# MATH-3012-QHS
# Homework 5

Taiyun

Professor Dr. Kalila Lehmann

**Note**: These problems are often a little challenging the first time around so there is an extra point available on this homework. That is, 24 points are available but you'll still be graded out of 23.

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*For each part, please simplify your answer to a single integer; you may use a calculator. If it is helpful, you may use your answer to any part in any subsequent part. Please remember to give some verbal indication of your process in addition to just doing the necessary calculations.*

1. (5 points) In how many ways can the letters of the word ARRANGEMENT be arranged so that there are exactly two pairs of consecutive identical letters?

   **Answer**:

   [Your answer here.]

2. (4 points) Determine an ordinary generating function that gives the number of integer solutions to the equation

   $$c_1 + c_2 + c_3 + c_4 + c_5 = n,$$

   where $2 \leq c_1 \leq 4, c_2$ is a multiple of 3 , and $3 \leq c_i \leq 8$ for $3 \leq i \leq 5$.
   *Which coefficient in your function gives the number of solutions to the equation* $c_1+c_2+c_3+c_4+c_5 = 30$ *? You do not need to explicitly find the coefficient, just identify which one it is.*

   **Answer**:

   [Your answer here.]

3. (4 points)

   (a) Find the sequence generated by each of the following ordinary generating functions.

   i. $f(x) = \frac{4x^2}{2-6x}$.
   **Answer**:

   [Your answer here.]

   ii. $g(x) = \frac{2}{1+x^2} - e^{3x}$.
   **Answer**:

   [Your answer here.]

   (b) Find, in closed form, each of the following:

i. The ordinary generating function for the sequence $(2, 0, 2, 0, 2, 0, \ldots)$
**Answer**:

[Your answer here.]

ii. The ordinary generating function for the sequence $(6, 27, 128, 629, \ldots) = \left(1 + 5, 2 + 5^2, 3 + 5^3, 4 + 5^4, \ldots\right)$
**Answer**:

[Your answer here.]

4. (4 points) Find the coefficient of $x^5$ in the power series expansion of the ordinary generating function $f(x) = \frac{2}{3x^2 - 4x + 1}$. Your answer should be an integer.
**Answer**:

[Your answer here.]

5. Find the closed form of the ordinary generating function for the convolution of the sequences $\left(n^2\right)_{n \geq 0}$ and $(n)_{n \geq 0}$.
**Answer**:

[Your answer here.]