IMPORTANT!

Cite all the theorem whenever I use it.
Be specific and check all the conditions correctly.
Understand concept profoundly.
Understand question correctly.

- Deducing the autonomou or not autonomous ODE
- Deducing linear or non-linear ODE
- Deducing Homogeneous or non-homogeneous ODE
- Deduing separable or not separable ODE

This seems hard to do, but it is not.
Following is the step-by-step process to deduce the ODE.

Definition

Autonomouse ODE is an ODE that does not depend on the independent variable explicitly.
Generally, look at if t is in the equation. If t is not in the equation, then it is autonomouse ODE.

- Example: $\frac{dy}{dt} = 2y$
- Example: $\frac{dy}{dt} = y^2$

Definition

Non-autonomouse ODE is an ODE that depend on the independent variable explicitly.
Generally, look at if t is in the equation. If t is in the equation, then it is non-autonomouse ODE.

For dealing with autonomous ODE, we can use the following method.

IMPORTANT!

As $y' = f(y)$ doesn't depend on t, the **slope of direction field for autonomous ODE only vary in vertical direction.**
Therefore, the **slope of each horizontal line is constant as** $y = c$ where c is constant.

Or in shorten, as time goes, if horizontal slope doesn't change, then it is autonomous ODE. (as it doesn't depend on time)

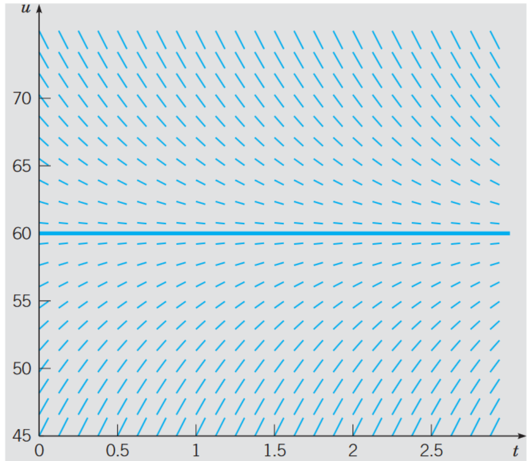


Figure 1: Autonomous ODE

As you can see from figure 1, the slope of each horizontal line is constant, and vertical line is not constant. Therefore, this is autonomous ODE.

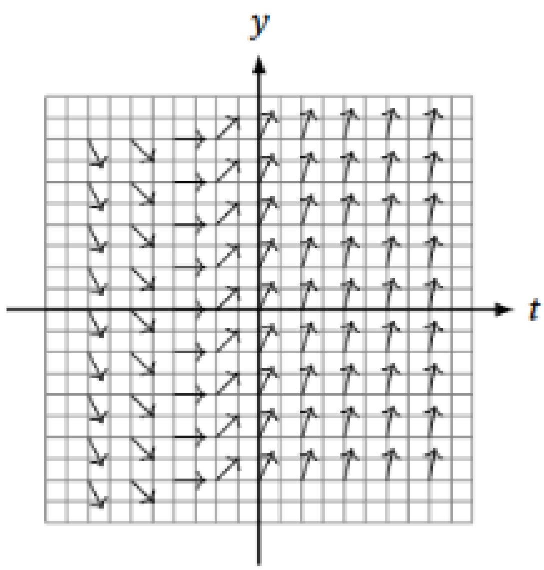


Figure 2: Non-Autonomous ODE

As you can see from figure 2, the slope of each horizontal line is not constant, and vertical line is constant. Therefore, this is non-autonomous ODE.

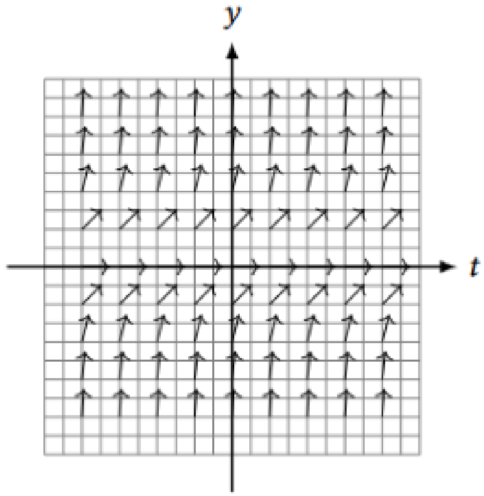


Figure 3: Homogeneous and autonomous ODE

As you can see from figure 3, the slope of each horizontal is constant, this is autonomous ODE.
Also, at y = 0, slope is 0, therefore, this is homogeneous.

Definition

Linear ODE is the equation that can be reduced to the form $Ly = f$ where L is linear operator.
There are few examples with explanation.

- Example: $y'' + y = 0$ is linear
- Example: $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} = sinx$ is linear. The reason behind this is dependent variable and its derivatives are linear.
- Example: $y' + y^2 = 0$ is non-linear. The reason behind this is dependent variable and its derivatives are not linear. (which is y^2)
- Example: $yy'' + y = 0$ is non-linear. The reason behind this is dependent variable and its derivatives are not linear. (which is yy'')
- Example: $xy'' + y = 0$ is linear, as x is not dependent variable.
- Example: $\frac{d^2y}{dx^2} + sin(y) = 0$ is non - linear, as $siny$ is not linear.

Example

During midterm, there was few equations that are linear or non-linear.

- Example: $x'(t) + expt^2x(t) = sin(t^3)$. This is linear, as $x'(t)$ and $x(t)$ are linear.
- Example: $x'(t) = x^2sin(t) + exptx(t)$. This is non-linear, as $x^2sin(t)$ and $exptx(t)$ are not linear.
- Example: $\frac{dx_1}{dt} = -x_1 + 2x_2 + t$. This is linear, as x_1 and x_2 are linear.
- Example: $\frac{dx_2}{dt} = 3x_1 - x_2 + t^2$. This is linear, as x_1 and x_2 are linear.

IMPORTANT!

In first ODE, if there is **monotonic is shown in direction fields, then it is linear.** (also constant) through column.
If in the direction field, there is no monotonic, then it is non-linear.

IN SHORTEN, if the direction field doesn't depend on y, then it is linear.

By doing that we can deduce the linear or non-linear ODE from 2 and 3.
In figure 2, we can see that slope is constant through out the column, therefore, this is linear.
In figure 3, we can see that slope is increasing and then decreasing thorughout the column, therefore, this is non-linear.

Definition

The homogenous ODE is when $A\frac{du^2}{dx^2} + B\frac{du}{dx} + Fu = G(x)$ where $G(x) = 0$

The non-homogenous ODE is when $A\frac{du^2}{dx^2} + B\frac{du}{dx} + Fu = G(x)$ where $G(x) \neq 0$

IMPORTANT!

To deduce the homogenous ODE, we can see the directoin field.
Based on the what we have defined homogenous, if $G(x) = 0$, then it is homogenous.
Therefore, if it is autonomous, then it is homogenous.
However, if it is non-autonomous, then it is non-homogenous.

This can be shown in figure 2 and 3.

Lec 1 summary

- Big Idea: Differential equations model physical situations.
- What is a **differential equation?**

Definition

A **differential equation** contains the derivatives of one or more dependent variables, with respect to one or more independent variables.

- What is **ODE?**

Definition

If the involved derivatives are ordinary and with respect to a single variable.

- What is **PDE?**

Definition

If it involves derivatives of one or more dependent variables of two or more independent variables.

- Here are following examples of differential equations:
  (a) $y' = f(t, y)$
  (b) $y'' + y' + y = 0$
  (c) $y'' + y' + y = \sin(t)$
  (d) $y'' + y' + y = \sin(y)$
  (e) $y'' + y' + y = \sin(y')$

- What is **order of DE?**

Definition

The order of a DE is **the highest derivatives** that appear in that equation.

Therefore, the order of the above DEs are:

(a) 1
(b) 2
(c) 2
(d) 2
(e) 2

- We can understand ODEs without solving them.
- Idea behind ODE is we can understand the behavior of a function by looking at its derivatives without solving it.
- n-th order ODEs in one dependent variable can be expressed as

$$F(t, y, y', y'', \ldots, y^{(n)}) = 0$$

where t is the independent variable, and y is the dependent variable.
Any function that satisfies the equation is called a **solution**

Definition

The ODE is linear if F is linear in all n+2 variables $t, y, y', y'', \ldots, y^{(n)}$.
Which can be written as

$$F(t, y, y', y'', \ldots, y^{(n)}) = f(t) + a_0(t)y + a_1(t)y' + a_2(t)y'' + \cdots + a_n(t)y^{(n)} = 0$$

(as all of coefficent are linear, F is also linear)

- F will be non-linear if dependent variable y or its derivatives appear in a non-linear way. Example of Linear and Non-Linear ODEs:

(a) $y'' + y' + y = 4y$ (Linear)
(b) $y' = 4e^x$ (Non-Linear)

Example

Let's consider a cup of coffee in a room. We want to model its change in temperature over time.

Let $u(t)$ = temp of the coffee at time t.
Let t_0 = Constant ambient temperature.
Find out why each of ODE is wrong.

(a) $u' = u^2$
(b) $u' = u'' + 2u$
(c) $u' = u - (t_0)$
(d) $u' = t_0 - u$

(a) is wrong because, change of temperaure is always non-negative, which is not true in real life.
(t0 was not considered)
(b) is wrong because, t0 was not considered.
(c) is wrong because, when $u > t_0$, the rate of temp change is positive, but when $u < t_0$, the rate of temp change is negative. Which is reciprocal to the real life.
(d) is right since if $t_0 > u$, the temp of coffee is expected to heat up. If $t_0 < u$, the temp of coffee is expected to

cool down.

Definition

Newton's law of cooling: The rate of change of the temperature is negatively proportional to the difference between the temperature of the object and the temperature of the surrounding.

- Newton's law of cooling can be expressed as:

$$u' = -k(u - t_0)$$

where k is a constant.

Lec 2 summary

- Big Idea: Solving methods for ODEs.

Example

Let h = arbitrary real-valued function

- $y'(t) = h(t)$
- Integrating both sides, we get $y = \int h(t)dt + C$
- This is not unique, therefore, initial condition is needed to find the value of C.

- General solution of $y'(t) = te^t$ is $y(t) = e^t(t-1) + C$
- Now, let's say that h is also dependent of y, too.

Example

- $y'(t) = h(t, y)$

  We know that $h(t, y) = a(t)y(t) + f(t)$ from the lecture 1.

  Hence, $y'(t) - a(t)y(t) = f(t)$

- Can be solved by integrating factor or Variation of Parameters.

- What is **Integrating Factor?**

Definition

Let I is not equal to 0.

- Multily I(t) to both sides of the equation.

  $y'(t)I(t) - I(t)a(t)y(t) = I(t)f(t)$

- Now, we can write the left side as $[I(t)y(t)]'$

  If $I'(t) = -a(t)I(t)$, then $[I(t)y(t)]' = y'(t)I(t) - a(t)I(t)y(t)$

  Which can be represented as $\frac{dI}{I(t)} = -a(t)dt$

- Integrating both sides, we get $I(t) = Ce^{-\int a(t)dt}$
  where C = $\pm e^c$
- Function I(t) is called **Integrating Factor**
- Going back to the first equation,
- this $y'(t)I(t) - I(t)a(t)y(t) = I(t)f(t)$ can be written as $\frac{d}{dt}[I(t)y(t)] = I(t)f(t)$
- Integrating both sides, we get $I(t)y(t) = \int I(t)f(t)dt + C$
- Therefore, $y(t) = \frac{1}{I(t)}[\int I(t)f(t)dt + C]$

  $y(t) = \frac{1}{I(t)}[\int I(t)f(t)dt + C]$

  This is called **General Solution** of the ODE.

Example

Newton's law of cooling with integrating factor.
$u'(t) = -k(u - t_0)$

- Integrating factor is $I(t) = e^{kt}$ as $a(t) = -k$
- Multiplying both sides by $I(t)$, we get $u'(t)e^{kt} + ku(t)e^{kt} = ke^{kt}t_0$
- $[u(t)e^{kt}]' = ke^{kt}t_0$
- $u(t)e^{kt} = t_0 k \int e^{kt} dt + C$
- $u(t)e^{kt} = t_0 e^{kt} + C$
- $u(t) = t_0 + Ce^{-kt}$
  Therefore, $u(t) = t_0 + Ce^{-kt}$ is the general solution of the ODE.

- What is **Variation of Parameters?**

Definition

- It doesn't matter if we use integrating factor or variation of parameters, we will get the same solution.
- What is **integral curve?**

Definition

- For each constant, we have a particular solution.
- Geometrical representation in tu-plane is called **integral curve**.(in this case)

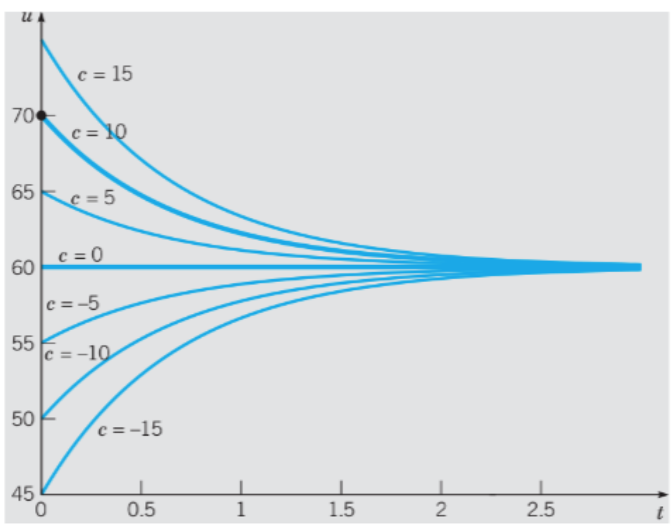


Figure 2: The integral curves of $u(t) = t_0 + ce^{-kt}$ for $k = 1.5$, $t_0 = 60$.

Figure 4: Integral Curve for newton's law of cooling

- As you can see, as time approaches infinity, the temperature is approaching the ambient temeperature. This is because the e^{-kt} is approaching 0 as t approaches infinity.
- Also, when t is fixed, bigger k means faster approaching to the ambient temperature.
- What is **autonomous ODE?**

Definition

Autonomouse ODE is an ODE that does not depend on the independent variable explicitly.

- Example: $\frac{dy}{dt} = 2y$
- Example: $\frac{dy}{dt} = y^2$

- Intro to separable ODE

Definition

- $y'(t) = h(t, y)$ can be $A(t, y) + B(t, y)y' = 0$
- This will be **separable** if $A(t, y)$ is function of t only and $B(t, y)$ is function of y only.
- Then can be integrated as $\int A(t, y)dt + \int B(t, y)dy = C$ where C is arbitary constant.
- Sol'n can be written as $F(t, y) = C$ where this is implicit solution.

Example

- Equation of $sintdt - y^2dy = 0$ is separable
- Sol'n is $y(t) = \sqrt[3]{3(cost + c)}$ where c is arbitary constant.
- Is the equation $t^2(e^y + 1)dt + y^2dy = 0$ separable?
  Yes, this is seperable.
  As t^2 is function of t only and $e^y + 1$ and y^2 is function of y only.
  This can be re-written as $t^2dt + \frac{y^2}{e^y+1}dy = 0$

Lec 3 summary

- Big Idea: Finalizing solvability methods for first ODE.
- Consider following DE.

$A(t, y)dt + B(t, y)dy = 0$

Definition

if $\frac{\partial A}{\partial y} = \frac{\partial B}{\partial t}$, then the DE is **exact**.
Exact tells that the existence of a function f s.t. $\frac{\partial f(t,y)}{\partial t} = A(t, y)$ and $\frac{\partial f(t,y)}{\partial y} = B(t, y)$

- Integrate the first one, we get $f(t, y) = \int A(t, y)dt + a(y)$ where $a(y)$ is arbitary function of y which is independent
- This is similar to calculate the potential function in vector field which can be solved easily.

IMPORTANT!

- If we use integrating factor, we can say that is also exact.
- ANY SEPERABLE ODE IS EXACT, **BUT EXACT ODE IS NOT NECESSARILY SEPERABLE**
- This is trival (Implicit function theorem), therefore, we can say as f(t,y) = C, this is implicit solution.

Example

Exact tells that the existence of a function f s.t. $\frac{\partial f(t,y)}{\partial t} = A(t, y)$ and $\frac{\partial f(t,y)}{\partial y} = B(t, y)$
Integrating $A(t, y)$, the it gives the $f(t, y) = \int A(t, y)dt + a(y)$
Then, $a'(y) = B(t, y) - \frac{\partial}{\partial y}(\int A(t, y)dt)$
As $f(t, y) = \int A(t, y)dt + a(y)$
This gives, $f(t, y) = \int A(t, y)dt + \int B(t, y) - (\int A(t, y)dt)$

Example

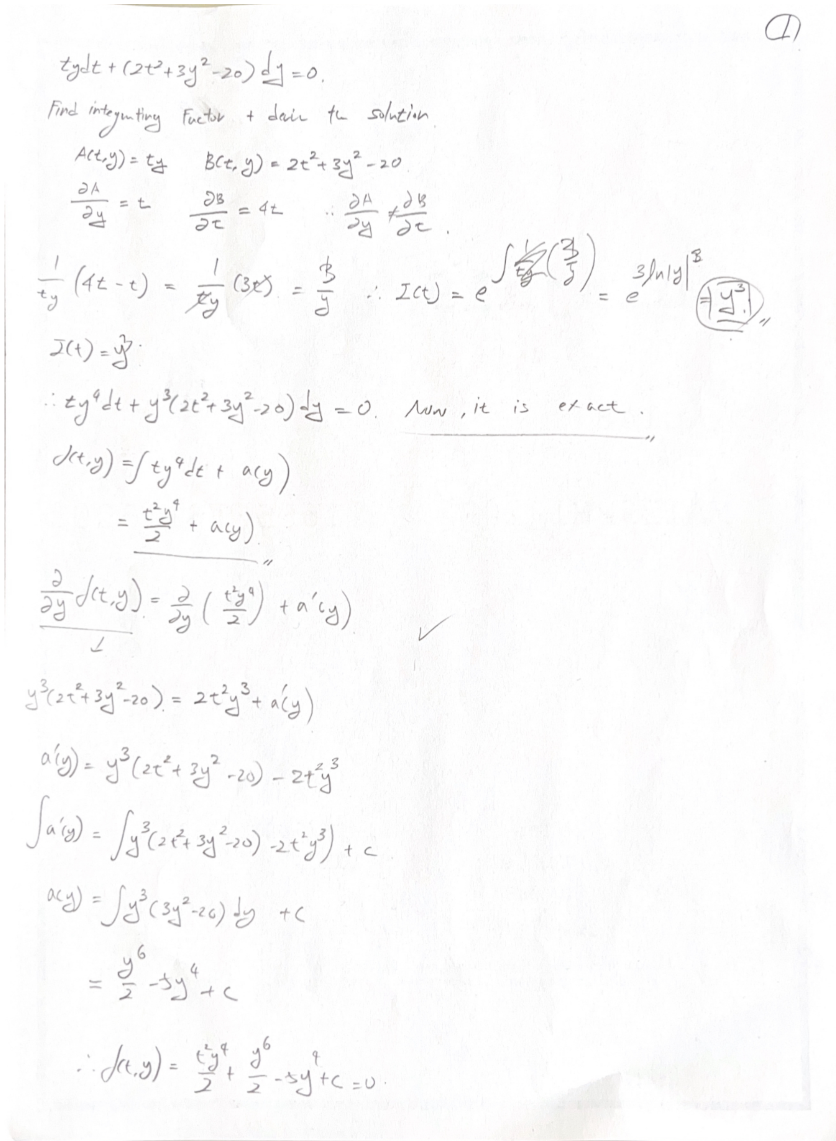
Verify if the following ODE is exact. $(2ty)dt + (t^2 - 1)dy = 0$
As $A = 2ty$ and $B = t^2 - 1$, $\frac{\partial A}{\partial y} = 2t$ and $\frac{\partial B}{\partial t} = 2t$
Therefore, $\frac{\partial A}{\partial y} = \frac{\partial B}{\partial t}$, this is exact.
$f(t, y) = \int 2tydt + a(y) = t^2y + a(y)$
$\frac{\partial f(t,y)}{\partial y} = t^2 + a'(y)$
As $\frac{\partial f(t,y)}{\partial y} = B(t, y)$, therefore, $t^2 - 1 = t^2 + a'(y)$, $a'(y) = -1$
Therefore, $a(y) = -y$
$f(t, y) = t^2y - y = C$
$y(t^2 - 1) = C$
$y = \frac{C}{t^2-1}$. This shows that soluntion will be different corresponding to different C.

Theorem

When the ODE is not exact, we can make it exact by multiplying integrating factor.

- Assume $\left(\frac{1}{B}\right)\left(\frac{\partial A}{\partial y} - \frac{\partial B}{\partial t}\right)$ is a function of t only.
- Then, the integrating factor is $I(t) = e^{\int \left(\frac{1}{B}\right)\left(\frac{\partial A}{\partial y} - \frac{\partial B}{\partial t}\right)dt}$
- Assume $\left(\frac{1}{A}\right)\left(\frac{\partial B}{\partial t} - \frac{\partial A}{\partial y}\right)$ is a function of y only.
- Then, the integrating factor is $I(t) = e^{\int \left(\frac{1}{A}\right)\left(\frac{\partial B}{\partial t} - \frac{\partial A}{\partial y}\right)dy}$

- Following figure shows the process of solving exact ODE.



1

$ty\,dt + (2t^2+3y^2-20)\,dy = 0.$

Find integrating factor + derive the solution

$A(t,y) = ty \quad B(t,y) = 2t^2+3y^2-20$

$\frac{\partial A}{\partial y} = t \quad \frac{\partial B}{\partial t} = 4t \quad \therefore \frac{\partial A}{\partial y} \neq \frac{\partial B}{\partial t}.$

$\frac{1}{ty}(4t - t) = \frac{1}{ty}(3t) = \frac{3}{y} \quad \therefore I(t) = e^{\int \frac{3}{y}} = e^{3\ln|y|} = \boxed{y^3}$

$I(t) = y^3$

$\therefore ty^4\,dt + y^3(2t^2+3y^2-20)\,dy = 0.$ Now, it is exact.

$\psi(t,y) = \int ty^4\,dt + a(y)$

$= \frac{t^2y^4}{2} + a(y)$

$\frac{\partial}{\partial y}\psi(t,y) = \frac{\partial}{\partial y}\left(\frac{t^2y^4}{2}\right) + a'(y)$

$y^3(2t^2+3y^2-20) = 2t^2y^3 + a'(y)$

$a'(y) = y^3(2t^2+3y^2-20) - 2t^2y^3$

$\int a'(y) = \int y^3(2t^2+3y^2-20) - 2t^2y^3) + c$

$a(y) = \int y^3(3y^2-20)\,dy + c$

$= \frac{y^6}{2} - 5y^4 + c$

$\therefore \psi(t,y) = \frac{t^2y^4}{2} + \frac{y^6}{2} - 5y^4 + c = 0.$

Figure 5: Using I.F. to solve exact ODE

Definition

- Bernoulli ODE is the ODE that can be written as $y' + p(t)y = q(t)y^n$ where $n \geq 2$
- Use $u = y^{1-n}$ to make it linear since this is for non-linear equation.
- $u' = (1-n)y^{-n}y'(t)$
- Multiply $(1-n)y^{-n}$ to both sides.
- $y'(1-n)y^{-n} + (1-n)p(t)y^{1-n} = (1-n)q(t)$
- As $y^{-n+1} = u(t)$
- $u' + (1-n)p(t)u(t) = (1-n)q(t)$
- Now, using I.F. or variation of parameters, we can solve this.

Lec 4 summary

- Big Idea: Behavior of Sol'n near certain points(w/o solving)

Example

- t: time
- x(t): Size of group haven't exposed at time
- y(t): Size of group exposed at time t
- N is fixed population and one person entered community to spread disease.
- Therefore, $x(t) + y(t) = N + 1$ (Rate at which the disease spreads corresponding to number of interactions)
- This can be re-wrriten as $x(t) = N + 1 - y(t)$
- $\frac{dy}{dt} = kxy = k(N + 1 - y)y$ where k is constant
- $y' = k(N + 1)y - ky^2$
- $y' - ky(N + 1) = -ky^2$ which is bernoulli ODE
- As n = 2, this is non-linear.
- $u = y^{-1}$
- Multiply by $-y^{-2}$ to both sides. (since $(1 - n)y^{-n}$)
- Then $u' + k(N + 1)u = k$
- Also, we know the integrating factor, $I(t) = Ce^{k(N+1)t}$
- Therefore, multiply both sides to solve.
- $u'Ce^{k(N+1)t} + k(N + 1)uCe^{k(N+1)t} = kCe^{k(N+1)t}$
- $(uCe^{k(N+1)t})' = kCe^{k(N+1)t}$
- $uCe^{k(N+1)t} = \int kCe^{k(N+1)t}dt$
- $uCe^{k(N+1)t} = \frac{k}{k(N+1)}e^{k(N+1)t} + C$
- $u = \frac{1}{N+1} + Ce^{-k(N+1)t}$
- $u^-1 = y = \frac{1}{\frac{1}{N+1}+Ce^{-k(N+1)t}}$
- Therefore, $y = \frac{N+1}{1+C(N+1)e^{-k(N+1)t}}$
  From this, we can see as t increases, this converges to $N + 1$.
  As there is no one is recovering, or dying, this is the expected result.

- Phase Portrait + Direction Field

Definition

Assume f and f' are continuous. A real number c^* is a critical point (stationary point or e.q point) of autonomous ODE $y' = f(y)$ if $f(c*) = 0$

Definition

- Stationary point: fnc's dervitative is 0 at that point. $y' = f(c^*) = 0$
- e.q point: sol'n that doesn't change over time. $y' = f(c^*) = 0$

IMPORTANT!

- Stationary point, e.q point, and critical point are all same thing.
- Equlibria are only constant sol'n of ODE($\frac{dy}{dt} = 0$)
- $y(t) = c^*$ is a constant sol'n that doesnt change over time.

IMPORTANT!

General Step of drawing phase portrait

- Find all e.q points of ODE
- Make table
- $f'(c^*)$ sign to observe behaviour. Then we can also draw sol'n curves.

- Now let's consider the disease example.

Example

- Find $\frac{dy}{dt} = ky(N+1-y) = 0$ which is e.q points
- When $y_1 = 0$ and $y_2 = N+1$ is e.q points

Check to draw phase portrait			
Interval	Sign of f(y)	y(t)	Arrow
$(-\infty, 0)$	$-$	decreasing	$\downarrow$
$(0, N+1)$	$+$	increasing	$\uparrow$
$(N+1, \infty)$	$-$	decreasing	$\downarrow$

This will generate the following phase portrait.

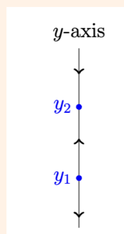


Figure 6: Phase Portrait

From this, we can draw the sol'n curve.

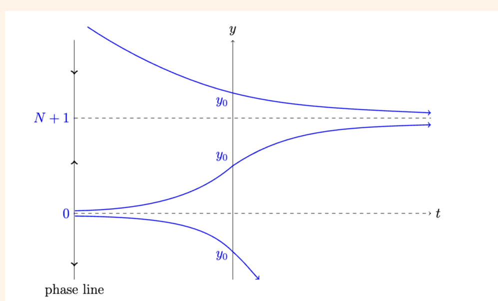


Figure 7: Sol'n Curve

IMPORTANT!

At e.q points, there is asymptote, since they don't change over time.

- Following figure will show what we need to see from this graph.

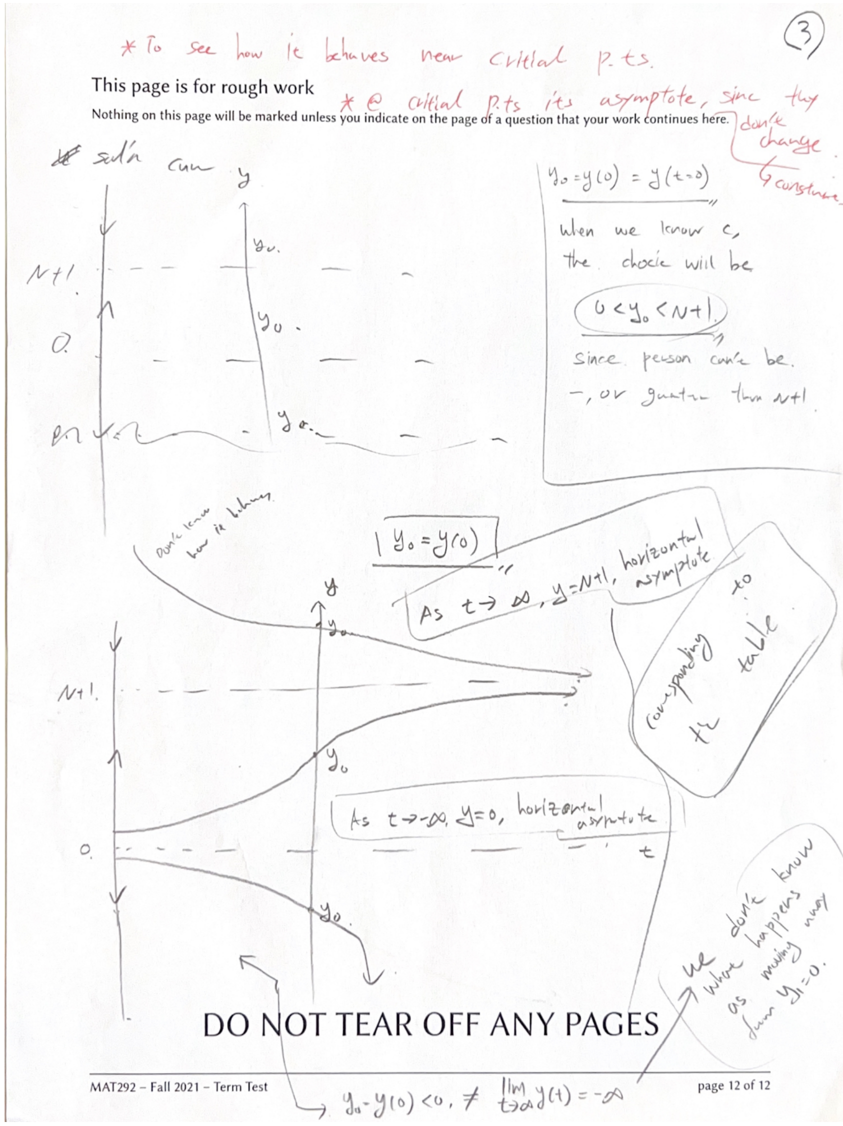


Figure 8: details

Lec 5 summary

- Big Idea: Behaviour of sol'n cuves near e.q points
- Let y^* be e.q point of $y' = f(y)$(autonomous)

Theorem

There are 4 types of behaviors of sol'n curves near e.q points.

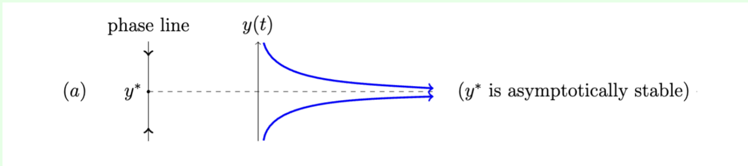


Figure 9: y^* is Asymptotically Stable / Attractor

- Figure 6 is when $\lim_{t\to\infty} y(t) = y^*$

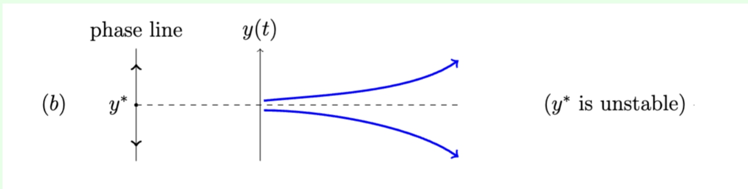


Figure 10: y^* is Unstable / Repeller

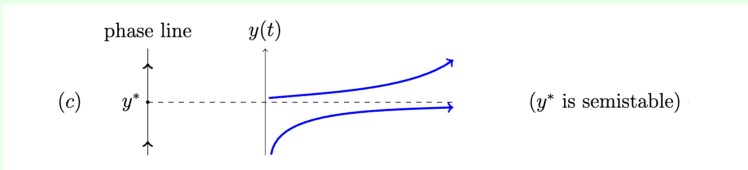


Figure 11: y^* is Semi-Stable

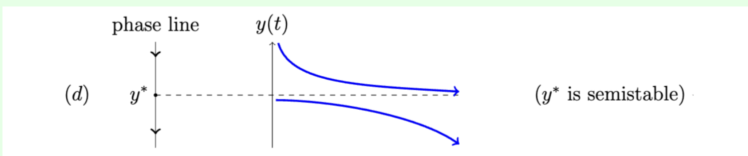


Figure 12: y^* is Semi-Stable

IMPORTANT!

Figure 11 and 12 shows both attractor and repeller at the same time.

Example

Consider $y_1 = 0$ and $y_2 = N + 1$ are e.q points of $\frac{dy}{dt} = ky(N + 1 - y)$
y_1 is repeller and y_2 is attractor.

Theorem

Let y^* be e.q point of autonomous ODE $y' = f(y)$ and assume that f has a continuous first order derivative in neighbourhood of y^*.
Then,

- If $f'(y^*) < 0$, then y^* is asymptotically stable.
- If $f'(y^*) > 0$, then y^* is unstable.

Example

This can be applied to disease example.
As $f(y) = ky(N+1-y)$, $f'(y) = k(N+1-2y)$
When $y^* = 0, f'(y^*) = k(N+1) > 0$, therefore, y^* is unstable.
When $y^* = N+1, f'(y^*) = -k(N+1) < 0$, therefore, y^* is asymptotically stable.

This is corresponding to the phase portrait.

IMPORTANT!

Sometimes, you need to check the two different interval to see the behaviour of the sol'n curve before drawing the phase portrait.
Example can be found for $y' = f(y) = (y-1)^2$ which can be found in lec note 5.

IMPORTANT!

Solution curve has no discontinuity.

- $\frac{dy}{dt} = f(t, y)$ in interval I of R
- Then y is differentiable, therefore, continuous.

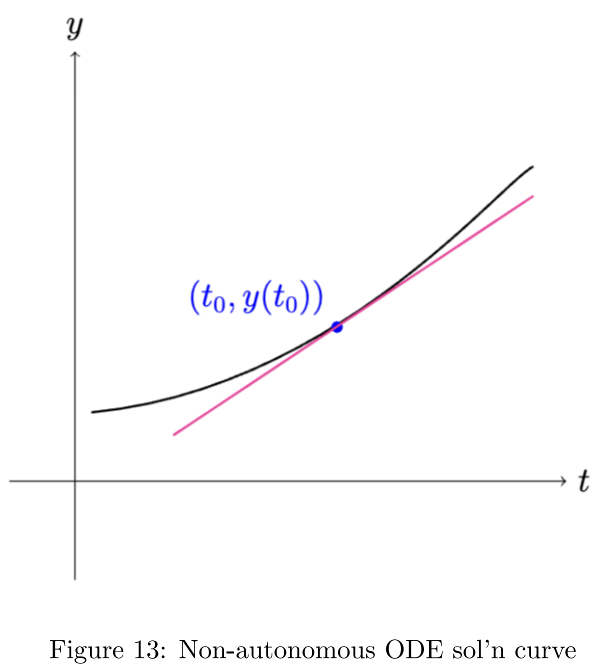


Figure 13: Non-autonomous ODE sol'n curve

Definition

Direction field of $\frac{dy}{dt} = f(t, y)$ is the collection of all tangent lines in rectangular grid of the plane, passing at each point (t,y) of the grid with slope $f(t, y)$.

- Direction field is approximation of sol'n curve, and detect the behaviour of sol'n curve.

Lec 6+7 summary

- Big Idea: Modeling first order ODE linear and non-linear

Step-By-Step

- **Process of modeling**
- Identify the variables (dependent, independent) and the parameters likely to change the phenomenon (physical, sociological,economic,...) we wish to describe.
- Make reasonable assumptions and hypotheses which are as close as possible to the observations (use empirical laws). In many situations, we want to understand the rate of change of the principal dependent variable with respect to one or more independent variables. This involves derivatives and leads to differential equations.
- Solve the resulting equation (analytically, graphically) and verify that the properties of solutions agree with the model predictions.

Lec 8 summary

- Big Idea: IVP

Definition

$\frac{dy}{dt} = f(t, y)$ with $y(t_0) = y_0$ for

$$t_0 \in I$$

and y_0 is given constant, is called initial value problem.

- Solving $y' = f(t, y)$ may lead to one parameter family of solutions, then what happen when we have initial condition?
- Following is the geometrical interpretation of IVP.

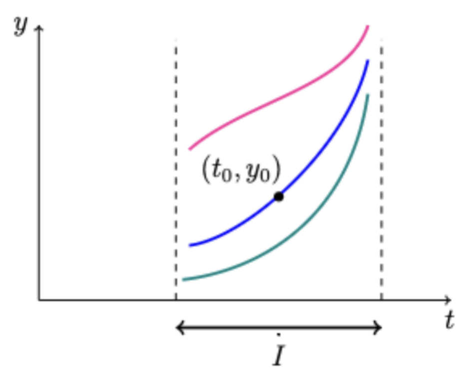


Figure 14: Geometrical INterpretation of IVP

Theorem

Let R be rectangular in the $t - y$ plane defined $t \in (\alpha, \beta)$ and $y \in (\mu, \gamma)$ and containing the point (t_0, y_0). If $f(t, y)$ **and** $\frac{\partial f}{\partial y}$ **are continuous** in R, then there exists $h > 0$ such that $I_0 = (t_0 - h, t_0 + h)$ is contained in (α, β) and there exists a unique solution $y(t)$ of the IVP $y' = f(t, y)$, $y(t_0) = y_0$ for $t_0 \in I$ on I_0.

IMPORTANT!

- This is for general case, including, the non-linear case.
- Two sol'ns cannot intersect each other.
- **Even though this theorem fails, we don't know if there is unique, infinite, or no sol'n.**
- A.K.A **Sufficent but not necessary condition**

Theorem

$f(t, y) = a(t)y(t) + b(t)$ If $I \in (\alpha, \beta)$ is that a and b are continuous on I, then there exists a unique solution $y(t)$ of the IVP $y' = f(t, y)$, $y(t_0) = y_0$ for $t_0 \in I$ on I_0.

IMPORTANT!

- This is for linear case.
- Two sol'ns cannot intersect each other.
- **Even though this theorem fails, we don't know if there is unique, infinite, or no sol'n.**
- A.K.A **Sufficent but not necessary condition**

Lec 9 summary

- Big Idea: Systems of linear first order differential equations