

Shinja19

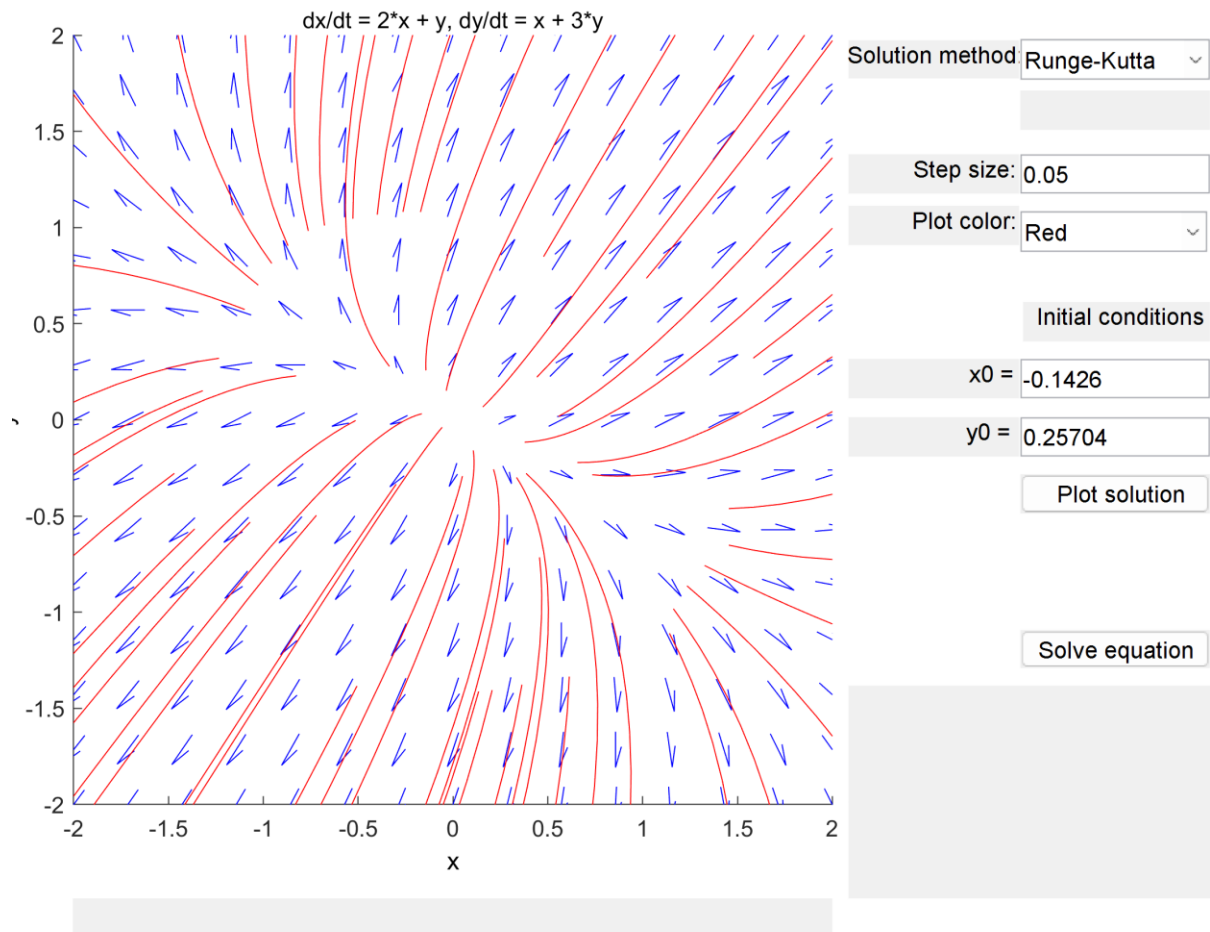
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(c) Compute the eigenvalues of the matrix (you do not need to show your calculations). Using the eigenvalues you computed, justify part (b).

To avoid numerical error, you should use Runge-Kutta solver with a step size of 0.05. Change the display parameters, if necessary, to best understand the phase portrait.

4.1. $\frac{dx}{dt} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} x$

a)



b)

e.g. on Asymptotic Stability: Unstable

Behaviour (sink, source, saddle-point, spiral, center, proper node, improper node): Nodal Source

Clockwise or Counter-clockwise movement: N/A

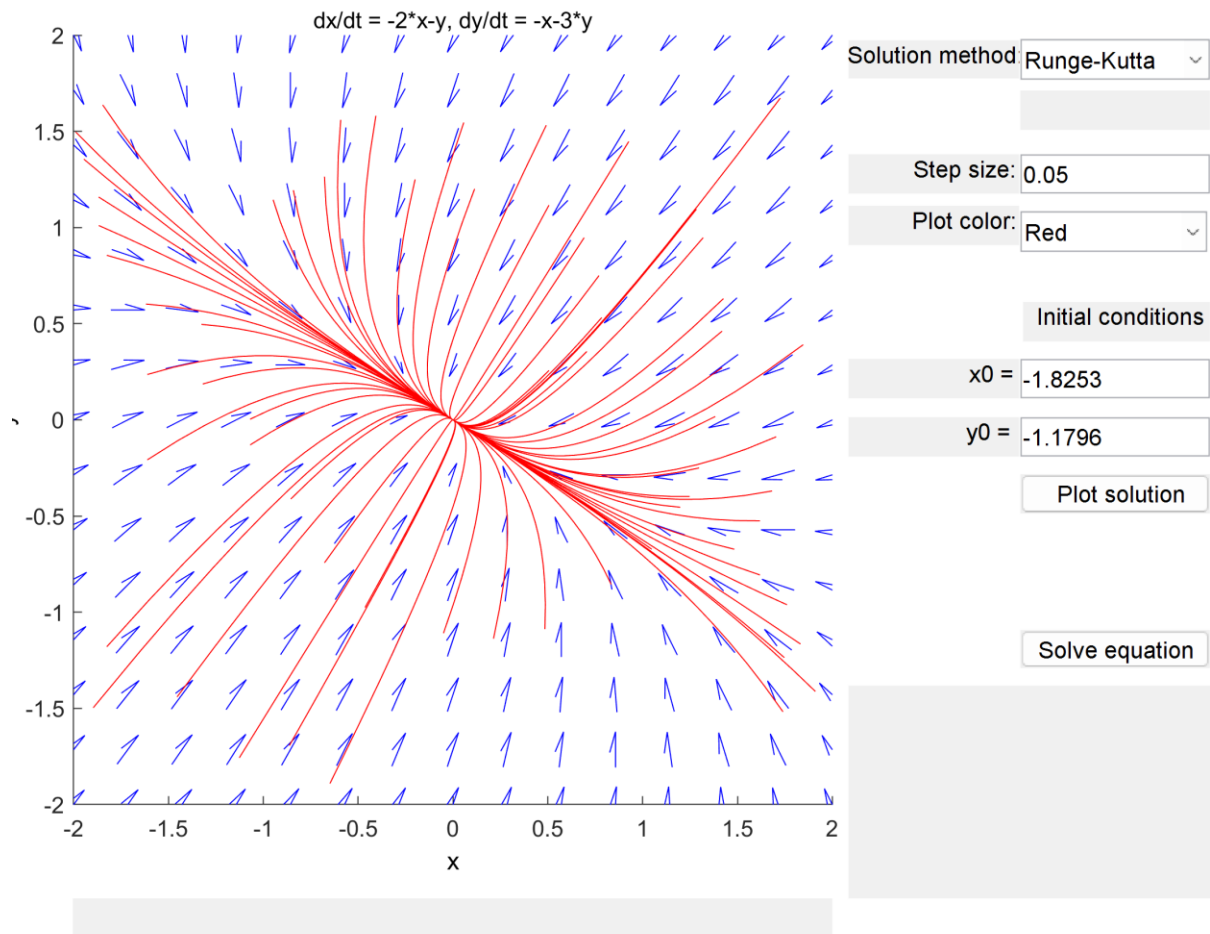
c)

Eigenvalues of the matrix: $0.5(5+\sqrt{5})$, $0.5(5-\sqrt{5})$

Justification: Eigenvalues are real, not equal, and both are positive. Therefore, nodal source, and unstable is promising result.

4.2. $\frac{dx}{dt} = \begin{bmatrix} -2 & -1 \\ -1 & -3 \end{bmatrix} x$

a)



b)

1. e.q. on Asymptotic Stability: Asymptotically stable

2. Behaviour (sink, source, saddle-point, spiral, center, proper node, improper node): Nodal Sink

3. Clockwise or Counter-clockwise movement: N/A

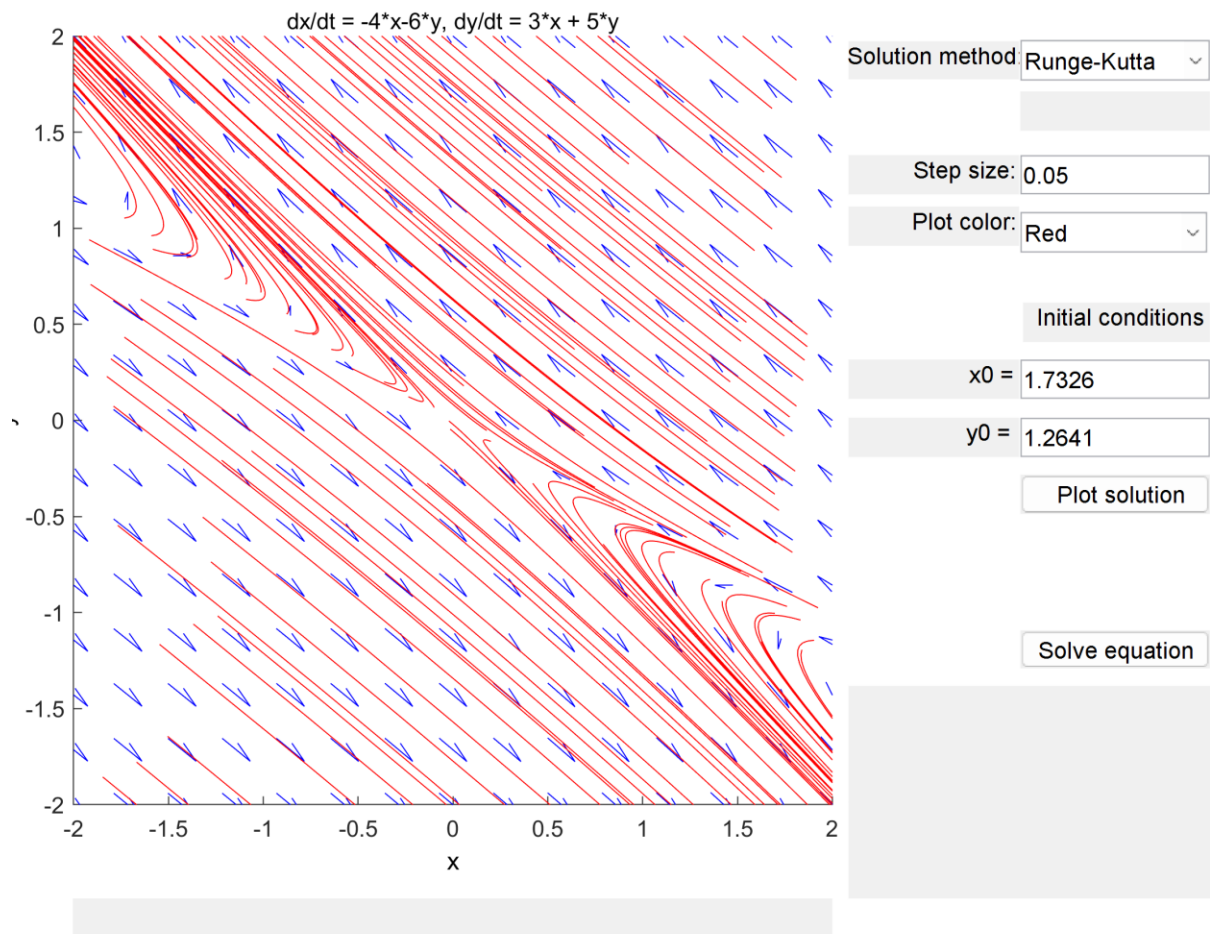
c)

Eigenvalues of the matrix: $0.5(-5-\sqrt{5})$, $0.5(-5+\sqrt{5})$

Justification: This is also promising result, as eigenvalues are both real, both negative, and they are not the same. Therefore, this has asymptotically stable and nodal sink.

$$4.3. \frac{dx}{dt} = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix} x$$

a)



b)

1. e.q. on Asymptotic Stability: Unstable

2. Behaviour (sink, source, saddle-point, spiral, center, proper node, improper node): Saddle-point

3. Clockwise or Counter-clockwise movement: N/A

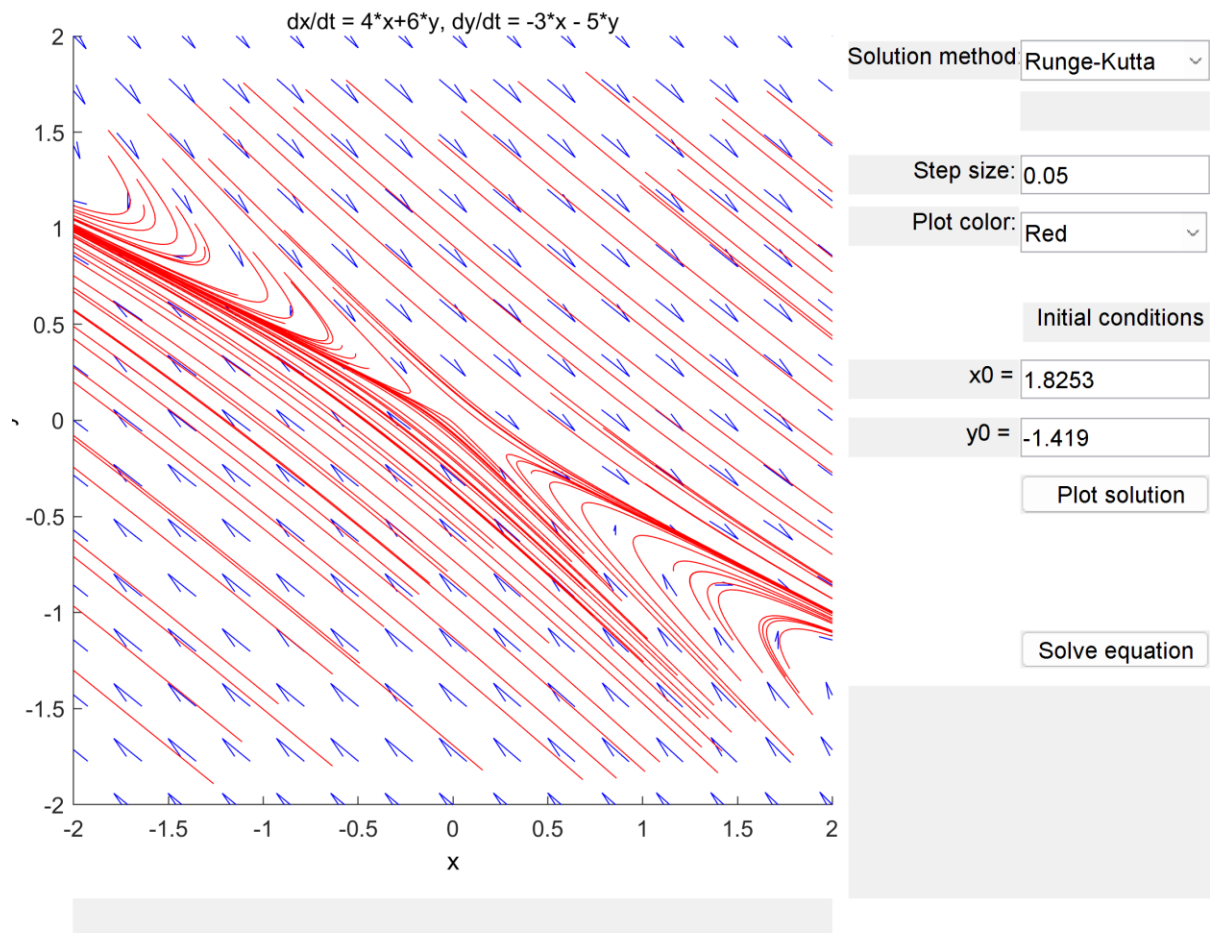
c)

Eigenvalues of the matrix: 2, -1

Justification: This is promising result since two eigenvalues are different, opposite sign and both are real which leads to saddle point with unstable.

4.4. $\frac{dx}{dt} = \begin{bmatrix} 4 & 6 \\ -3 & -5 \end{bmatrix} x$

a)



b)

1. e.q. on Asymptotic Stability: Unstable

2. Behaviour (sink, source, saddle-point, spiral, center, proper node, improper node): Saddle Point

3. Clockwise or Counter-clockwise movement: N/A

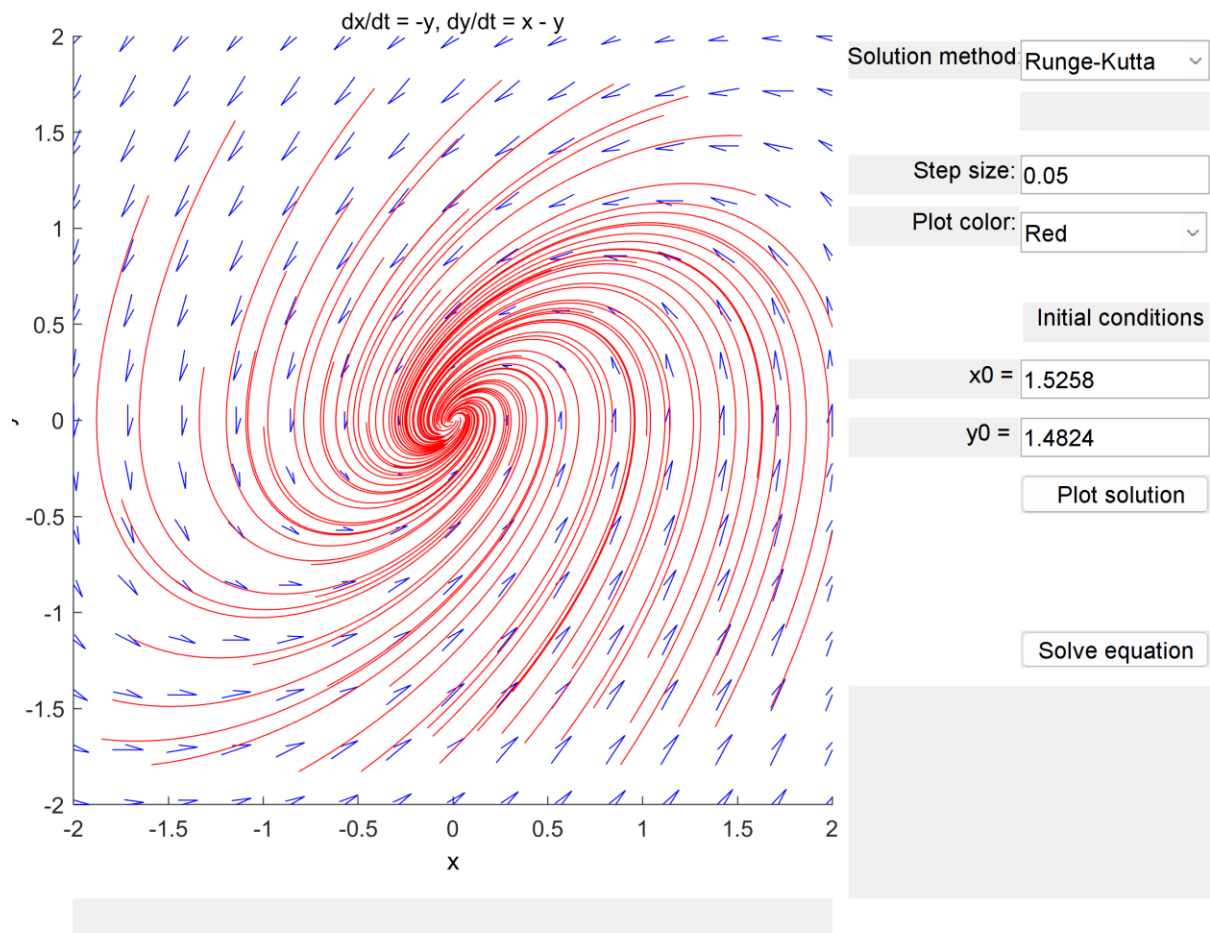
c)

Eigenvalues of the matrix: -2, 1

Justification: This is also promising result since two eigenvalues are different, opposite sign and both are real which leads to saddle point with unstable.

4.5. $\frac{dx}{dt} = \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix} x$

a)



b)

1. e.q. on Asymptotic Stability: Asymptotically Stable
2. Behaviour (sink, source, saddle-point, spiral, center, proper node, improper node): Spiral Sink
3. Clockwise or Counter-clockwise movement: C.C.W

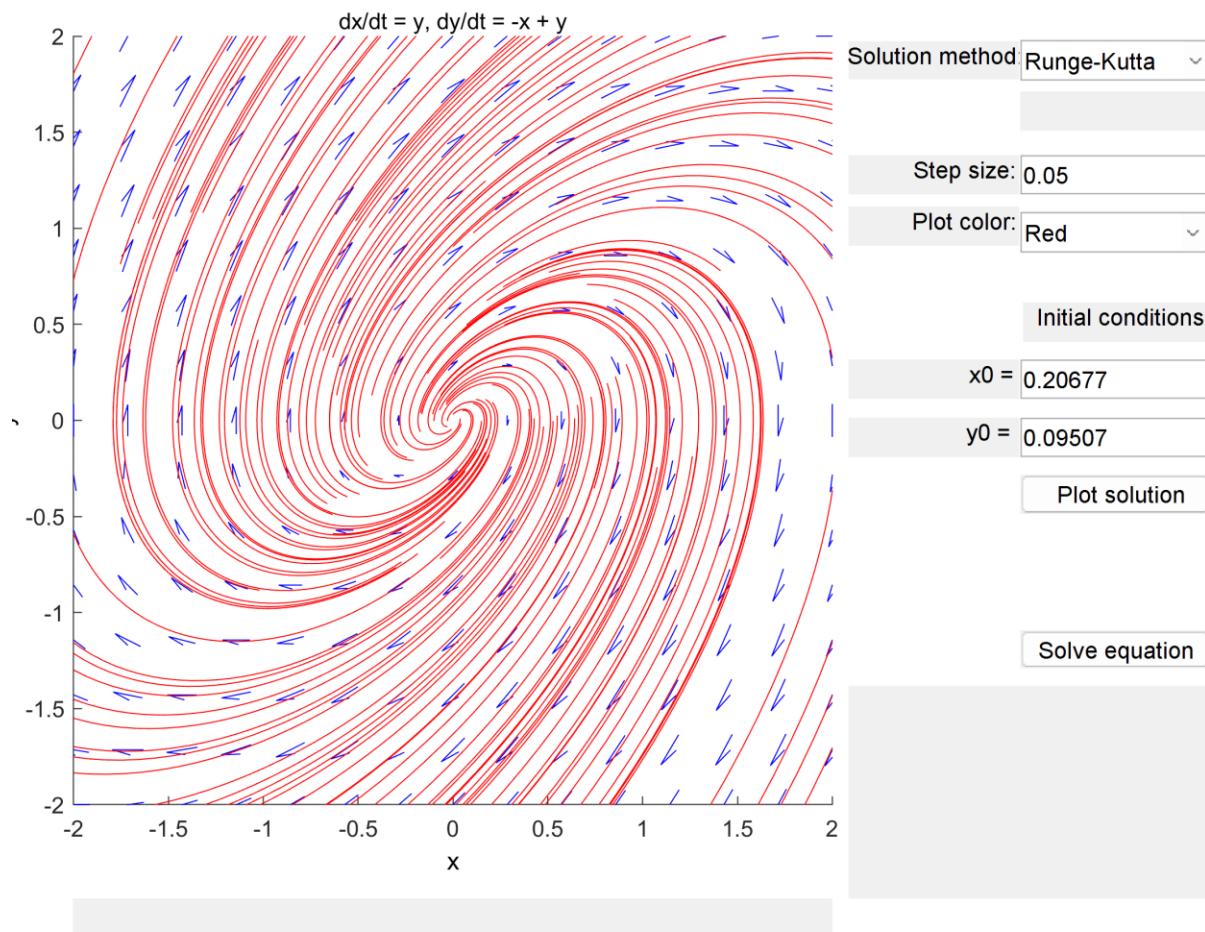
c)

Eigenvalues of the matrix: $0.5(-1 + i\sqrt{3})$, $0.5(-1 - i\sqrt{3})$

Justification: As two eigenvalues are not real (complex) with negative real parts. This should be C.C.W with spiral sink and asymptotically stable. This is promising result. To see if it is C.C.W, looking at the matrix is the best way. By multiplying by $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ gives $\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$, therefore, we can see its C.C.W. (looking at the direction between two matrices)

4.6. $\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} x$

a)



b)

1. e.q. on Asymptotic Stability: Unstable
2. Behaviour (sink, source, saddle-point, spiral, center, proper node, improper node): Spiral Source
3. Clockwise or Counter-clockwise movement: C.W.

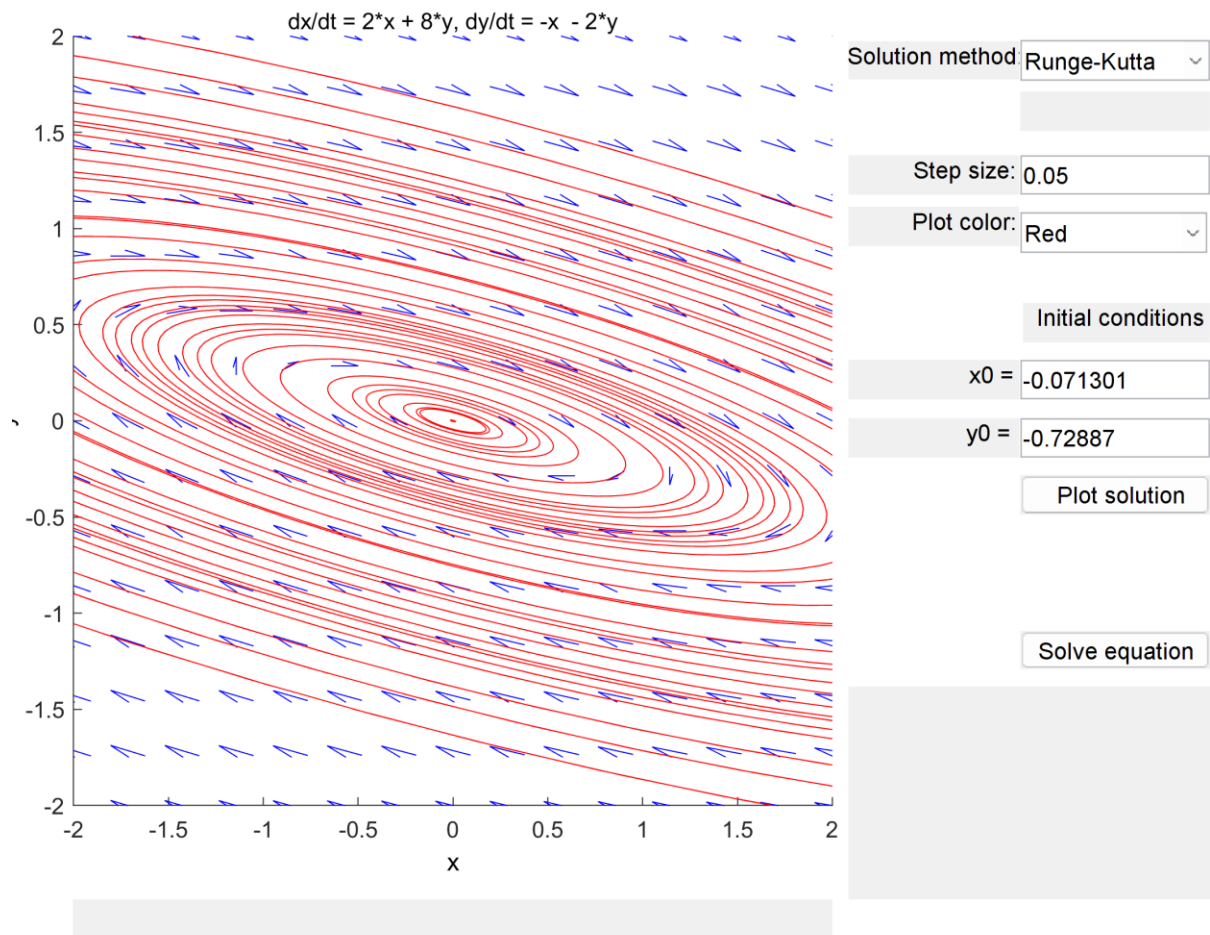
c)

Eigenvalues of the matrix: $0.5 + i\sqrt{3}/2$, $0.5 - i\sqrt{3}/2$

Justification: As the two eigenvalues are complex, not equal, with positive real parts, therefore, this should be unstable with spiral source. Also, with the C.W. Therefore, this is also promising result. To see if it is C.W, looking at the matrix is the best way. By multiplying by $\begin{bmatrix} 1; 0 \end{bmatrix}$ gives $\begin{bmatrix} 0; -1 \end{bmatrix}$, therefore, we can see its C.W. (looking at the direction between two matrices)

4.7. $\frac{dx}{dt} = \begin{bmatrix} 2 & 8 \\ -1 & -2 \end{bmatrix} x$

a)



b)

1. e.q. on Asymptotic Stability: Stable
2. Behaviour (sink, source, saddle-point, spiral, center, proper node, improper node): Centre
3. Clockwise or Counter-clockwise movement: C.W

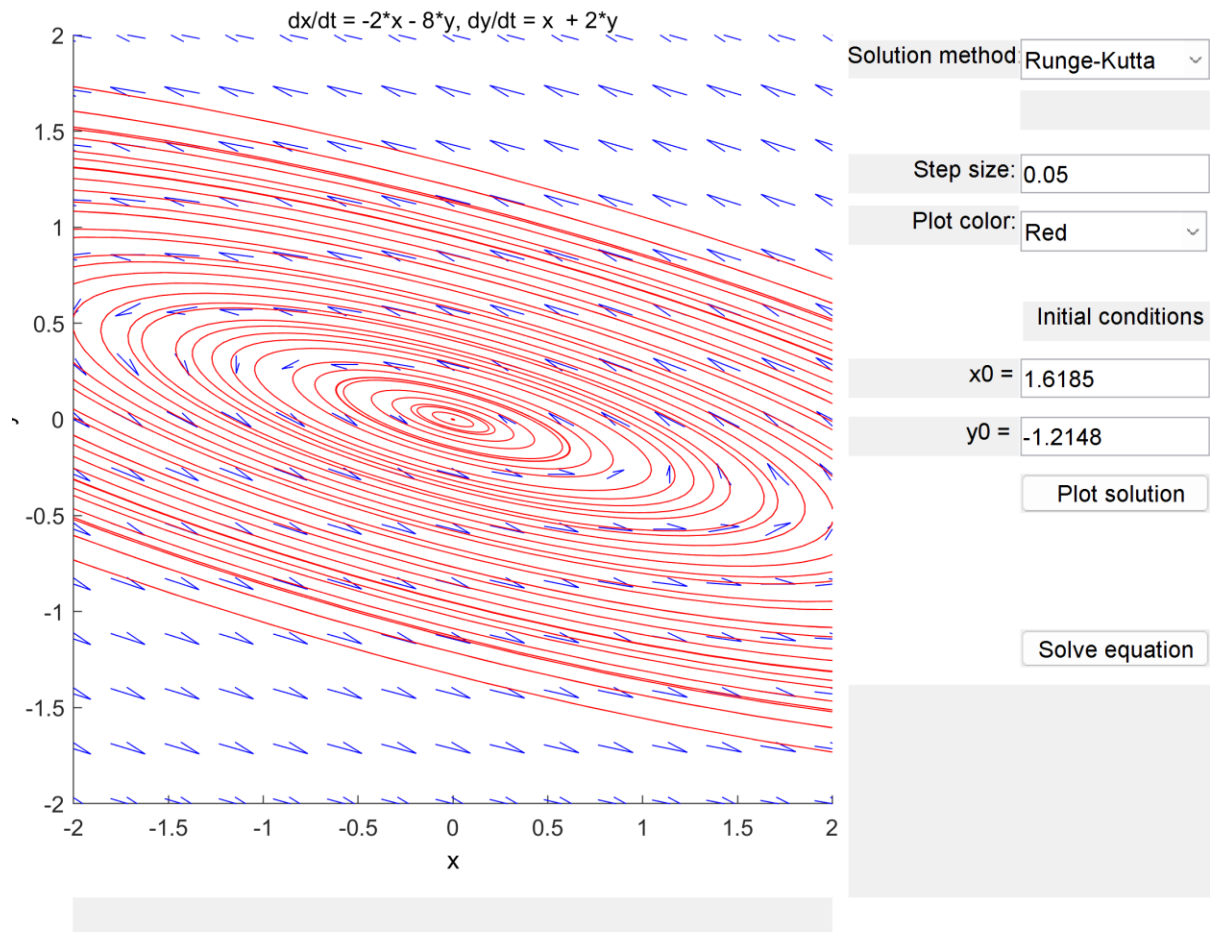
c)

Eigenvalues of the matrix: $2i$, $-2i$

Justification: This has no real parts with complex eigenvalues, therefore, this is stable and centre. To see if it is C.W, looking at the matrix is the best way. By multiplying by $\begin{bmatrix} 1; 0 \end{bmatrix}$ gives $\begin{bmatrix} 2; -1 \end{bmatrix}$, therefore, we can see its C.W. (looking at the direction between two matrices)

4.8. $\frac{dx}{dt} = \begin{bmatrix} -2 & -8 \\ 1 & 2 \end{bmatrix} x$

a)



b)

1. e.q. on Asymptotic Stability: Stable
2. Behaviour (sink, source, saddle-point, spiral, center, proper node, improper node): Centre
3. Clockwise or Counter-clockwise movement: C.C.W.

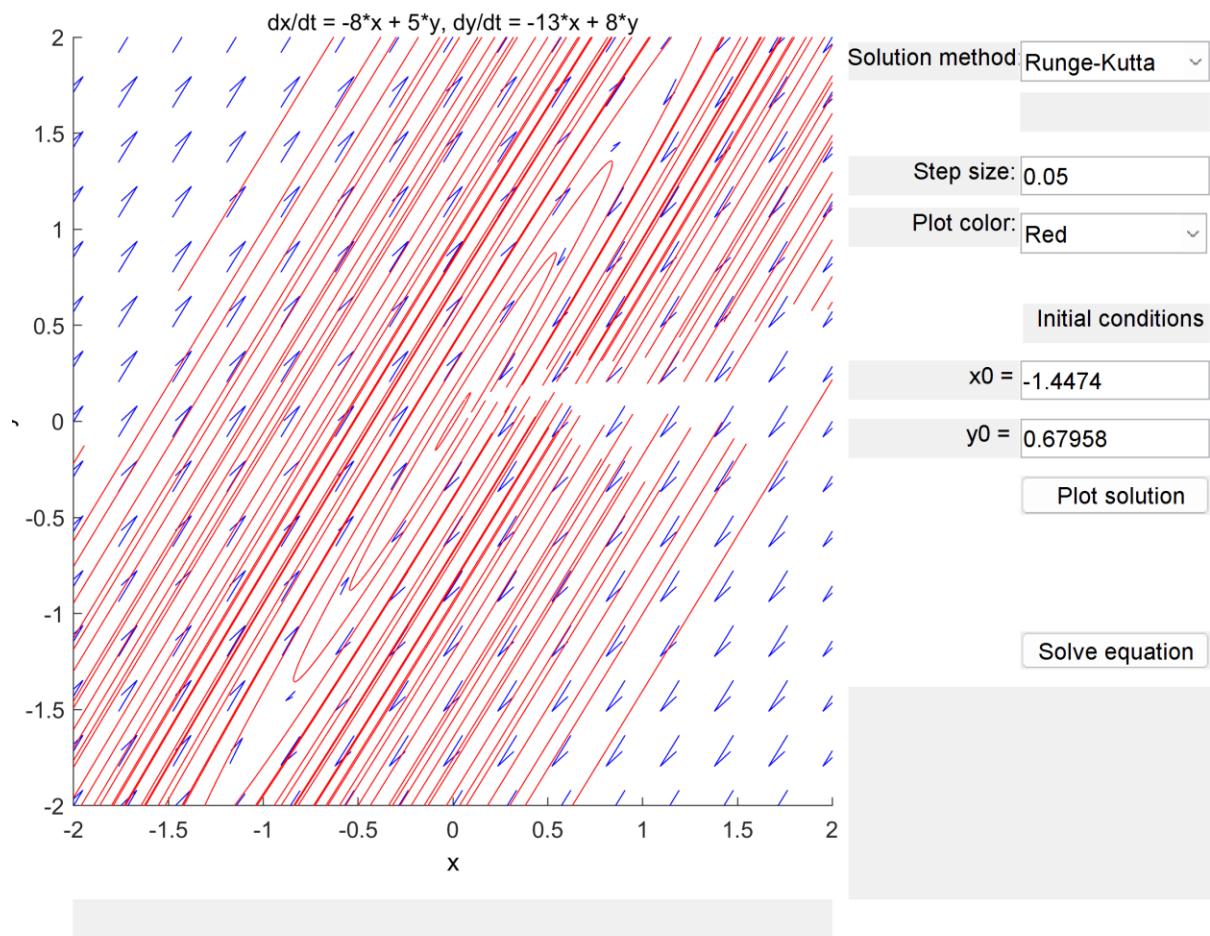
c)

Eigenvalues of the matrix: $2i$, $-2i$

Justification: This has no real parts with complex eigenvalues, therefore, this is stable and centre. To see if it is C.C.W, looking at the matrix is the best way. By multiplying by $[1;0]$ gives $[-2;1]$, therefore, we can see its C.C.W. (looking at the direction between two matrices)

$$4.9. \frac{dx}{dt} = \begin{bmatrix} -8 & 5 \\ -13 & 8 \end{bmatrix} x$$

a)



b)

1. e.q. on Asymptotic Stability: Stable
2. Behaviour (sink, source, saddle-point, spiral, center, proper node, improper node): Center
3. Clockwise or Counter-clockwise movement: C.W.

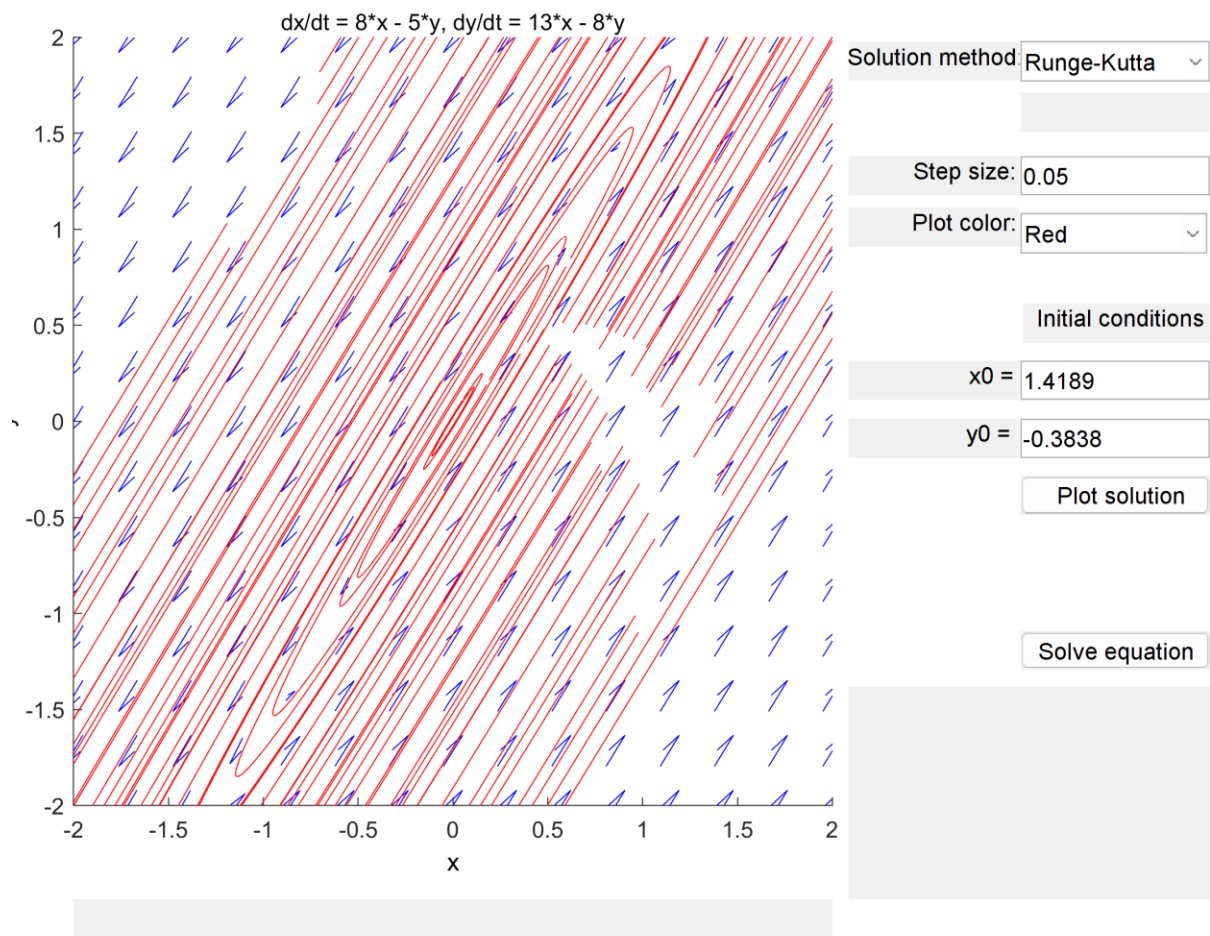
c)

Eigenvalues of the matrix: i , $-i$

Justification: This has no real parts with complex eigenvalues; therefore, this is stable and centre. To see if it is C.W, looking at the matrix is the best way. By multiplying by $[1;0]$ gives $[-8;-13]$, therefore, we can see its C.W. (looking at the direction between two matrices)

$$4.10. \frac{dx}{dt} = [8 \ -5; 13 \ -8] x$$

a)



b)

1. e.q. on Asymptotic Stability: Stable
2. Behaviour (sink, source, saddle-point, spiral, center, proper node, improper node): Center
3. Clockwise or Counter-clockwise movement: C.C.W

c)

Eigenvalues of the matrix: i , $-i$

Justification: This has no real parts with complex eigenvalues, therefore, this is stable and centre. To see if it is C.C.W, looking at the matrix is the best way. By multiplying by $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ gives $\begin{bmatrix} 8 \\ 13 \end{bmatrix}$, therefore, we can see its C.C.W. (looking at the direction between two matrices)