

Loss of information

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1 Introduction

Suppose that we know the parametric model of the probability distribution function. When we can only access the restricted versions of observed values, we can still construct the estimator of parameters with given information. In this situation, we can compare the amount of information in the estimators using the inverse of their variances. We can also check the amount of information loss using Cramer-Rao bound (Information inequality). We follow the given procedures to compare the quantities of information.

2 Situation

Let X_1, \dots, X_n be the random samples from the normal distribution $N(\mu, 1)$ with an unknown $\mu \in (-\infty, \infty)$. Suppose that we can only observe

$$Z_i = \lfloor X_i \rfloor, i = 1, \dots, n.$$

- 1) Obtain the maximum likelihood estimator (MLE) of μ , $\hat{\mu}^{MLE}$.
- 2) In this case, obtain the variance of $\hat{\mu}^{MLE}$.
- 3) Suppose we observe

$$Z_i = \frac{\lfloor 10X_i \rfloor}{10}, i = 1, \dots, n,$$

namely, up to the first decimal point of each observation. Repeat 1) and 2).

- 4) Compare the inverse of the variance in each case and check the information inequality (Hint: Use bootstrap method.).

3 Theoretical Approach & Simulation Result

Attached in **hand-written file & simulation report**.

4 Conclusion

When estimating the parameter with restricted information, we can still make a reasonable estimation. In this example, we could quantify the amount of information loss using the Cramer-Rao bound.