CSE 12 – Basic Data Structures: Running Time Analysis

Prof Joe Politz

[Slides borrowed from

Marina Langlois and Christine Alvarado]

Big-O

We say a function f(n) is **"big-O"** of another function g(n), and write f(n) is **O**(g(n))*, if there are positive constants c and n_0 such that:

- $f(n) \le c\, g(n)$ for all $n \ge n_0$.

In other words, for large n, can you multiply g(n) by a constant and have it always be bigger than or equal to f(n)

n is the "size of your problem"
Some notations write f(n) = O(g(n)) rather than f(n) **is** O(g(n))

Steps for calculating the Big O bound on code or algorithms

1. Count the number of instructions in your code (or algorithm) as precisely as possible as a function of n, which represents the size of your input (e.g. the length of the array). This is your f(n).
   - Make sure you know if you are counting best case, worst case or average case – could be any of these!
2. Simplify your f(n) to find a simple g(n) such that f(n) = O(g(n))

Almost always, what we care about is the WORST CASE or the AVERAGE CASE. Best case is usually not that interesting, unless we can prove it's slow!

In CSE 12 when we do analysis, we are doing **WORST CASE** analysis unless otherwise specified.

Analyzing the worst case

```
boolean find( String[] theList, String toFind ) {
  for ( int i = 0; i < theList.length; i++ ) {
    if ( theList[i].equals(toFind) )
      return true;
  }
  return false;
}

boolean fastFind( String[] theList, String toFind ) {
  return false;
}
```

Which method is faster?

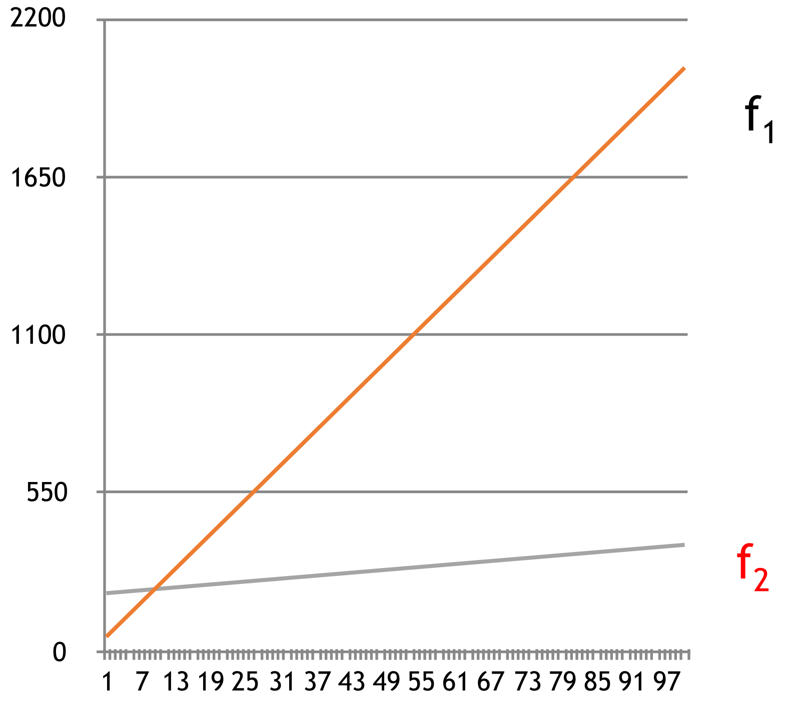
A. find
B. fastFind
C. They are about the same

f_2 is* $O(f_1)$

$f(n)$ is $\mathbf{O}(g(n))$, if there are positive constants c and n_0 such that $f(n) \leq c * g(n)$ for all $n \geq n_0$.

A. TRUE
B. FALSE

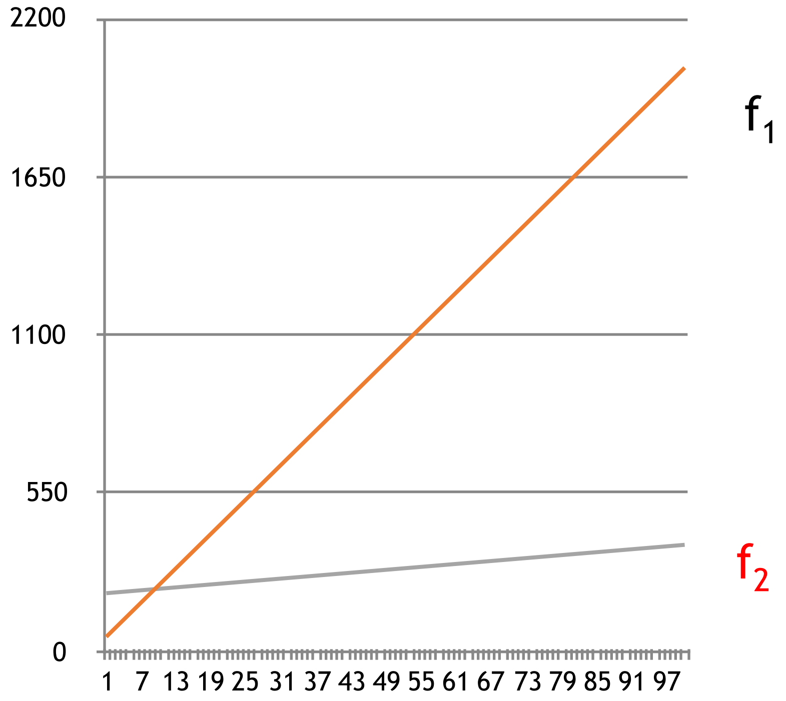
Why or why not?



* You can't actually tell if you don't know the function, because it could do something crazy just off the graph, but we'll assume it doesn't.

f(n) is O(g(n)), if there are positive constants c and n_0 such that $f(n) \leq c * g(n)$ for all $n \geq n_0$.

- Obviously $f_2 = O(f_1)$ because $f_1 > f_2$ (after about n=10, so we set $n_0 = 10$)
  - f_1 is clearly an *upper bound* on f_2 and that's what big-O is all about



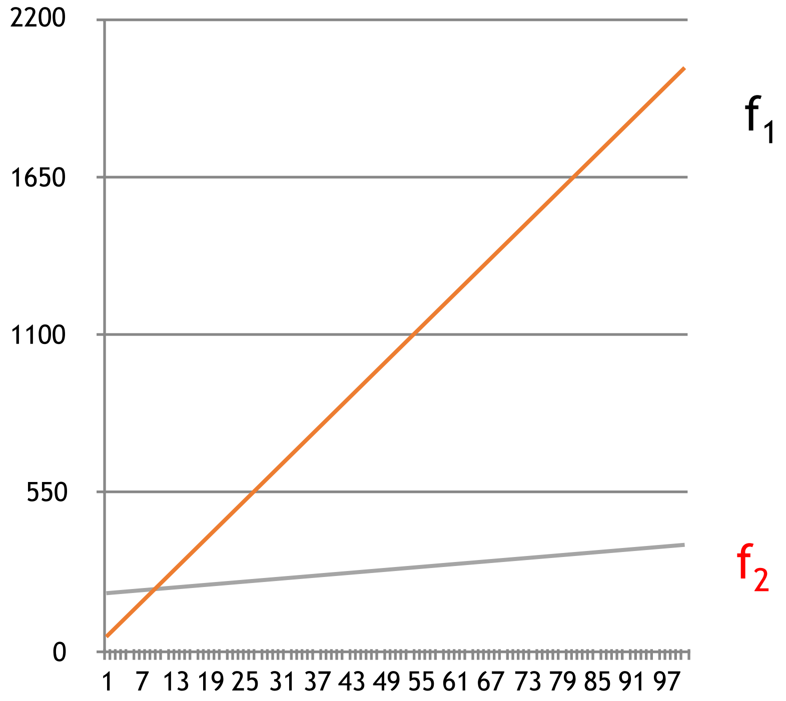
f_1 is $O(f_2)$

$f(n) = \mathbf{O}(g(n))$, if there are positive constants c and n_0 such that $f(n) \leq c * g(n)$ for all $n \geq n_0$.

A. TRUE
B. FALSE

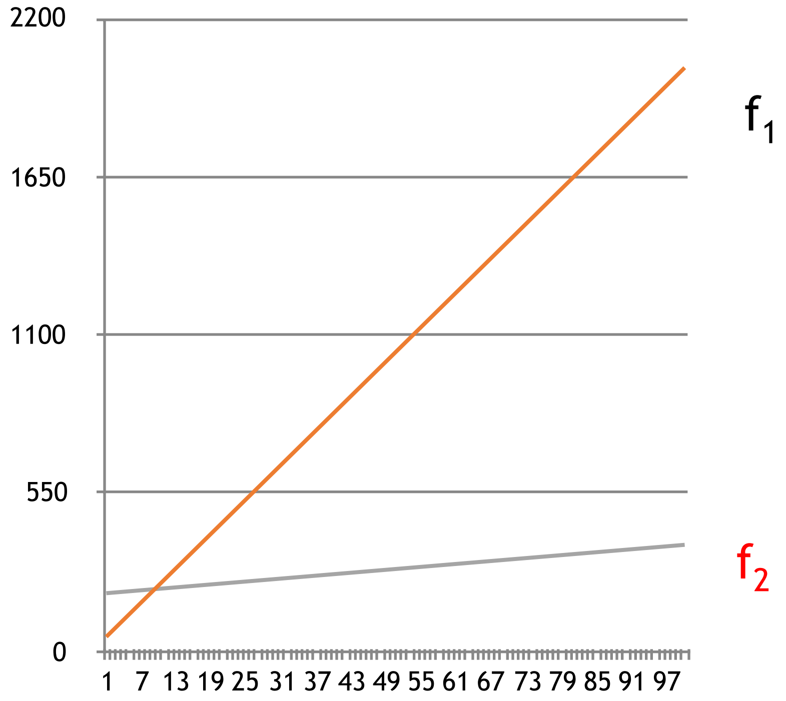
Why or why not?

In other words, for large n, can you multiply f_2 by a constant and have it always be bigger than f_1 for large enough n?



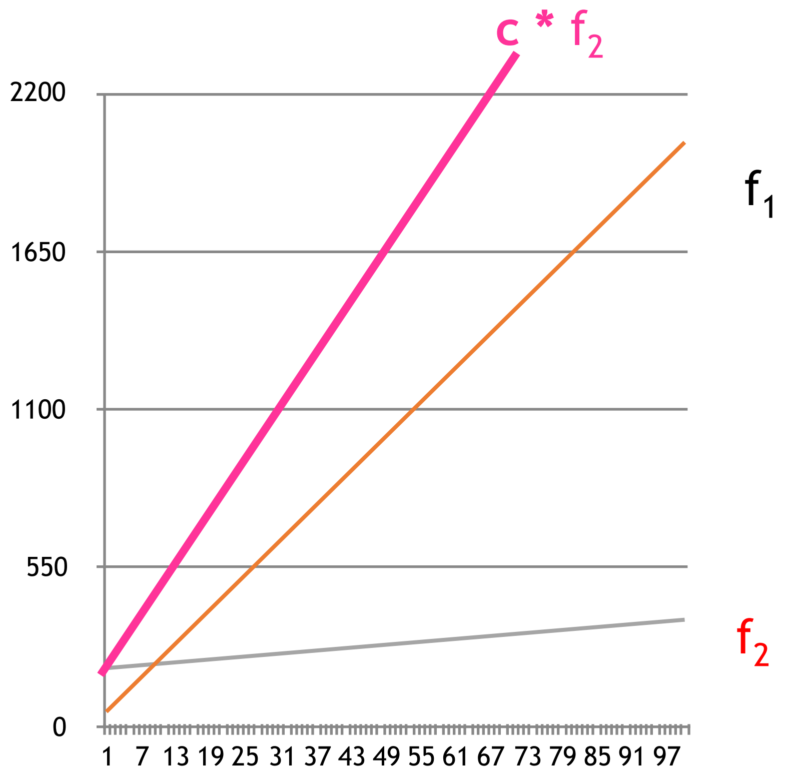
$f(n)$ is $O(g(n))$, if there are positive constants c and n_0 such that $f(n) \leq c * g(n)$ for all $n \geq n_0$.

- Obviously $f_2 = O(f_1)$ because $f_1 > f_2$ (after about n=10, so we set $n_0 = 10$)
  - f_1 is clearly an *upper bound* on f_2 and that's what big-O is all about
- But $f_1 = O(f_2)$ as well!
  - We just have to use the "c" to adjust so f_2 that it moves above f_1



f(n) is O(g(n)), if there are positive constants c and n_0 such that $f(n) \leq c * g(n)$ for all $n \geq n_0$.

- Obviously $f_2 = O(f_1)$ because $f_1 > f_2$ (after about n=10, so we set $n_0 = 10$)
  - f_1 is clearly an *upper bound* on f_2 and that's what big-O is all about
- But $f_1 = O(f_2)$ as well!
  - We just have to use the "c" to adjust so f_2 that it moves above f_1

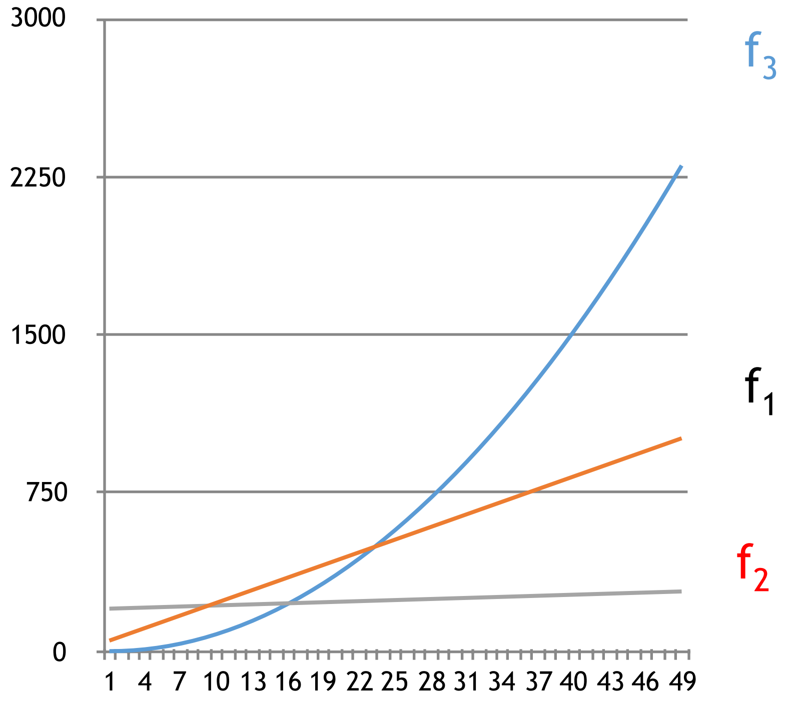


f(n) is **O**(g(n)), if there are positive constants c and n_0 such that $f(n) \leq c * g(n)$ for all $n \geq n_0$.

f_1 is $O(f_3)$

A. TRUE
B. FALSE

Why or why not?

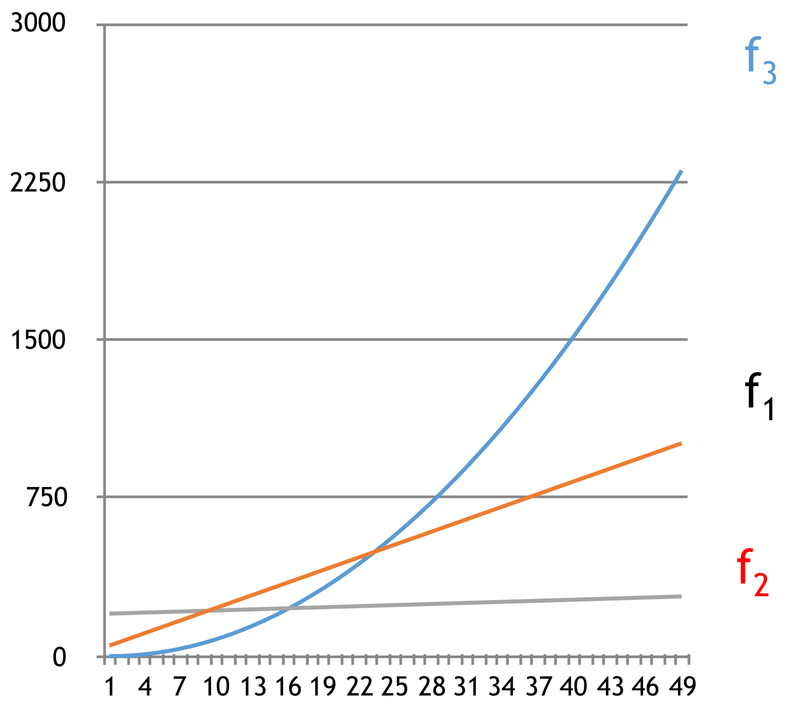


$f(n)$ is $\mathbf{O}(g(n))$, if there are positive constants c and n_0 such that $f(n) \le c * g(n)$ for all $n \ge n_0$.

f_3 is $O(f_1)$

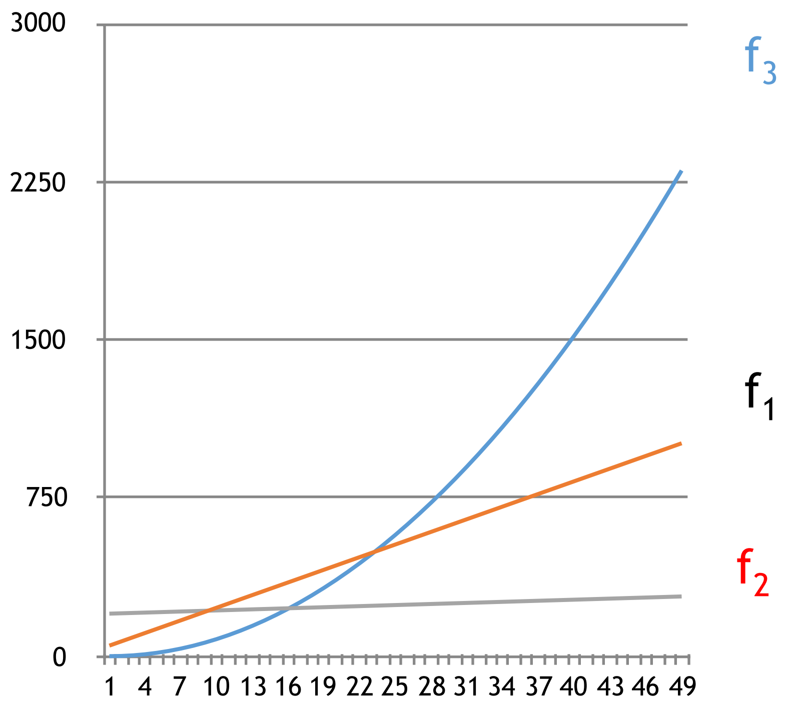
A. TRUE
B. FALSE

Why or why not?



f_1 **is** $O(f_3)$ *but* f_3 **is not** $O(f_1)$

- There is no way to pick a c that would make an O(n) function (f_1) stay above an O(n^2) function (f_3).



Common Big-O confusions when trying to argue that f_2 is $O(f_1)$:

- What if we multiply f_2 by a large constant, so that $c*f_2$ is larger than f_1 ? Doesn't that mean that f_2 is not $O(f_1)$?
  No, because we get to control the constants to our advantage, and only on f1.
- What about when n is less than 10? Isn't f2 larger than f1?
  Remember, we get to pick our n0, and only consider n larger than n0.