

# MATH-3012-QHS

## Homework 5

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**Note:** These problems are often a little challenging the first time around so there is an extra point available on this homework. That is, 24 points are available but you'll still be graded out of 23.

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*For each part, please simplify your answer to a single integer; you may use a calculator. If it is helpful, you may use your answer to any part in any subsequent part. Please remember to give some verbal indication of your process in addition to just doing the necessary calculations.*

1. (5 points) In how many ways can the letters of the word ARRANGEMENT be arranged so that there are exactly two pairs of consecutive identical letters?

**Answer:**

[Your answer here.]



2. (4 points) Determine an ordinary generating function that gives the number of integer solutions to the equation

$$c_1 + c_2 + c_3 + c_4 + c_5 = n,$$

where  $2 \leq c_1 \leq 4$ ,  $c_2$  is a multiple of 3, and  $3 \leq c_i \leq 8$  for  $3 \leq i \leq 5$ .

Which coefficient in your function gives the number of solutions to the equation  $c_1 + c_2 + c_3 + c_4 + c_5 = 30$ ? You do not need to explicitly find the coefficient, just identify which one it is.

3. (4 points)

(a) Find the sequence generated by each of the following ordinary generating functions.

i.  $f(x) = \frac{4x^2}{2-6x}$ .

ii.  $g(x) = \frac{2}{1+x^2} - e^{3x}$ .

(b) Find, in closed form, each of the following:

i. The ordinary generating function for the sequence  $(2, 0, 2, 0, 2, 0, \dots)$

ii. The ordinary generating function for the sequence  $(6, 27, 128, 629, \dots) = (1 + 5, 2 + 5^2, 3 + 5^3, 4 + 5^4, \dots)$

4. (4 points) Find the coefficient of  $x^5$  in the power series expansion of the ordinary generating function  $f(x) = \frac{2}{3x^2-4x+1}$ . Your answer should be an integer.

5. Find the closed form of the ordinary generating function for the convolution of the sequences  $(n^2)_{n \geq 0}$  and  $(n)_{n \geq 0}$ .