Homework 7

1. (4 points) Determine an ordinary generating function that gives the number of integer solutions to the equation

$$c_1 + c_2 + c_3 + c_4 + c_5 = n,$$

   where $2 \leq c_1 \leq 4$, c_2 is a multiple of 3, and $3 \leq c_i \leq 8$ for $3 \leq i \leq 5$.

   Which coefficient in your function gives the number of solutions to the equation $c_1 + c_2 + c_3 + c_4 + c_5 = 30$? *You do not need to explicitly find the coefficient, just identify which one it is.*

2. (4 points) (a) Find the sequence generated by each of the following ordinary generating functions.
   i. $f(x) = \frac{4x^2}{2-6x}$.
   ii. $g(x) = \frac{2}{1+x^2} - e^{3x}$.

   (b) Find, *in closed form,* each of the following:
   i. The ordinary generating function for the sequence $(2, 0, 2, 0, 2, 0, \ldots)$
   ii. The ordinary generating function for the sequence $(6, 27, 128, 629, \ldots) = (1+5, 2+5^2, 3+5^3, 4+5^4, \ldots)$

3. (4 points) Find the coefficient of x^5 in the power series expansion of the ordinary generating function $f(x) = \frac{2}{3x^2-4x+1}$. *Your answer should be an integer.*

4. Find the closed form of the ordinary generating function for the convolution of the sequences $(n^2)_{n\geq 0}$ and $(n)_{n\geq 0}$.