PHY 474 Notes

Cosmology

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1 Introduction to Cosmology

1.1 Isotropic and Homogeneous Space-times
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The universe is homogenous and isotropic.

⇒ **Homogenous**: Looks the same everywhere

⇒ **Isotropic**: Looks the same in all directions

Observational evidence that support this assumption are from the Cosmic Microwave Background (CMB) and large-scale structure observations.

1.2 Expansion of the Universe
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Redshift (z): The doppler shift of the light from galaxies observed through emission/absorption lines in spectra.

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}} \tag{1}$$

For an expanding universe, $z_{\text{galaxy}} > 0$.

The larger z is, the further the distance the galaxy is.

1.3 Hubble Constant

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Imagine 3 galaxies. Let the relative distances, at t = t_0, be $r_{12}(t_0)$, $r_{23}(t_0)$, and $r_{13}(t_0)$. In an expanding universe, the distances increase by a scale factor $a(t)$. Thus,

$$\begin{aligned} r_{12}(t) &= a(t)\ r_{12}(t_0) \\ r_{23}(t) &= a(t)\ r_{23}(t_0) \\ r_{13}(t) &= a(t)\ r_{13}(t_0) \end{aligned}$$

Imagine you are an observer in galaxy 1. You will see galaxies 2 and 3 moving with velocity:

$$\begin{aligned} v_{12} &= \frac{\mathrm{d}r_{12}}{\mathrm{d}t} \\ &= \dot{a}\ r_{12}(t_0) \\ &= \frac{\dot{a}}{a}\ r_{12}(t) \end{aligned}$$

A similar relation is true for v_{23} and v_{13}.

The factor $\frac{\dot{a}}{a}$ relates the velocity of galaxies to their distance away from us. It is called the Hubble constant ($H_0 \equiv \frac{\dot{a}}{a}$). Thus for galaxies:

$$v = H_0 r \tag{2}$$

This relation can be described with the Hubble diagram. It is a graph of velocity vs. distance of a bunch of galaxies. There is a linear trend with slop of H_0.

Doppler shift is $z = \frac{v}{c}$, so

$$z = \frac{H_0}{c} r \tag{3}$$

1.4 Age of the Universe
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H_0 is measured to be around 70 km/s /Mpc. If we assume a constant H_0, we can calculate an age of the universe:

$$t_0 = \frac{r}{v} = \frac{r}{H_0 r} = \frac{1}{H_0}$$

For $H_0 \approx 70$ km/s/Mpc, $t_0 \approx 14$ Gyr.

1.5 Contents of the Universe
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Dark matter	25%
Dark energy	75%
Stars	0.2%
All baryons	5%
Radiation (primarily CMB)	0.01%

1.6 Timeline of the Universe
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$t = 0$	Big Bang
$t = 10^{-39}$ s	Classical gravity breaks down
$t = 10^{-35}$ s	Inflation
	(density fluctuations formed)
$t = 10^{-14}$ s	Baryogenesis
	(formation of protons/electrons and matter dominates over anti-matter)
$t = 100$ s	Big Bang nucleosynthesis
	(formation of Hydrogen nuclei)
$t = 5 \times 10^4$ s	Matter dominates over radiation
$t = 4 \times 10^5$ yr	Recombination
	(formation of neutron atoms and the CMB was emitted)
$t = 100$ Myr	First population III stars form
$t = 500$ Myr	First galaxies form
$t =$ 0.5-1 Gyr	Reionization or "Cosmic Dawn"
$t =$ 2-3 Gyr	"Cosmic Noon"
	(peak of stellar formation and SMBH accretion)
$t = 10$ Gyr	Dark energy dominates
$t = 14$ Gyr	Present day

1.7 Geometries of isotropic and homogeneous space-times Sept. 13, 2021

In cosmology, we often need to measure distances in an expanding (and sometimes curved) space-time.

Space can be flat(Euclidean) with ($k = 0$), positively curved ($k = +1$), or negatively curved ($k = -1$).

1.7.1 In 2D

In flat space,