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PHY 180 Pendulum Lab

Figure 1. Front View of the Pendulum

1. Introduction

Lab 1 ultimately elaborates on the idealness of the pendulum. Analyzing the relationship between angular amplitude and period demonstrates that the pendulum is ideal by generating a slope close to 0. This also indicates that the period is dependent on the angular amplitude of the pendulum out of range of small angle. This lab also demonstrates how small angle effects on pendulum. Also, the Q factor is calculated through two different procedures to identify if this pendulum can be considered an ideal model. As this pendulum generates a high Q factor, it doesn't lose energy quickly which is ideal. Even though this lab is operated in non-ideal circumstances, results from two experiments prove that this pendulum is close to ideal model. Lab 2 ultimately conveys how the length of the pendulum affects the period and quality factor. Q factor depends on the length of the pendulum. Also, the period depends on the length of the pendulum. Tau acts as how much off the result from the theoretical. As tau has a specific ratio relationship with the length of the pendulum, this demonstrates that the Q factor and period are dependent on each other with tau.

2. Methods and Procedures

2.1 The design/justification of pendulum

Previous pendulum set up and justification is written in Appendix A.

The perfectly symmetrical ball is used for the mass and is fixed to the thread tightly by the tape. Tape reduces the shakiness while oscillating, preventing the three-dimensional motion that the pendulum possibly generates.

The fishing wire is selected to reduce the possibility of the pendulum being affected by the mass and friction of the thread.

Put the carrier on the closet and make the hand bar come out. Since there is shallow cite that fishing wire can perfectly fit, therefore, this reduce the 3D motion that can be generated.

For using the app "tracker," the length of a tile is necessary to calibrate the data. Therefore, the width of the tile is measured, which is 14.7cm.

2.2 Experimental Process

Lab 1:

The mass is set at the equilibrium position, and then mass is lifted to 30° (π/6 rad) with the equilibrium state. Then, the water bottle lead is released from that position. The only force applied to the mass is the weight itself, caused by gravity.

As taking the video, I taped the phone to the mirror in the washroom to avoid the shakiness.

After the pendulum is stopped completely, the program "Tracker" is used for analyzing and extracting data from the video. There is a feature that tracks the mass automatically. While monitoring the mass, it also provides the distance the mass travels on the x-axis and y-axis as the time elapsed.

With the x-axis and y-axis value, the "Python" code calculates the angular displacement for plotting the graph.

Lab 2:

The angle is adjusted to 20° (π/9 rad). How I justified for the small angle is written in Appendix B. The length of the pendulum is altered from 20cm to 110cm, and interval increases by 10cm.

Taking the video of pendulum with each length until it reaches 46% of its initial amplitude. The method of calculating Q factor will be discussed in section 3.4.

For evaluating for the period with different length, taking the video of pendulum with each length until it reaches to the point where it starts from for ten times.

3. Results and Analysis

3.1 Graphs

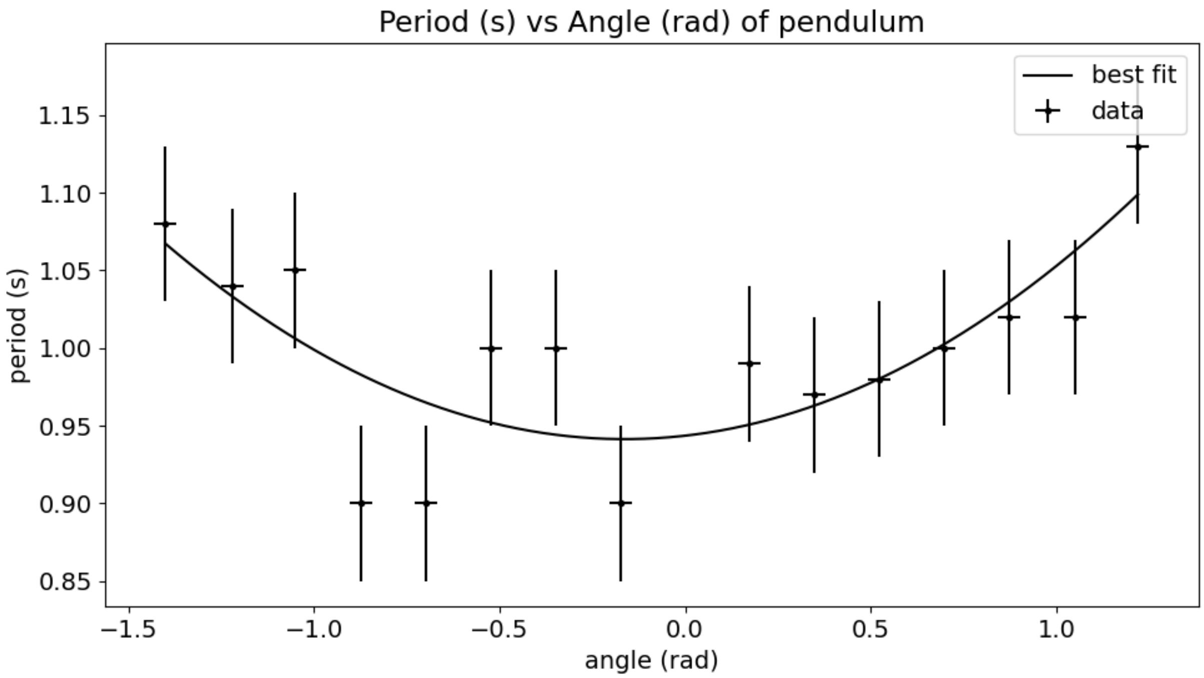

Figure 2: Period (s) vs. Angle (rad) of a pendulum (Quadratic Equation)

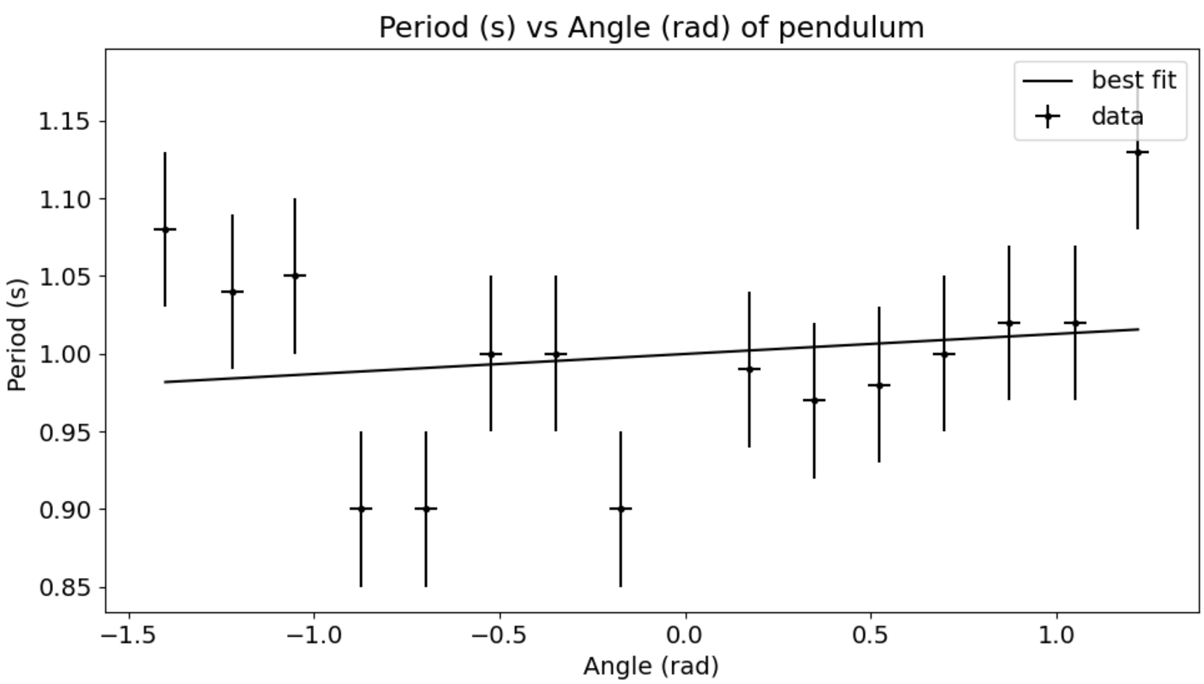

Figure 3: Period (s) vs. Angle (rad) of a pendulum (Linear Equation)

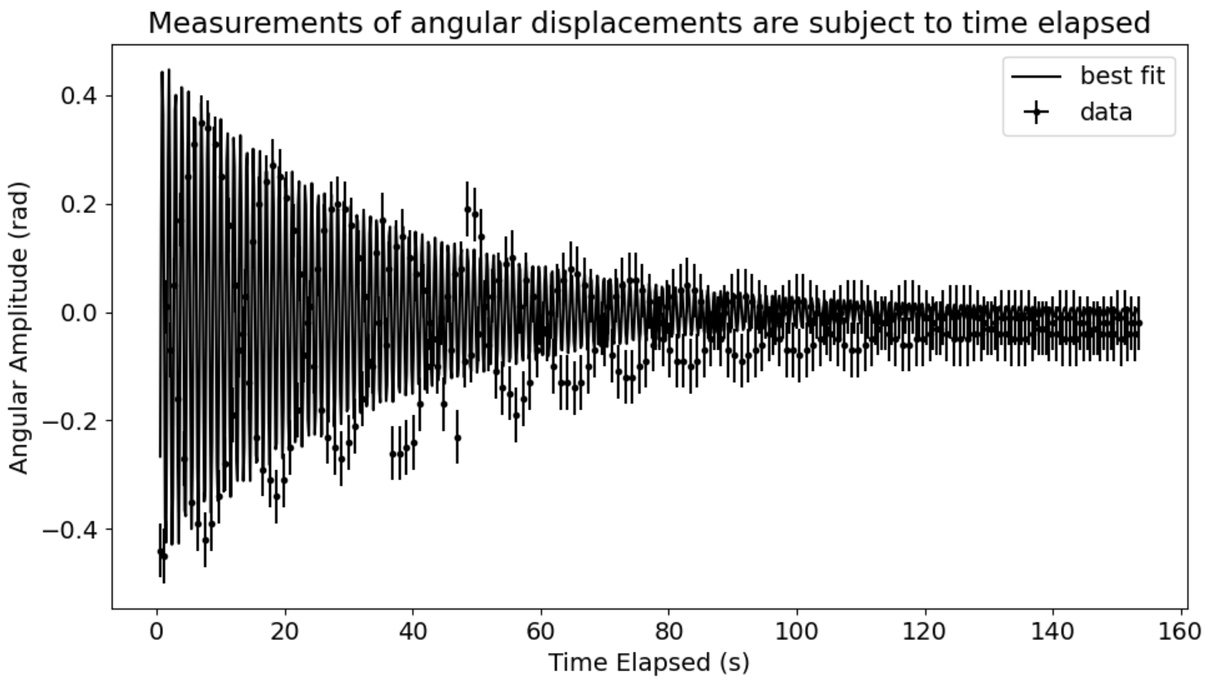

Figure 4: Measurements of angular displacements are subject to the time elapsed

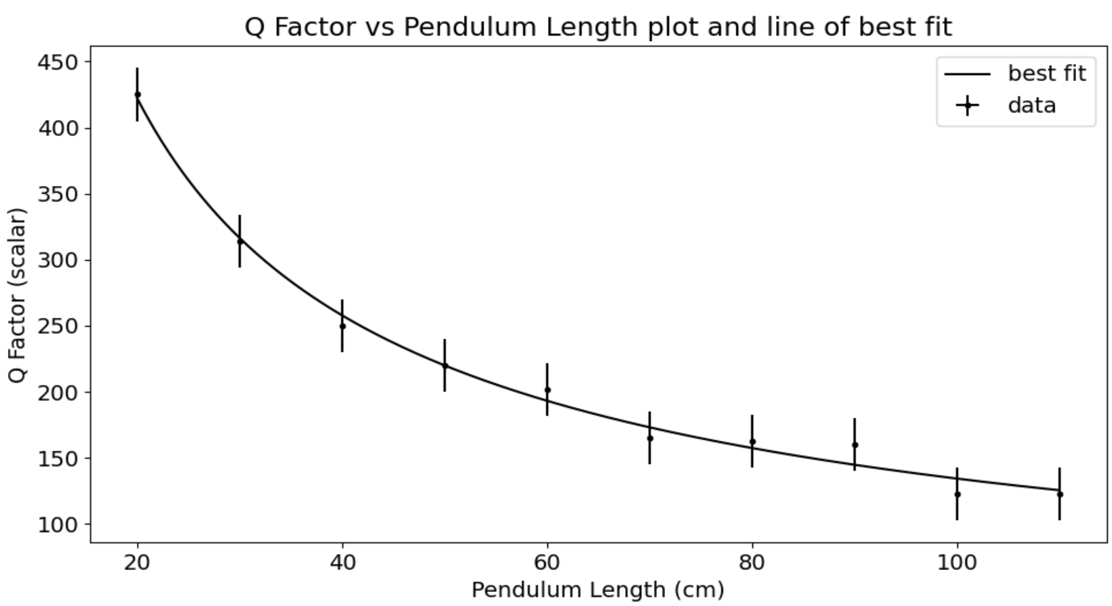

Figure 5: Measurement of Q factor while changing the length of the pendulum

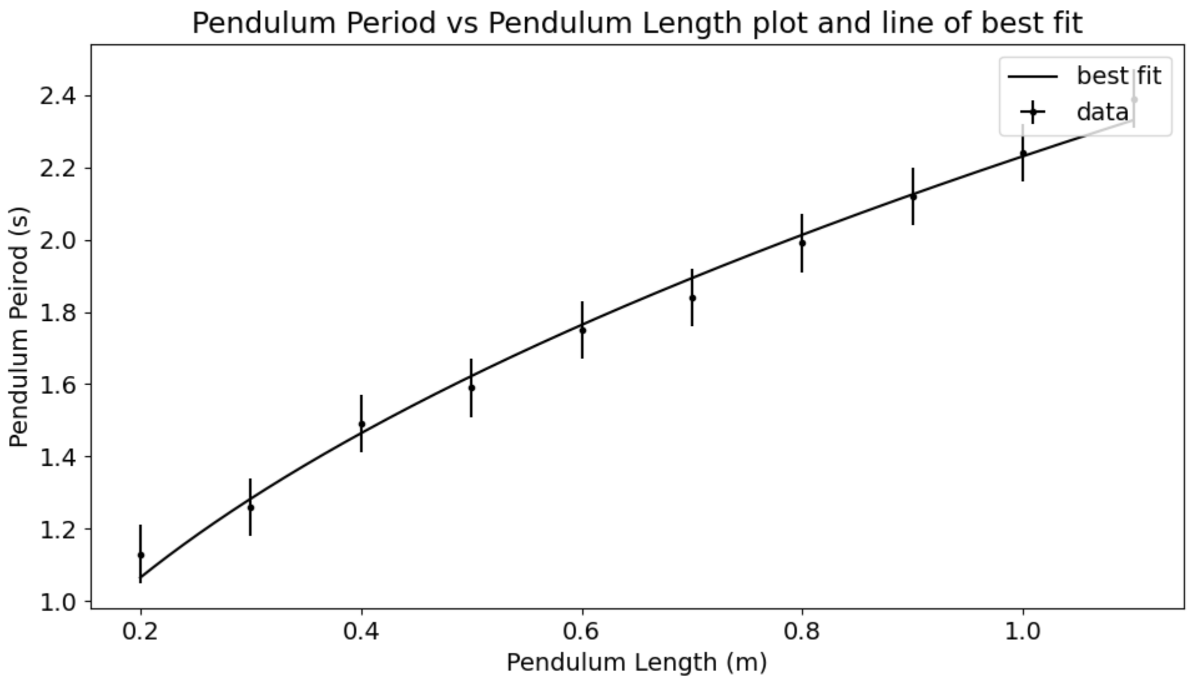

Figure 6: Measurement of period while changing the length of the pendulum

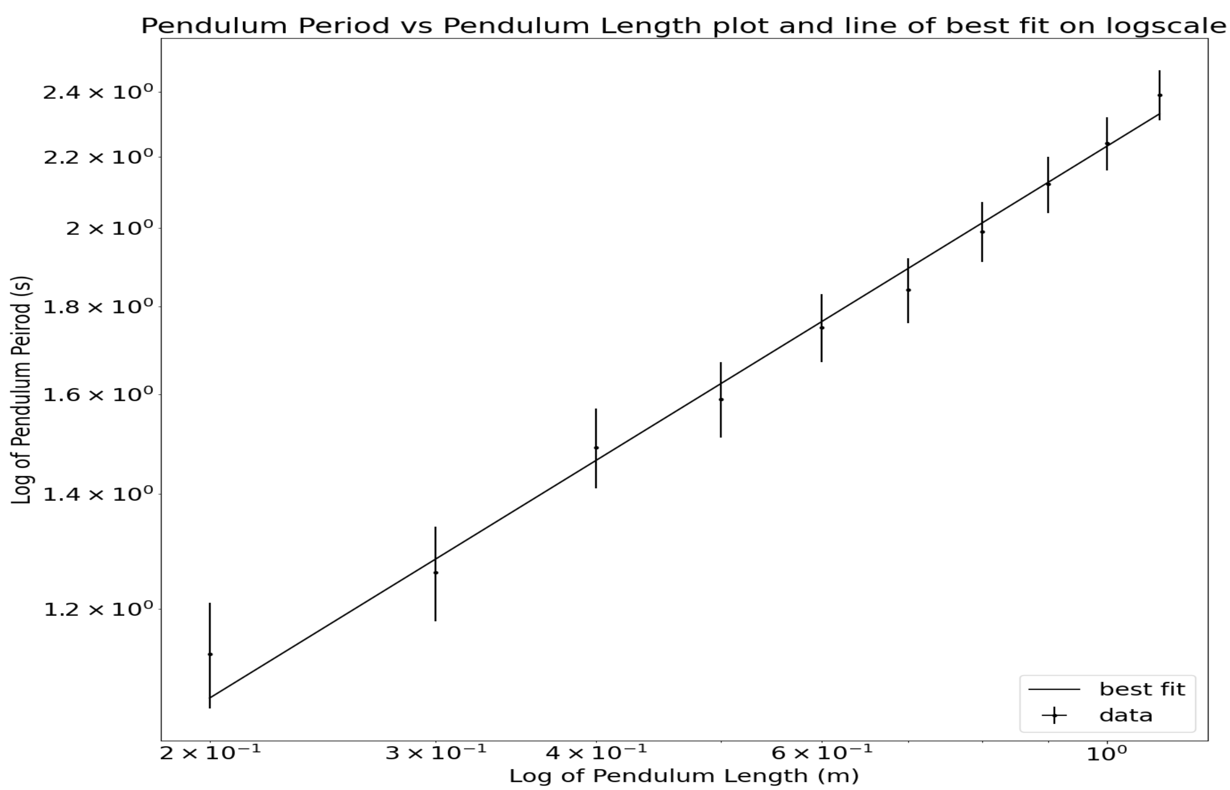

Figure 7: Figure 6 in a log scale

3.2 Observation / Brief Analysis of Graphs

Figure 2’s data shows if the pendulum is symmetrical by plotting the data. This Figure 2’s data indicates that the period is dependent on the angle. When there is a simple pendulum with the small angle, it can be assumed that period is independent on the angle, however, since this is damped pendulum with not negligible angle, angle is considered.

Figure 3's data demonstrates that my pendulum is symmetrical by the slope itself. The slope of the graph is generated by "Python" which is 0.0129 ± 0.001, close to zero slopes. Based on this data, the pendulum can be described as an ideal.

Figure 4's data conveys that the pendulum's period decays exponentially. This graph provides the values of tau and period for calculating the Q factor.

Figure 5’s data conveys that the Q factor depends on the length of the pendulum. Specific relationship will be discussed in section 3.7.

Figure 6 demonstrates that the equation of $T = 2\sqrt{L}$ (3) is consistent. As the graph is shown the relationship of square root, this demonstrates that the small angle approximation is correct. Therefore, this conveys that the period of the pendulum depends on the length of the pendulum. This graph also generates k and n and this will be discussed in section 3.8.

Figure 7’s data is generated by taking log-log plot on bot x, and y axis of the figure 6’s data. As this graph generates the positive linear graph, this ultimately proves that figure 6 is a power function.

3.4 Calculation of the Q factor

Lab 1

- **Method 1: Using the equation through python**

The program is written through "Python" to calculate the Q - factor with uncertainties.
The "Python" script is written in Appendix C.
$Q=\pi\frac{\tau}{T}$ [1]
This is the equation that is written in the lab manual.

Q factor with error rate; Q = 109 ± 8

- **Method 2: Counting the oscillation**

$$2N = Q \quad [2]$$

N = Number of oscillations until it reaches 20% of its initial amplitude

Counting how many oscillations the pendulum performs before getting to 20% of its initial amplitude provides the value of Q/2. Derivation of Q is written in Appendix D.
Q factor with error rate; Q = 151 ± 1
Overall, the first Q factor is 109 ± 8, and the second Q factor is 151 ± 1.

Lab 2

- **Method: Counting the oscillation**

$$4N = Q \quad \text{(Equation provided in lab manual)}$$

N = Number of oscillations until it reaches 46% of its initial amplitude

Counting how many oscillations the pendulum performs before getting to 46% of its initial amplitude provides the value of Q/4. Therefore, multiplying by 4 provides the quality factor of the pendulum by the length. As the experiments are proceeded ten different lengths, this method generates 10 different Q factors.

As Lab 1 conveys, the magnitude of uncertainties of Q factor from the equation is bigger than counting the oscillation. Therefore, the method of counting the oscillation is chosen.

However, there is a small adjustment from last method. The way of calculating of Q factor is altered from 2N = Q to 4N = Q. More data points will generate more accurate values.

3.5 Uncertainties

Uncertainty of Period/Tau

The uncertainty of Period/Tau is calculated by the program “Python” which is discussed in section 3.4.

Human Error

Even though the “Tracker” automatically recorded the data, it requires me to click the mass again when they can’t find it. It happened one time per 1000 frames. As the number of total frames is 8997 for the experiment of getting the Q factor, these nine errors can be negligible throughout the 8997 frames.

Also, since human counts the number of oscillations, there can be an error while counting the number of oscillations which is ± 0.5. However, since we multiply by 2 for lab 1, and multiply by 4 for lab 2 for calculating the value of a Q factor. Therefore, the uncertainties of lab 1 is ± 1, and the uncertainties of lab 2 is ± 2.

Friction/mass of the thread

There were two main aspects: massless and frictionless. However, the ideal thread does not exist; therefore, the military-grade thread is chosen to minimize the effect. Thus, the mass and friction of military-grade thread are ignored throughout the whole experiments.

Tracker

To reduce the uncertainties, the calibration stick is used in “Tracker”, and the x and y-axis sticks are used. As the error of the calibration stick is ± 2mm, and the x and y data are used to calculate the angles, this error rate is also applied to the angle as ± 0.02 rad.

Air Resistance

As the face of mass that encounters the air is not infinitely thin, air resistance must be through the pendulum. Other than operating the pendulum in vacuum circumstances, this will never get perfectly resolved.

Meter Stick

Meter stick is used to calculate the length of the pendulum. The uncertainty of ± 0.0005m is used to calculate the length of pendulum. ± 0.001m / 2 = ± 0.0005m. The reason why dividing is because there is a ± sign.

Camera frame with the period

Since the video is filmed by the camera with 30 frames per second, the uncertainty of period becomes ± 1/60 seconds. As 30 FPS means 1 frame is done in 1/30 seconds, and there is also ± sign, therefore, it is divided by 2.

3.6 Analysis of the different two Q factors

The Q factor generated from my pendulum conveys that my pendulum is close to ideal and efficient as my first Q factor is 109 ± 8, and second Q factor is 151 ± 1. However, two Q factors determined by two different methods indicate that they don't agree with each other. The error range doesn't meet each other in the specific point. The primary reason is how I calculate the different uncertainties between two procedures. For the first one, period is the biggest uncertainties. More digits timer will reduce the uncertainties. For the second one, measuring angle is the biggest uncertainties. As trigonometry is used to calculate for the angle, more accurate x, and y lengths will reduce the uncertainties. Two reducing methods will generate more closer Q factors.

3.7 Analysis of the relationship between Q factors and length of the pendulum

Theoretical relationship between Q factors and length of the pendulum can be derived by using two equations: $Q = \pi\frac{\tau}{T}$[1], and $T = 2\sqrt{L}$[3]. This equation provides the derivation of $Q = \frac{\pi\tau}{2\sqrt{L}}$.

First, there is the limitation of this equation, since the period used for derivation is for the small angle approximation, therefore, the derivation is also limited to small angle only.
However, as justification of small angle approximation applied, this will not be likely the case.

Second, this is significant to analyze what is the role of tau. This can be analyzed by the result generated by "python". Python generated the n value as -0.71 ± 0.02, however, n should be close to -0.50 if tau is constant. Tau is the tool to capture how much off it was from the predicted value. This conveys that tau is not constant throughout the experiment, and this provides the relationship of $Q \propto \frac{\tau}{\sqrt{L}}$. To calculate tau value with the experimentally collected Q value data and length, this generates the relationship of tau with length.

$L^{-0.71} \propto \frac{\tau}{L^{0.5}}$

$\tau \propto L^{-0.21}$ for this specific experiment.

If this experiment is run in vacuum system, since tau is related with the air resistance, tau would be constant, then this would generate the relationship of $Q \propto \frac{1}{\sqrt{L}}$. Tau should be independent of the length of the pendulum. One factor that gives arise the length dependence of the tau is an air resistance. The Q factor is affected by those two relations with specific ratio. Therefore, length and the Q factors are dependent each other.

3.8 Analysis of the period and length of the pendulum

As the function $T = kL^n$ provided in the lab manual, the "python" generates the value of n and k from the collected data; k = 2.23 ± 0.02, and n = 0.46 ± 0.02.
K should be equal to 2 and n should be equal to 0.5 theoretically.
However, experimental k value and theorical k doesn't match within the uncertainties. Since the experiment is run with the initial angle within small angle, theoretical k and experimental k should be equal.
In this case, tau is the biggest factor that alters the data of k. As being discussed in 3.7, length depends on the tau. This demonstrates that the value of k is altered due to tau. If the tau is not presented through

whole experiment, this will generate the ideal k value. For instance, if the experiment is operated in vacuum system, this would generate the perfect value for k.

4 Conclusion

The ultimate purpose of these pendulum experiments is to make a symmetrical and ideal pendulum for further investigation. The slope generated from the angle and period and the Q factor indicates that the pendulum is close to ideal and symmetrical. However, data demonstrates that the initial angle of the pendulum is significant factor of pendulum, since at out of range of small angle, the period becomes dependent on the angular amplitude of the pendulum. Lab 2 ultimately conveys on how the length of pendulum affects the period and quality factor. This lab also demonstrates how the small angle approximation affects to period and Q factor. Tau acts as how much off the experimental from the theoretical value. Due to presents of tau, the experimental value is slightly away from the theoretical. However, tau has a specific ratio with the length of pendulum, this demonstrates that Q factor, and period are dependent each other with tau.

Appendix A

Previous Pendulum Setting and Design Justification

Even though the lab 1 conveys that the pendulum is an ideal model with high Q factor, the new pendulum is set up, so that more accurate result can be produced.

The water bottle lid is used for the mass and is fixed to the thread tightly by the tape. Tape reduces the shakiness while oscillating, preventing the three-dimensional motion that the pendulum possibly generates.

The military-grade thread is selected to reduce the possibility of the pendulum being affected by the mass and friction of the thread.

This pendulum is operated in the washroom, and the pole for the shower curtain is used to adjust the thread length more quickly for further experiments.

For using the app "tracker," the length of a tile is necessary to calibrate the data. Therefore, the width of the tile is measured, which is 14.7cm.

In terms of the length of the pendulum, the shorter length of the pendulum is chosen. This is tested in both ways, experimentally and theoretically. While trying for the variable lengths of the pendulum, the shorter length generates the higher Q factor. Also, the one equation is derived from Q = $\pi\frac{\tau}{T}$[1], and T = $2\sqrt{L}$[3] and it demonstrates that as the pendulum's length increases, the Q factor will decrease exponentially.

Appendix B

Justification of Small Angle Approximation

As the mass of the lab 2 has changed from lab 1, the lab 1 has repeated with new set up to find out the new range of the small angle. By operating number of experiments to find out the range of the small angle, it is acknowledged that until 20°can be considered as small angle. Since T = $2\sqrt{L}$[3] is for small angle approximation, simply, the point before when the period changes can be considered as the small angle. From the angle when the period is similar to the C value until when the period drastically changes.

Appendix C

```
# This is for determining the Q factor
import math
tau = 35.19179439685943 # error: +/-
tau_error = 2.6449351864311166
T = 1.0139715477971942 # error: +/-
T_error = 0.00032335891589299224
pi = math.pi
Q = int(pi * (tau / T))

error_rate = max((tau_error / tau), (T_error / T))
error = int( Q * error_rate )

print ("Q: " + str(Q) + " error: +/-" + str(error))
```

Appendix D

Derivation and Calculation of Q factor

Counting the number of changing signs before reaching 20% of its initial amplitude divided by two will provide the number of oscillations. For these reasons, the Number of changing signs indicates the Q factor since Q / 2 = (the number of changing signs) / 2, which leads to Q = the Number of changing signs.

As the initial amplitude of the pendulum is 0.43 ± 0.02 rad, and (0.43 rad) * (0.2) is 0.086 ± 0.02 rad. As counting the number of times, it changes signs; this gives the Q factor of 151 ± 1.

Appendix E

Q factor means how the pendulum is efficient in terms of energy. Generally, when the Q factor is equal to 1, this is considered a low Q factor as the pendulum's amplitude will decay rapidly. Therefore, the higher Q factors demonstrate that this will not likely lose energy, which will be decayed more slowly compared to the pendulum with the lower Q factor.

Appendix F

Analysing the graph of tau and length to apply on Q factor and length

As tau can be described as the exponent of -0.21 to the function of length, this will generate an inverse function. Therefore, tau is the biggest factor that can alter the relation between Q factor and length. Since as length increases, tau also gets decreased in the ratio of $\tau \propto L^{-0.21}$.

Appendix G

Improvements for Next Experiments

The pendulum should be perfectly symmetrical. Regarding uncertainties, air resistance can be improved by modifying the shape of the mass. The air resistance is one of the factors for the errors; therefore, controlling this will extract perfect data. Therefore, by reducing the surface area of the mass, the air resistance will be decreased. Thus, the Period vs. Angle slope becomes closer to 0.

Citation

[1] M. W. MacCall, "3.7," in Classical mechanics: From Newton to einstein, Oxford: Wiley-Blackwell, 2010.
[2] B. Crowell, "18," in Light and matter, Fullerton, CA: B. Crowell, 2019.

[3] "Physics - simple harmonic motion," *University of Birmingham*. [Online]. Available: https://www.birmingham.ac.uk/teachers/study-resources/stem/physics/harmonic-motion.aspx#:~:text=Pendulum%20%2D%20Where%20a%20mass%20m,g%20is%20the%20gravitational%20acceleration. [Accessed: 30-Sep-2022].