Emergency Note
Lecture Notes

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Polynomials: Even vs. Odd

Definition 1. A function $f(x)$ is said to be **even** if for all values of x in the domain, we have:

$$f(-x) = f(x)$$

Definition 2. A function $f(x)$ is said to be **odd** if for all values of x in the domain, we have:

$$f(-x) = -f(x)$$

Example 1. Consider the function $f(x) = x^2 + 4$.
To check if it is even:

$$f(-x) = (-x)^2 + 4 = x^2 + 4 = f(x)$$

Since $f(-x) = f(x)$, the function is **even**.

Example 2. Consider the function $f(x) = x^3 - 3x$.
To check if it is odd:

$$f(-x) = (-x)^3 - 3(-x) = -x^3 + 3x = -(x^3 - 3x) = -f(x)$$

Since $f(-x) = -f(x)$, the function is **odd**.

Problem 1. Determine if the following function is even, odd, or neither:

$$f(x) = x^3 + x^2$$

Answer:

To check if $f(x) = x^3 + x^2$ is even or odd:

$$f(-x) = (-x)^3 + (-x)^2 = -x^3 + x^2$$

Since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$, the function is **neither even nor odd**. $\blacksquare$

Rational Functions

Definition 3. A **rational function** is a function of the form

$$f(x) = \frac{g(x)}{k(x)},$$

where $g(x)$ and $k(x)$ are polynomial functions and $k(x) \neq 0$.

1. Domain Restrictions

A rational function is **undefined** whenever the denominator equals zero.
To determine the domain:

1. Set the denominator equal to zero.
2. Solve for the values of x.
3. Exclude these values from the domain.

Example 3. Determine the domain of

$$f(x) = \frac{3x+2}{x^2-5x-6}.$$

Factor the denominator:

$$x^2 - 5x - 6 = (x-6)(x+1).$$

Thus, the domain excludes:

$$x \neq 6, \ -1.$$

Example 4. Determine the domain of

$$v(x) = \frac{x}{x^2-3x+2}.$$

Factor the denominator:

$$x^2 - 3x + 2 = (x-2)(x-1).$$

Thus, the domain excludes:

$$x \neq 2, \ 1.$$

Problem 2. Determine the domain of

$$h(x) = \frac{x^2-9}{x^2-4x-5}.$$

Answer:

Factor the denominator:

$$x^2 - 4x - 5 = (x-5)(x+1).$$

Thus, the domain excludes the values that make the denominator zero:

$$x \neq 5, \ -1.$$

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2. Zeros of Rational Functions

Definition 4. A rational function equals zero when the **numerator** is zero and the **denominator is nonzero**.

Example 5. Find the zeros of

$$f(x) = \frac{3x+2}{x^2-5x+6}.$$

Solve the numerator:

$$3x + 2 = 0 \quad \Rightarrow \quad x = -\frac{2}{3}.$$

Since this value does not make the denominator zero, it is a valid zero.

Example 6. Find the zeros of

$$g(x) = \frac{x^2 - 4}{x^2 - 3x + 2}.$$

Factor the numerator:

$$x^2 - 4 = (x - 2)(x + 2),$$

so potential zeros are $x = \pm 2$.
However, the denominator

$$x^2 - 3x + 2 = (x - 2)(x - 1)$$

excludes $x = 2$ and $x = 1$.
Thus the only valid zero is:

$$x = -2.$$

Problem 3. Find the zeros of

$$p(x) = \frac{2x^2 - 8x}{x^2 - x - 6}.$$

Answer:

Factor both numerator and denominator:

$$2x^2 - 8x = 2x(x - 4), \qquad x^2 - x - 6 = (x - 3)(x + 2).$$

The numerator is zero when $x = 0$ or $x = 4$. Neither value makes the denominator zero, so both are valid zeros:

$$x = 0,\ 4.$$

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3. Key Ideas

- A rational function is undefined where the denominator equals zero.
- A rational function is zero where the numerator equals zero (and denominator is nonzero).
- End behavior depends on how the numerator and denominator behave for large values of x.