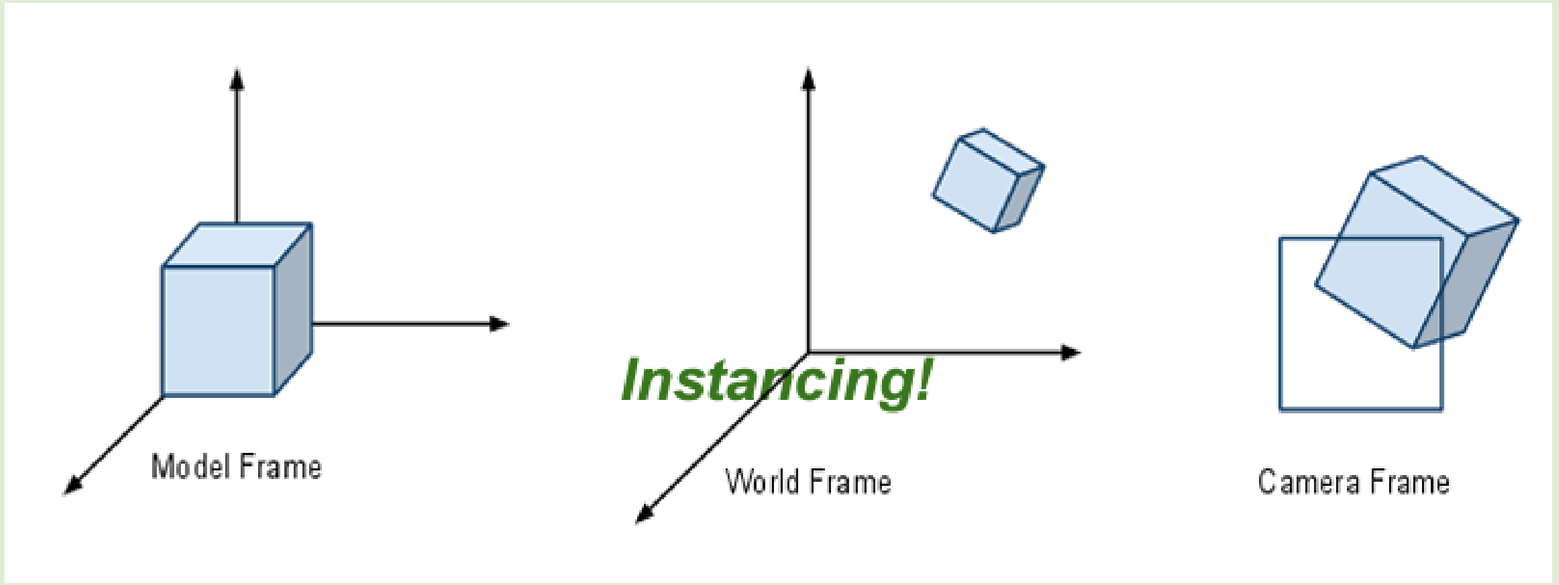
TRANSFORMATIONS

OpenGL Fundamentals

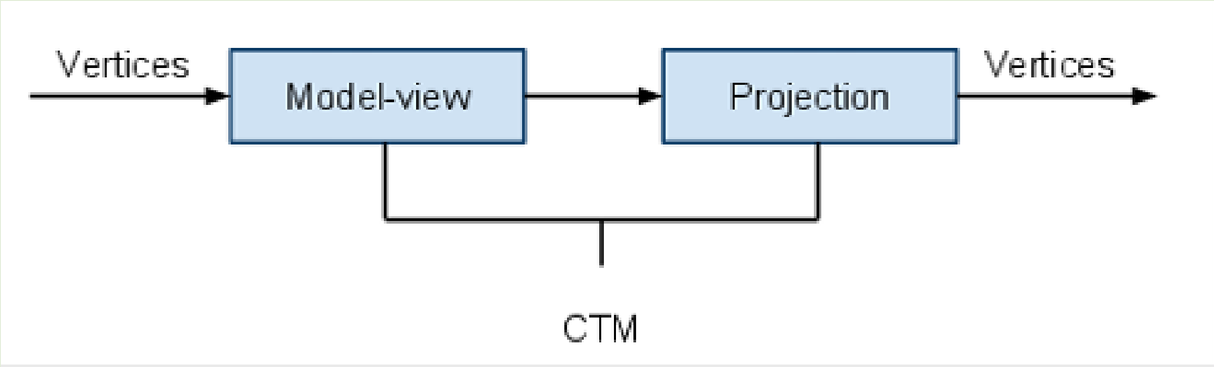
OVERALL OBJECTIVE

- Define object in object frame
- Move object to world/scene frame
- Bring object into camera/eye frame



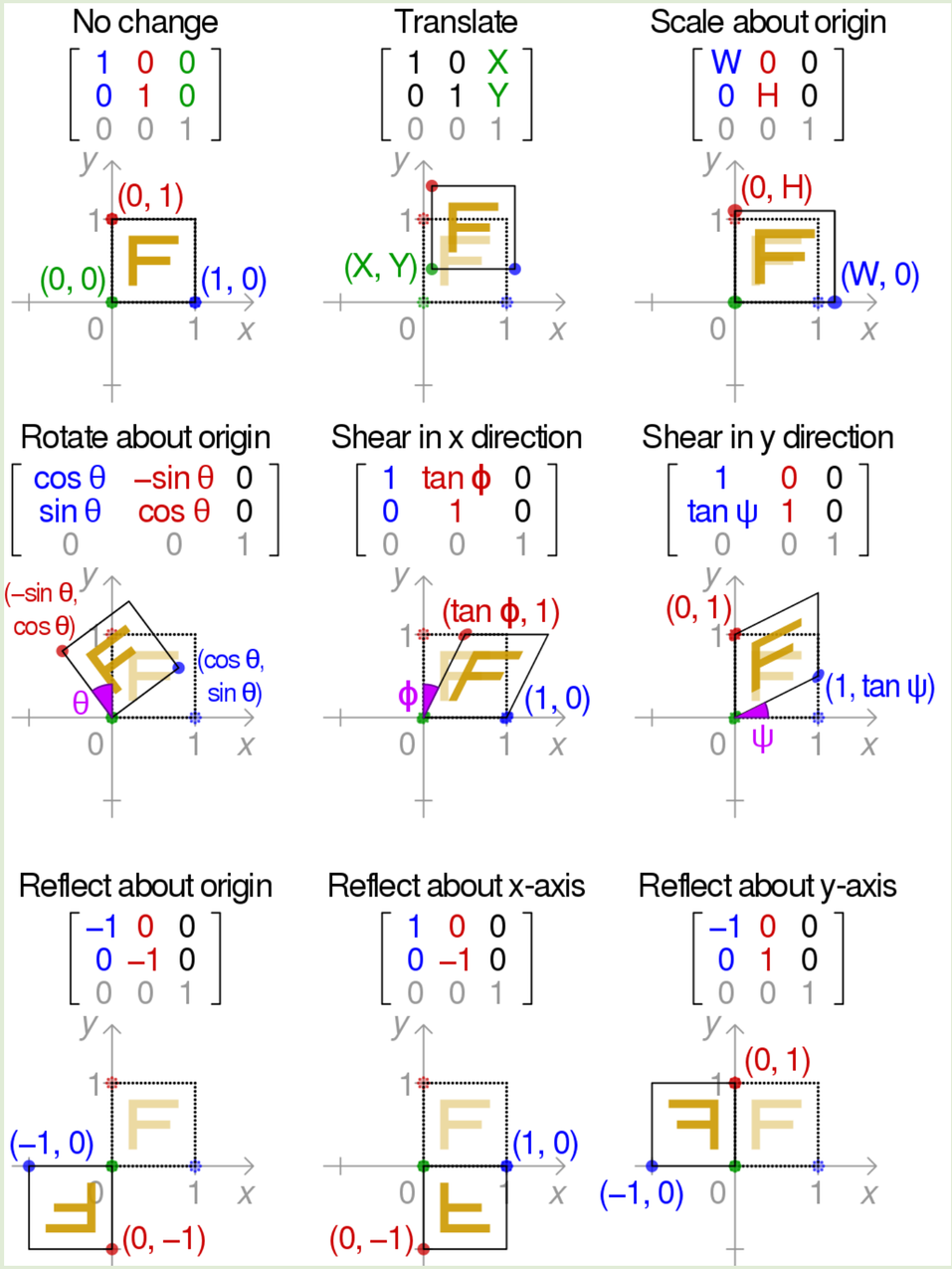
GRAPHICS... HOW DOES IT WORK?

- Linear Algebra and geometry (magical math)
- Frames are represented by tuples and we change frames (representations) through the use of matrices. In OpenGL, vertices are modified by the Current Transformation Matrix (CTM)
- 4x4 homogeneous coordinate matrix that is part of the state and applied to all vertices that pass down the pipeline.



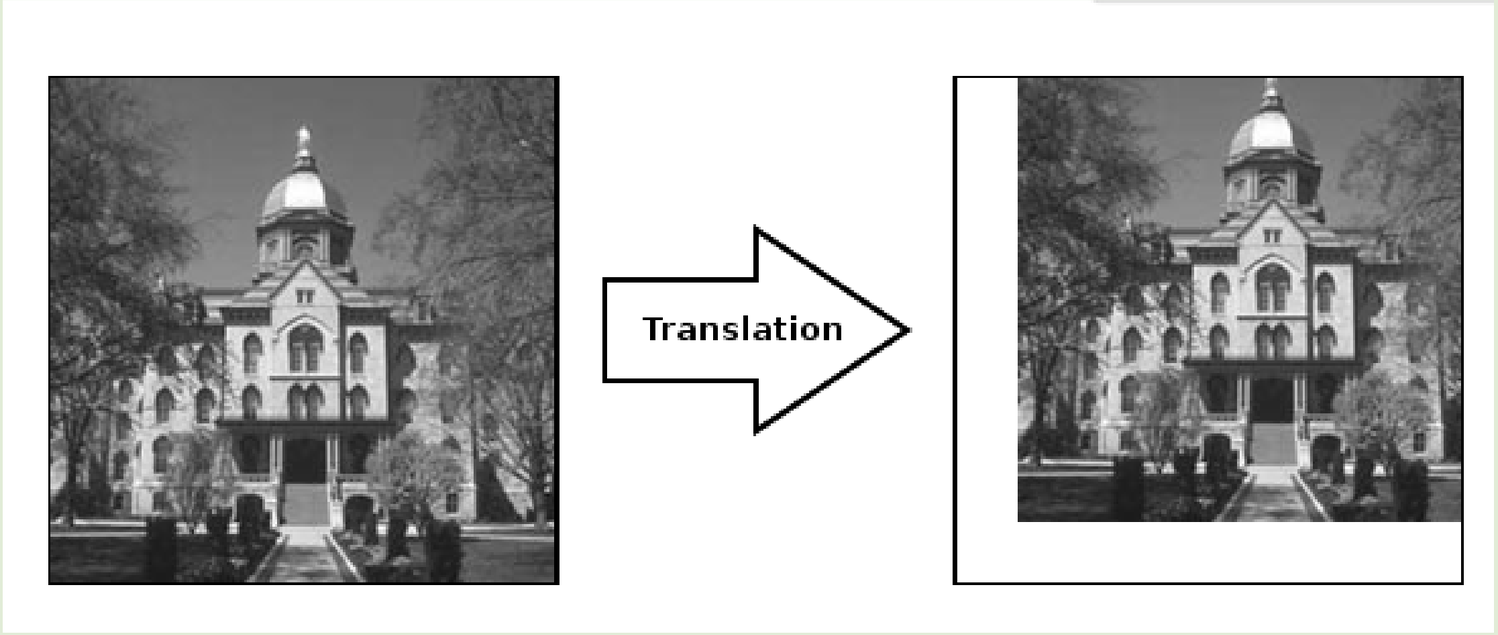
AFFINE TRANSFORMATIONS

- Affine transformation is a linear mapping method that preserves points, straight lines, and planes. Sets of parallel lines remain parallel after an affine transformation. The affine transformation technique is typically used to correct for geometric distortions or deformations that occur with non-ideal camera angles.



TRANSLATION

- Move (translate, displace) a point to a new location
- Openl function: glTranslatef(x, y, z)



TRANSLATION MATRIX

- Move (translate, displace) a point to a new location

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

resulting coordinate

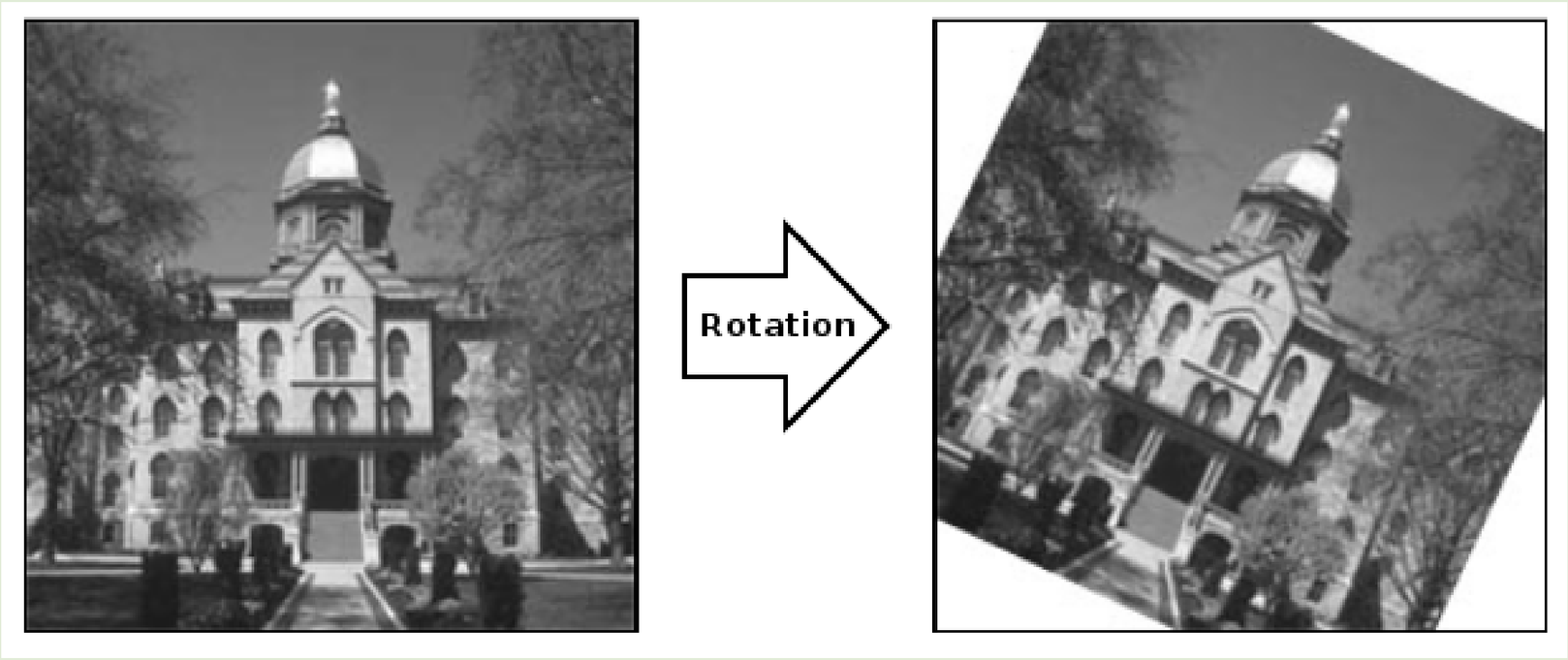
3d translation matrix

original coordinate

$$\begin{pmatrix} x + T_x \\ y + T_y \\ z + T_z \\ 1 \end{pmatrix}$$

ROTATION

- Rotation about z axis leaves all points with the same z:
- Openl function: glRotatef(x, y, z)



ROTATION MATRIX

X axis

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} x \\ \cos\theta \cdot y - \sin\theta \cdot z \\ \sin\theta \cdot y + \cos\theta \cdot z \\ 1 \end{pmatrix}$$

Y axis

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$\begin{pmatrix} \cos\theta \cdot x + \sin\theta \cdot z \\ y \\ -\sin\theta \cdot x + \cos\theta \cdot z \\ 1 \end{pmatrix}$$

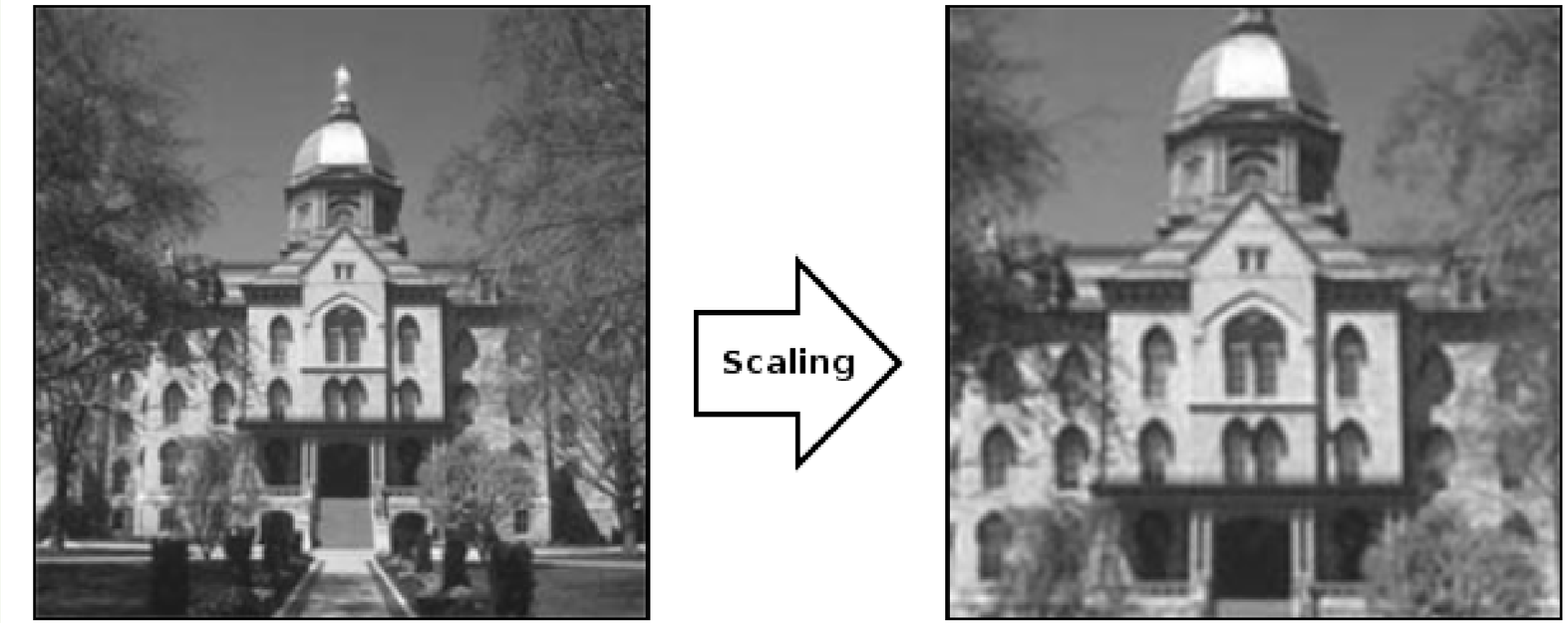
Z axis

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} \cos\theta \cdot x - \sin\theta \cdot y \\ \sin\theta \cdot x + \cos\theta \cdot y \\ z \\ 1 \end{pmatrix}$$

SCALING

- Expand or contract along each axis (fixed point of origin)
- Openl function: gScalef(x, y, z)



SCALING MATRIX

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

resulting coordinate

3d scaling matrix

original coordinate

$$\begin{pmatrix} S_1 \cdot x \\ S_2 \cdot y \\ S_3 \cdot z \\ 1 \end{pmatrix}$$

LEARNING LINKS

- https://en.wikipedia.org/wiki/Affine_transformation
- https://en.wikipedia.org/wiki/Transformation_matrix