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9/26/2025

ECON-4570 Econometrics

ECON4570 : Problem Set 2

Due by 11:59pm on 09/28/2025 [1]

1. Assume that you are in charge of the central monetary authority in a mythical country. You are given the following historical data on the quantity of money (X) and national income (Y)(both in million of dollars). Also assume **Assumptions 1, 2, 3 hold and the variance is (conditional) homoskedasticity**:

Year	Quantity of Money (X)	National Income (Y)
1989	2.0	5.0
1990	2.5	5.5
1991	3.2	6.0
1992	3.6	7.0
1993	3.3	7.2
1994	4.0	7.7
1995	4.2	8.4
1996	4.6	9.0
1997	4.8	9.7
1998	5.0	10.0

(a) Estimate the regression of national income Y on the quantity of money X and provide estimates of their standard errors (Assuming Assumption 4 (homoskedastic conditional variance) holds).

Estimated regression model: $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$

$\hat{Y} = 1.168 + 1.716X$:

- $\hat{\beta}_1 = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{\sum(X_i - \bar{X})^2} \approx 1.716$
  - Mean X $\bar{X} = \frac{1}{n}\sum(X_i)$ = 3.72
  - Mean Y: $\bar{Y} = \frac{1}{n}\sum(Y_i)$ = 7.55
  - Covariance: $\sum(X_i - \bar{X})(Y_i - \bar{Y})$ = 15.09
  - Variance X: $\sum(X_i - \bar{X})^2$ = 8.796

- $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} \approx 1.168$
- $s.e(\hat{\beta}_0) = \sqrt{\frac{s^2}{n} \frac{\sum X_i^2}{\sum (X_i - \bar{X})^2}} = 0.483$
  - $s^2 = \frac{1}{n-2} \sum (Y_i - \hat{Y}_i)^2 \approx 0.1394$
- $s.e(\hat{\beta}_1) = \sqrt{\frac{s^2}{\sum (X_i - \bar{X})^2}} = 0.126$

(b) How do you interpret the intercept and the slope of the regression line?

The slope of the regression line tells us about the relationship and effect the independent variable X has on the dependent variable Y. For every 1 unit increase in X, Y will be predicted to increase by 1.716.

The intercept tells us that when X = 0, Y is estimated to be at 1.168. For our dataset, it's not realistic that the quantity of money ever is 0 at any point. The intercept in this context is just a mathematical construct to position the regression line correctly.

(c) Test the significance of X at 5% significance level by assuming Assumptions 0-5) are satisfied.

$H_0 : \beta_1 = 0$

$H_1 : \beta_1 \neq 0$

$t = \frac{\hat{\beta}_1 - \beta_1}{s.e(\hat{\beta}_1)} = 13.62$

$df = n - k - 1 = 8$

$\alpha/2 = 0.025$

$t_c = 2.306$

$|t| > t_c \implies |13.62| > 2.306$

Since the calculated t-statistic is larger than the critical value, we reject the null hypothesis that $\beta_1 = 0$. There is strong evidence that Y is correlated and dependent on X.

(d) If you had sole control over the money supply and wished to achieve a level of national income of 12.0 in 1999, at what level would you set the money supply? Explain.

Using our regression model, we have

$12.0 = 1.168 + 1.716X$

$=> X = 6.312$

I would set the money supply (X) at 6.312, although it's important to consider that we do not have any historical data beyond X = 5.0, and thus we are meeely extrapolating.

(e) Compute the coefficient of determination R^2. Explain the meaning of the estimated R^2.

$$R^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum e_i^2}{\sum (Y_i - \bar{Y})^2} = 1 - \frac{(n-2)s^2}{\sum (Y_i^2) - n\bar{Y}^2} = 0.959$$

(f) Construct 95% confidence interval for the parameter of X. Interpret this confidence interval.

$$CI = \hat{\beta}_1 \pm (t_c * s.e(\hat{\beta}_1)) = 1.716 \pm (2.306 * 0.126)$$

$$\implies [1.425, 2.007]$$

This confidence interval means that we are 95% confident that the parameter of X (β_1) lies between 1.425 and 2.007. If we repeated the sampling process 100 times, 95 of those processes would be expected to have β_1 lie between that interval. We are 95% confident that for every 1 unit increase in X, Y will increase by an interval between 1.425 and 2.007.