## iter   5 value -6.091319
## iter   6 value -6.095255
```

```
## iter   7 value -6.095886
## iter   8 value -6.095982
## iter   9 value -6.095984
## iter  10 value -6.095987
## iter  11 value -6.095993
## iter  12 value -6.096006
## iter  13 value -6.096024
## iter  14 value -6.096036
## iter  15 value -6.096039
## iter  16 value -6.096040
## iter  17 value -6.096040
## iter  18 value -6.096040
## iter  19 value -6.096040
## iter  19 value -6.096040
## iter  19 value -6.096040
## final  value -6.096040
## converged
## initial  value -6.075372
## iter   2 value -6.077112
## iter   3 value -6.080624
## iter   4 value -6.083981
## iter   5 value -6.086061
## iter   6 value -6.091109
## iter   7 value -6.094569
## iter   8 value -6.096817
## iter   9 value -6.097887
## iter  10 value -6.099778
## iter  11 value -6.100684
## iter  12 value -6.101115
## iter  13 value -6.101118
## iter  14 value -6.101118
## iter  15 value -6.101118
## iter  16 value -6.101124
## iter  17 value -6.101136
## iter  18 value -6.101149
## iter  19 value -6.101171
## iter  20 value -6.101187
## iter  21 value -6.101190
## iter  22 value -6.101192
## iter  23 value -6.101197
## iter  24 value -6.101201
## iter  25 value -6.101209
## iter  26 value -6.101219
## iter  27 value -6.101230
## iter  28 value -6.101233
## iter  29 value -6.101237
## iter  29 value -6.101237
## iter  29 value -6.101237
## final  value -6.101237
## converged
```

```
## Warning in sqrt(diag(fitit$var.coef)): NaNs produced
```

```
## Warning in sqrt(diag(fitit$var.coef)): NaNs produced
```




```
coeff_table <- as.data.frame(model4$ttable)
coeff_table <-coeff_table %>% mutate(ci_lower = Estimate-1.96*SE,ci_upper =  Estimate+1.96*SE)
coeff_table # show estimated coefficient and its ci
```

```
##        Estimate     SE t.value p.value  ci_lower  ci_upper
## ar1     -0.3458    NaN     NaN     NaN       NaN       NaN
## ar2      0.0214    NaN     NaN     NaN       NaN       NaN
## ma1      0.4374    NaN     NaN     NaN       NaN       NaN
## sar1     0.9498 0.0488 19.4654  0.0000  0.854152  1.045448
## sma1    -0.7278 0.1344 -5.4138  0.0000 -0.991224 -0.464376
## xmean   -0.0002 0.0006 -0.3604  0.7191 -0.001376  0.000976
```

Evaluation Matrix

1. The first two models, SARIMA(2,0,0)(1,0,1)12 , SARIMA(1,0,0)(1,0,1)12 gives the lowest AIC,AICc,BIC, hence theses models are the potential best two parametric models

```
# AIC, AICc, BIC
eval<- function(model){
  return (c(model$AIC, model$AICc,model$BIC))
}

m1_evaludation = eval(model1)
```

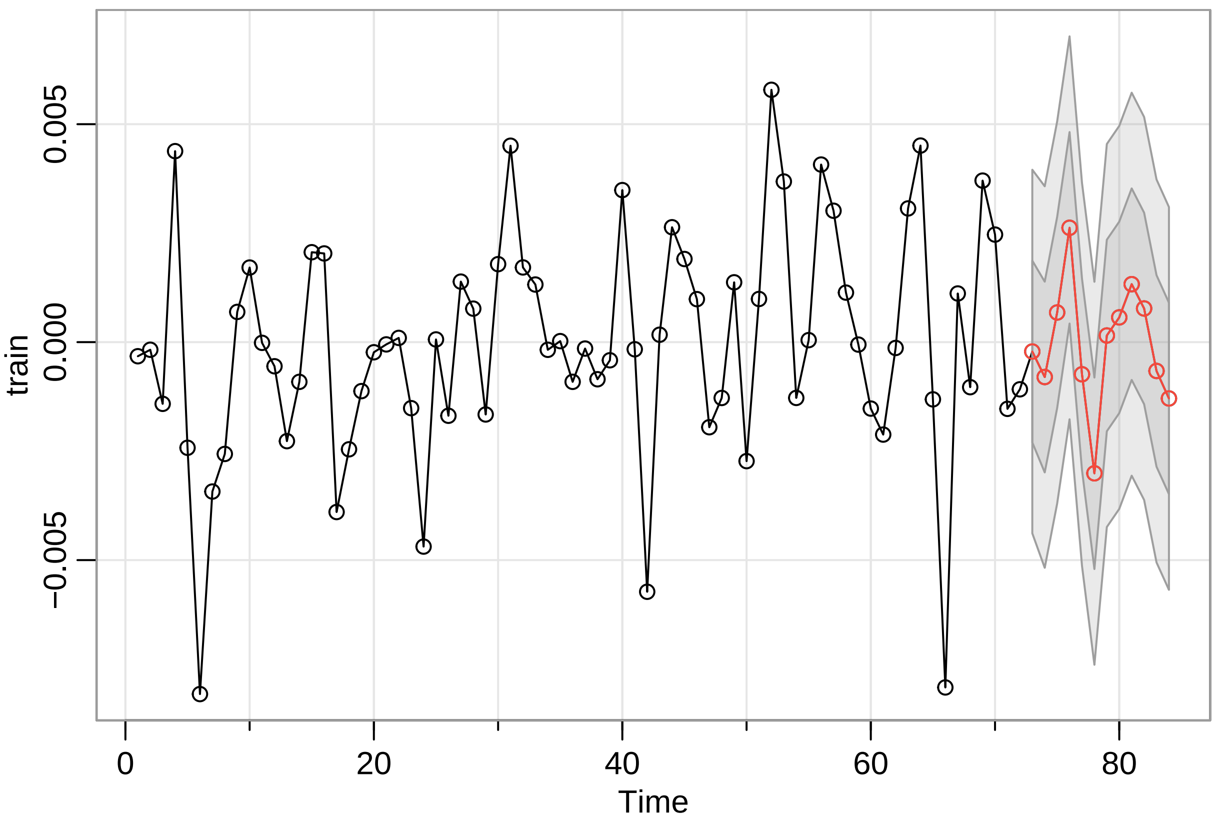
```r
m2_evaludation = eval(model2)
m3_evaludation = eval(model3)
m4_evaludation = eval(model4)

eval_matrix = rbind(m1_evaludation,m2_evaludation,m3_evaludation,m4_evaludation)
rownames(eval_matrix) = c("SARIMA(2,0,0)(1,0,1)12","SARIMA(1,0,0)(1,0,1)12",
                          "SARIMA(2,1,1)(1,0,1)12 ","SARIMA(2,0,1)(1,0,1)12 ")
colnames(eval_matrix) = c("AIC","AICc","BIC")
eval_matrix
```

```
##                               AIC      AICc       BIC
## SARIMA(2,0,0)(1,0,1)12  -9.281128 -9.278109 -9.157386
## SARIMA(1,0,0)(1,0,1)12  -9.294789 -9.292790 -9.191670
## SARIMA(2,1,1)(1,0,1)12  -9.262661 -9.258342 -9.117626
## SARIMA(2,0,1)(1,0,1)12  -9.267376 -9.263118 -9.123010
```

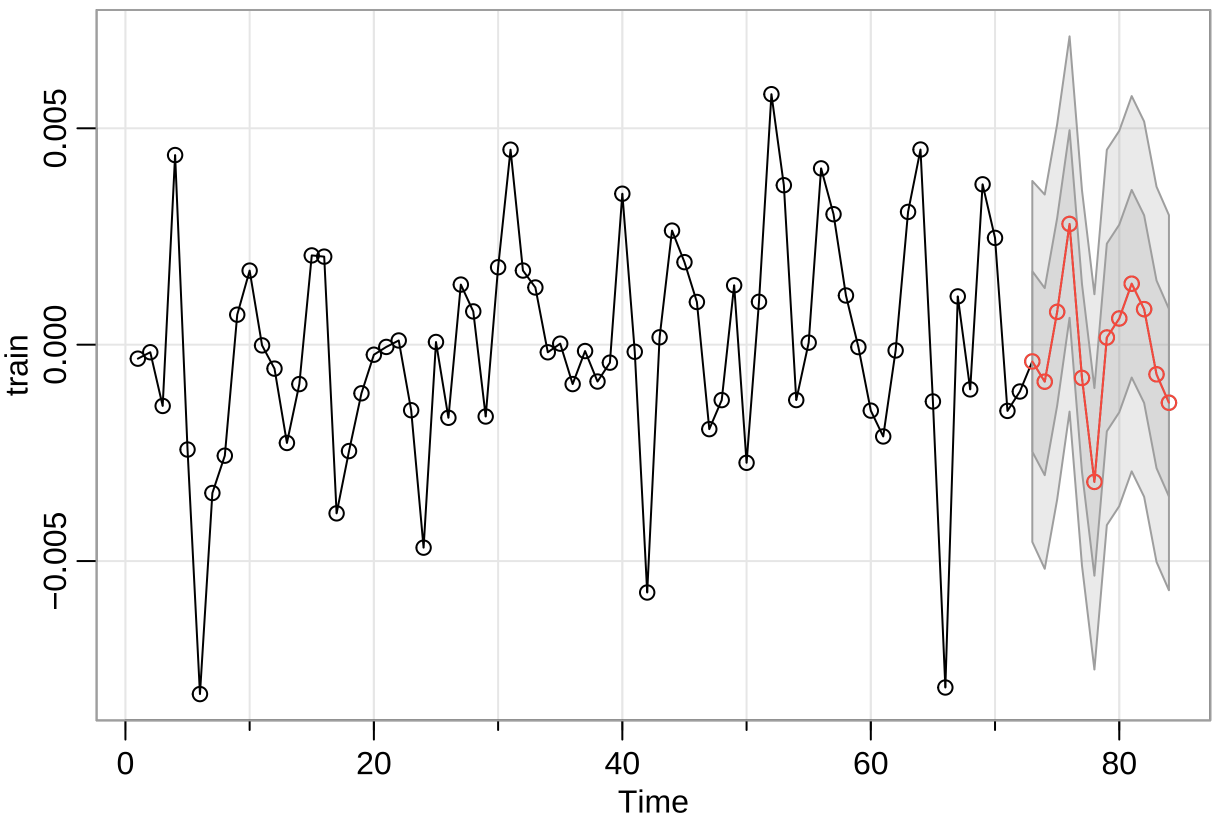
Cross Validation

1. To determine best two models, use crovalidations and find two models that gives the lowest SSEs. Train : 2004 ~ 2011 Test : 2012 - 2015
2. The first two models SARIMA(2,0,0)(1,0,1)12 , SARIMA(1,0,0)(1,0,1)12 gives the lowest SSEs.
3. Overall, the first two models gives the lowest values on both (AIC,AICc,BIC) and the SSE. Hence, the best two parametric models are SARIMA(2,0,0)(1,0,1)12, SARIMA(1,0,0)(1,0,1)12

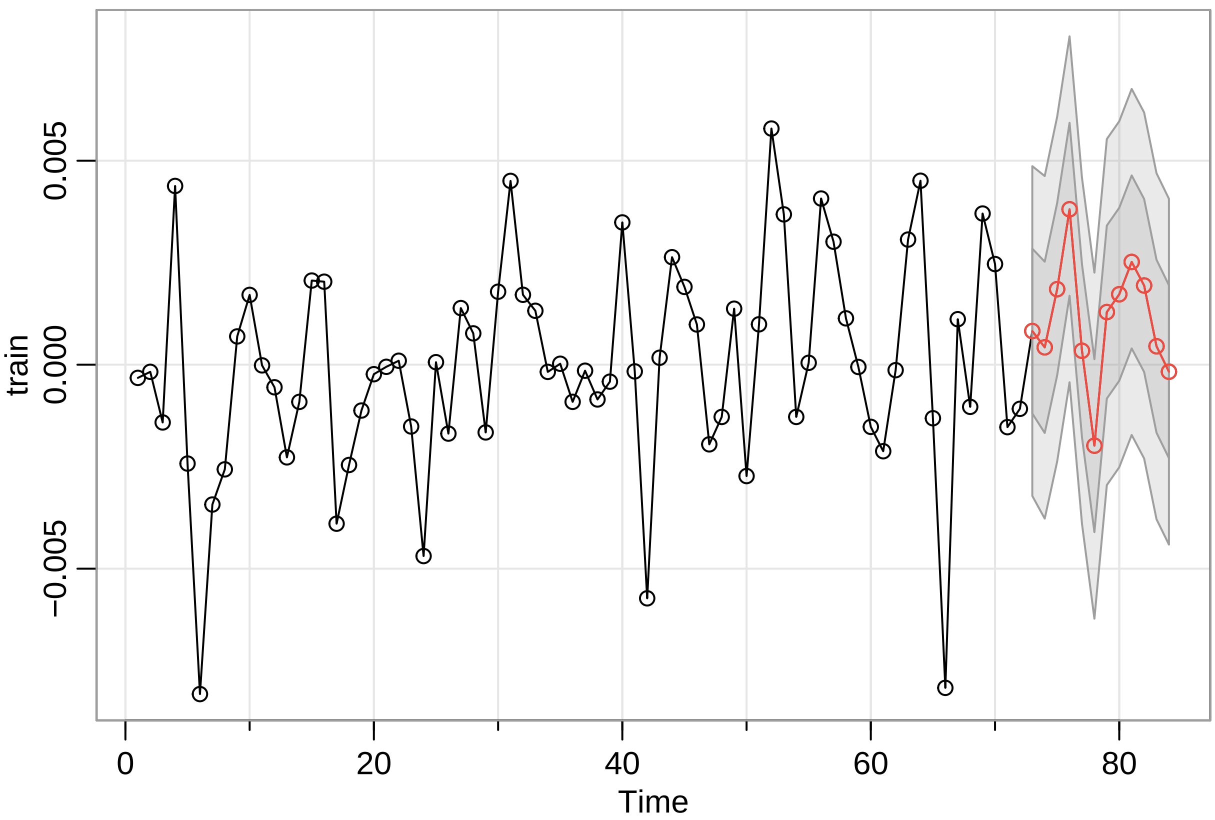
```r
sse1 = c()
sse2 = c()
sse3 = c()
sse4 = c()
test_years = seq(10,15,1)
for (year in test_years) {

train_index = 1:(12*(year-4))
test_index = (12*(year-4)+1):(12*(year-4+1))

train <- ozone_sinusoid_residual[train_index]
test <- ozone_sinusoid_residual[test_index]
m1_forecast <- sarima.for(train, p=2, d=0, q=0, P=1, D=0, Q=1, S=12, n.ahead=12)$pred
m2_forecast <- sarima.for(train, p=1, d=0, q=0, P=1, D=0, Q=1, S=12, n.ahead=12)$pred
m3_forecast <- sarima.for(train, p=2, d=1, q=1, P=1, D=0, Q=1, S=12, n.ahead=12)$pred
m4_forecast <- sarima.for(train, p=2, d=0, q=1, P=1, D=0, Q=1, S=12, n.ahead=12)$pred

sse1 = c(sse1,sum((m1_forecast - test)^2))
sse2 = c(sse2,sum((m2_forecast - test)^2))
sse3 = c(sse3,sum((m3_forecast - test)^2))
sse4 = c(sse4,sum((m4_forecast - test)^2))
}
```

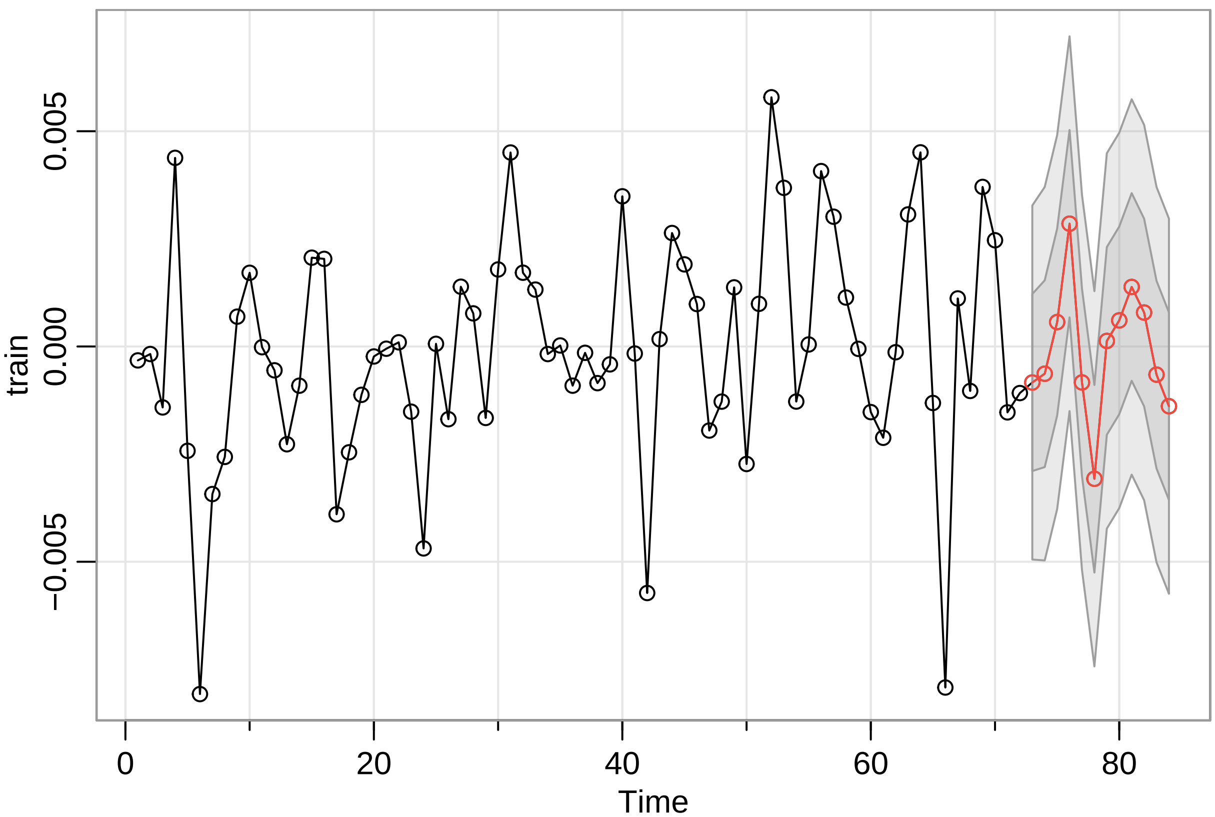


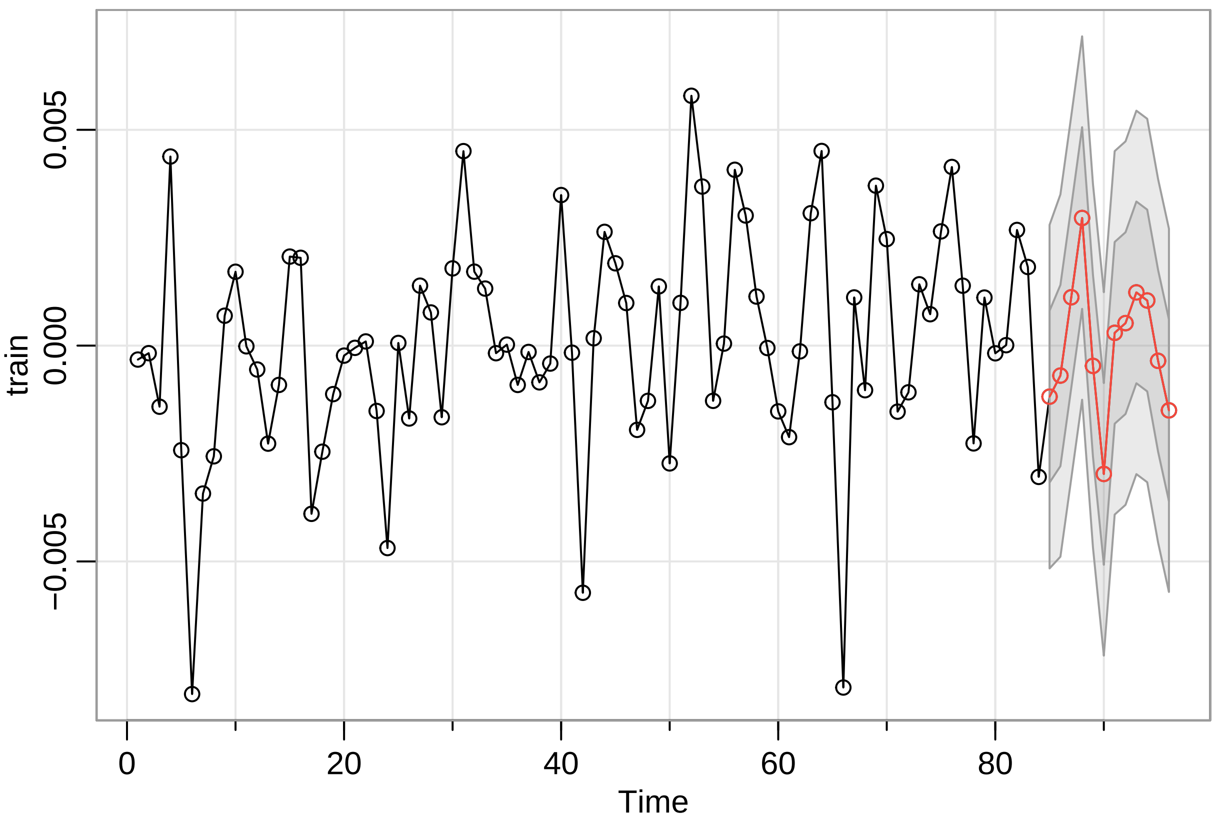
0.005
0.000
−0.005
train
0
20
40
60
80
Time

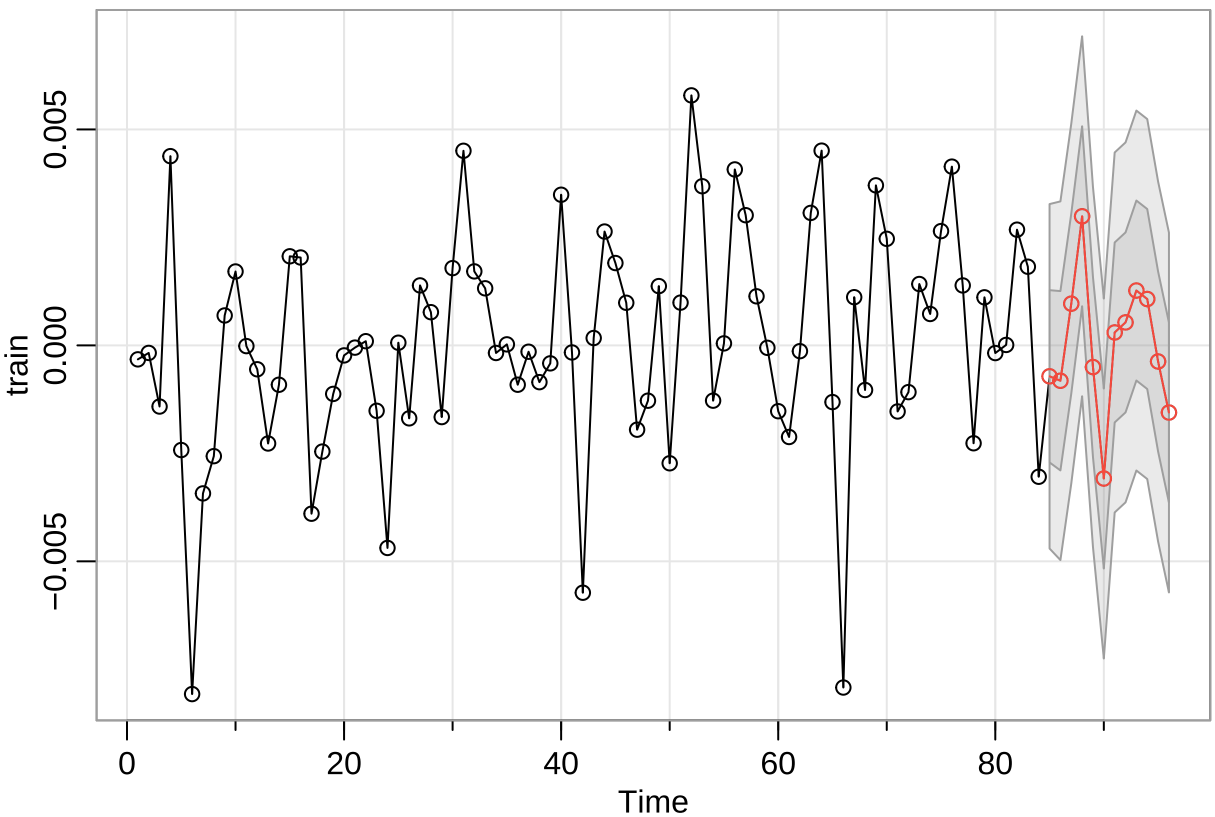


0.005
0.000
−0.005
train
0
20
40
60
80
Time

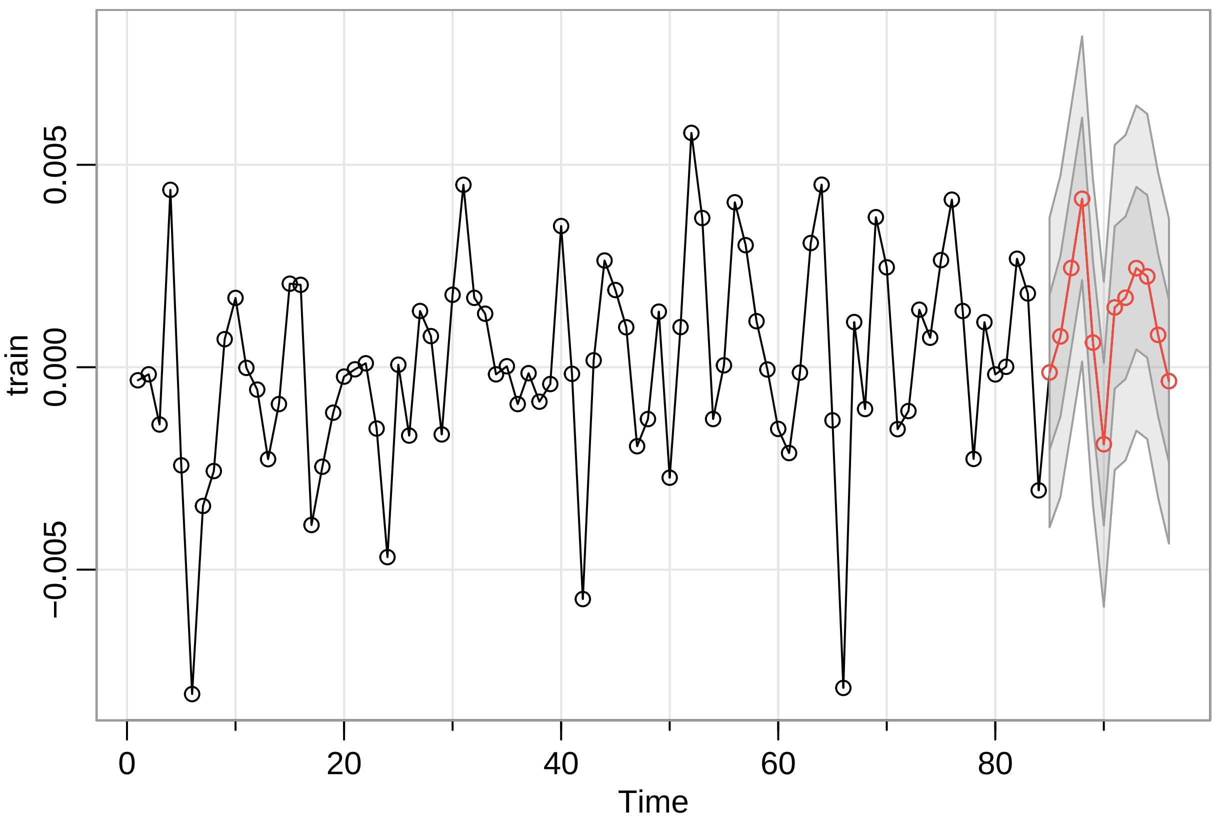

train
0.005
0.000
−0.005
0
20
40
60
80
Time


0.005
0.000
−0.005
train
0
20
40
60
80
Time

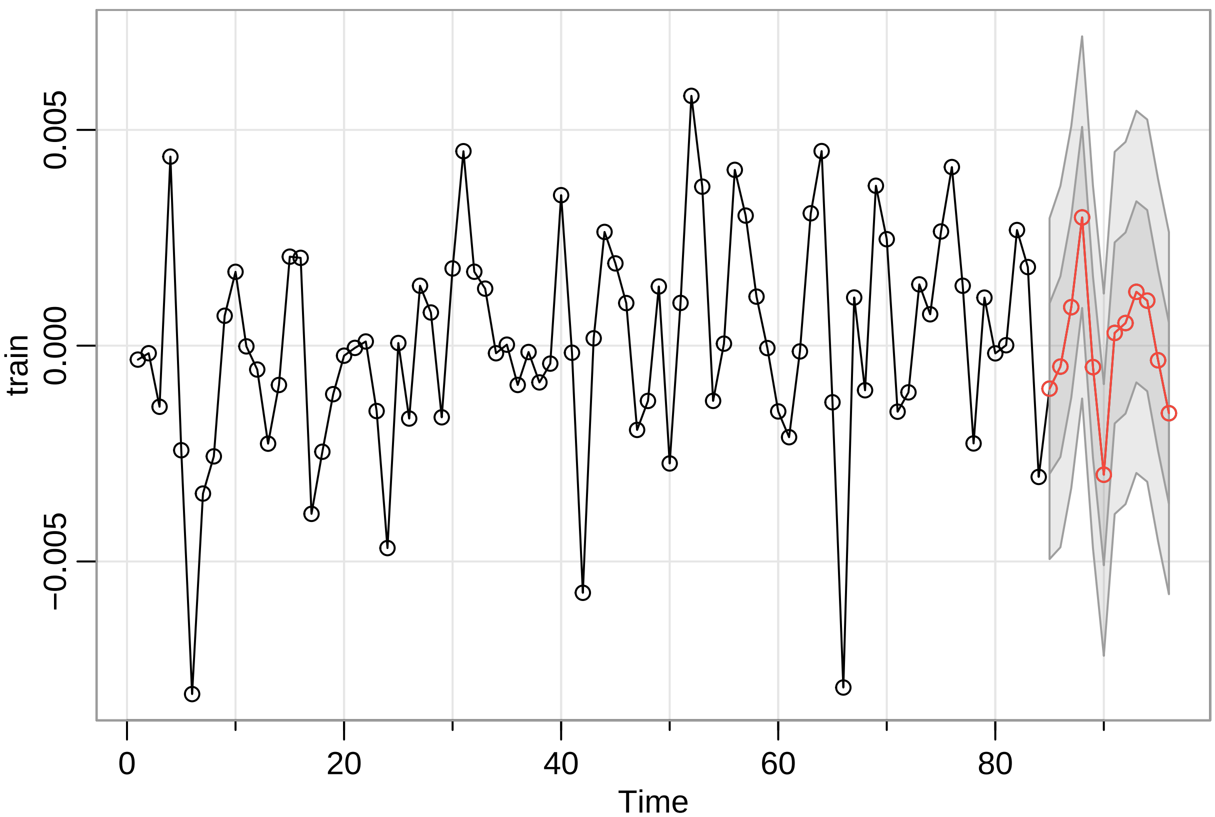

0.005
0.000
−0.005
train
0
20
40
60
80
Time

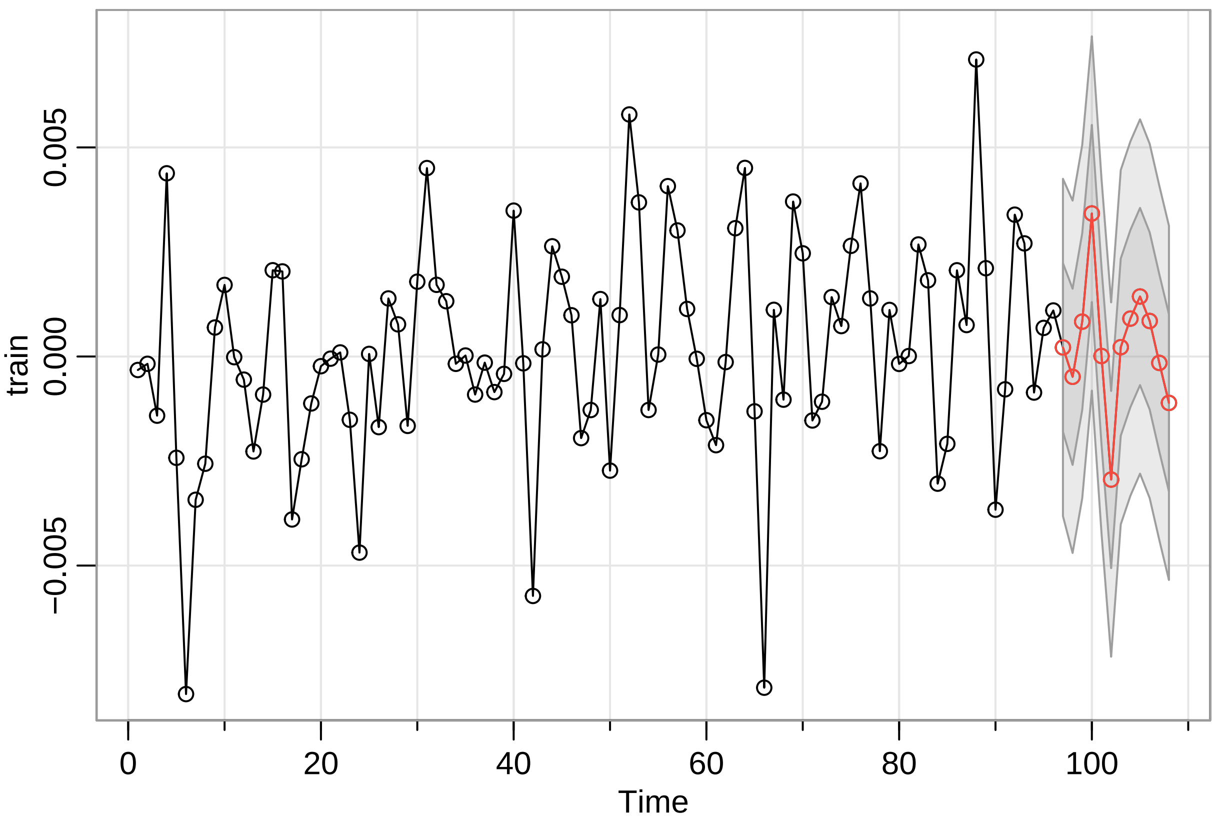


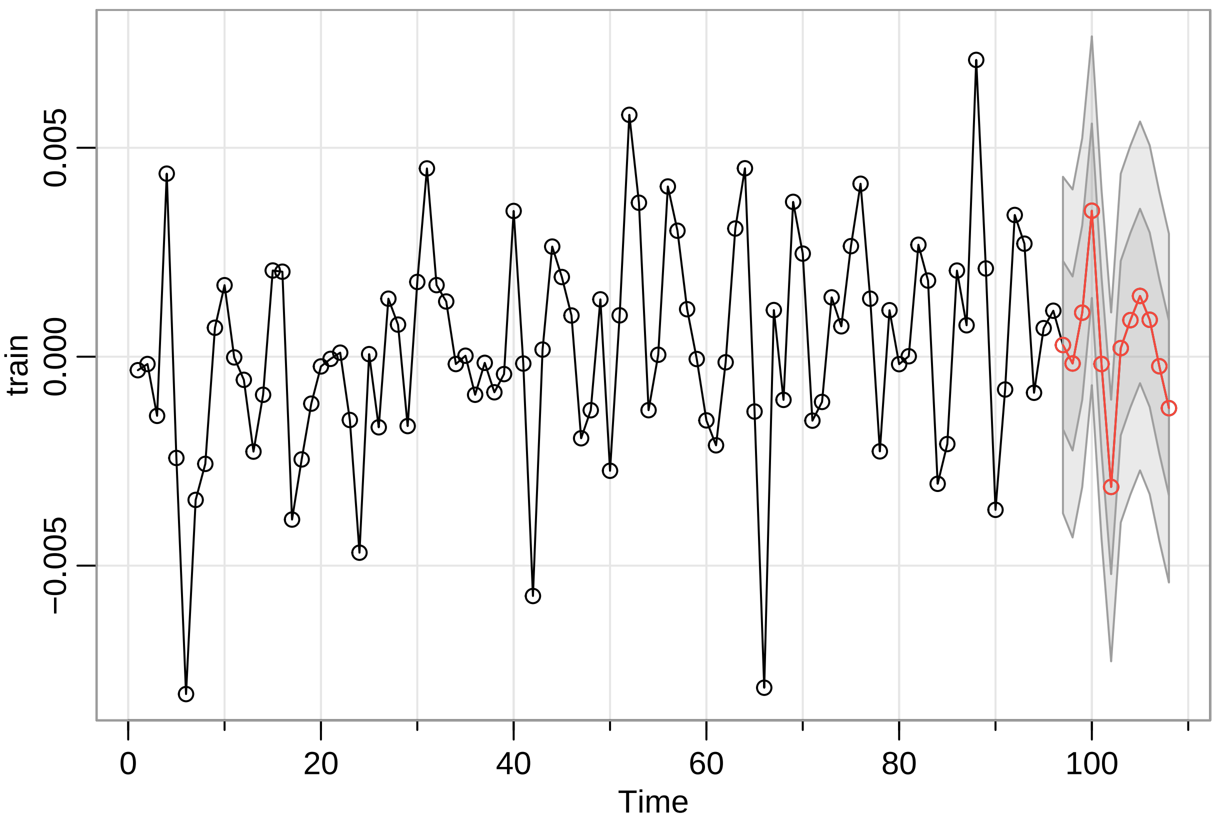
0.005
0.000
−0.005
train
0
20
40
60
80
Time

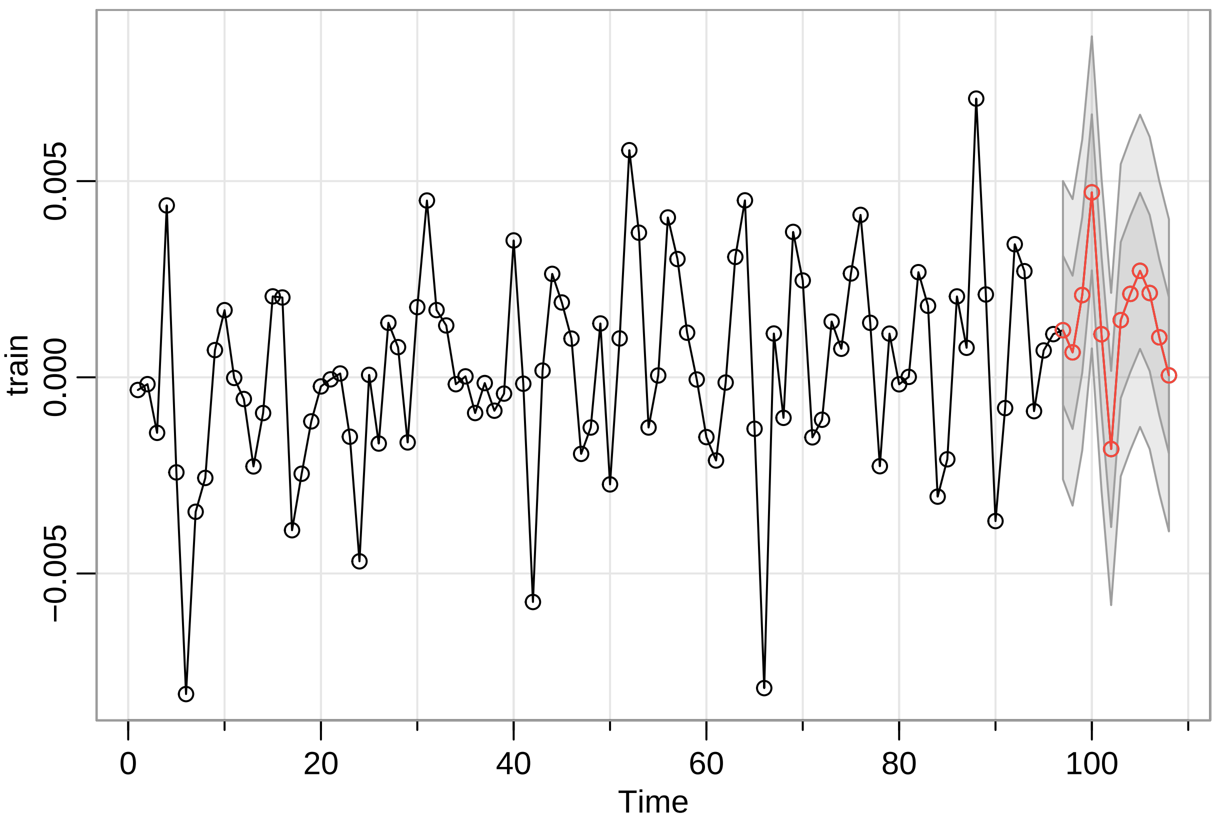


train
0.005
0.000
−0.005
0
20
40
60
80
Time

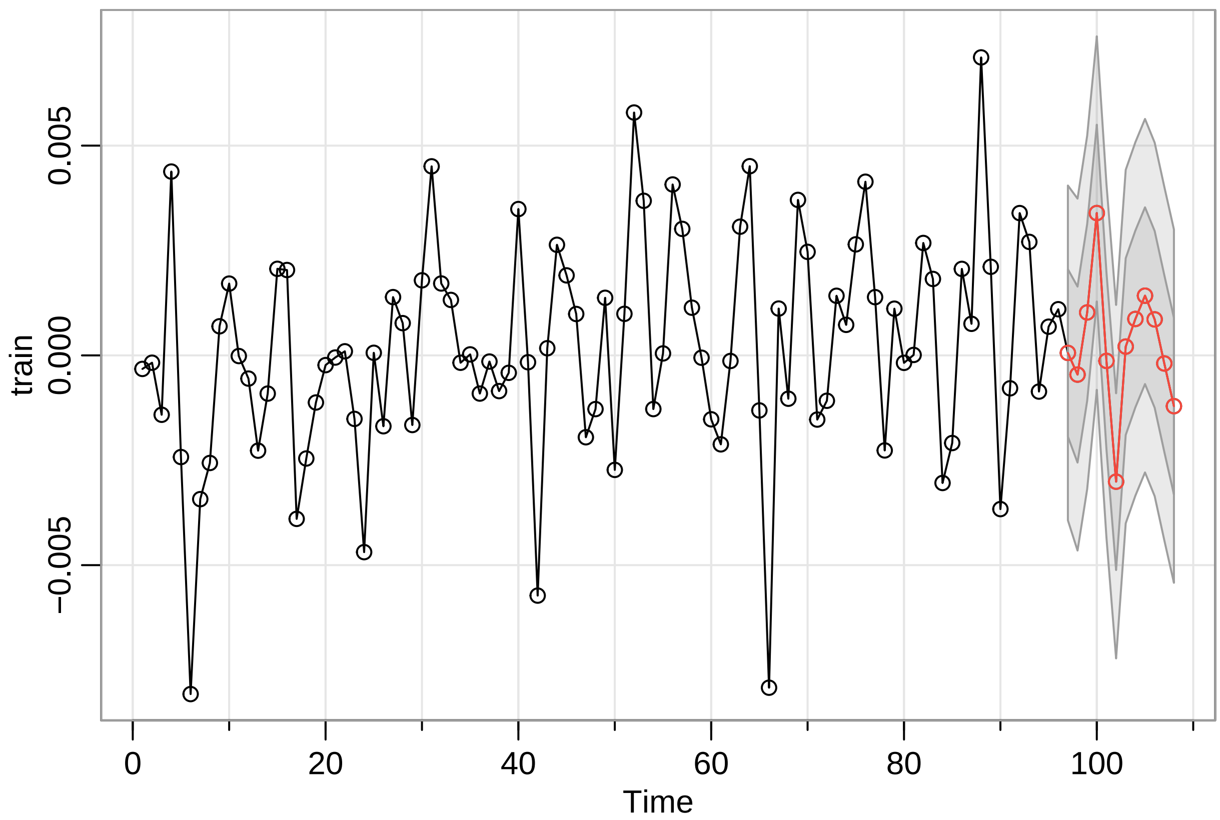

0.005
0.000
−0.005
train
0
20
40
60
80
Time


train
0.005
0.000
−0.005
0
20
40
60
80
100
Time

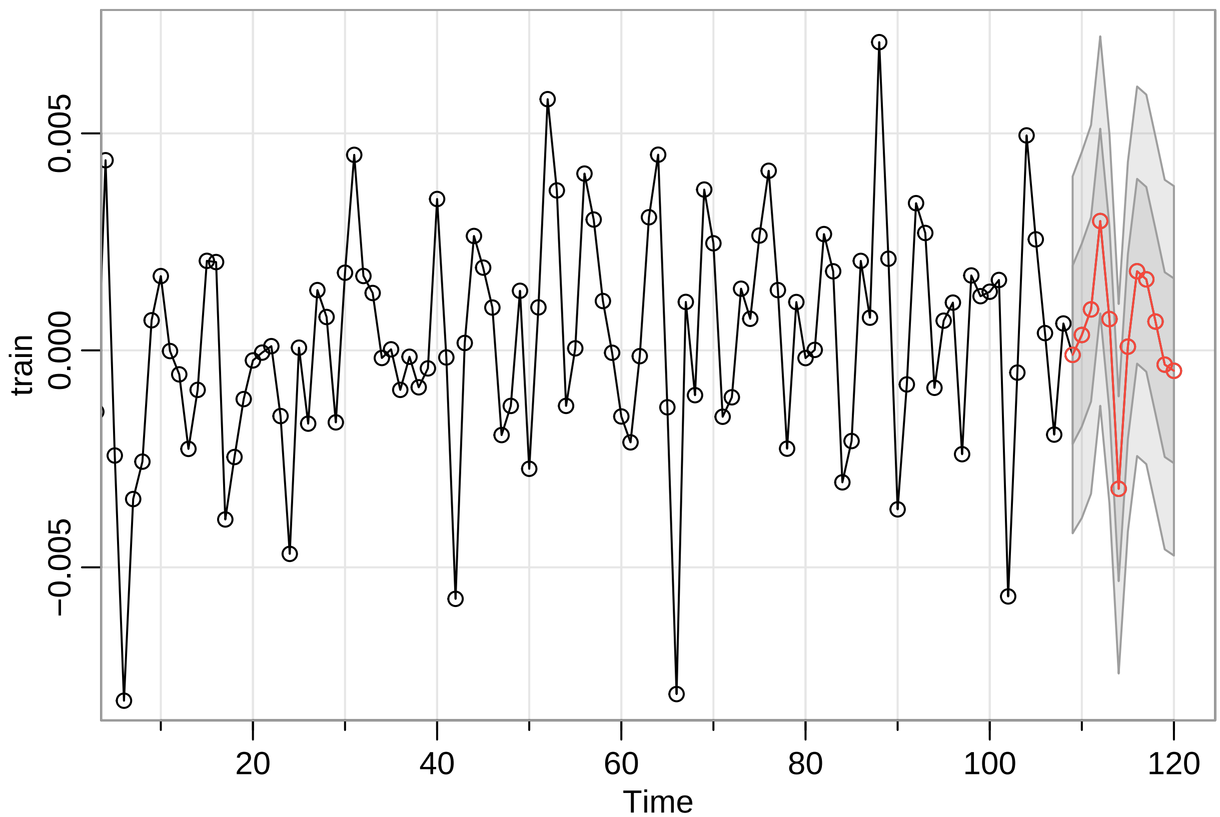

train
0.005
0.000
−0.005
0
20
40
60
80
100
Time



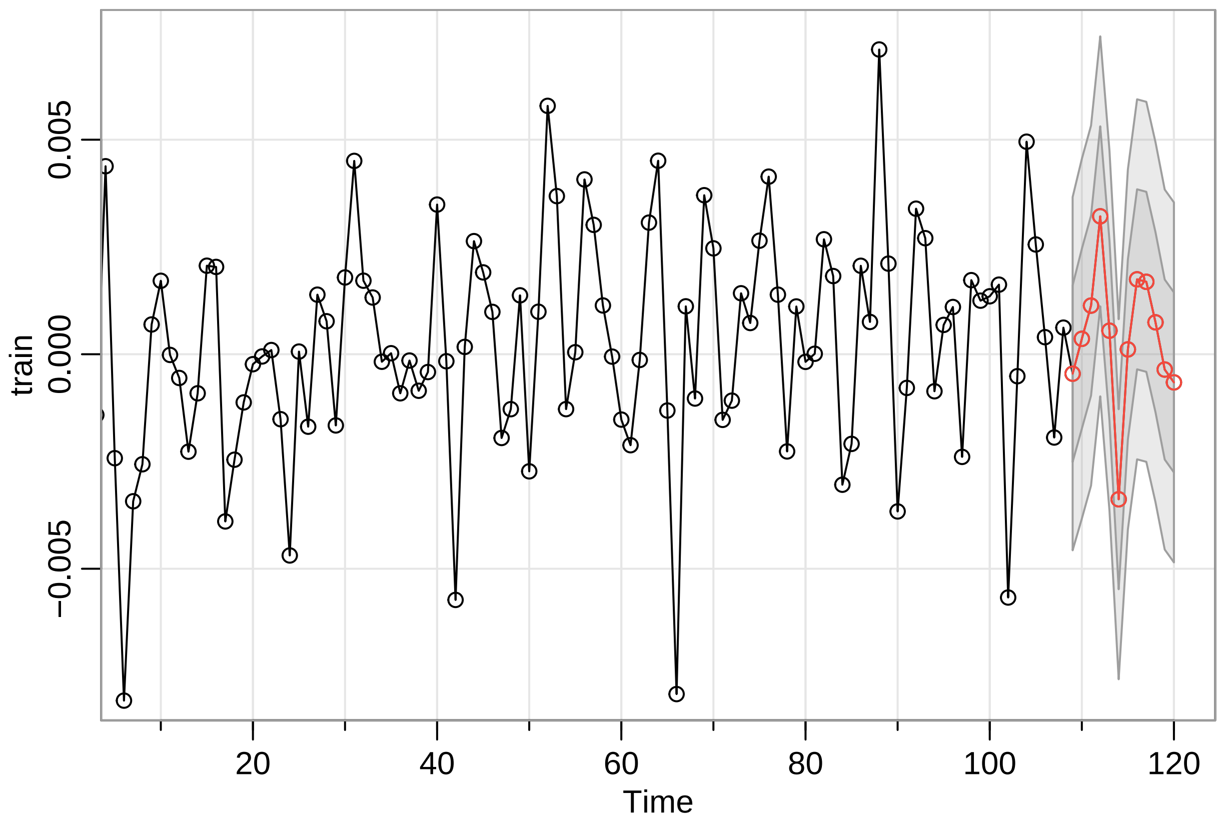
train
0.005
0.000
−0.005
0
20
40
60
80
100
Time



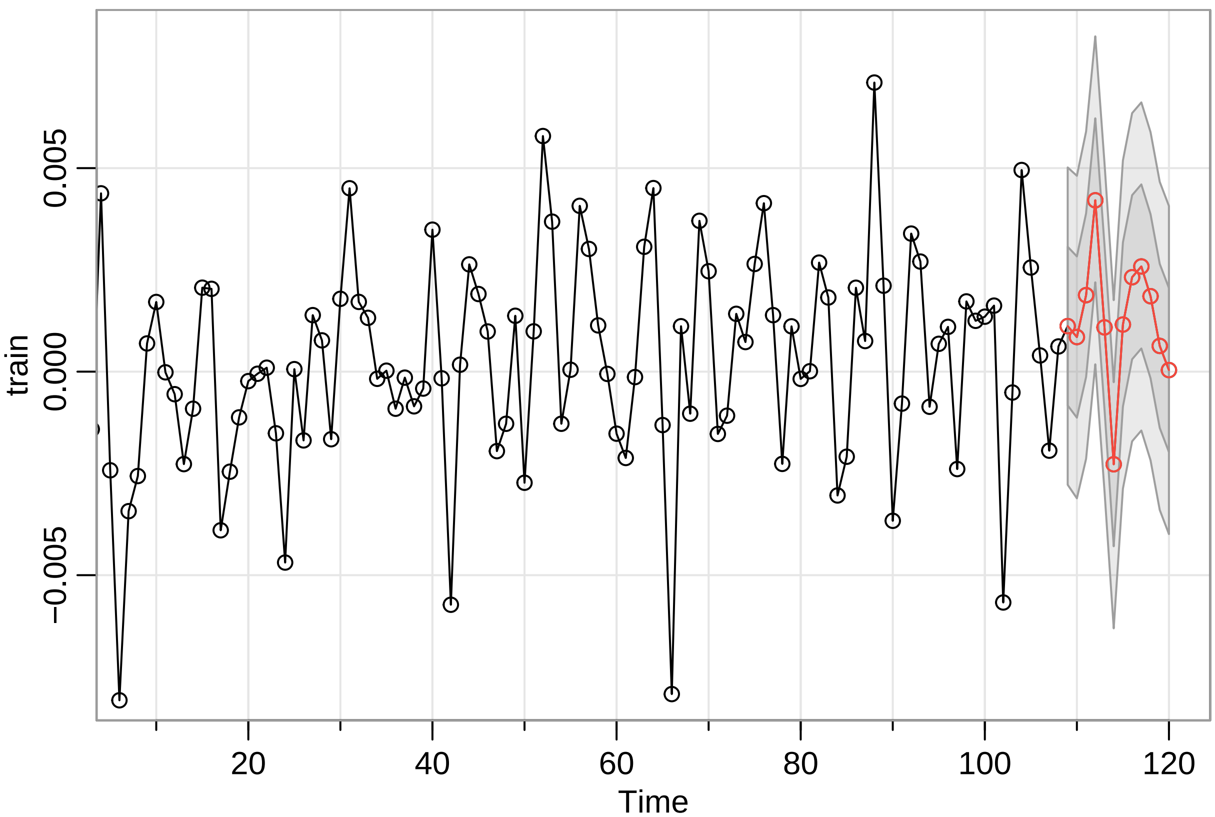
train
0.005
0.000
−0.005
0
20
40
60
80
100
Time



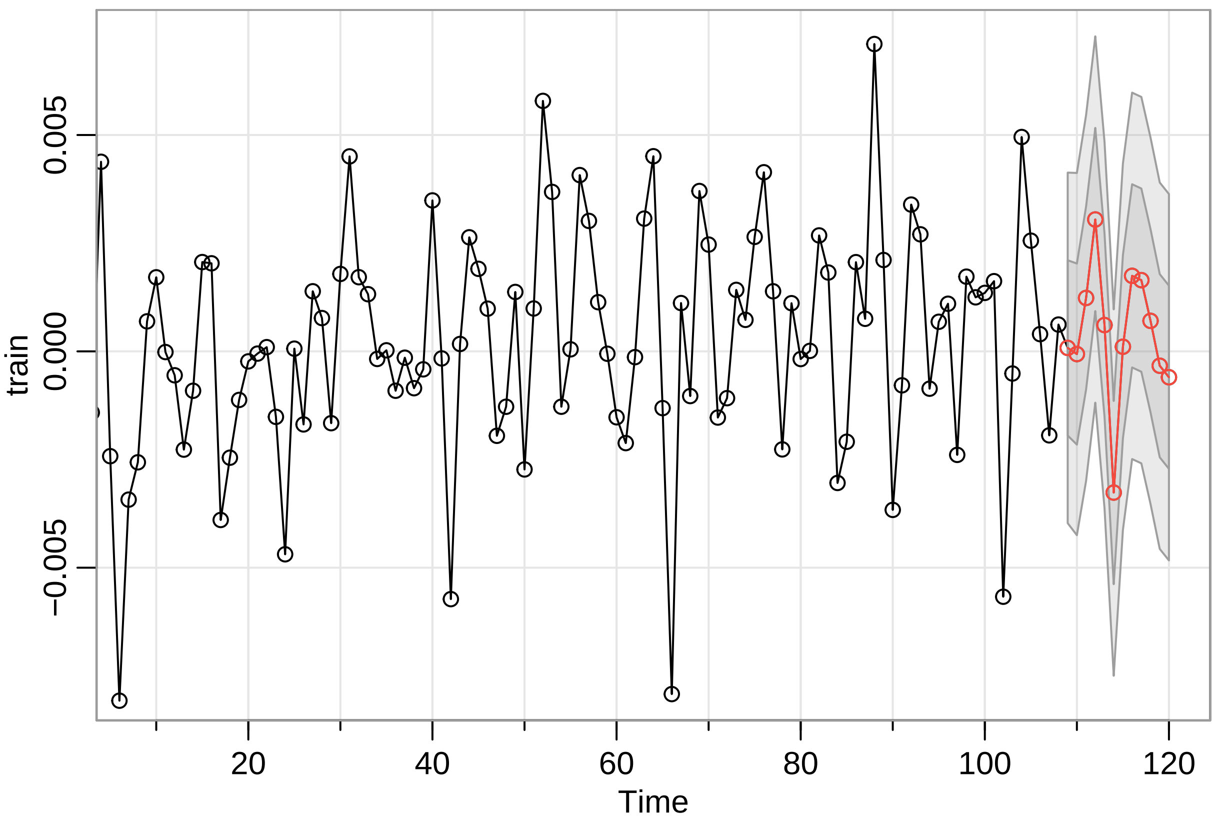
train
0.005
0.000
−0.005
20
40
60
80
100
120
Time



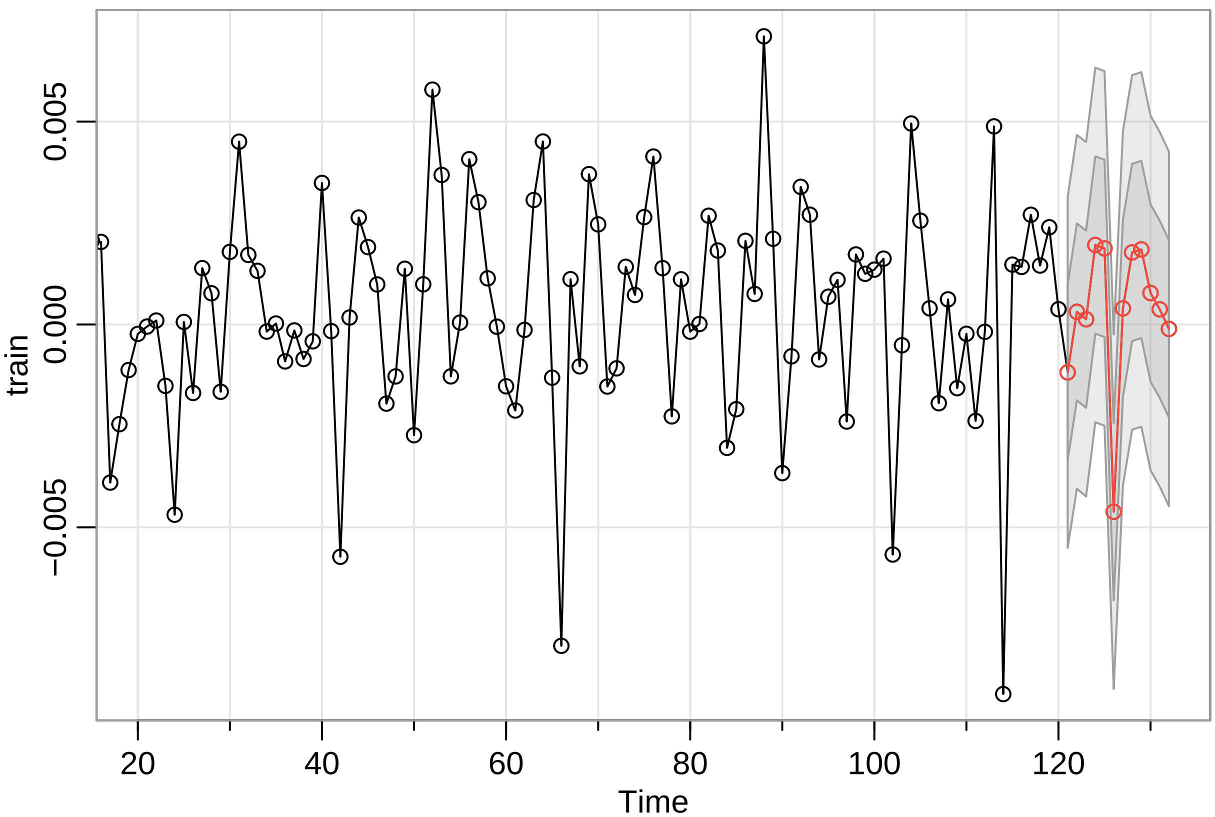
0.005
0.000
−0.005
train
20
40
60
80
100
120
Time

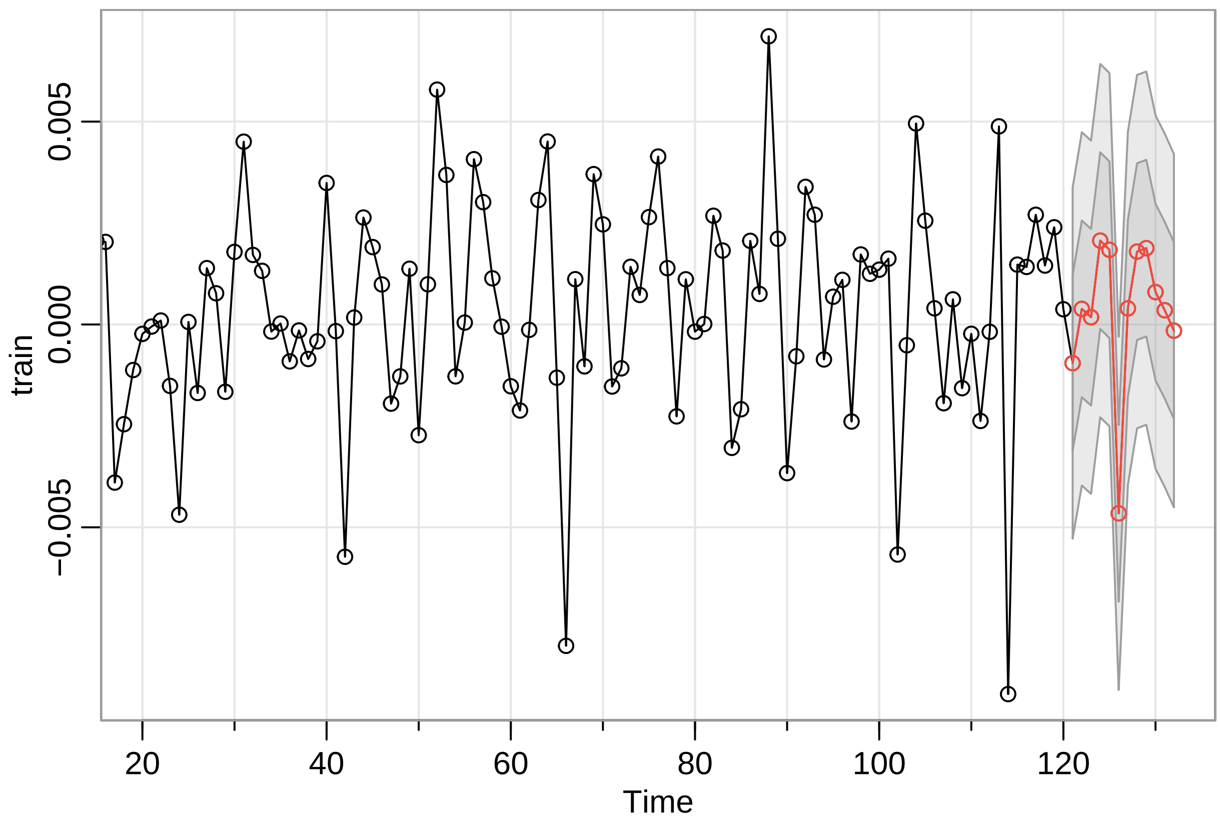


0.005
0.000
−0.005
train
20
40
60
80
100
120
Time

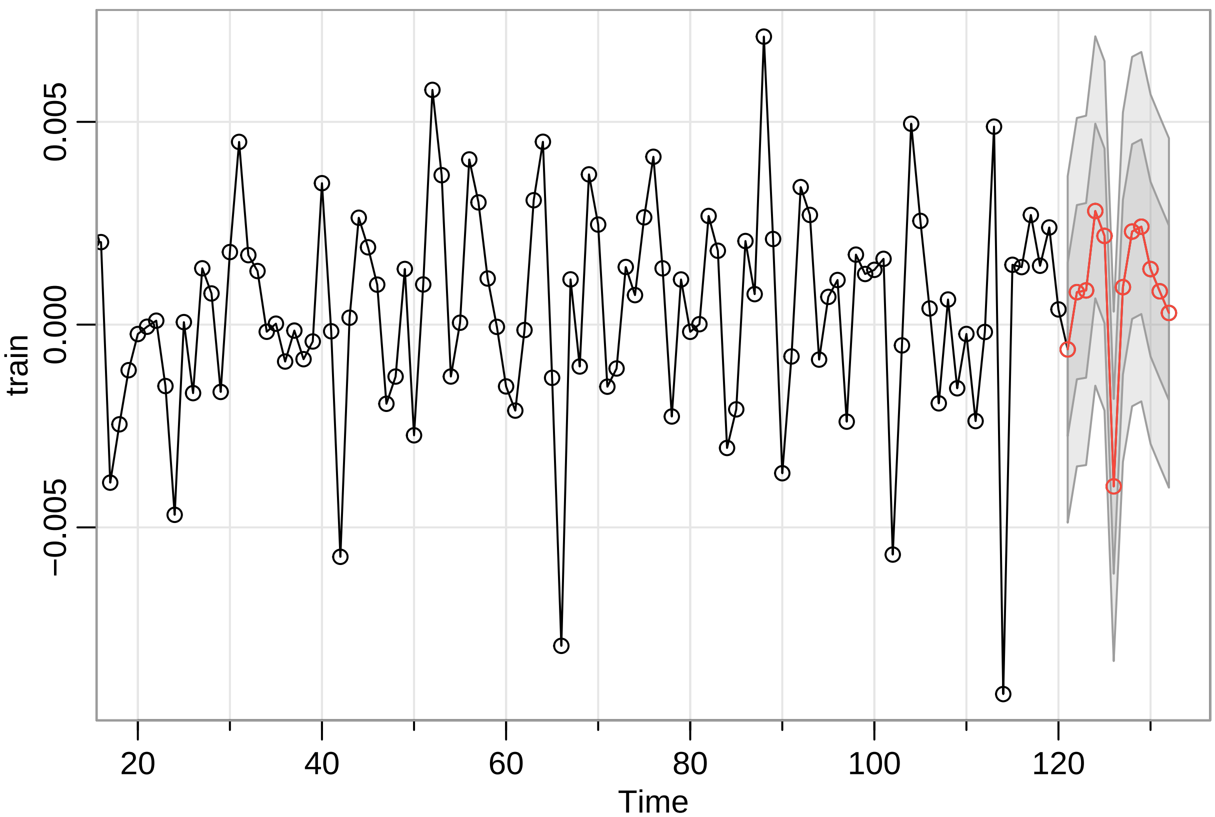


0.005
0.000
−0.005
train
20
40
60
80
100
120
Time

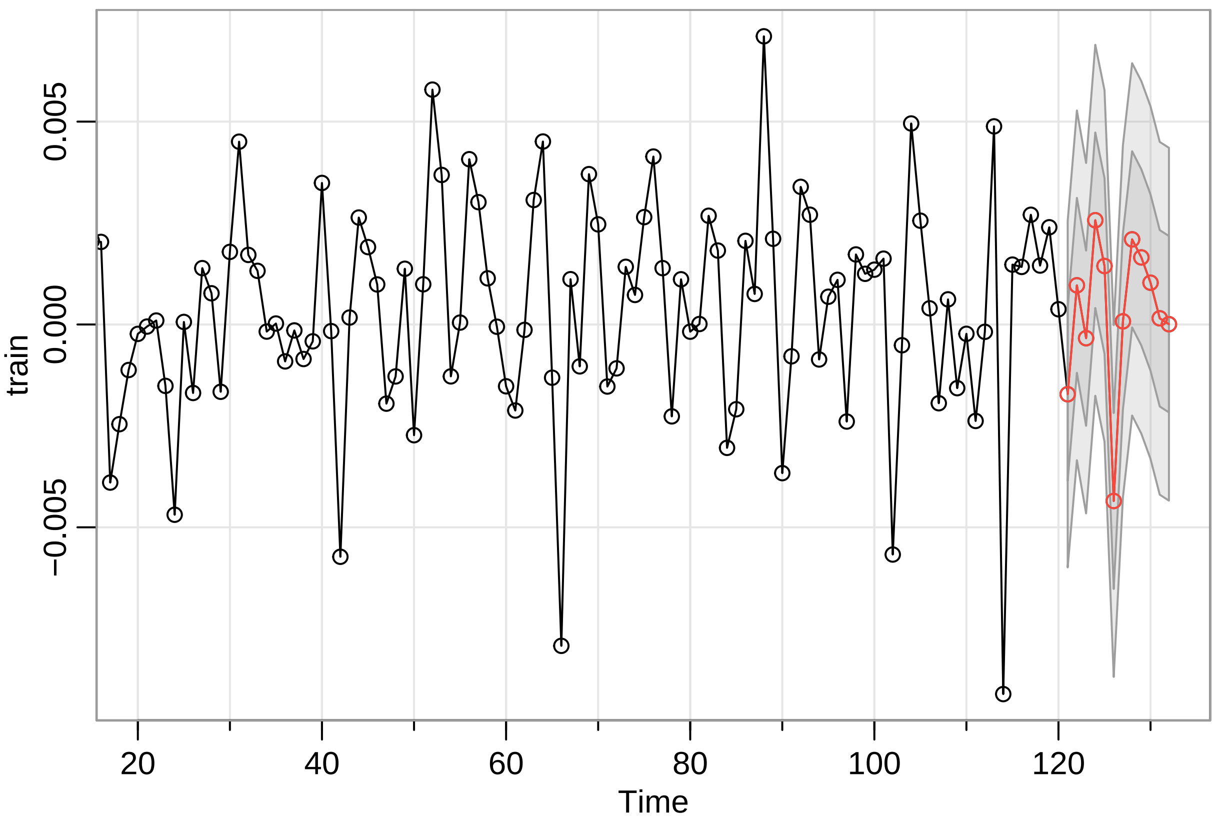

0.005
0.000
−0.005
train
20
40
60
80
100
120
Time



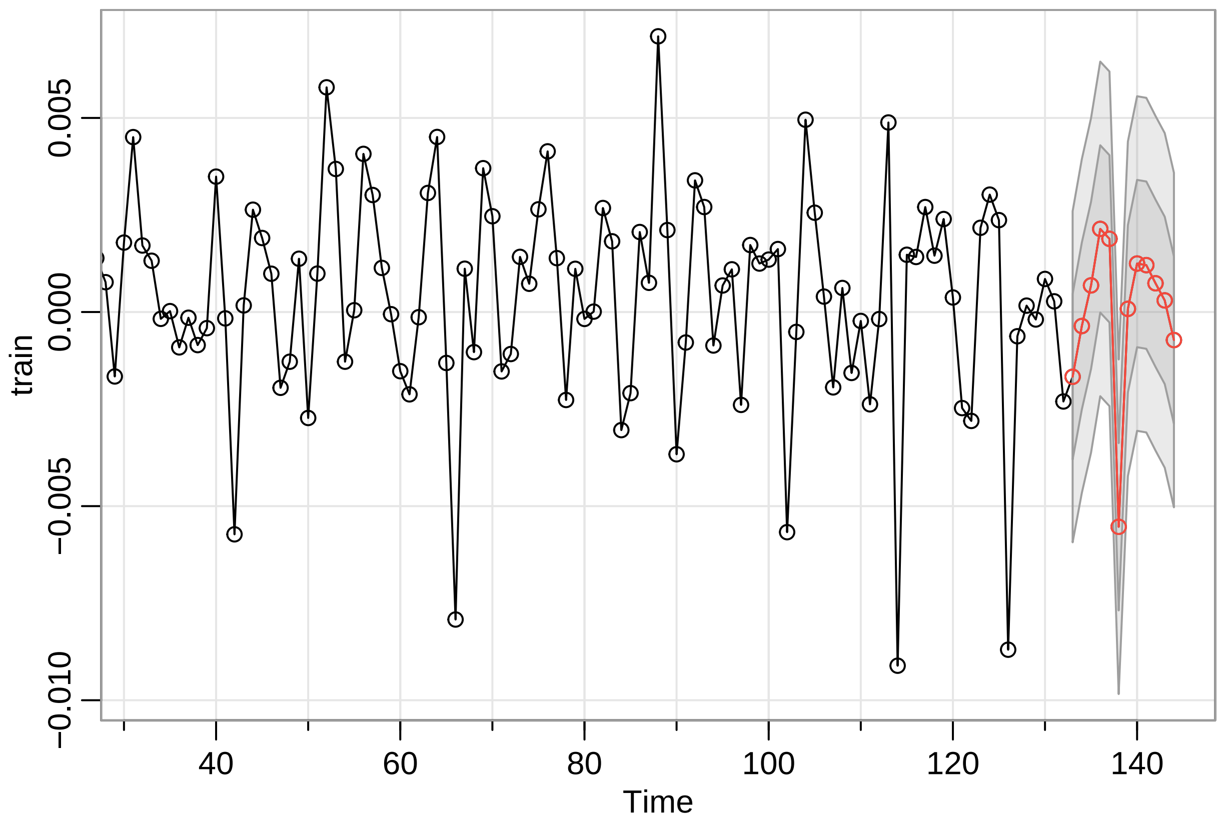
0.005
0.000
−0.005
train
20
40
60
80
100
120
Time

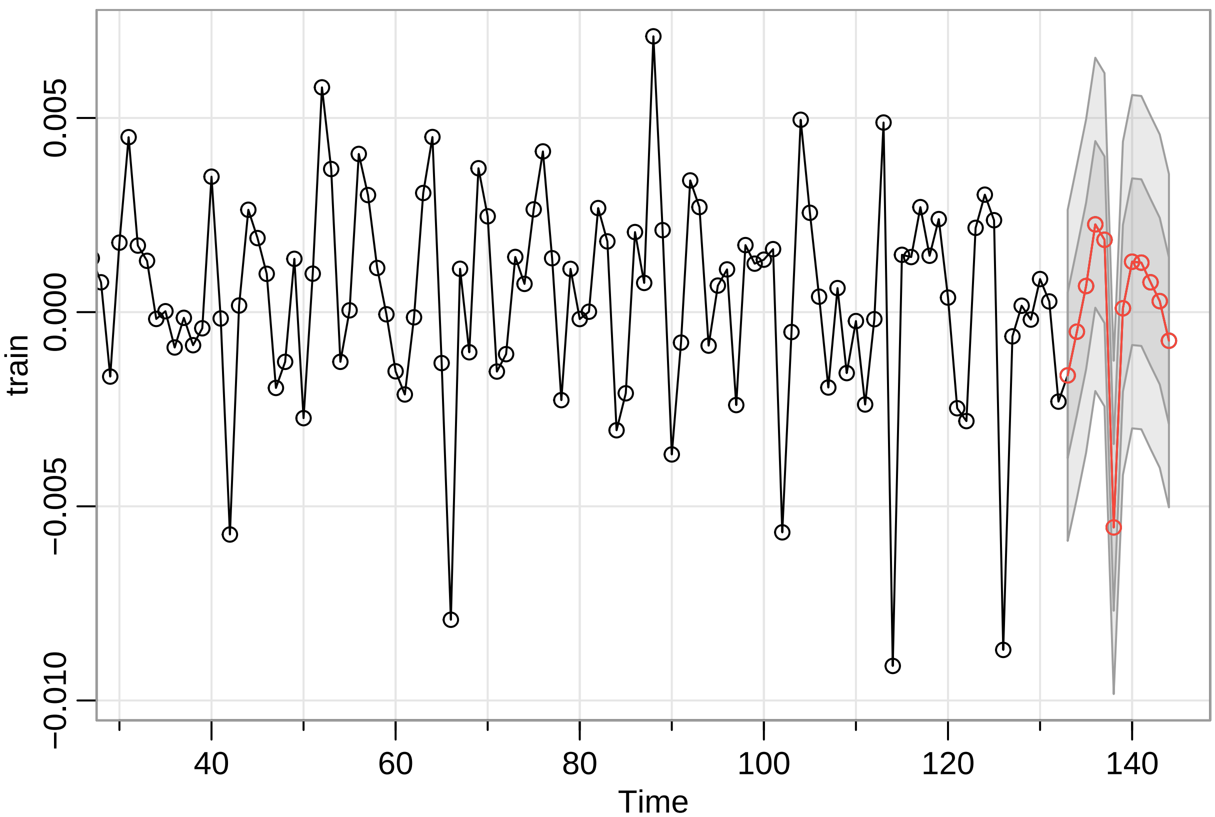


0.005
0.000
−0.005
train
20
40
60
80
100
120
Time

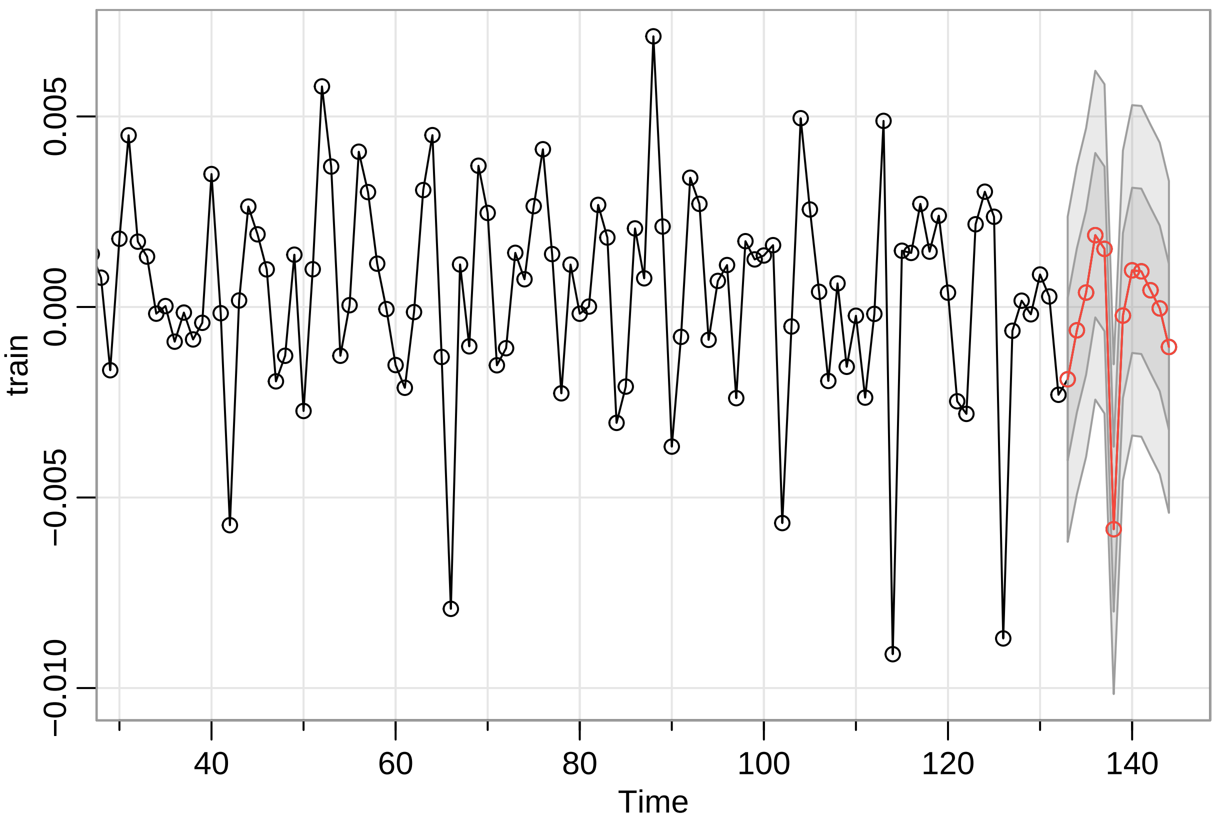


0.005
0.000
−0.005
train
20
40
60
80
100
120
Time

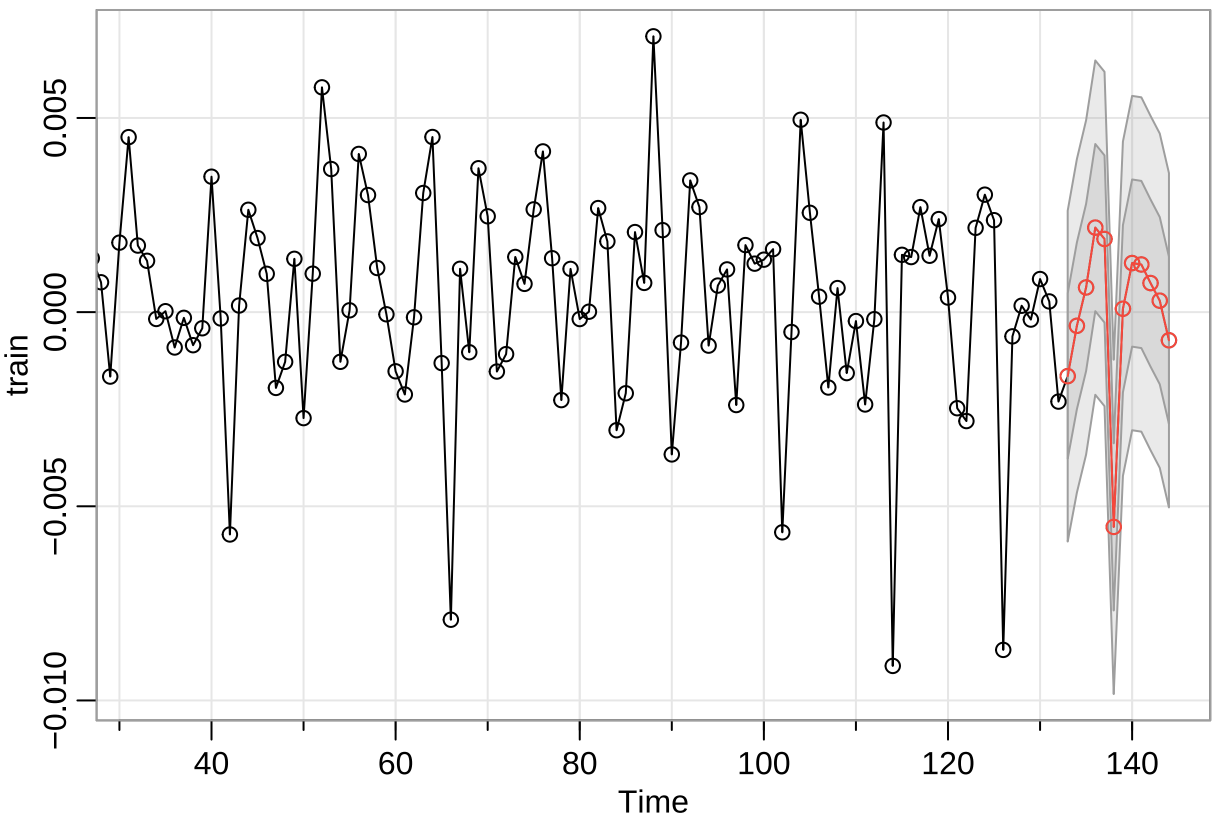

train
0.005
0.000
−0.005
−0.010
40
60
80
100
120
140
Time



train
0.005
0.000
−0.005
−0.010
40
60
80
100
120
140
Time



train
0.005
0.000
−0.005
−0.010
40
60
80
100
120
140
Time



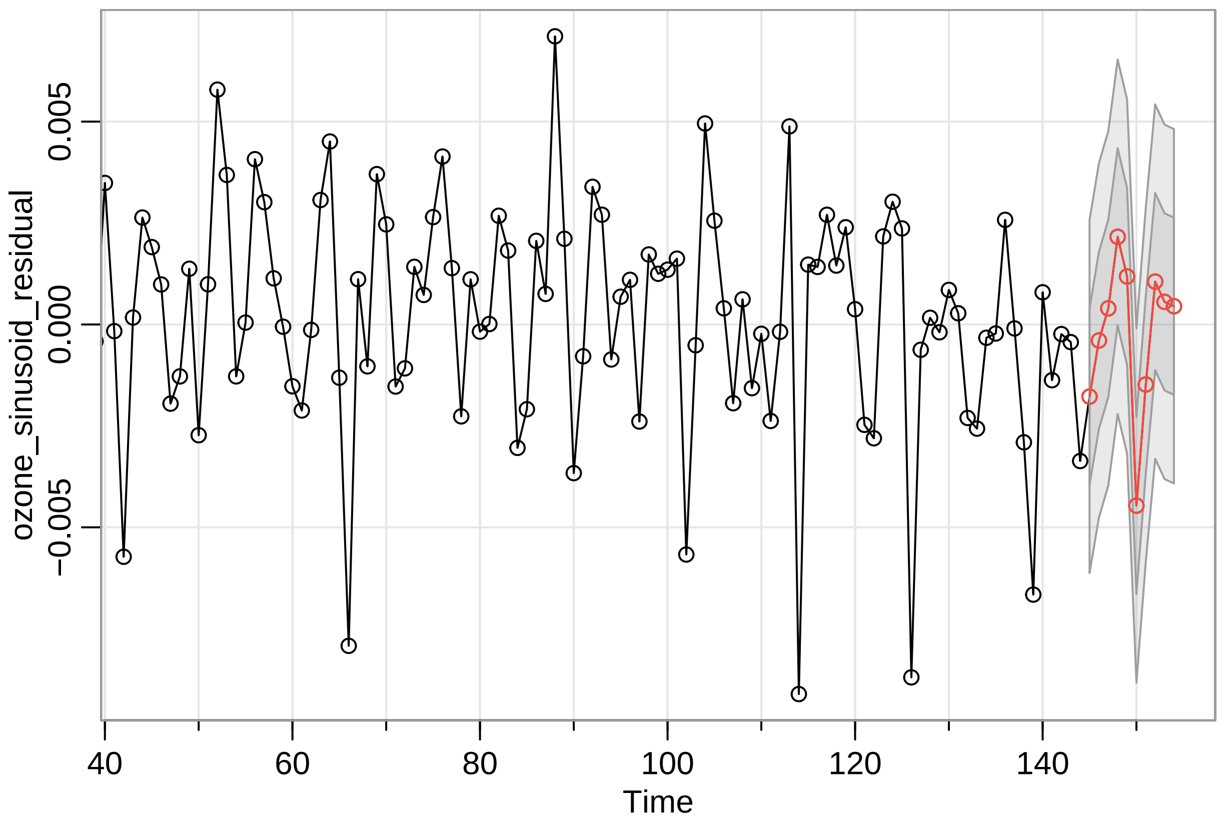

```
sse = rbind(mean(sse1),mean(sse2),mean(sse3),mean(sse4))
rownames(sse) = c("SARIMA(2,0,0)(1,0,1)12","SARIMA(1,0,0)(1,0,1)12","SARIMA(2,1,1)(1,0,1)12","SARIMA(2,0
colnames(sse) = c("SSE")
print(sse)
```

```
##                                 SSE
## SARIMA(2,0,0)(1,0,1)12 5.776333e-05
## SARIMA(1,0,0)(1,0,1)12 5.837593e-05
## SARIMA(2,1,1)(1,0,1)12 6.140208e-05
## SARIMA(2,0,1)(1,0,1)12 5.971999e-05
```

The SSE for both model are low, but SARIMA(2,0,0)(1,0,1)12 is slighly better

Future 10 values prediction

```
m1_forecast <- sarima.for(ozone_sinusoid_residual, p=2, d=0, q=0, P=1, D=0, Q=1, S=12, n.ahead=10)$pred
```




```
ts = as.data.frame(145:154)
predict_t = 145:154
prediction = c()
coef = ozone_sinusoid_model$coefficients

for (t in predict_t){
  pred = coef[1]+coef[2]*sin(t) + coef[3]*t+ coef[4]*cos(t)+ coef[5]*sin(t)*t + coef[6]*cos(t)*t
  prediction = c(prediction,pred)
}
prediction
```

```
##  (Intercept)  (Intercept)  (Intercept)  (Intercept)  (Intercept)  (Intercept)
## -0.009641503  0.001033070  0.013597515  0.016528621  0.007152463 -0.005881246
##  (Intercept)  (Intercept)  (Intercept)  (Intercept)
## -0.010543420 -0.002492147  0.010919518  0.017393830
```

```
m1_forecast = as.vector(m1_forecast)
prediction = as.vector(prediction)
predictions = m1_forecast+prediction

ozone2 = c(ozone,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA)

ts2 = 1:154
preds = rep(NA, 144)
preds = c(preds,predictions)
```

```r
plot(ozone2,type ="l")
lines(ts2,preds,col = "red")
```