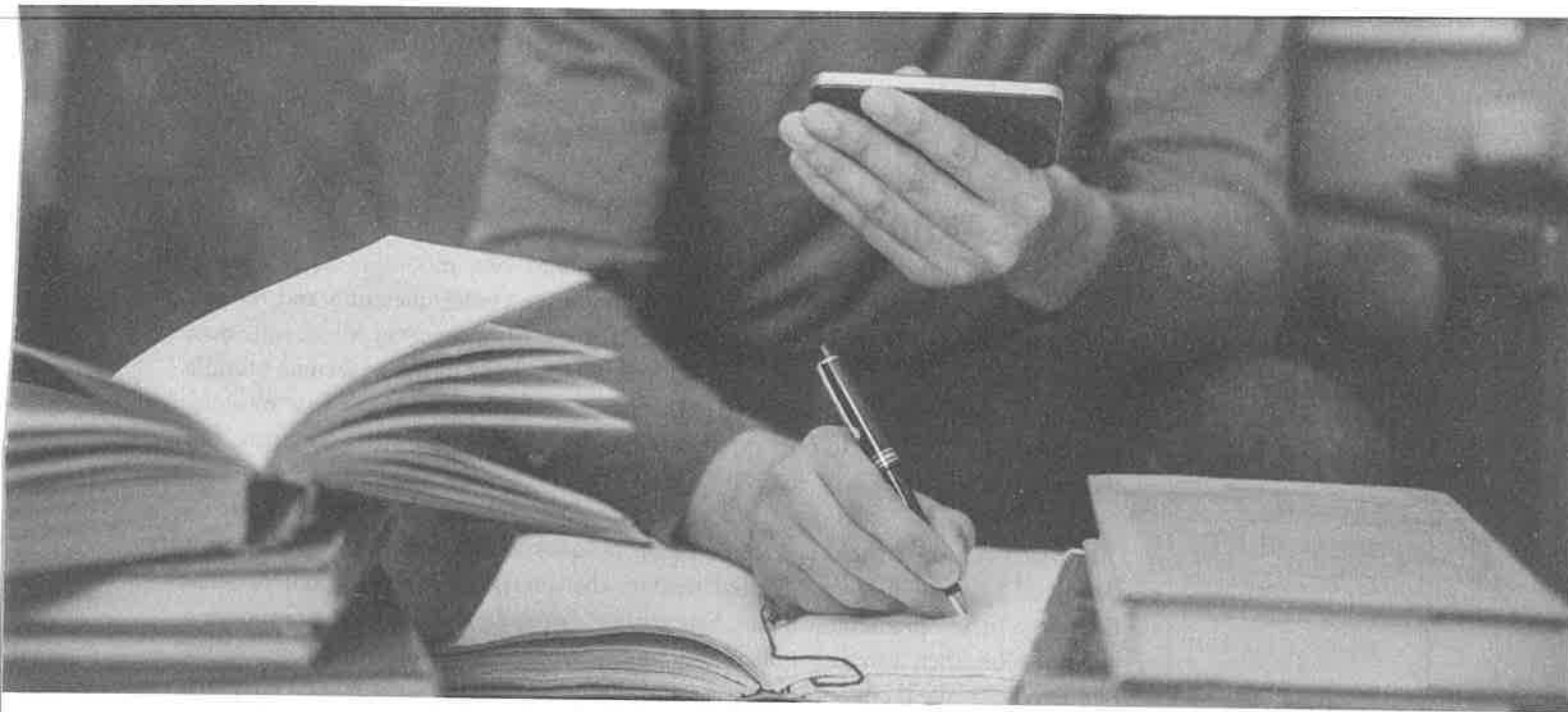
Part III
How to Crack the Math Section



Chapter 18
Math Introduction

As you learned in Chapter 4, the Digital SAT isn't your typical school test. This goes for the Math section of the Digital SAT as well as the Reading and Writing section. This chapter will give you an overview of the Math section, show you what kind of math to expect, talk about applying the test-taking strategies you learned in Chapter 4 to the Math section, and introduce you to the only questions on the Digital SAT that are not multiple-choice.

THE MATH BREAKDOWN

The Math section of the Digital SAT is split into two modules. Each module contains 22 questions, of which 16 or 17 are multiple-choice questions and the rest are student-produced response questions (SPR), meaning that you fill in your own answer instead of choosing from four answers. Questions on the second module are, on average, easier or harder based on your performance on the first module. Each module has two "pre-test" questions that do not count towards your score, but they are not identified, so treat every question as if it counts.

No Need to Know
Here are a few things you won't need to know to answer Digital SAT Math questions: calculus, logarithms, and matrices. Essentially, the Digital SAT tests a whole lot of algebra and some arithmetic, statistics, and geometry.

The Math section is further broken down by question type and content area, as described below. Unlike the Verbal section, the question types and content areas do not go in any predictable order. Everything is mixed together, so you could see a trig question, then a word problem about averages, then a question about the vertex of a parabola. We'll cover these topics and many more in the next several chapters, so you'll be ready for all of it.

Question Type Breakdown	
70% Problem Solving	15–16 questions per module
30% Word Problems	6–7 questions per module
Content Breakdown	
35% Algebra	7–8 questions per module
35% Advanced Math	7–8 questions per module
15% Problem-Solving & Data Analysis	3–4 questions per module
15% Geometry and Trigonometry	3–4 questions per module

Fill-In Questions

Approximately 25% of the questions on the Math section of the Digital SAT are what College Board calls Student-Produced Response questions, or SPRs. These are the only questions on the test that are not multiple-choice. Instead of selecting the correct answer from among several choices, you will have to find the answer on your own and type it into a box. We call these fill-ins because you fill in your answer. The fill-in questions cover the same math topics as the multiple-choice questions do, and they fit within the order of difficulty of the module. The fill-in format has special characteristics, and we'll tell you more about them in Chapter 26.

You Don't Have to Finish

We've all been taught in school that when you take a test, you have to finish it. If you answered only two-thirds of the questions on a high school math test, you probably wouldn't get a very good grade. But as we've already seen, the Digital SAT is not at all like the tests you take in school. Most students don't know about the difference, so they make the mistake of trying to work all of the questions on both Math modules of the Digital SAT.

Quick Note
Remember, this is not a math test in school! It is not scored on the same scale your math teacher uses. You don't need to get all the questions right to get an above-average score.

Because they have only a limited amount of time to answer all the questions, most students rush through the questions they think are easy in order to get to the harder ones as soon as possible. At first, it seems reasonable to save more time for the more challenging questions, but think about it this way. When students rush through a Math section, they're actually spending too little time on the easier questions (which they have a good chance of getting right), just so they can spend more time on the harder questions (which they have less chance of getting right). Does this make sense? Of course not.

Here is the secret: on the Math section, you don't have to answer every question in each module. In fact, unless you are aiming for a top score, you should intentionally skip some harder questions. Most students can raise their Math scores by concentrating on correctly answering all of the questions that they think are easy or medium difficulty. In other words...

Slow Down!

Most students do considerably better on the Math section when they slow down and spend less time worrying about the more complex questions (and more time working carefully on the more straightforward ones). Haste causes careless errors, and careless errors can ruin your score. In most cases, you can actually raise your score by answering fewer questions. That doesn't sound like a bad idea, does it? If you're shooting for an 800, you'll have to answer every question correctly. But if your target is 550, you should ignore the hardest questions in each module and use your limited time wisely.

POOD and the Math Section

The questions in both modules of the Digital SAT Math section are arranged in a loose order of difficulty. The earlier questions are generally easier and the last few are harder, but the level of difficulty may jump around a little. Furthermore, the second module might start with a question that feels much easier than the last question of the first module. Assessing the difficulty of a question is also complicated by the fact that in College Board's view, "hard" on the Digital SAT means that a higher percentage of students tend to get it wrong, often due to careless errors or lack of time.

The two experimental questions in each module can also alter the order of difficulty. If you encounter a question that seems surprisingly easy or surprisingly hard based on the questions before and after, use your POOD to decide whether to do it, mark it for later, or enter a guess.

Because difficulty levels can go up and down a bit, don't worry too much about how hard the test-writers think a question is. Focus instead on the questions that are easiest for you, and do your best to get those right—no matter where they appear—before moving on to the tougher ones. So which will be the easy ones for you? It is *personal* order of difficulty, but here are some things to consider:

- **Math knowledge:** Do you know the topic cold? Do you see exactly how to start solving it? Then the question is worth attempting, but read and work carefully!
- **SAT knowledge:** Is there a Princeton Review technique from this book that would be perfect for this question? Now is the time to put your skills to use.
- **Self-knowledge:** Do your eyes glaze over halfway through a word problem? Do you think, "More like trigoNOmetry" when you see a trig question? Then come back to that question later or just pick a random answer to select.
- **Take the first bite:** A great way to decide whether a question deserves your time is to think about Bite-Sized Pieces. If you know immediately how to start a question, there's a good chance you'll be able to finish it and get it right.

> Don't forget: Fill in answers for questions you decide to skip, use the Mark for Review tool to mark questions to come back to later, and enter an answer for every question.

USING THE ONLINE TOOLS AND SCRATCH PAPER

Online Tools

Several of the built-in features of the Digital SAT will be useful on the Math section.

- Mark for Review tool to mark questions to come back to later
- Built-in calculator, which can be accessed at any time
- Reference sheet with common geometry formulas, which can be accessed at any time
- The Annotate tool is NOT available on the Math section, so you will not be able to underline or highlight parts of the question.

Scratch Paper

The proctor at the test center will hand out three sheets of scratch paper, and you can use your own pen or pencil. Plan ahead about how to use the scratch paper in combination with what's on the screen.

Use the Tools Effectively!

Online Tools	Scratch Paper
Eliminate wrong answers	Rewrite key parts of the question
Work steps on the calculator	Write out every calculation
Look up geometry formulas	Redraw geometric figures and label them
	Rewrite answer choices as needed

Here's a question from your first practice test with an image of the testing app screen on the left and scratch paper on the right. On a question like this, use your scratch paper to rewrite parts of the question and translate them into math, and then use the Answer Eliminator tool to cross out answers that don't match that piece. Always include the question number next to your work on the scratch paper to keep yourself organized.

16

Stella had 211 invitations to send for an event. She has already sent 43 invitations and will send them all if she sends 24 each day for the next d days. Which of the following equations represents this situation?

~~A $24d - 43 = 211$~~ Undo

B $24d + 43 = 211$ B

~~C $43d - 24 = 211$~~ Undo

~~D $43d + 24 = 211$~~ Undo

16)

24 a day for d days
24d

add to 43 sent,
not subtract

Calculators

Calculators are permitted on every Math question on the Digital SAT. In addition, the testing app includes a built-in calculator with many, many features. Practice with the built-in calculator or the one you're planning to bring with you. We'll tell you more about calculators as we go along.

The Princeton Review Approach

We're going to give you the tools you need in order to handle the easier questions on the Digital SAT Math section, along with several great techniques to help you crack some of the more difficult ones. But you must concentrate first on getting the easier questions correct. Don't worry about the questions you find difficult on the Math section until you've learned to work carefully and accurately on the easier questions.

When it does come time to look at some of the harder questions, use Process of Elimination to help you avoid trap answers and to narrow down your choices if you have to guess. You will learn to use POE to improve your odds of finding the answer by getting rid of answer choices that can't possibly be correct.

Generally speaking, each chapter in this section begins with the basics and then gradually moves into more advanced principles and techniques. If you find yourself getting lost toward the end of the chapter, don't worry. Concentrate your efforts on principles that are easier to understand but that you still need to master.

Chapter 19
Digital SAT Math: The Big Picture

In this chapter, you'll see a few ways you can eliminate bad answer choices, avoid traps, improve your odds of answering correctly if you have to guess, and maximize your Math score. You'll also learn how to best make use of the built-in calculator, should you choose to use it instead of bringing your own.

THE BIG PICTURE AND IMPORTANT STRATEGIES

The Reading and Writing section of this book describes various ways to eliminate wrong answers. That idea comes into play on the Digital SAT Math section, as well. There are also ways to break down math questions and avoid trap answers. This chapter provides an overview of the strategies you should know in order to maximize your Math score.

BALLPARKING STRATEGY

One way to eliminate answers on the Math section is by looking for ones that are the wrong size, or that are not "in the ballpark." We call this strategy **Ballparking.** Although you can use your calculator on the following question, you can also eliminate one answer without doing any calculations.

1 Mark for Review

In a garden, the corn on the north edge of the garden is 30% shorter than that on the south edge. If the corn on the south edge of the garden is 50 inches tall, how tall is the corn on the north edge of the garden, in inches?

(A) 30

(B) 33

(C) 35

(D) 65

Here's How to Crack It

The question asks for the height of the corn on the north edge of the garden and states that the corn there is shorter than the corn on the south edge, which is 50 inches tall. You are asked to find the height of the corn on the north edge, so the correct answer must be less than 50. Eliminate (D), which is too high. Often, one or more of the bad answers on these questions is the result you would get if you applied the percentage to the wrong value. To find the right answer, take 30% of 50 by multiplying 0.3 by 50 to get 15; then subtract that from 50. The corn on the north edge is 35 inches tall. The correct answer is (C).

READ THE FINAL QUESTION STRATEGY

It's a bad idea to assume you know what a question is going to ask you to do. Make sure to always read the final question *before* starting to work on the question. Write key words or the entire final question on the scratch paper. Then, try to ballpark before you solve.

2 Mark for Review

If $16x - 2 = 30$, what is the value of $8x - 4$?

Ⓐ 12

Ⓑ 15

Ⓒ 16

Ⓓ 28

Here's How to Crack It

The question asks for the value of an expression, but don't just dive in and solve for the variable. First, see if you can eliminate answers by Ballparking, which can also work on algebra questions. To go from $16x$ to $8x$, you would just divide by 2. Dividing 30 by 2 gives you 15, so 28 is way too big. Eliminate it. The correct answer is not likely to be 15, either, because that ignores the -2 and the -4 in the question.

To solve this one, add 2 to each side of the equation to get $16x = 32$. Divide both sides by 2, which gives you $8x = 16$. But don't stop there! The final question asks for $8x - 4$, so (C) is a trap answer. You have to take the last step and subtract 4 from both sides to find that $8x - 4 = 12$. The correct answer is (A).

RTFQ:
Read
The
Final
Question

Get started faster and avoid trap answers by reading and rewriting the actual question being asked.

BITE-SIZED PIECES STRATEGY

When dealing with complicated math questions, take it one little piece at a time. We call this strategy **Bite-Sized Pieces**. If you try to do more than one step at a time, especially if you do it in your head, you are likely to make mistakes or fall for trap answers. After each step, take a look at the answer choices and determine whether you can eliminate any.

Try the following question.

3 Mark for Review

A paper airplane is thrown horizontally from the top of a hill. It travels in a straight line, and as it moves forward, it also descends. The plane moves horizontally at 9 feet per second while descending one foot for every 3 feet traveled horizontally. After 5 seconds of travel, how many feet has the plane descended from the height at which it was thrown?

(A) 3

(B) 10

(C) 15

(D) 20

Here's How to Crack It

The question asks how many feet the plane has descended after 5 seconds. There are a few things going on here. The plane is traveling horizontally, and it is also descending. Start by figuring out how far it travels horizontally. It moves in that direction at 9 feet per second for 5 seconds, so it moves horizontally $9 \times 5 = 45$ feet. It descends 1 foot for every 3 traveled horizontally. If it goes 45 feet horizontally, it will descend more than 3 feet, so eliminate (A). Now figure out how many "3 feet" are in 45 feet—for each one of them, the plane will descend 1 foot. Since $45 \div 3 = 15$, the plane descends 15 feet. The correct answer is (C).

You may also have noticed that all the numbers in the question are odd. This makes it unlikely that the answer would be 10 or 20, which are even. If you see things like that, use them as opportunities to eliminate.

Bite-Sized Pieces: Do one small, manageable piece at a time and keep writing things down.

Here's another example.

4 Mark for Review

$$(5ck^2 + 5c^2 - 2c^2k) - (ck^2 + 2c^2k + 5c^2)$$

Which of the following is equivalent to the expression above?

Ⓐ $4ck^2$

Ⓑ $4ck^2 - 4c^2k$

Ⓒ $5c^2k^4 - 10c^4k$

Ⓓ $8c^2k^3 + 7c^2k - 5c^2$

Here's How to Crack It

The question asks for an expression that is equivalent to the difference of two polynomials. In math class, your teacher would want you to combine all like terms and show your work, but this isn't math class. Start with one tiny piece of this intimidating question. The first set of parentheses starts with a term containing ck^2. Check the second set of parentheses for the same combination of variables and exponents. The first term there matches, so the first step to take is $5ck^2 - ck^2 = 4ck^2$. There are no other terms with ck^2, so the correct answer must contain $4ck^2$. Eliminate (C) and (D). Now you have a fifty-fifty chance of getting it right, so you can guess and go, *or* you can do one more step to determine whether the answer is (A) or (B). The difference between the two answers is the $-4c^2k$ term, so focus on the terms in the expression that contain c^2k. In the first set of parentheses, you have $-2c^2k$, and then you subtract the $2c^2k$ term in the second set of parentheses to get $-2c^2k - 2c^2k = -4c^2k$. The correct answer is (B).

WORD PROBLEMS

The two strategies we just showed you—RTFQ and Bite-Sized Pieces—are a large part of the approach to word problems on the Digital SAT. The test-writers will try to make things difficult to understand by making a story out of the math. To make sure you have the best shot at reaching the correct answer quickly and accurately, follow this basic approach.

Word Problem Basic Approach

1. **Read the Final Question (RTFQ)**—Understand the actual question being asked. Write down key words.
2. **Let the answers point the way**—Use the answer type to help determine how to start working on the question.
3. **Work in bite-sized pieces**—Find one piece to start with, then work piece-by-piece until the final question has been answered.
4. **Use POE**—Check to see whether any answers can be eliminated after each bite-sized piece.

This approach can be helpful on questions that are "just" math, but they are vital on word problems. Here are some details about the word problem basic approach.

RTFQ—The final question will start with something like *Which of the following, What is,* or *How many.* Find the final question (it's usually at the end) and write down key words. If the question asks for the value of a variable or the measure of an angle, write down which variable or which angle. If it asks for a specific part of a graph or a word problem, write down which part. Terms and units, such as *median, positive, minutes,* or *miles,* also go on the scratch paper.

Let the answers point the way—On multiple-choice questions, the answer type often gives a clue about how to approach the question. Do the answers have numbers? variables? equations? graphs? a bunch of words? Use that information to get started.

Work in bite-sized pieces—Rather than trying to plan the entire question up front, just get started. Work the question one bite-sized piece at a time, reading more along the way and making notes on the scratch paper. The final question and the answer types usually reveal the best approach. If it's not obvious, either mark the question to come back to later or enter a guess.

Use POE—On some questions, it's possible to eliminate answers along the way while working in bite-sized pieces. If the question asks about an equation representing a situation, for example, an answer that gets any piece of the equation wrong can be eliminated. Eliminate answers that don't work when you plug them in, answers that are clearly too big, too small, or have the wrong sign, and answers that contradict information given in the question.

THE CALCULATOR

You are allowed to use a calculator on every question in the Math section, although it doesn't help on every question. You have two calculator options:

- Use the built-in Desmos calculator within the testing app.
- Bring your own approved scientific or graphing calculator.

Whether you use the built-in calculator or your own, practice, practice, practice! A calculator can make some questions much easier to answer and will save you time on other questions. To practice with the built-in calculator, download the Bluebook app or just use the free version on the Desmos website.

Calculator Guide

Head to your Student Tools to read our Guide for the built-in calculator. There you will find basic information about opening and using the calculator, details about how to use the keypads and advanced functions, and instructions for getting the most out of the graphing calculator. This Guide also includes a number of questions from your first practice test with detailed instructions and screenshots to show you how to solve them using the built-in calculator.

Even if you are planning to use your own calculator, this Guide might help you think of ways to use it that you hadn't considered before.

Call on the Calculator
Practice using the built-in calculator in your Student Tools or your own throughout your test preparation. Use it for example questions in this book, use it while taking practice tests, and use it any time you're doing practice questions for the Digital SAT.

Personal Calculator

If you have a good calculator that you are familiar with and like using, you may take it with you and use it on the test. Make sure that your calculator is either a scientific or a graphing calculator and can perform the order of operations correctly. To test your calculator, try the following problem, typing it in exactly as written without hitting the ENTER or "=" key until the end: $3 + 4 \times 6 =$. The calculator should give you 27. If it gives you 42, it's not a good calculator to use.

If you do decide to use your own graphing calculator, keep in mind that it *cannot* have a QWERTY-style keyboard (like the TI-95). Most of the graphing calculators have typing capabilities, but because they don't have typewriter-style keyboards, they are perfectly legal. To see the full College Board calculator policy, visit satsuite.collegeboard.org/digital/what-to-bring-do/calculator-policy.

Also, you *cannot* use the calculator on your phone. In fact, on test day, you will have to turn your phone off and put it away.

The only danger in using a calculator on the Digital SAT is that you may be tempted to use it in situations in which it won't help you. Some students believe that their calculator will solve many difficulties they have with math. This type of thinking may even occasionally cause students to miss a question they might have otherwise answered correctly on their own.

Remember that your calculator is only as smart as you are. But if you practice and use a little caution, you will find that your calculator will help you a great deal.

What a Calculator Is Good at Doing

Here's a list of some of the things a calculator is good for on the Digital SAT:

- arithmetic
- decimals
- fractions
- square roots
- percentages
- graphs (if it is a graphing calculator)

We'll discuss the calculator's role in most of these areas in the next few chapters.

Calculator Arithmetic

Adding, subtracting, multiplying, and dividing integers and decimals is easy on a calculator. But, you need to be careful when you key in the numbers. A calculator will give you an incorrect answer to an arithmetic calculation if you press the wrong keys.

Calculators Don't Think for You
A calculator crunches numbers and often saves you a great deal of time and effort, but it is not a substitute for your problem-solving skills.

The main thing to remember about a calculator is that it can't help you find the answer to a question you don't understand. If you wouldn't know how to solve a particular problem using pencil and paper, you won't know how to solve it using a calculator either. Your calculator will help you, but it won't take the place of a solid understanding of basic Digital SAT mathematics.

Use Your Paper First

When you decide to use a calculator to help answer a question, the first step should be to set up the problem or equation on paper; this will keep you from getting lost or confused. This is especially important when solving the problem involves a number of separate steps. The basic idea is to use the scratch paper to make a plan, and then use your calculator to execute it.

Write Things Down
You have scratch paper, so make the most of it. Keep track of your progress through each question by writing down each step.

Working on paper first will also give you a record of what you have done if you change your mind, run into trouble, or lose your place. If you suddenly find that you need to try a different approach to a question, you may not have to go all the way back to the beginning. This will also make it easier for you to check your work, if you have time to do so.

Don't use the memory function on your calculator (if it has one). Because you can use your scratch paper, you don't need to juggle numbers within the calculator itself. Instead of storing the result of a calculation in the calculator, write it on your scratch paper, clear your calculator, and move to the next step of the question. A calculator's memory is fleeting; scratch paper is forever.

Order of Operations

In the next chapter, we will discuss the proper order of operations for solving equations that require several operations to be performed. Be sure you understand this information, because it applies to calculators as much as it does to pencil-and-paper computations. You may remember PEMDAS from school. PEMDAS is the order of operations. You'll learn more about it and see how questions on the Digital SAT require you to know the order of operations. You must always perform calculations in the proper order.

Fractions

Most scientific calculators have buttons that will automatically simplify fractions or convert fractions from decimals. (For instance, on the TI-81, TI-83, and TI-84, hitting "Math" and then selecting the first option, "Answer → Fraction," will give you the last answer calculated as a fraction in the lowest terms.) Find out if your calculator has this function! If it does, you can use it to simplify messy fractions. This function is also very useful when you get an answer as a decimal, but the answer choices given are all fractions.

The Digital SAT Calculator Guide has details about working with fractions, along with other types of math and graphs. Pull up the Guide in your Student Tools and become an expert!

Batteries

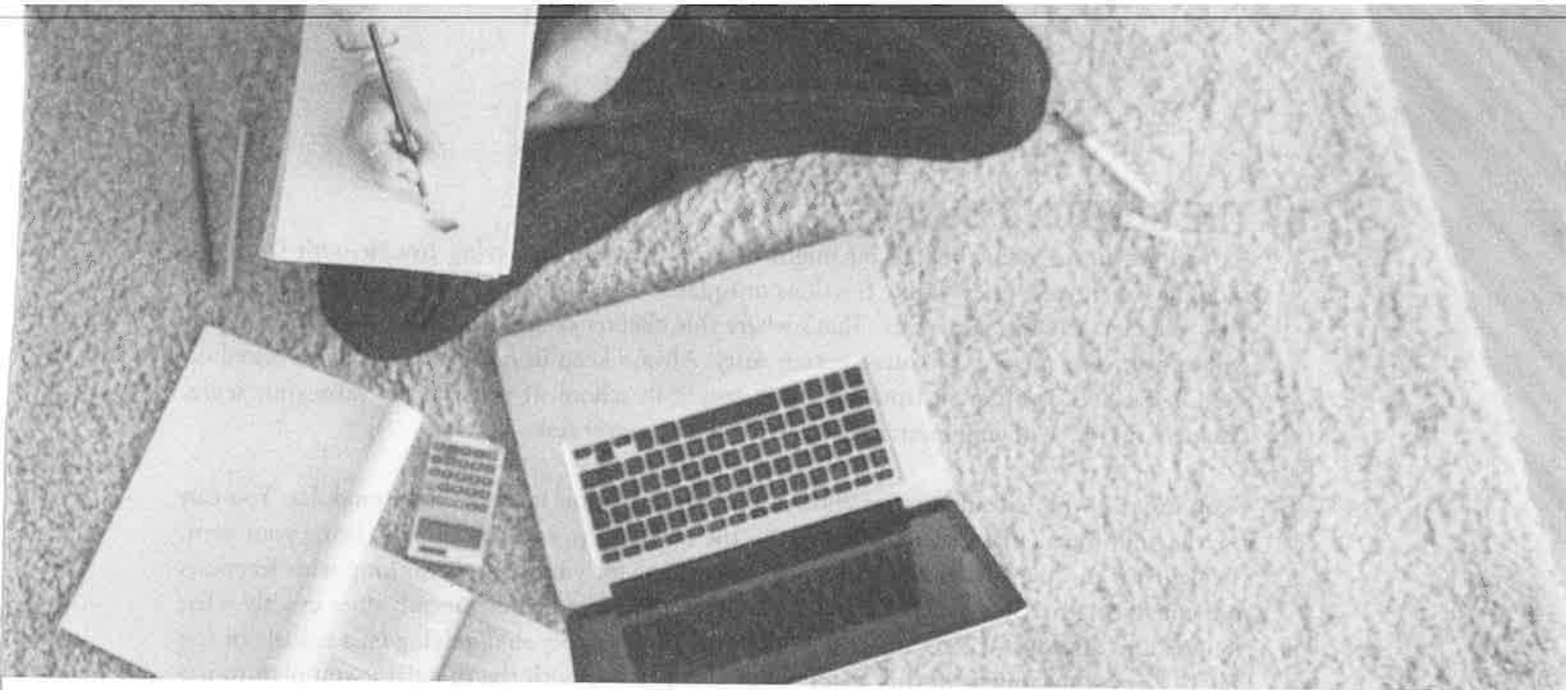
Change the batteries on your calculator a week before the Digital SAT so that you know your calculator won't run out of power halfway through the test. You can also take extra batteries with you, just in case. Although it isn't very likely that the batteries will run out on your calculator on the day of the test, it could happen—so you want to be prepared.

Final Words on the Calculator

Remember that the test-writers are trying to test your ability to use your calculator wisely. As such, they have purposely created many questions in which the calculator is worthless. So be sure to read the final question—there may be some serious surprises in there. Practice your math skills *and* your calculator skills so you have options about how you solve questions.

Summary

- Look for ways to eliminate answer choices that are too big or too small. Ballparking can help you find the right answer without extensive calculations, avoid trap answers, and improve your chances of getting the question right even if you have to guess.
- Use the built-in tools as much as possible. The most useful ones on the Math section are the Calculator, Reference Sheet, and Answer Eliminator tool.
- Use your scratch paper constantly: number the work for each question, write down key words from the final question, redraw geometric figures, and write down every step of math. Even if you use a calculator, it's worth setting up the math on your scratch paper to stay organized and avoid mistakes.
- Utilize the Word Problem Basic Approach: Read the Final Question (RTFQ), Let the Answers Point the Way, Work in Bite-Sized Pieces, and use Process of Elimination (POE).
- Practice with the calculator you plan to use for the test: either the built-in Desmos calculator or your personal scientific or graphing calculator.
- If you are going to use the built-in calculator, read the Digital SAT Calculator Guide in your Student Tools to maximize its effectiveness.
- If you are going to use your own calculator, make sure it is on the approved list and has fresh batteries.
- Set up the question on the scratch paper before entering anything into a calculator. By doing so, you will eliminate the possibility of getting lost or confused.
- A calculator can't help you find the answer to a question you don't understand. Be sure to use your calculator as a tool, not a crutch.
- Whether you are using a calculator or not, you must always perform calculations in the proper order (PEMDAS).



Chapter 20
Fun with Fundamentals

We'll show you which mathematical concepts are most important to know for the Digital SAT. However, this book relies on your knowledge of basic math concepts. If you're a little rusty, this chapter is for you. Read on for a quick review of the math fundamentals you'll need to know.

THE BUILDING BLOCKS

As you go through this book, you might discover that you're having trouble with stuff you thought you already knew—like fractions or square roots. If this happens, it's probably a good idea to review the fundamentals. That's where this chapter comes in. Our drills and examples will refresh your memory if you've gotten rusty. Always keep in mind that the math tested on the Digital SAT is different from the math taught in school. If you want to raise your score, don't waste time studying math that the Digital SAT never tests.

Keep in mind that calculators are allowed on every question in both Math modules. You can use the built-in calculator available to you in the testing app, and you can also bring your own. A calculator can perform many basic math operations for you, such as working with fractions and converting between fractions and decimals. However, a calculator only does exactly what you tell it to do, so read carefully and write things down rather than relying on a calculator too much. Learn the content in this chapter, but also practice with the calculator you plan to use on test day. Your brain, pencil, and calculator all work together to get questions right efficiently.

Let's talk first about what you should expect to see on the test.

The Instructions

Both of the Math modules on the Digital SAT begin with the same set of instructions. The clock starts right away, so you want to know the instructions cold before you go in. That way, you can click Close and start working right away. If you forget something about the instructions while you're working on the test, click the Directions button in the upper left corner. We've reprinted the instructions below. Be sure to familiarize yourself with them ahead of time.

The questions in this section address a number of important math skills.

Use of a calculator is permitted for all questions. A reference sheet, calculator, and these directions can be accessed throughout the test.

Unless otherwise indicated:

- All variables and expressions represent real numbers.
- Figures provided are drawn to scale.
- All figures lie in a plane.
- The domain of a given function f is the set of all real numbers x for which $f(x)$ is a real number.

For **multiple-choice questions**, solve each problem and choose the correct answer from the choices provided. Each multiple-choice question has a single correct answer.

For **student-produced response questions**, solve each problem and enter your answer as described below.

- If you find **more than one correct answer**, enter only one answer.
- You can enter up to 5 characters for a **positive** answer and up to 6 characters (including the negative sign) for a **negative** answer.
- If your answer is a **fraction** that doesn't fit in the provided space, enter the decimal equivalent.
- If your answer is a **decimal** that doesn't fit in the provided space, enter it by truncating or rounding at the fourth digit.
- If your answer is a **mixed number** (such as $3\frac{1}{2}$), enter it as an improper fraction (7/2) or its decimal equivalent (3.5).
- Don't enter **symbols** such as a percent sign, comma, or dollar sign.

Examples

Answer	Acceptable ways to enter answer	Unacceptable: will NOT receive credit
3.5	3.5 3.50 7/2	31/2 3 1/2
$\frac{2}{3}$	2/3 .6666 .6667 0.666 0.667	0.66 .66 0.67 .67
$-\frac{1}{3}$	–1/3 –.3333 –0.333	–.33 –0.33

The instructions have a lot to say about the fill-in questions. Don't worry: we'll show you everything you need to know about that question format in Chapter 26.

Standard Symbols

The following standard symbols are used frequently on the Digital SAT:

Formulas and Definitions
Go to your online Student Tools for a complete list of the math terms and formulas that you'll need to know for the Digital SAT.

SYMBOL	MEANING
$=$	is equal to
$\neq$	is not equal to
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to

THE SIX ARITHMETIC OPERATIONS

There are only six arithmetic operations that you will ever need to perform on the Digital SAT:

1. Addition ($3 + 3$)
2. Subtraction ($3 - 3$)
3. Multiplication (3×3 or $3 \cdot 3$)
4. Division ($3 \div 3$ or $3/3$)
5. Raising to a power (3^3)
6. Finding a root ($\sqrt{9}$ and $\sqrt[3]{8}$)

If you're like most students, you probably haven't paid much serious attention to these topics since junior high school. You'll need to learn about them again if you want to do well on the Digital SAT. By the time you take the test, using them should be automatic. All the arithmetic concepts are fairly basic, but you'll have to know them cold. You'll also have to know when and how to use your calculator, which will be quite helpful.

What Do You Get?

You should know the following arithmetic terms:

- The result of addition is a *sum* or *total.*
- The result of subtraction is a *difference.*
- The result of multiplication is a *product.*
- The result of division is a *quotient.*
- In the expression 5^2, the 2 is called an *exponent.*

ORDER OF OPERATIONS

The Six Operations Must Be Performed in the Proper Order

Very often, solving an equation on the Digital SAT will require you to perform several different operations, one after another. These operations must be performed in the proper order. In general, the questions are written in such a way that you won't have trouble deciding what comes first. In cases in which you are uncertain, you need to remember only the following sentence:

Please Excuse My Dear Aunt Sally;
she walks from *left* to *right.*

That's **PEMDAS**, for short. It stands for Parentheses, Exponents, Multiplication, Division, Addition, and Subtraction. First, do any calculations inside the parentheses; then take care of the exponents; then perform all multiplication and division, from *left* to *right,* followed by addition and subtraction, from *left* to *right.*

Do It Yourself
The Digital SAT's built-in calculator follows the order of operations. It will also rewrite division into fractions—pretty neat! However, as with all calculators, the built-in calculator will only calculate *exactly* what you put in. Furthermore, if you decide to use your own calculator, be sure to practice and use parentheses whenever necessary. Write out the steps on your scratch paper first to avoid entry errors.

The following exercise will help you learn the order in which to perform the six operations. First, set up the equations on paper. Then, use your calculator for the arithmetic. Make sure you perform the operations in the correct order.

Exercise 1

Solve each of the following problems by performing the indicated operations in the proper order. Answers can be found on page 385.

1. $107 + (109 - 107) =$ ______________
2. $(7 \times 5) + 3 =$ ______________
3. $6 - 3(6 - 3) =$ ______________
4. $2 \times [7 - (6 \div 3)] =$ ______________
5. $10 - (9 - 8 - 6) =$ ______________

Whichever Comes First
For addition and subtraction, solve from left to right. The same is true of multiplication and division. And remember: if you don't solve in order from left to right, you could end up with the wrong answer!
Example:
$24 \div 4 \times 6 = 24 \div 24 = 1$ wrong
$24 \div 4 \times 6 = 6 \times 6 = 36$ right

Parentheses Can Help You Solve Equations

Using parentheses to regroup information in arithmetic problems can be very helpful. In order to do this, you need to understand a basic law that you have probably forgotten since the days when you last took arithmetic—the **Distributive Law.** You don't need to remember the name of the law, but you do need to know how to use it to help you solve problems.

The Distributive Law

If you're multiplying the sum of two numbers by a third number, you can multiply each number in your sum individually. This comes in handy when you have to multiply the sum of two variables.

If a question gives you information in "factored form"—$a(b + c)$—then you should distribute the first variable before you do anything else. If you are given information that has already been distributed—$(ab + ac)$—then you should factor out the common term, putting the information back in factored form. Very often on the Digital SAT, simply doing this will enable you to spot the answer.

Here are some examples:

Distributive: $6(53) + 6(47) = 6(53 + 47) = 6(100) = 600$

Multiplication first: $6(53) + 6(47) = 318 + 282 = 600$

You get the same answer each way, so why get involved with complicated arithmetic? If you use the Distributive Law for this problem, you don't even need to use your calculator.

The exercise below illustrates the Distributive Law.

Exercise 2

Rewrite each problem by either distributing or factoring and then solve. (Hint: For questions 1, 2, 4, and 5, try factoring.) Questions 3, 4, and 5 have no numbers in them; therefore, they can't be solved with a calculator. Answers can be found on page 385.

1. $(6 \times 57) + (6 \times 13) =$ ______________
2. $51(48) + 51(50) + 51(52) =$ ________
3. $a(b + c - d) =$ ______________
4. $xy - xz =$ ______________
5. $abc + xyc =$ ______________

FRACTIONS

A Fraction Is Just Another Way of Expressing Division

The expression $\frac{x}{y}$ is exactly the same thing as $x \div y$. The expression $\frac{1}{2}$ means nothing more than $1 \div 2$. In the fraction $\frac{x}{y}$, x is known as the **numerator,** and y is known as the **denominator.**

Fractions and Calculators

It can be tempting to try to directly enter the information from the question into the calculator. The writers of the Digital SAT know this, so they write questions that will lead you to errors if you do so. Always start by writing out what you need to calculate on your scratch paper; only afterwards should you move to use the calculator.

On the Digital SAT's built-in calculator, fractions will appear as fractions. You can use arrow keys or a mouse to navigate between the numerator and denominator of a fraction.

The results will be given as a decimal. To get the results as a fraction, click the button next to the entry box that looks like this:

Adding and Subtracting Fractions with the Same Denominator

To add two or more fractions that all have the same denominator, simply add the numerators and put the sum over the common denominator. Consider the following example:

$$\frac{1}{100}+\frac{4}{100}=\frac{1+4}{100}=\frac{5}{100}$$

Subtraction works exactly the same way:

$$\frac{4}{100}-\frac{1}{100}=\frac{4-1}{100}=\frac{3}{100}$$

Call on the Calculator

Whether you are adding, subtracting, multiplying, or dividing fractions, a calculator can make it a snap. Keep the following things in mind:

1. Practice with whichever calculator you plan to use on test day.
2. Follow the order of operations and use parentheses to make sure the calculator does exactly what you want.
3. Learn the rules anyway. You might need them if, for example, the fractions include variables.

Adding and Subtracting Fractions with Different Denominators

In school you were taught to add and subtract fractions with different denominators by finding the common denominator. To do this, you have to multiply each fraction by a number that makes all the denominators the same. Most students find this process annoying.

Fortunately, we have an approach to adding and subtracting fractions with different denominators that simplifies the entire process. Use the example below as a model. Just multiply in the direction of each arrow, and then either add or subtract across the numerator. Lastly, multiply across the denominator.

$$\frac{1}{3}+\frac{1}{2}=$$

$$\begin{array}{ccc} \mathbf{2} & \mathbf{3} & \\ \frac{1}{3} & \frac{1}{2} & \mathbf{6} \end{array}$$

$$\frac{2+3}{6}=\frac{5}{6}$$

We call this procedure the **Bowtie** because the arrows make it look like a bowtie. Use the Bowtie to add or subtract any pair of fractions without thinking about the common denominator, just by following the steps above.

Multiplying All Fractions

Multiplying fractions is easy. Just multiply across the numerator; then multiply across the denominator.

Here's an example.

$$\frac{4}{5}\times\frac{5}{6}=\frac{20}{30}$$

When you multiply fractions, all you are really doing is performing one multiplication problem on top of another.

You should never multiply two fractions before looking to see if you can reduce either or both. If you reduce first, your final answer will be in the form that the test-writers are looking for. Here's another way to express this rule: *Simplify before you multiply.*

$$\frac{63}{6} \times \frac{48}{7} = \frac{\overset{9}{\cancel{63}}}{6} \times \frac{48}{\underset{1}{\cancel{7}}} = \frac{\overset{9}{\cancel{63}}}{\underset{1}{\cancel{6}}} \times \frac{\overset{8}{\cancel{48}}}{\underset{1}{\cancel{7}}} =$$

$$\frac{9}{1} \times \frac{8}{1}$$

$$\frac{72}{1} = 72$$

Dividing All Fractions

To divide one fraction by another, flip over (or take the reciprocal of) the second fraction and multiply.

Here's an example.

$$\frac{2}{3} \div \frac{4}{3} =$$

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$$

Be careful not to cancel or reduce until after you flip the second fraction. You can even do the same thing with fractions whose numerators and/or denominators are fractions. These problems look quite frightening, but they're actually easy if you keep your cool.

Just Flip It
Dividing by a fraction is the same thing as multiplying by the reciprocal of that fraction. So just flip over the fraction you are dividing by and multiply instead.

Here's an example.

$$\frac{4}{\frac{4}{3}} =$$

$$\frac{4}{1} \div \frac{4}{3} =$$

$$\frac{4}{1} \times \frac{3}{4} =$$

$$\frac{\cancel{4}}{1} \times \frac{3}{\cancel{4}} =$$

$$\frac{3}{1} = 3$$

Reducing Fractions

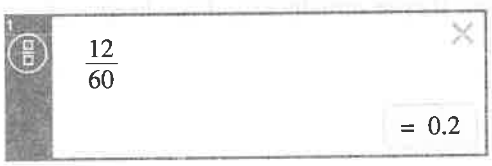
When you add or multiply fractions, you will very often end up with a big fraction that is hard to work with. You can almost always reduce such a fraction into one that is easier to handle.

Start Small
It is not easy to see that 26 and 286 have a common factor of 13, but it's pretty clear that they're both divisible by 2. So start from there.

To reduce a fraction, divide both the numerator and the denominator by the largest number that is a factor of both. For example, to reduce $\frac{12}{60}$, divide both the numerator and the denominator by 12, which is the largest number that is a factor of both. Dividing 12 by 12 yields 1; dividing 60 by 12 yields 5. The reduced fraction is $\frac{1}{5}$.

If you can't immediately find the largest number that is a factor of both, find any number that is a factor of both and divide both the numerator and denominator by that number. Your calculations will take a little longer, but you'll end up in the same place. In the previous example, even if you don't see that 12 is a factor of both 12 and 60, you can no doubt see that 6 is a factor of both. Dividing numerator and denominator by 6 yields $\frac{2}{10}$. Now divide both numbers by 2. Doing so yields $\frac{1}{5}$. Once again, you have arrived at the answer.

Reducing fractions can be pretty easy on a calculator. Using the built-in calculator, click the button to the left of the entry field that toggles between fractions and decimals. When in fraction mode, the result will appear in its most reduced form. Take a look at the same fraction in decimal mode and fraction mode:



Converting Mixed Numbers to Fractions

A **mixed number** is a number such as $2\frac{3}{4}$. It is the sum of an integer and a fraction. When you see mixed numbers on the Digital SAT, you should usually convert them to ordinary fractions.

> Here's a quick and easy way to convert mixed numbers.
>
> - Multiply the integer by the denominator.
> - Add this product to the numerator.
> - Place this sum over the denominator.

For practice, let's convert $2\frac{3}{4}$ to a fraction. Multiply 2 (the integer part of the mixed number) by 4 (the denominator). That gives you 8. Add that to the 3 (the numerator) to get 11. Place 11 over 4 to get $\frac{11}{4}$.

The mixed number $2\frac{3}{4}$ is exactly the same as the fraction $\frac{11}{4}$. We converted the mixed number to a fraction because fractions are easier to work with than mixed numbers.

Just Don't Mix
For some reason, the test-writers think it's okay to give you mixed numbers as answer choices. On fill-in questions, however, if you use a mixed number, you won't get credit. You can see why. In your fill-in box, $3\,1/4$ will look like $\frac{31}{4}$. See Chapter 26 for more about fill-in questions.

Exercise 3

Try converting the following mixed numbers to fractions. Answers can be found on page 385.

1. $8\frac{1}{3}$

2. $2\frac{3}{7}$

3. $5\frac{4}{9}$

4. $2\frac{1}{2}$

5. $6\frac{2}{3}$

Fractions Behave in Peculiar Ways

Fractions don't always behave the way you might want them to. For example, because 4 is obviously greater than 2, it's easy to forget that $\frac{1}{4}$ is less than $\frac{1}{2}$. It's particularly confusing when the numerator is something other than 1. For example, $\frac{2}{7}$ is less than $\frac{2}{5}$. Finally, you should keep in mind that when you multiply one fraction by another, you'll get a fraction that is smaller than either of the first two. Study the following example:

$$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

$$\frac{1}{8} < \frac{1}{2}$$

$$\frac{1}{8} < \frac{1}{4}$$

A Final Word About Fractions and Calculators

Throughout this section, we've given you some hints about using a calculator to work with fractions. The built-in calculator or your own can be a tremendous help if you know how to use it properly. Make sure you practice with the calculator you plan to use on test day so that working fractions on it becomes second nature before the test.

Exercise 4

Work these examples with the techniques you've read about in this chapter so far. Then work them again using a calculator. If you have any problems, go back and review the information just outlined. Answers can be found on page 385.

1. Reduce $\frac{18}{6}$. ____________________

2. Convert $6\frac{1}{5}$ to an improper fraction. ____________

3. $2\frac{1}{3} - 3\frac{3}{5} =$ ____________________

4. $\frac{5}{18} \times \frac{6}{25} =$ ____________________

5. $\frac{3}{4} \div \frac{7}{8} =$ ______

6. $\frac{\frac{2}{5}}{5} =$ ______

7. $\frac{\frac{1}{3}}{\frac{3}{4}} =$ ______

DECIMALS

A Decimal Is Just Another Way of Expressing a Fraction

Fractions can be expressed as decimals. To find a fraction's decimal equivalent, simply divide the numerator by the denominator. (You can do this easily with a calculator.)

$$\frac{3}{5} =$$

$$3 \div 5 = 0.6$$

Adding, Subtracting, Multiplying, and Dividing Decimals

Manipulating decimals is easy with a calculator. Simply punch in the numbers—being especially careful to get the decimal point in the right place every single time—and read the result from the display. A calculator makes these operations easy. In fact, working with decimals is one area on the Digital SAT in which your calculator will prevent you from making careless errors. You won't have to line up decimal points or remember what happens when you divide. The calculator will keep track of everything for you, as long as you punch in the correct numbers to begin with. Just be sure to practice carefully before test day.

Exercise 5

Calculate each of the answers to the following questions on paper with your pencil, rounding any awkward numbers to make the math easier to handle. Sometimes, estimating the answer is all you need to do to answer a multiple-choice question. Then use a calculator to find the exact answer. Answers can be found on page 385.

1. $0.43 \times 0.87 =$ ____________

2. $\dfrac{43 + 0.731}{0.03} =$ ____________

3. $3.72 \div 0.02 =$ ____________

4. $0.71 - 3.6 =$ ____________

EXPONENTS AND SQUARE ROOTS

Exponents Are a Kind of Shorthand

Warning #1
The rules for multiplying and dividing exponents do not apply to addition or subtraction:
$2^2 + 2^3 = 12$
$(2 \times 2) + (2 \times 2 \times 2) = 12$
It does not equal 2^5 or 32.

Many numbers are the product of the same value multiplied over and over again. For example, $32 = 2 \times 2 \times 2 \times 2 \times 2$. Another way to write this would be $32 = 2^5$, or "thirty-two equals two to the fifth power." The little number, or **exponent,** denotes the number of times that 2 is to be used as a factor. In the same way, $10^3 = 10 \times 10 \times 10$, or 1,000, or "ten to the third power," or "ten cubed." In this example, the 10 is called the **base** and the 3 is called the **exponent.** (You won't need to know these terms on the Digital SAT, but you will need to know them in order to understand our explanations.)

Multiplying Numbers with Exponents

To multiply two numbers with the same base, simply add the exponents. For example, $2^3 \times 2^5 = 2^{3+5} = 2^8$.

Dividing Numbers with Exponents

To divide two numbers with the same base, simply subtract the exponents. For example, $\frac{2^5}{2^3} = 2^{5-3} = 2^2$.

Raising a Power to a Power

When you raise a power to a power, you multiply the exponents. For example, $(2^3)^4 = 2^{3 \times 4} = 2^{12}$.

MADSPM

To remember the exponent rules, all you need to do is remember the acronym **MADSPM**. Here's what it stands for:

- Multiply → Add
- Divide → Subtract
- Power → Multiply

Whenever you see an exponent question, you should think MADSPM. The three MADSPM rules are the only rules that apply to exponent operations.

Here's a typical Digital SAT exponent question.

Warning #2
Parentheses are very important with exponents because you must remember to distribute powers to everything within them.

For example, $(3x)^2 = 9x^2$, not $3x^2$. Similarly, $\left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2}$, not $\frac{9}{2}$. But the Distributive Law applies only when you multiply or divide: $(x+y)^2 = x^2 + 2xy + y^2$, not $x^2 + y^2$.

1 Mark for Review

For the equations $\frac{a^x}{a^y} = a^{10}$ and $(a^y)^3 = a^x$, if $a > 1$, what is the value of x?

Ⓐ 5

Ⓑ 10

Ⓒ 15

Ⓓ 20

Here's How to Crack It

The question asks for the value of x, but it looks pretty intimidating with all those variables. In fact, you might be about to cry "POOD" and go on to the next question. That might not be a bad idea, but before you skip the question, pull out those MADSPM rules.

For the first equation, you can use the Divide-Subtract Rule: $\frac{a^x}{a^y} = a^{x-y} = a^{10}$. In other words, the first equation tells you that $x - y = 10$.

For the second equation, you can use the Power-Multiply Rule: $\left(a^y\right)^3 = a^{3y} = a^x$. So, that means that $3y = x$.

Now, it's time to substitute: $x - y = 3y - y = 10$. So, $2y = 10$ and $y = 5$. Be careful, though! Don't choose (A). That's the value of y, but the question wants to know the value of x. Since $x = 3y$, $x = 3(5) = 15$. The correct answer is (C).

You could also do this question by using Plugging In the Answers, or PITA, which will be discussed in more detail later in this book. Of course, you still need to know the MADSPM rules to do the question that way.

Exponents and Your Calculator

The built-in calculator has both an a^2 button (for squaring a value) and an a^b button (for other exponents). As with fractions, the built-in calculator shows you the operations in the same way you'd write them out on paper.

The Peculiar Behavior of Exponents

Raising a number to a power can have quite peculiar and unexpected results, depending on what sort of number you start out with. Here are some examples.

- If you square or cube a number greater than 1, it becomes larger. For example, $2^3 = 8$.
- If you square or cube a positive fraction smaller than one, it becomes smaller. For example, $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$.
- A negative number raised to an even power becomes positive. For example, $(-2)^2 = 4$.
- A negative number raised to an odd power remains negative. For example, $(-2)^3 = -8$.

See the Trap

The test-writers may hope you won't know these strange facts about exponents and throw them in as trap answers. Knowing the peculiar behavior of exponents will help you avoid these tricky pitfalls in a question.

You should also have a feel for relative sizes of exponential numbers without calculating them. For example, 2^{10} is much larger than 10^2 ($2^{10} = 1{,}024$; $10^2 = 100$). To take another example, 2^5 is twice as large as 2^4, even though 5 seems only a bit larger than 4.

Square Roots

The radical sign ($\sqrt{\ }$) indicates the **square root** of a number. For example, $\sqrt{25} = 5$. Note that square roots cannot be negative. If the test-writers want you to think about a negative solution, they won't use the radical sign; instead they'll say $x^2 = 25$ because then $x = 5$ or $x = -5$.

The Only Rules You Need to Know

Here are the only rules regarding square roots that you need to know for the Digital SAT.

1. $\sqrt{x}\sqrt{y} = \sqrt{xy}$. For example, $\sqrt{3}\sqrt{12} = \sqrt{36} = 6$.

2. $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$. For example, $\sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{\sqrt{4}} = \frac{\sqrt{5}}{2}$.

3. $\sqrt{x}$ = positive root only. For example, $\sqrt{16} = 4$.

Note that rule 1 works in reverse: $\sqrt{50} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$. This is really a kind of factoring. You are using rule 1 to factor a large, clumsy radical into numbers that are easier to work with. Rule 2 works in reverse as well. $\sqrt{75}$ divided by $\sqrt{3}$ looks complicated, but $\sqrt{\frac{75}{3}} = \sqrt{25} = 5$. And remember that radicals are just fractional exponents, so the same rules of distribution apply. We'll get to fractional exponents below.

Careless Errors

The Digital SAT will try to confuse you with the behavior of roots. Remember that the square root of a number between 0 and 1 is *larger* than the original number. For example, $\sqrt{\frac{1}{4}} = \frac{1}{2}$, and $\frac{1}{2} > \frac{1}{4}$.

Negative and Fractional Exponents

So far we've dealt with only positive integers for exponents, but they can be negative integers as well as fractions. The same concepts and rules apply, but the numbers just look a little weirder. Keep these concepts in mind:

- Negative exponents are a fancy way of writing reciprocals:

$$x^{-n} = \frac{1}{x^n}$$

- Fractional exponents are a fancy way of taking roots and powers:

$$x^{\frac{y}{z}} = \sqrt[z]{x^y}$$

Roots and the Calculator

The Digital SAT's built-in calculator has a button for square roots right on the main ABC keypad. The calculator can do other roots, but College Board has hidden that function. Click the "funcs" button and scroll all the way down to the "NUMBER THEORY" section. There you'll find the $\sqrt[n]{}$ button. As with other functions on the built-in calculator, both radicals will look the same as they would if you wrote them out yourself on paper.

If you don't want to scroll through a menu, you can also use fractional exponents using the a^b button to do roots other than square roots, as well.

Here's an example.

2 Mark for Review

If $x > 0$, which of the following is equivalent to $\sqrt{x^3}$?

I. $x + x^{\frac{1}{2}}$

II. $\left(x^{\frac{1}{2}}\right)^3$

III. $\left(x^2\right)\left(x^{-\frac{1}{2}}\right)$

Ⓐ None

Ⓑ I and II only

Ⓒ II and III only

Ⓓ I, II, and III

Here's How to Crack It

The question asks for an equivalent form of a root and gives three expressions with exponents, so it really tests your knowledge of exponents. First, convert $\sqrt{x^3}$ into an exponent to more easily compare it to the choices in the Roman numeral statements. (Plus, exponents are easier to work with because they have those nice MADSPM rules.) According to the definition of a fractional exponent, $\sqrt{x^3} = x^{\frac{3}{2}}$. You want the items in the Roman numerals to equal $x^{\frac{3}{2}}$.

Now, it's time to start working with the Roman numerals. In (I), the test-writers are trying to be tricky. (There's a surprise.) There's no exponent rule for adding exponent expressions with like bases. So, $x + x^{\frac{1}{2}}$ does *not* equal $x^{\frac{3}{2}}$. (If you want to be sure, you could try a number for x: if $x = 4$, then $\sqrt{4^3} = 8$, but $4 + 4^{\frac{1}{2}} = 4 + 2 = 6$.) So, cross off any answer that includes (I): (B) and (D) are gone.

Now, since you are down to either (A) or (C), all you really need to do is try either (II) or (III). If either one works, the answer is (C). Try (II). Use the Power-Multiply Rule: $\left(x^{\frac{1}{2}}\right)^3 = x^{\left(\frac{1}{2}\right)(3)} = x^{\frac{3}{2}}$. Since (II) works, (C) is the correct answer.

Notice that you didn't even need to check (III). Using POE on a Roman numeral question often means that you won't need to check all of the Roman numerals.

HOW TO READ CHARTS AND GRAPHS

Another basic math skill you will need for the Digital SAT is the ability to interpret data from charts, graphs, tables, and more. This section will cover the basics of reading these figures. How to answer questions related to charts and other figures in detail will be discussed in Chapter 24.

What's Up with All of These Figures?

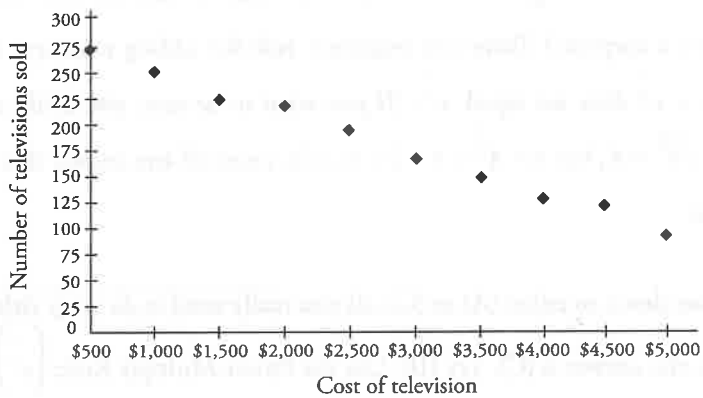
The Digital SAT includes charts, graphs, and tables throughout the test to present data for students to analyze. College Board believes this will better reflect what students learn in school and what they need to understand in the real world. Questions will typically include real-life scenarios, such as finance and business situations, social science issues, and scientific matters.

Since you'll be seeing graphics throughout the test, let's look at the types you may encounter and the skills you'll need to work with to analyze charts and graphs.

Types of Graphs

The Scatterplot

A **scatterplot** is a graph with distinct data points, each representing one piece of information. On the scatterplot below, each dot represents the number of televisions sold at a certain price point.



Here's How to Read It

To find the cost of a television when 225 televisions are sold, start at 225 on the vertical axis and look to the right until you hit a data point. Use the edge of your scratch paper as a ruler or, if you have a steady hand, drag the mouse pointer. Once you hit a point, visualize (again using your scratch paper or the mouse pointer to help) a straight line down from the point to the horizontal axis and read the number the line hits, which should be $1,500. To determine the number of televisions sold when they cost a certain amount, reverse the steps—start at the bottom, look up until you hit a point, and then look left until you intersect the vertical axis.

Now try a question based on that scatterplot.

---

3 Mark for Review

A certain store sells televisions ranging in price from $500 to $5,000 in increments of $500. The scatterplot graph shows the total number of televisions sold at each price during the last 12 months. Approximately how much more revenue did the store collect from the televisions it sold priced at $3,500 than it did from the televisions it sold priced at $1,000?

(A) $175,000

(B) $250,000

(C) $275,000

(D) $350,000

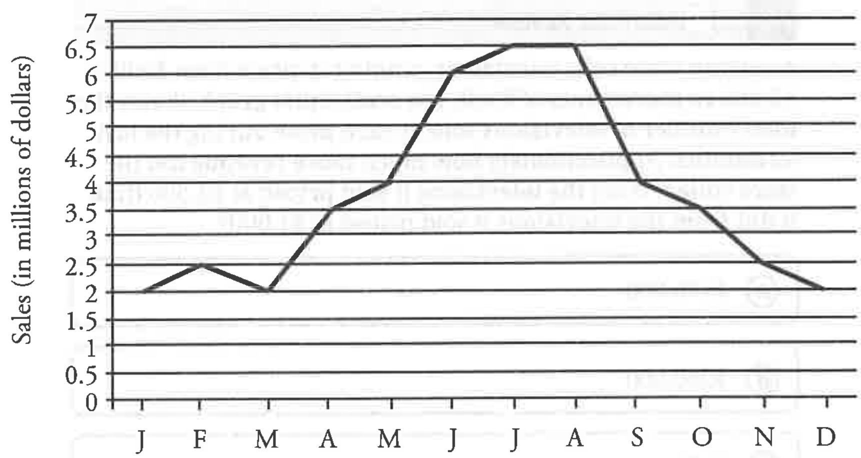
Here's How to Crack It

The question asks for the difference in revenue from selling televisions at two different prices. The revenue is the *cost of television* × *number of televisions sold*. You need the information from the graph only for the television that costs $3,500 and for the television that costs $1,000 in order to determine how much more revenue the $3,500 television produced. There were 150 of the $3,500 televisions sold, for a revenue of $525,000. There were 250 of the $1,000 televisions sold, for a revenue of $250,000. The difference between the two is $525,000 – $250,000 = $275,000. The correct answer is (C).

A scatterplot may also include a **line of best fit**. This is the line that best represents the data.

The Line Graph

A **line graph** is similar to a scatterplot in that it shows different data points that relate the two variables. The difference with a line graph, though, is that the points have been connected to create a continuous line.



Here's How to Read It

Reading a line graph is very similar to reading a scatterplot. Start at the axis that represents the data given, and use scratch paper or the mouse pointer to visualize a straight line up or to the right until you intersect the graph line. Then move left or down until you hit the other axis. For example, in February, indicated by an F on the horizontal axis, there were $2.5 million in sales. Be sure to notice the units on each axis. If February sales were only $2.50, rather than $2.5 million, then this company wouldn't be doing very well!

Let's look at a question about this line graph.

4 Mark for Review

The forecasted monthly sales of sunscreen are presented in the line graph. For which period are the forecasted monthly sales figures strictly decreasing and then strictly increasing?

Ⓐ January to March

Ⓑ February to April

Ⓒ June to August

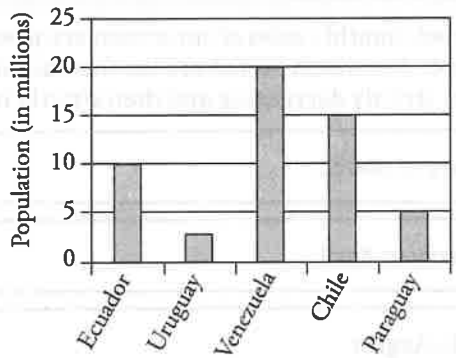
Ⓓ September to November

Here's How to Crack It

The question asks for a period during which the forecasted sales are decreasing and then increasing. Look up the values for each period in question and use Process of Elimination to get rid of those that don't fit. For (A), January sales are forecasted to be $2 million, February $2.5 million, and March $2 million. This is an increase followed by a decrease, not the other way around, so eliminate (A). For (B), you already know sales decreased from February to March, so check for a following increase in April. The figure for April is $3.5 million, which is an increase over the March figure. The correct answer is (B).

The Bar Graph (or Histogram)

Instead of showing a variety of different data points, a **bar graph** shows the number of items that belong to a particular category. If the variable at the bottom is given in ranges instead of distinct items, the graph is called a **histogram,** but you read it the same way.



Here's How to Read It

The height of each bar corresponds to a value on the vertical axis. In this case, the bar above Chile hits the line that intersects with 15 on the vertical axis, so there are 15 million people in Chile. Again, watch the units to make sure you know what the numbers on the axes represent. On this graph, horizontal lines are drawn at 5-unit intervals, making the graph easier to read. If these lines do not appear on a bar graph, use your scratch paper to determine the height of a given bar.

Here's an example of a bar graph question, which is based on the Populations of Countries graph above.

---

5 Mark for Review

The populations of five countries are shown in the bar graph. If population density is defined as $\frac{\text{population}}{\text{area}}$, and the area of Paraguay is 400,000 square kilometers, what is the population density of Paraguay, in people per square kilometer?

Ⓐ 0.08

Ⓑ 0.8

Ⓒ 1.25

Ⓓ 12.5

Here's How to Crack It

The question asks for the population density of Paraguay. Start by determining the population of Paraguay. The bar hits right at the horizontal line for 5, which is in millions, so there are 5 million people in Paraguay. Now use the definition of population density in the question.

$$\frac{\text{population}}{\text{area}} = \frac{5{,}000{,}000}{400{,}000}$$

Be very careful with the number of zeroes you put in the fraction—the answer choices are pairs that vary by a factor of 10, meaning the test-writers expect you to miss a zero. The answer must be greater than 1, since your numerator is bigger than your denominator, so eliminate (A) and (B). Choice (C) also seems too small, but check the math on your calculator (carefully). You should get 12.5 people per square kilometer. The correct answer is (D).

The Two-Way Table

A **two-way table** is another way to represent data without actually graphing it. Instead of having the variables represented on the vertical and horizontal axes, the data will be arranged in rows and columns. The top row will give the headings for each column, and the left-most column will give the headings for each row. The numbers in each box indicate the data for the category represented by the row and the column the box is in. This two-way table, for example, shows computer production arranged by days of the week and shift times.

	Morning Shift	Afternoon Shift
Monday	200	375
Tuesday	245	330
Wednesday	255	340
Thursday	250	315
Friday	225	360

Here's How to Read It

If you want to find the number of computers produced on Tuesday morning, you can start in the Morning Shift column and look down until you find the number in the row that says Tuesday, or you can start in the row for Tuesday and look to the right until you find the Morning Shift column. Either way, the result is 245. Some tables will give you totals in the bottom row and/or the right-most column, but sometimes you will need to find the totals yourself by adding up all the numbers in each row or in each column. More complicated tables will have more categories listed in rows and/or columns, or the tables may even contain extraneous information.

Give this one a try.

6 Mark for Review

	Morning Shift	Afternoon Shift
Monday	200	375
Tuesday	245	330
Wednesday	255	340
Thursday	250	315
Friday	225	360

Computer production at a factory occurs during two shifts, as shown in the chart above. If computers are produced only during the morning and afternoon shifts, on which of the following pairs of days is the greatest total number of computers produced?

(A) Monday and Thursday

(B) Tuesday and Thursday

(C) Wednesday and Friday

(D) Tuesday and Friday

Here's How to Crack It

The question asks for the pair of days on which the greatest number of computers was produced. This is a perfect calculator question. Just add the Morning Shift and the Afternoon Shift for each day to see which total is the greatest. Write each day and total on your scratch paper, so you don't have to keep track of it all in your head. Monday is 200 + 375 = 575, Tuesday is 245 + 330 = 575, Wednesday is 255 + 340 = 595, Thursday is 250 + 315 = 565, and Friday is 225 + 360 = 585. According to these calculations, Wednesday and Friday have the two greatest totals, so the greatest number of computers is produced on those two days together. The correct answer is (C).

Figure Facts

Every time you encounter a figure or graphic on the Digital SAT, you should make sure you understand how to read it by checking the following:

- What are the variables for each axis or the headings for the table?
- What units are used for each variable?
- Are there any key pieces of information (numbers, for example) in the legend of the chart that you should note?
- What type of relationship is shown by the data in the chart? For instance, if the chart includes curves that show an upward slope, then the graph shows a **positive association,** while curves that show a downward slope show a **negative association.**
- You can use the edge of your scratch paper as a ruler to help you make sure you are locating the correct data in the graph. The mouse pointer on the screen can also help, as long as you trust yourself to move it in a straight line.

Surveys on the Digital SAT

Some Digital SAT math questions will appear to be about data, but there's no figure and the question has a lot of words about survey results. These questions are usually about a biased sample, meaning that the group doing the survey asked people who are likely to already be in favor of or opposed to the issue, but then drew a conclusion about a larger group. For example, we couldn't survey people who read this book to ask whether they read the final question and take bite-sized pieces and then assume that everyone who takes the Digital SAT does the same thing.

The best way to perform a survey is to ask a random sample rather than asking a group that's probably already picked a side. Questions about surveys might include numbers, and there's a chance that there is a problem with the sample size, meaning the survey asked too few people to get an accurate result. But usually the number is a distraction, so focus on using POE and crossing out answers that don't fit the information given in the question.

Fundamentals Drill

Work these questions using the skills you've learned so far. Be sure to use a calculator when necessary to avoid careless calculation errors. Don't forget, though, that using it may slow you down when doing the math on paper would be faster. Answers and explanations can be found starting on page 386.

1 Mark for Review

If 7 times a number is 84, what is 4 times the number?

(A) 16

(B) 28

(C) 48

(D) 56

2 Mark for Review

If $3x = 12$, what is the value of $\frac{24}{x}$?

(A) $\frac{1}{6}$

(B) $\frac{2}{3}$

(C) 4

(D) 6

3 Mark for Review

Which of the following represents the statement "the sum of the squares of x and y is equal to the square root of the difference of x and y"?

(A) $x^2 + y^2 = \sqrt{x - y}$

(B) $x^2 - y^2 = \sqrt{x + y}$

(C) $(x + y)^2 = \sqrt{x} - \sqrt{y}$

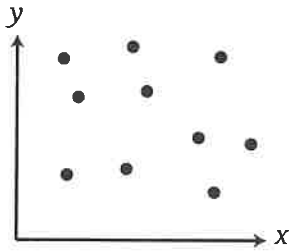
(D) $\sqrt{x + y} = (x - y)^2$

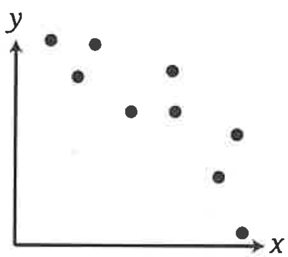
4 Mark for Review

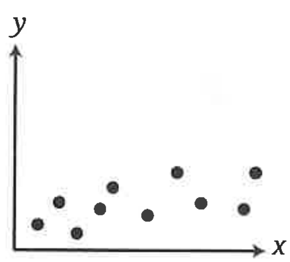
If $a = -2$, then $a + a^2 - a^3 + a^4 - a^5 =$

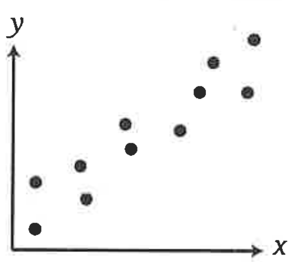
5 Mark for Review

Which of the following graphs shows a strong positive association between x and y?

Ⓐ


Ⓑ


Ⓒ


Ⓓ


6 Mark for Review

If $9^{-2} = \left(\frac{1}{3}\right)^x$, what is the value of x?

Ⓐ 1

Ⓑ 2

Ⓒ 4

Ⓓ 6

7 Mark for Review

If $\sqrt{x} + 22 = 38$, what is the value of x?

Ⓐ 4

Ⓑ 16

Ⓒ 32

Ⓓ 256

8 Mark for Review

$$\frac{1}{8}+\frac{1}{10}=\frac{a}{b}$$

In the equation above, if a and b are positive integers and $\frac{a}{b}$ is in its simplest reduced form, what is the value of a?

Ⓐ 2

Ⓑ 9

Ⓒ 18

Ⓓ 40

9 Mark for Review

If $4^x \cdot n^2 = 4^{x+1} \cdot n$ and x and n are both positive integers, what is the value of n?

Answers to Chapter Exercises

Exercise 1

1. 109
2. 38
3. –3
4. 10
5. 15

Exercise 2

1. $6(57 + 13) = 6 \times 70 = 420$
2. $51(48 + 50 + 52) = 51(150) = 7{,}650$
3. $ab + ac - ad$
4. $x(y - z)$
5. $c(ab + xy)$

Exercise 3

1. $\frac{25}{3}$
2. $\frac{17}{7}$
3. $\frac{49}{9}$
4. $\frac{5}{2}$
5. $\frac{20}{3}$

Exercise 4

1. 3
2. $\frac{31}{5}$
3. $-\frac{19}{15}$
4. $\frac{1}{15}$
5. $\frac{6}{7}$
6. $\frac{2}{25}$
7. $\frac{4}{9}$

Exercise 5

	Estimated Answer	Calculator Answer
1.	$0.4 \times 0.9 = 0.36$	0.3741
2.	$44 \div 0.03 = 1{,}466$	1,457.7
3.	$3.7 \div 0.02 = 185$	186
4.	$0.7 - 3.6 = -2.9$	–2.89

FUNDAMENTALS DRILL ANSWERS AND EXPLANATIONS

1. **C** The question asks for the value of 4 times an unknown number. Translate the English into math, calling the number n, to get $7n = 84$. Divide both sides by 7 to get $n = 12$. Finally, $4n = 4(12) = 48$. The correct answer is (C).

2. **D** The question asks for the value of $\frac{24}{x}$. First, solve for x. Divide both sides of the equation by 3, and you get $x = 4$. Then divide 24 by 4, which gives you 6. The correct answer is (D).

3. **A** The question asks for an algebraic expression. Translate the English into math by taking it one phrase at a time. "Sum" means you will add two things. The "squares of x and y" means to square x and square y, or x^2 and y^2. Add these to get $x^2 + y^2$. Cross out any choice that does not have $x^2 + y^2$ as the first part of the equation. Eliminate (B), (C), and (D). The correct answer is (A).

4. **58** The question asks for the value of an expression for a certain value of the variable. Plug in the number given for a in the expression to find the value: $-2 + (-2)^2 - (-2)^3 + (-2)^4 - (-2)^5$. Remember PEMDAS, the order of operations. The first thing to do here is deal with the **E**xponents, then take care of the **A**ddition and **S**ubtraction: $-2 + 4 - (-8) + 16 - (-32)$, which simplifies to $-2 + 4 + 8 + 16 + 32 = 58$. The correct answer is 58.

5. **D** The question asks for the graph that shows a strong positive association between x and y. A "strong positive association" means that as one variable increases, the other one increases. This will be shown as a line that angles through the graph from the lower left to the upper right. These scatterplots don't have any lines of best fit drawn on them, so imagine the line that would go through most of the points on each graph. In (A), the points are all over the place, so no line of best fit can even be drawn. Eliminate (A). In (B), the line that hits most of the points would go from the upper left to the lower right. This is a negative association, not a positive one, so eliminate (B). In (C), the line would go straight across, parallel to the x-axis. This is not a positive association, so eliminate (C). The correct answer is (D).

6. **C** The question asks for the value of x in an equation with exponents. A negative exponent means to take the reciprocal and apply the positive exponent. So $9^{-2} = \left(\frac{1}{9}\right)^2 = \frac{1}{81}$. Now find what power of $\frac{1}{3}$ equals $\frac{1}{81}$. Because $3^4 = 81$, $\left(\frac{1}{3}\right)^4 = \frac{1}{81}$, and x must be 4. The correct answer is (C).

7. **D** The question asks for the value of x. To solve this equation, get $\sqrt{x}$ by itself by subtracting 22 from both sides. The result is $\sqrt{x} = 16$, so square both sides: $(\sqrt{x})^2 = 16^2$, so $x = 256$. The correct answer is (D).

8. **B** The question asks for the value of a in an equation with fractions. The lowest number that both 8 and 10 are factors of is 40. Convert the fractions to a denominator of 40: $\frac{5}{40} + \frac{4}{40} = \frac{9}{40}$. There is no factor that 9 and 40 have in common, so the fraction cannot be reduced. The number in place of a in $\frac{a}{b}$ is 9. Be careful not to choose (D), which contains the value of b. The correct answer is (B).

9. **4** The question asks for the value of n. First, simplify the equation $4^x \cdot n^2 = 4^{x+1} \cdot n$ by dividing both sides by n to get $4^x \cdot n = 4^{x+1}$, and then try an easy number for x. If $x = 2$, then $4^2 \cdot n = 4^{2+1}$. Since $16n = 4^3$, $16n = 64$ and $n = 4$. The correct answer is 4.

Summary

- There are only six arithmetic operations tested on the Digital SAT: addition, subtraction, multiplication, division, exponents, and square roots.
- These operations must be performed in the proper order (PEMDAS), beginning with operations inside parentheses.
- Apply the Distributive Law whenever possible. This is often enough to find the answer.
- A fraction is just another way of expressing division.
- You must know how to add, subtract, multiply, and divide fractions. Don't forget that you can also use your calculator or the built-in calculator on all questions.
- If any questions involving large or confusing fractions appear, try to reduce the fractions first. Before you multiply two fractions, for example, see if it's possible to reduce either or both of the fractions.
- If you know how to work out fractions on your calculator or the built-in calculator, use it to help you with questions that involve fractions. If you intend to use your calculator for fractions, be sure to practice. You should also know how to work with fractions the old-fashioned way with paper and pencil.
- A decimal is just another way of expressing a fraction.
- Use your calculator or the built-in calculator to add, subtract, multiply, and divide decimals.
- Exponents are a kind of shorthand for expressing numbers that are the product of the same factor multiplied over and over again.
- To multiply two exponential expressions with the same base, add the exponents.
- To divide two exponential expressions with the same base, subtract the exponents.
- To raise one exponential expression to another power, multiply the exponents.

- To remember the exponent rules, think MADSPM.
- When you raise a positive number greater than 1 to a power greater than 1, the result is larger. When you raise a positive fraction less than 1 to an exponent greater than 1, the result is smaller. A negative number raised to an even power becomes positive. A negative number raised to an odd power remains negative.
- When you're asked for the square root of any number, $\sqrt{x}$, you're being asked for the positive root only.
- Here are the only rules regarding square roots that you need to know for the Digital SAT:

$$\sqrt{x} \times \sqrt{y} = \sqrt{xy}$$

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

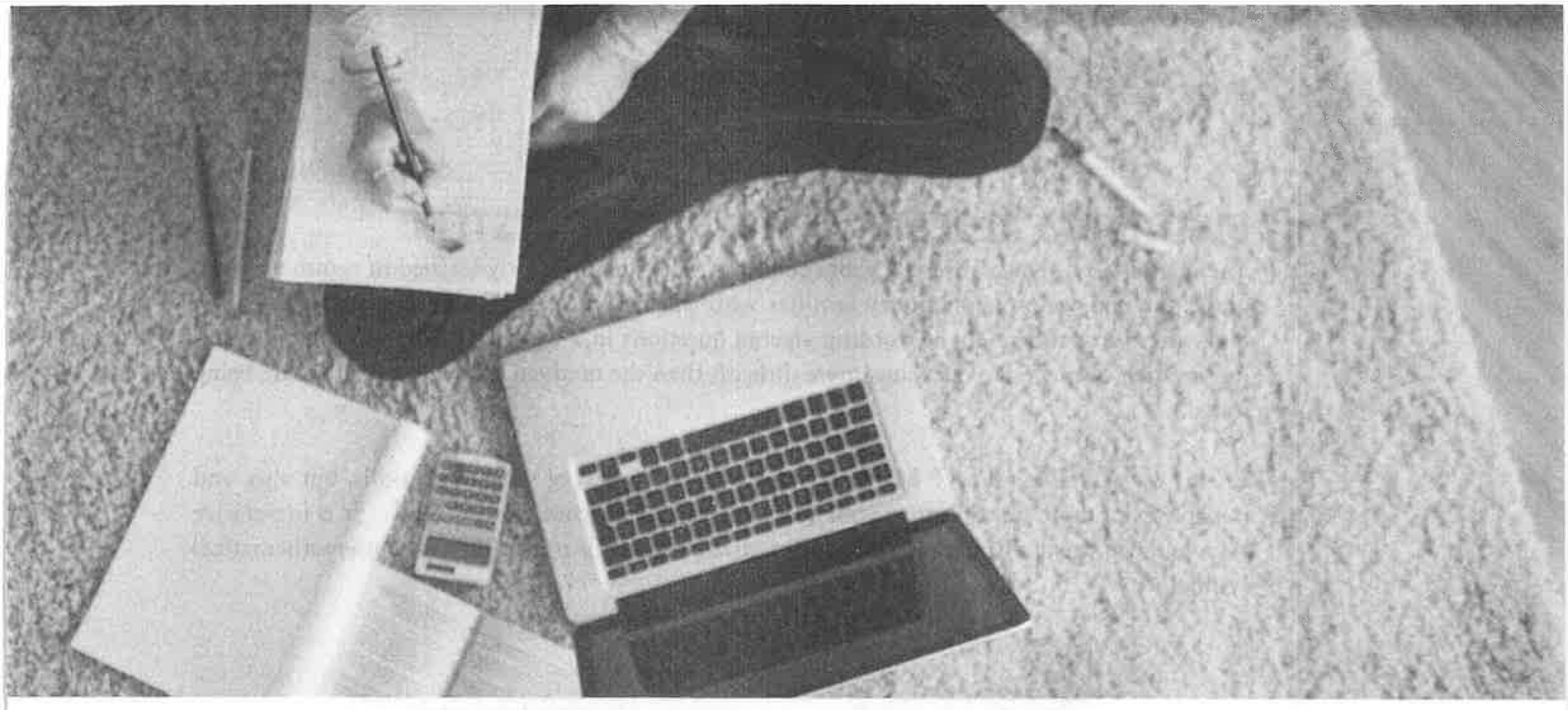
- The rule for fractional exponents is this:

$$x^{\frac{y}{z}} = \sqrt[z]{x^y}$$

- The rule for negative exponents is this:

$$x^{-n} = \frac{1}{x^n}$$

- When you encounter questions with charts, carefully check the chart for important information. Remember that you can use your scratch paper or the mouse pointer to help yourself locate the information.



Chapter 21
Algebra: Cracking the System

In the last chapter, we reviewed some fundamental math concepts featured on the Digital SAT. Many questions raise the difficulty by replacing numbers with variables, or letters that stand for unknown quantities. This chapter covers multiple ways to answer algebra questions, while the next chapter provides some ways to turn algebra questions back into arithmetic.

DIGITAL SAT ALGEBRA: CRACKING THE SYSTEM

The Digital SAT generally tests algebra concepts that you most likely learned in eighth or ninth grade. So, you are probably pretty familiar with the level of algebra on the test. However, the test-writers are fairly adept at wording algebra questions in a way that is confusing or distracting in order to make the questions more difficult than the mathematical concepts that are being tested.

In this way, the Digital SAT Math section is not only a test of your math skills, but also, and possibly even more important to your score improvement, your reading skills. It is imperative that you read the questions carefully and translate the words in the question into mathematical symbols.

ENGLISH	MATH EQUIVALENTS
is, are, were, did, does, costs	=
what (or any unknown value)	*any variable* (x, y, n)
more, sum	+
less, difference	–
of, times, product	× (*multiply*)
ratio, quotient, out of, per	÷

A Little Terminology

Here are some words that you will need to know to understand the explanations in this chapter. These words may even show up in the text of a question, so make sure you are familiar with them.

Term: An equation is like a sentence, and a **term** is the equivalent of a word. It can be just a number, just a variable, or a number multiplied by a variable. For example, 18, $-2x$, and $5y$ are the terms in the equation $18 - 2x = 5y$.

Expression: If an equation is like a sentence, then an **expression** is like a phrase or a clause. An expression is a combination of terms and mathematical operations with no equals or inequality sign. For example, $9 \times 2 + 3x$ is an expression.

Polynomial: A **polynomial** is any expression containing two or more terms. Binomials and trinomials are both examples of polynomials. Binomials have two terms, and trinomials have three terms.

FUNDAMENTALS OF DIGITAL SAT ALGEBRA

Many questions on the Digital SAT require you to work with variables and equations. In your math classes, you probably learned to solve equations by "solving for x" or "solving for y." To do this, you isolate x or y on one side of the equals sign and put everything else on the other side. The good thing about equations is that to isolate the variable you can do anything you want to them—add, subtract, multiply, divide, square—provided you perform the same operation to both sides of the equation.

Thus, the golden rule of equations:

> Whatever you do to the terms on one side of the equals sign, you must do to the terms on the other side of it as well.

Let's look at a simple example of this rule, without the distraction of answer choices.

Problem: If $2x - 15 = 35$, what is the value of x?

Solution: The question asks for the value of x, so you want to isolate the variable. First, add 15 to each side of the equation. Now you have the following:

$$2x = 50$$

Divide each side of the equation by 2. Thus, x equals 25.

The skills for algebraic manipulation work just as well for more complex equations. The following question is another example of the way the Digital SAT may ask you to manipulate equations. Don't panic when you see a question like this; just use the skills you already have and work carefully so you don't make an avoidable mistake in your algebra.

Find the Math
Word problems on the Digital SAT are full of, well, words. To stay focused on the math, always read the final question, work in bite-sized pieces, and use your scratch paper to write down important information. If there are words that don't help you work the problem—such as the explanation of the variables in this question—ignore them.

1 Mark for Review

The wave velocity of a vibrating string can be determined using the formula $v = \sqrt{\frac{T}{\frac{m}{L}}}$, where T is the tension of the string, m is the mass of the string, and L is the length of the string. Which of the following expresses the length of the string in terms of v, T, and m?

Ⓐ $L = \frac{T}{v^2 m}$

Ⓑ $L = \frac{v^2 m}{T}$

Ⓒ $L = v\sqrt{\frac{m}{t}}$

Ⓓ $L = \sqrt{\frac{T}{vm}}$

Here's How to Crack It

The question asks for an equation that expresses the length of a string, which is represented by L, so the goal is to get L by itself. Anything you do to one side of the equation, you must also do to the other side of the equation. Start by squaring both sides of the equation to get rid of the square root on the right side.

The equation becomes

$$v^2 = \frac{T}{\frac{m}{L}}$$

Next, multiply both sides by $\frac{m}{L}$ to get the fraction out of the denominator.

$$\frac{v^2 m}{L} = T$$

To finish isolating L, multiple both sides by L to get

$$v^2 m = TL$$

Now divide both sides by T to get L by itself.

$$\frac{v^2m}{T} = L$$

The correct answer is (B).

SOLVING RADICAL EQUATIONS

Radical equations are just what the name suggests: an equation with a radical ($\sqrt{\ }$) in it. Not to worry, just remember to get rid of the radical first by raising both sides to that power.

Here's an example.

2 Mark for Review

If $7\sqrt{x} - 24 = 11$, what is the value of x?

Ⓐ $\sqrt{5}$

Ⓑ $\sqrt{7}$

Ⓒ 5

Ⓓ 25

Here's How to Crack It

The question asks for the value of x, so start by adding 24 to both sides to get $7\sqrt{x} = 35$. Now, divide both sides by 7 to find that $\sqrt{x} = 5$. Finally, square both sides to find that $x = 25$. The correct answer is (D).

SOLVING RATIONAL EQUATIONS

The built-in calculator or a calculator you bring with you can help on a lot of algebra questions. However, there are times when you will need to solve an equation algebraically by hand. Even when the calculator might help, you may find it more efficient to use your mathematical skills to answer a question. When a question asks you to solve for an expression, algebraic manipulation will often be the best way to answer the question.

Here's an example.

3 Mark for Review

If $\frac{18}{r+10} = \frac{3}{r}$, what is the value of $\frac{r}{3}$?

Ⓐ $\frac{2}{3}$

Ⓑ $\frac{3}{2}$

Ⓒ 2

Ⓓ 3

Here's How to Crack It

The question asks for the value of $\frac{r}{3}$. Cross-multiply to get $18r = 3(r + 10)$ or $18r = 3r + 30$. Subtracting $3r$ from both sides gives you $15r = 30$, so $r = 2$. Finally, $\frac{r}{3} = \frac{2}{3}$. The correct answer is (A).

Extraneous Solutions

Sometimes solving an equation with a rational or radical expression makes funny things happen. Look at the following example.

4 Mark for Review

$$\sqrt{t+4} = t - 2$$

Which of the following contains all possible solutions to the equation above?

Ⓐ 0, 5

Ⓑ 0, 4, 5

Ⓒ 0

Ⓓ 5

Extra Answers
Any time you are solving for a variable, make sure your solutions actually work. If they do not, they are *extraneous,* or extra.

Here's How to Crack It

The question asks for the solution set to the equation, so solve it for t. Start by squaring both sides of the equation to get rid of the radical. The equation becomes

$$t + 4 = (t - 2)^2$$

Use FOIL (First, Outer, Inner, Last) to multiply the right side of the equation to get $t^2 - 2t - 2t + 4$ or $t^2 - 4t + 4$. Now the equation is

$$t + 4 = t^2 - 4t + 4$$

FOIL:
First
Outer
Inner
Last

Subtract t and 4 from both sides to get

$$0 = t^2 - 5t$$

The right side factors to $t(t-5)$, so $t = 0$ or 5. Eliminate (B), since 4 is not a solution at all, extraneous or otherwise. Now plug 0 and 5 back into the original equation to see if they work. If both do, the answer is (A). If one of them does not, that one is an extraneous solution.

$\underline{t=0}$	$\underline{t=5}$
$\sqrt{0+4}=0-2$	$\sqrt{5+4}=5-2$
$\sqrt{4}=-2$	$\sqrt{9}=3$
$2\neq-2$	$3=3$

Since the equation is false when $t = 0$, eliminate (A) and (C). The correct answer is (D).

SOLVING FOR EXPRESSIONS

Some algebra questions on the Digital SAT ask you to find the value of an expression rather than the value of a variable. In most cases, you can find the value of the expression without finding the value of the variable.

5 Mark for Review

If $4x + 2 = 4$, what is the value $4x - 6$?

Ⓐ -6

Ⓑ -4

Ⓒ 4

Ⓓ 8

Here's How to Crack It

The question asks for the value of an expression. This is where reading the final question (RTFQ) can save time. Since the question doesn't ask for the value of x, there may be a shortcut. The term $4x$ is in both expressions, so instead of solving for x, you can solve for $4x$. Subtract 2 from both sides of $4x + 2 = 4$ to get $4x = 2$. Now, plug $4x = 2$ into $4x - 6$ to get $(2) - 6 = -4$. The correct answer is (B).

This approach will save you time—provided that you see it quickly. So, while you practice, you should train yourself to look for these sorts of direct solutions whenever you are asked to solve for the value of an expression.

Here's another example.

6 Mark for Review

If $\sqrt{5} = x - 2$, what is the value of $(x-2)^2$?

Ⓐ $\sqrt{5}$

Ⓑ $\sqrt{7}$

Ⓒ 5

Ⓓ 25

Here's How to Crack It

The question asks for the value of an expression. If you were to attempt the math class way, you'd find that $x = \sqrt{5} + 2$ and then you would have to substitute that into the provided expression. There's got to be an easier way!

The question is much easier if you read the final question and look for a direct solution. Then, you notice that all the question wants you to do is to square the expression on the right of the equals sign. Well, if you square the expression on the right, then you'd better square the expression on the left too. Therefore, $(\sqrt{5})^2 = 5 = (x-2)^2$, and the correct answer is (C). That was pretty painless by comparison.

SOLVING SYSTEMS OF EQUATIONS

Some Digital SAT questions will give you two or more equations involving two or more variables and ask for the value of an expression or one of the variables. These questions are very similar to the questions containing one variable. The test-writers would like you to spend extra time trying to solve for the value of each variable, but that is not always necessary.

Here's an example of this type of question as a fill-in. We'll look at fill-ins in more detail in Chapter 26.

7 Mark for Review

If $4x + y = 14$ and $3x + 2y = 13$, what is the value of $x - y = ?$

Watch Us Crack It
Check out the Video Walk-throughs in your online Student Tools to watch a Princeton Review teacher work through this question step-by-step.

Here's How to Crack It

The question asks for the value of an expression. You've been given two equations here. But read the final question: instead of being asked to solve for a variable (x or y), you've been asked to solve for $x - y$. Why? Because there must be a direct solution.

Rather than solving for one variable and then substituting it to solve for the other variable, see if there's a faster way. Try stacking the two equations on top of each other and then adding or subtracting the two equations. There's a good chance that this shortcut will take you right to the answer. Let's try it.

Adding the two equations gives you this:

$$\begin{array}{r} 4x + y = 14 \\ +\ 3x + 2y = 13 \\ \hline 7x + 3y = 27 \end{array}$$

Unfortunately, that doesn't get you anywhere, so try subtracting:

$$\begin{array}{r} 4x + y = 14 \\ -(3x + 2y = 13) \end{array}$$

When you subtract equations, just change the signs of the second equation and add. So the equation above becomes

$$\begin{array}{r} 4x + y = 14 \\ +(-3x - 2y = -13) \\ \hline x - y = 1 \end{array}$$

The value of $(x - y)$ is precisely what you are looking for. The correct answer is 1.

Solving for Variables in Systems of Equations

Shortcuts are awesome, so take them whenever you can on the Digital SAT. But occasionally, you won't have the option of using a shortcut with a system of equations, so knowing how to solve for a variable is crucial.

Here's an example.

8 Mark for Review

If $3x + 2y = 17$ and $5x - 4y = 21$, what is the value of y?

Here's How to Crack It

The question asks for the value of y. Look for the most direct way to get there. In this case, the stack and solve method doesn't give you an immediate answer:

IF YOU ADD:

$$\begin{array}{r} 3x + 2y = 17 \\ \underline{+\ 5x - 4y = 21} \\ 8x - 2y = 38 \end{array}$$

IF YOU SUBTRACT:

$$\begin{array}{r} 3x + 2y = 17 \\ \underline{+\ (-5x + 4y = -21)} \\ -2x + 6y = -4 \end{array}$$

Neither of these methods gives you the value of y. The best way to approach this question is to try to eliminate one variable. To do this, multiply one or both of the equations by a number that will cause the other variable to have a coefficient of 0 when the equations are added or subtracted.

Since the question is asking you to solve for y, try to make the x terms disappear. You want to make the coefficient of x zero so you can quickly find the value of y.

Use the coefficient of x in the second equation, 5, to multiply the first equation:

$$5(3x + 2y) = 5(17)$$
$$15x + 10y = 85$$

Then use the original coefficient of x in the first equation to multiply the second equation:

$$3(5x - 4y) = 3(21)$$
$$15x - 12y = 63$$

Now stack your equations and subtract (or flip the signs and add, which is less likely to lead to a mistake).

$$\begin{array}{r} 15x + 10y = \ \ 85 \\ \underline{+ (-15x + 12y = -63)} \\ 0x \ + 22y = \ \ 22 \end{array}$$

Simplify your equation and you have your answer.

$$22y = 22$$
$$y = 1$$

Number of Solutions to a System of Equations

Some Digital SAT Math questions won't even ask you to solve for the solution(s) to a system of equations: they'll simply ask you how many solutions there are. A solution to a system is a point of intersection when the system is graphed, so the graphing calculator can help on these. If you know the rules of when two linear equations have zero, one, or infinitely many solutions, you can use that knowledge and a little algebra to answer the question.

Here's an example that can be solved algebraically or with a calculator.

9 Mark for Review

Which of the following systems of equations has an infinite number of solutions?

Ⓐ $x = -5$
$y = 10$

Ⓑ $x = 10y$
$y = 10x$

Ⓒ $y = -4x - 10$
$y = -4x - 15$

Ⓓ $y = \frac{1}{2}x + 5$
$4y = 2x + 20$

Here's How to Crack It

The question asks which system of equations has infinitely many solutions. A system of linear equations has infinitely many solutions when the two equations are identical. That means both equations represent the same line, so there are infinitely many points of intersection. The two equations in (A) do not represent the same line, so eliminate (A). The equations in (B) look similar, but rearrange them to check. Plug $y = 10x$ into the first equation to get $x = 10(10x)$, which becomes $x = 100x$. The only possible value of x is 0. Do the same thing with the second equation: substitute $x = 10y$ to get $y = 10(10y)$, which becomes $y = 100y$. The only possible value of y is 0. Thus, the two lines intersect only once, at (0, 0), not infinitely many times. Eliminate (B). The equations in (C) have the same y-term and the same x-term, but different constants. This means the lines defined by the equations are parallel and have zero solutions. Eliminate (C). The equations in (D) don't look the same, but do a little algebra to make them look similar. Make the y-terms the same by multiplying the first equation by 4 to get $4y = 2x + 20$. It's the same equation! That means the two equations describe the same line, and the system has infinitely many solutions.

The other way to answer a question like this is to use a graphing calculator. To use the built-in calculator, test one answer at a time by entering each equation in an entry field and looking in the graphing area to see what the graphs look like. Graphing the equations in (A) shows two perpendicular lines that intersect once; eliminate (A). Graphing the equations in (B) shows two lines that intersect once at the origin; eliminate (B). Graphing the equations in (C) shows two parallel lines that never intersect; eliminate (C). Graphing the equations in (D) shows only one line. Click the equations one after another to see the line change color. This confirms that the exact same line was graphed twice, so the two equations have infinitely many solutions.

Using either algebra or the calculator, the correct answer is (D).

SOLVING INEQUALITIES

In an equation, one side equals the other. In an **inequality,** one side does not equal the other. The following symbols are used in inequalities:

SYMBOL	MEANING
$>$	is greater than
$<$	is less than
$\geq$	is greater than or equal to; at least
$\leq$	is less than or equal to; no more than

Hungry Gator
Think of the inequality sign as the mouth of a hungry alligator. The alligator eats the bigger number.

Solving inequalities is pretty similar to solving equations. You can collect like terms, and you can simplify by performing the same operation to both sides. All you have to remember is that if you multiply or divide both sides of an inequality by a negative number, the direction of the inequality symbol changes.

For example, here's a simple inequality:

$$x > y$$

Now, just as you can with an equation, you can multiply both sides of this inequality by the same number. But if the number you multiply by is negative, you have to change the direction of the symbol in the result. For example, if you multiply both sides of the inequality above by –2, you end up with the following:

$$-2x < -2y$$

When you multiply or divide an inequality by a negative number, you must reverse the inequality sign.

Here's an example of how an inequality question may be framed on the test.

10 Mark for Review

If $-3x + 6 \geq 18$, which of the following must be true?

Ⓐ $x \leq -4$

Ⓑ $x \leq 8$

Ⓒ $x \geq -4$

Ⓓ $x \geq -8$

Here's How to Crack It

The question asks for a true statement based on the inequality, and the answers are all possible values of x. Isolate the x by simplifying the inequality as you would with any equation:

$$-3x + 6 \geq 18$$
$$-3x \geq 12$$

Remember to change the direction of the inequality sign!

$$x \leq -4$$

The correct answer is (A).

WRITING YOUR OWN EQUATIONS

Sometimes you'll be asked to take a word problem and create one or more equations or inequalities from that information. In general, you will not be asked to solve these equations/inequalities, so if you are able to locate and translate the information in the question, you have a good shot at getting the correct answer. Always start with the most straightforward piece of information. What is the most straightforward piece of information? Well, that's up to you to decide. Consider the following question.

11 Mark for Review

Max uses a humidifier in his son's bedroom. The humidifier must be filled with 0.5 gallons of water before it starts running, and it has to be refilled with 0.07 gallons of water each week. Which of the following equations models the total gallons of water, g, needed to run the humidifier for w weeks?

Ⓐ $g = 0.07 + 0.5w$

Ⓑ $g = 0.07(0.5 + w)$

Ⓒ $g = 0.5(0.07 + w)$

Ⓓ $g = 0.5 + 0.07w$

When to translate

Start translating words into math in bite-sized pieces when you see the following:

- The question asks what "models" or "represents" a situation.
- The answers contain one or more equations or inequalities.

Here's How to Crack It

The question asks for an equation that models a situation. Find a straightforward piece of information, translate it into math, and eliminate answers that don't match. One piece of information is that the *humidifier must be filled with 0.5 gallons of water before it starts running.* If 0.5 is the starting value and not something that is added every week, it should not be multiplied by anything. Only (D) has 0.5 by itself, so it's the right answer.

What if you started with a different piece of information? The other piece of information is that the humidifier *has to be refilled with 0.07 gallons of water each week.* The question also states that w represents weeks. The number of gallons added each week must be multiplied by the number of weeks, which translates to $0.07w$. Eliminate (A) and (C) because they do not include this term, even after distributing. Although (B) includes the term $0.07w$ after you distribute, the other term becomes 0.035. This does not match any of the information in the question, so eliminate (B). Only (D) is left.

It doesn't matter which piece of information you start with. Just be sure to eliminate after each piece. In this case, either piece alone was enough to eliminate 3 answers, so the remaining answer is right. The correct answer is (D).

Sometimes you will need more than one bite-sized piece in order to eliminate all but one answer. This will often happen when the answers have systems of equations or inequalities rather than a single equation or inequality.

Here's one of those.

12 Mark for Review

A tailor is ordering red and blue ribbon to use when creating a set of dresses. The tailor wants to include at least 200 meters of ribbon in her order, and she will order no more than 3 times as much blue ribbon as red ribbon. Each spool of red ribbon contains 22.86 meters, and each spool of blue ribbon contains 18.29 meters. If r and b are nonnegative integers and represent the number of spools of red and blue ribbon, respectively, that the tailor will order, which of the inequalities below best represents this scenario?

Ⓐ $22.86r + 18.29b \geq 200$
$3b \leq r$

Ⓑ $22.86r + 54.87b \geq 200$
$3b \leq r$

Ⓒ $22.86r + 18.29b \geq 200$
$b \leq 3r$

Ⓓ $22.86r + 54.87b \geq 200$
$b \leq 3r$

Here's How to Crack It

The question asks for a system of inequalities that describes the situation. Start with the most straightforward piece of information and translate it into math. In this case, the most straightforward information is about the total meters of ribbon, 200 meters. However, all of the answers include ≥ 200, so look for something else. The answers also all include $22.86r$ for the red ribbon, so work with the blue ribbon. The question states that *each spool of blue ribbon contains 18.29 meters* and that b represents the number of spools of blue ribbon. Therefore, the equation should include $18.29b$. Eliminate (B) and (D) because they have the wrong number multiplied by b.

Next, look at the relationship between the blue and red ribbon. The question states that *she will order no more than 3 times as much blue ribbon as red ribbon.* The phrase *no more than* is indicated by the symbol $\leq$, and the amount of blue ribbon is being compared to 3 times the amount of red ribbon. The correct inequality to depict this information is $b \leq 3r$. Eliminate (A). The correct answer is (C).

SIMPLIFYING EXPRESSIONS

If a question contains an expression that can be factored, it is very likely that you will need to factor it to solve the question. So, you should always be on the lookout for opportunities to factor. For example, if a question contains the expression $2x + 2y$, you should see if factoring it to produce the expression $2(x + y)$ will help you to solve the problem.

If a question contains an expression that is already factored, you should consider using the Distributive Law to expand it. For example, if a question contains the expression $2(x + y)$, you should see if expanding it to $2x + 2y$ will help.

Something to Hide
Because factoring or expanding is usually the key to finding the answer on such questions, learn to recognize expressions that could be either factored or expanded. This will earn you more points. The test-writers will try to hide the answer by factoring or expanding the result.

Here are five examples that we've worked out:

1. $4x + 24 = 4(x) + 4(6) = 4(x + 6)$

2. $\frac{10x-60}{2} = \frac{10(x)-10(6)}{2} = \frac{10(x-6)}{2} = 5(x-6) = 5x-30$

3. $\frac{x+y}{y} = \frac{x}{y} + \frac{y}{y} = \frac{x}{y} + 1$

4. $2(x + y) + 3(x + y) = (2 + 3)(x + y) = 5(x + y)$

5. $p(r + s) + q(r + s) = (p + q)(r + s)$

Here's how this might be tested on the Digital SAT.

13 Mark for Review

Which of the following is equivalent to $\frac{a^2}{b} + a$?

Ⓐ $a\left(\frac{a}{b}+1\right)$

Ⓑ $a\left(\frac{a}{b}+a\right)$

Ⓒ $a^2 + ab$

Ⓓ $a^2\left(\frac{1}{b}+1\right)$

Here's How to Crack It

The question asks for an equivalent form of an expression. Two of the answers have a factored, so try factoring a first. Rewrite each term as a times something else:

$$a\left(\frac{a}{b}\right)+a(1)$$

Then factor a out of both terms:

$$a\left(\frac{a}{b}+1\right)$$

This matches (A), which is the correct answer. You can check by distributing a in (A) to confirm that it returns the expression to its original form.

Don't be tempted by (C). It looks plausible because both terms from the original expression have been multiplied by b. However, you cannot multiply an expression by a value other than 1. That only works with equations because then you multiply both sides by the same value. Choice (C) is a trap answer based on not remembering the difference between an expression and an equation. The correct answer is (A).

Multiplying Binomials

Multiplying binomials is easy. Just be sure to use **FOIL** (First, Outer, Inner, Last).

$$\begin{aligned}(x+2)(x+4) &= (x+2)(x+4)\\ &= \underset{\text{FIRST}}{(x \times x)} + \underset{\text{OUTER}}{(x \times 4)} + \underset{\text{INNER}}{(2 \times x)} + \underset{\text{LAST}}{(2 \times 4)}\\ &= x^2 + 4x + 2x + 8\\ &= x^2 + 6x + 8\end{aligned}$$

Combine Like Terms First

When manipulating long, complicated algebraic expressions, combine all like terms before doing anything else. In other words, if one of the terms is $5x$ and another is $-3x$, simply combine them into $2x$. Then you won't have as many terms to work with. Here's an example:

$$(3x^2 + 3x + 4) + (2 - x) - (6 + 2x) =$$
$$3x^2 + 3x + 4 + 2 - x - 6 - 2x =$$
$$3x^2 + (3x - x - 2x) + (4 + 2 - 6) =$$
$$3x^2$$

TERMinology
Remember: A **term** is a number, variable, or a number *and* variable that are combined by multiplication or division. Consider the expression $6x + 10 - y$. In this expression, $6x$, 10, and y are all terms. $6x + 10$, however, is not a term. It is two terms added together, which makes it an *expression*.

SOLVING QUADRATIC EQUATIONS

To solve quadratic equations, remember everything you've learned so far: look for direct solutions and either factor or expand when possible.

Here's an example.

14 Mark for Review

If $(x - 3)^2 = (x + 2)^2$, what is the value of x?

Here's How to Crack It

The question asks for the value of x. Expand both sides of the equation using FOIL:

$$(x - 3)(x - 3) = (x + 2)(x + 2)$$
$$x^2 - 6x + 9 = x^2 + 4x + 4$$

Now you can simplify. Eliminate the x^2 terms because they are on both sides of the equals sign. Now you have $-6x + 9 = 4x + 4$, which simplifies to

$$-10x = -5$$
$$x = \frac{1}{2}$$

Factoring Quadratics

To solve a quadratic, you might also have to factor the equation. Factoring a quadratic basically involves doing a reverse form of FOIL.

For example, suppose you needed to know the factors of $x^2 + 7x + 12$. Here's what you would do:

1. Write down 2 sets of parentheses and put an x in each one because the product of the first terms is x^2.

$$x^2 + 7x + 12 = (x \quad)(x \quad)$$

2. Look at the number at the end of the expression you are trying to factor. Write down its factors. In this case, the factors of 12 are 1 and 12, 2 and 6, and 3 and 4.

3. To determine which set of factors to put into the parentheses, look at the coefficient of the middle term of the quadratic expression. In this case, the coefficient is 7. So, the correct factors will also either add or subtract to get 7. The only factors that work are 3 and 4. Write these factors in the parentheses.

$$x^2 + 7x + 12 = (x \_\_ 3)(x \_\_ 4)$$

Factoring
When factoring an equation like $x^2 + bx + c$, think "**A.M.**" Find two numbers that **A**dd up to the middle term (b) and **M**ultiply to give the last term (c).

4. Finally, determine the signs for the factors. To get a positive 12, the 3 and the 4 are either both positive or both negative. But, since 7 is also positive, the signs must both be positive.

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

You can always check that you have factored correctly by using FOIL on the factors to see if you get the original quadratic expression.

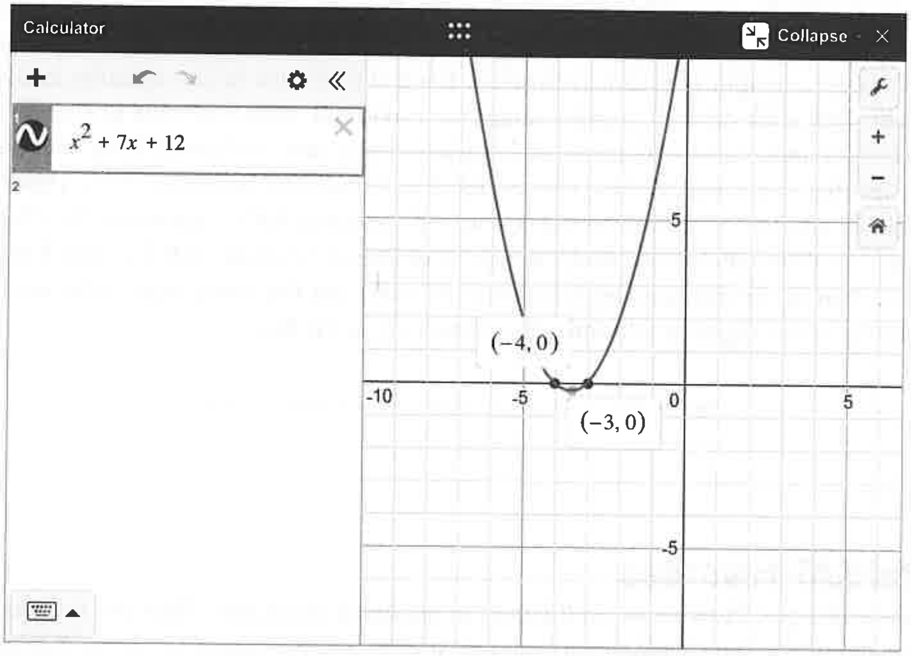
To find the solutions on paper, you would set each factor equal to zero and solve for x. This becomes $x + 3 = 0$ and $x + 4 = 0$, so $x = -3$ or -4.

The built-in graphing calculator or your own calculator can also quickly provide the solutions to a quadratic. We'll show you how to do it on the built-in calculator. If you're planning to use your own, make sure you know how to do this.

Open the built-in calculator, then enter the quadratic into the first entry field. Let's use the same one as above and enter $x^2 + 7x + 12$.

The graph shows three grey dots. One is the vertex, and the other two are the solutions, or x-intercepts. Hover over or click on one of the x-intercept dots to see the x-value for that point, and write it down. You can scroll and zoom to make it easier to see. Do the same for the other x-intercept dot.

The calculator will look like this on the screen:



The two points are (–4, 0) and (–3, 0), so the solutions to the quadratic are $x = -4$ and $x = -3$.

It's best to enter only the expression and *not* include "= 0" when you enter a quadratic in the built-in graphing calculator. Try entering $x^2 + 7x + 12 = 0$, and the resulting graph is two parallel lines at $x = -4$ and $x = -3$. This still shows the solutions, but it isn't as easy to interpret as the graph of the parabola.

Try a Digital SAT question that is much easier to solve using a graphing calculator.

15 Mark for Review

For the equation $12x^2 + 5x - 143 = 0$, what is one possible value of x?

Ⓐ $-\frac{13}{4}$

Ⓑ $\frac{5}{12}$

Ⓒ $\frac{13}{4}$

Ⓓ $\frac{11}{3}$

Here's How to Crack It

The question asks for a possible value of x in a quadratic equation. The values of x are also the solutions to the equation. This quadratic would not be fun to factor, even with the quadratic formula, so use a graphing calculator instead. Enter the left side of the equation into the first entry field, then click on each gray dot along the x-axis and write down the points. The points are (–3.667, 0) and (3.25, 0). Convert each answer choice into its decimal form, and eliminate any answer that is not one of these two values of x. Choice (A) becomes –3.25. This does not match either solution, so eliminate (A). Choice (B) becomes 0.417; eliminate (B). Choice (C) becomes 3.25. This matches one of the solutions, so keep (C). Choice (D) becomes 3.667; eliminate (D). Two of the trap answers have the right value but the wrong sign, so be very careful with positive versus negative solutions. The correct answer is (C).

Digital SAT Favorites

The test-writers play favorites when it comes to quadratic equations. There are three equations that they use often. You should memorize these and be on the lookout for them. Whenever you see a quadratic that contains two variables, it is frequently one of these three.

$$(x + y)(x - y) = x^2 - y^2$$

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

Here's an example of how these equations will likely be tested on the Digital SAT.

16 Mark for Review

If $2x - 3y = 5$, what is the value of $4x^2 - 12xy + 9y^2$?

Ⓐ $\sqrt{5}$

Ⓑ 12

Ⓒ 25

Ⓓ 100

Here's How to Crack It

The question asks for the value of an expression. Since the expression seems kind of random, see if there is a way to get from the given equation to the expression.

In this case, work with $2x - 3y = 5$. If you square the left side of the equation, you get

$$(2x - 3y)^2 = 4x^2 - 12xy + 9y^2$$

That's precisely the expression for which you need to find the value. It's also the third of the equations from the box on the preceding page. Now, since you squared the left side, all you need to do is square the 5 on the right side of the equation to discover that the expression equals 25. The correct answer is (C).

Did you notice that this question was just another version of being asked to solve for the value of an expression rather than for a variable? Quadratics are one of the test-writers' favorite ways to do that.

Solving Quadratics Set to Zero

Before factoring most quadratics, you need to set the equation equal to zero. Why? Well, if $ab = 0$, what do you know about a and b? At least one of them must equal 0, right? That's the key fact you need in order to solve most quadratics.

Here's an example.

17 Mark for Review

If $3 - \frac{3}{x} = x + 7$, and $x \neq 0$, which of the following is a possible value for x?

Ⓐ −7

Ⓑ −1

Ⓒ 1

Ⓓ 3

Here's How to Crack It

The question asks for a possible value of x. Here, the test-writers have tried to hide that the equation is actually a quadratic. Start by multiplying both sides of the equation by x to get rid of the fraction.

$$x\left(3-\frac{3}{x}\right)=x(x+7)$$

$$3x-3=x^2+7x$$

Rearrange the terms to set the quadratic equal to 0. You'll get $x^2 + 4x + 3 = 0$. Now it's time to factor:

$$x^2+4x+3=(x+1)(x+3)=0$$

At least one of the factors must equal 0. If $x + 1 = 0$, then $x = -1$. If $x + 3 = 0$, then $x = -3$. Only one of these values appears in the answer choices. The correct answer is (B).

The Quadratic Formula

In addition to asking you to solve easily factorable quadratics, the test-writers would also like to see you demonstrate your understanding of the quadratic formula. We know what you're thinking: "Not that thing *again!* Can't I just solve it with a graphing calculator?" Why yes, yes you can. However, once in a while there's a question that requires knowledge of the quadratic formula. Let's review it so you're prepared.

For a quadratic equation in the form $y = ax^2 + bx + c$, the quadratic formula is

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

To find the roots of a quadratic, or the points where $y = 0$, simply plug your values for a, b, and c into the quadratic formula.

Here's an example:

$$7x^2 - 5x - 17 = 0$$

So $a = 7$, $b = -5$, and $c = -17$. Plugging the constants into the quadratic equation, you get

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(7)(-17)}}{2(7)}$$

$$x = \frac{5 \pm \sqrt{25 + 476}}{14}$$

$$x = \frac{5 \pm \sqrt{501}}{14}$$

$$x = \frac{5}{14} + \frac{\sqrt{501}}{14} \text{ and } x = \frac{5}{14} - \frac{\sqrt{501}}{14}$$

The Signs, They Are a Changin'
The quadratic formula works for quadratics in the form $y = ax^2 + bx + c$. There is only addition in that form, so be careful when your quadratic has negative signs in it.

Let's put your quadratic skills to work with a question you may see on the Digital SAT.

18 Mark for Review

What is the product of all the solutions to the equation $3z^2 - 12z + 6 = 0$?

Ⓐ $\sqrt{2}$

Ⓑ 2

Ⓒ 4

Ⓓ $4\sqrt{2}$

Here's How to Crack It

The question asks for the product of the solutions to a quadratic. The radicals in the answer choices are a clue that the quadratic may be hard to factor. Simplify the equation first by dividing both sides by 3 to get

$$z^2 - 4z + 2 = 0$$

To find the solutions using the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, you would do the following:

Call on the calculator
Don't forget that a graphing calculator is a fast way to find the solutions to a quadratic. To answer this question using the built-in graphing calculator, enter the left side of the quadratic, find the values of the two x-intercepts, and multiply them.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 8}}{2} = \frac{4 \pm \sqrt{8}}{2}$$

$$x = \frac{4 \pm \sqrt{4 \times 2}}{2} = \frac{4 \pm 2\sqrt{2}}{2}$$

$$x = 2 \pm \sqrt{2}$$

So $x = 2 + \sqrt{2}$ *or* $2 - \sqrt{2}$. "Product" means to multiply, so use FOIL to multiply $\left(2 + \sqrt{2}\right) \times \left(2 - \sqrt{2}\right)$ to get $4 - 2\sqrt{2} + 2\sqrt{2} - \left(\sqrt{2}\right)^2 = 4 - 2 = 2$. The correct answer is (B).

The Root of the Problems

Sometimes you'll be asked to solve for the sum or the product of the roots of a quadratic equation. You can use the quadratic formula and then add or multiply the results, but it's quicker to just memorize these two expressions.

sum of the roots: $-\dfrac{b}{a}$ product of the roots: $\dfrac{c}{a}$

Wow, that was a lot of work! Wouldn't it be great if there were a shortcut? Actually, there is! When a quadratic is in the form $y = ax^2 + bx + c$, the product of the roots is equal to the value of c divided by the value of a. In this case, that's $6 \div 3 = 2$! It's the same answer for a lot less work. (See the inset "The Root of the Problems" for this and another handy trick—they're worth memorizing.)

Number of Solutions to a Quadratic Equation

There is one more useful thing that can be determined from just a piece of the quadratic formula. The part under the root symbol in the formula is called the *discriminant.* The value of the discriminant can tell you the number of roots the quadratic has.

> The discriminant, D, of a quadratic in the standard form $ax^2 + bx + c = 0$ is $b^2 - 4ac$.
>
> - If the discriminant is positive, the quadratic has 2 real solutions.
> - If the discriminant equals 0, the quadratic has 1 real solution.
> - If the discriminant is negative, the quadratic has no real solutions.

You saw earlier in this chapter how a question might ask about the number of solutions to a system of linear equations. When you see a similar question about a single quadratic, see if you can graph it or use the discriminant.

Take a look at a typical question.

19 Mark for Review

If the equation $12x = k - 4x^2$ has exactly one real solution, what is the value of the constant k?

Watch Us Crack It

Check out the Video Walk-throughs in your online Student Tools to watch a Princeton Review teacher work through this question step-by-step.

Here's How to Crack It

The question asks for the value of a constant in a quadratic equation. When a quadratic has exactly one real solution, the discriminant equals 0. First, put the quadratic in standard form, which is $ax^2 + bx + c$. Add $4x^2$ to both sides of the equation to get $4x^2 + 12x = k$, and then subtract k from both sides of the equation to get $4x^2 + 12x - k = 0$. Now that the quadratic is in standard form, $a = 4$, $b = 12$, and $c = -k$. Plug the values for a, b, and c into the discriminant, $D = b^2 - 4ac$, and set it equal to 0 to get $12^2 - (4)(4)(-k) = 0$. Simplify the left side of the equation to get $144 + 16k = 0$. Subtract $16k$ from both sides of the equation to get $144 = -16k$. Divide both sides of the equation by -16 to get $-9 = k$. The correct answer is -9.

GROWTH AND DECAY

There's one more equation with an exponent that's tested frequently on the Digital SAT: the **growth and decay** formula. Real-world examples include population growth, radioactive decay, and compound interest, to name a few. The growth or decay can be a percent or a multiple, which changes what's inside the parentheses.

When the growth or decay rate is a percent of the total population:

$$\textit{final amount} = \textit{original amount}\ (1 \pm \textit{rate})^{\textit{number of changes}}$$

When the growth or decay is a multiple of the total population:

$$\textit{final amount} = \textit{original amount}\ (\textit{multiplier})^{\textit{number of changes}}$$

Let's see how this formula can make quick work of an otherwise tedious question.

20 Mark for Review

Becca deposits \$100 into a bank account that earns an annual interest rate of 4%. If she does not make any additional deposits and makes no withdrawals, how long will it take her, in years, to increase the value of her account by at least 60%?

Ⓐ 12

Ⓑ 15

Ⓒ 25

Ⓓ 30

Here's How to Crack It

The question asks for the number of years it will take for Becca's account to reach a certain value. You could add 4% to the account over and over again until you get to the desired amount, but that would take a long time. Knowing the formula will make it a lot easier. First, set up the equation with the things you know. The original amount is 100, and the rate is 4%, or 0.04.

The account is increasing, so you add the rate, and you can put in "years" for the number of changes. The formula becomes

$$\textit{final amount} = 100(1 + 0.04)^{years}$$

Now you need to figure out what you want the final amount to be. Translate the English to math: the value of her account (100) will increase (+) by 60 percent (0.6) of the current value (×100). This becomes 100 + (0.6)(100) = 100 + 60 = 160. Now the formula is

$$160 = 100(1.04)^{years}$$

The answer choices represent the number of years Becca keeps her money in the account. Now you are all set to easily plug in the answers. Start with (B), so *years* = 15. Is $100(1.04)^{15}$ = 160? Use your calculator to check, making sure to follow PEMDAS rules and do the exponent before you multiply by 100. The result is \$180.09. That is a bit too much money, so the answer will likely be (A), but let's just check it. $100(1.04)^{12}$ = \$160.10, which is at least \$160. The correct answer is (A).

Here's an example of growth with a multiplier instead of a rate.

21 Mark for Review

An invasive species was discovered to have a population of 2,100 individuals after 10 years of uninhibited growth. The equation $P = G(3)^{\frac{t}{10}}$ gives the number of individuals in the population, where t is the number of years after the uninhibited growth began, P is the current number of individuals in the population, and G is the number of individuals in the population when the uninhibited growth began. What is the value of G?

Ⓐ 30

Ⓑ 700

Ⓒ 2,100

Ⓓ 6,300

Here's How to Crack It

The question asks for a value in an equation. The equation is in the form of the growth and decay formula, and recognizing that leads to the next step. Label each piece of the formula and fill in the number given for each piece. The question states that P *is the current number of individuals in the population*, and that the invasive species had *a population of 2,100 individuals*, so $P = 2,100$. The question also states that 2,100 was the final amount *after 10 years of uninhibited growth*, and that *t is the number of years after the uninhibited growth began*, so $t = 10$. Plug those values into the formula and solve for G: $2,100 = G(3)^{\frac{10}{10}}$. Simplify the right side of the equation to get $2,100 = G(3)^1$, and then $2,100 = 3G$. Divide both sides of the equation by 3 to get $700 = G$. The correct answer is (B).

WHEN VALUES ARE ABSOLUTE

Absolute value is a measure of the distance between a number and 0. Since distances are always positive, the absolute value of a number is also always positive. The absolute value of a number is written as $|x|$.

When solving for the value of a variable inside the absolute value bars, it is important to remember that the variable could be either positive or negative. For example, if $|x| = 2$, then $x = 2$ or $x = -2$, as both 2 and -2 are a distance of 2 from 0.

Here's an example.

Digital SAT Smoke and Mirrors

When you're asked to solve an equation involving an absolute value, it is very likely that the correct answer will involve the negative result. Why? Because the test-writers know that you are less likely to think about the negative result! Another way to avoid mistakes is to do all the math inside the absolute value symbols first, and then make the result positive.

22 Mark for Review

Which of the following is the value of $|y + z|$ if y and z are the solutions to the equation $|-4x - 2| = 6$?

Ⓐ -3

Ⓑ -2

Ⓒ 1

Ⓓ 3

Here's How to Crack It

The question asks for the value of an expression with an absolute value given an equation with an absolute value. Start with the equation and find the two solutions. Remember that the expression inside the absolute value symbols could be positive or negative and will still yield a positive result. Set that expression, $-4x - 2$, equal to 6 and -6, and solve for the two solutions.

$$-4x - 2 = 6 \qquad\qquad -4x - 2 = -6$$
$$-4x = 8 \qquad \textbf{and} \qquad -4x = -4$$
$$x = -2 \qquad\qquad x = 1$$

Thus, the solutions y and z are -2 and 1. There's no way to know which is y and which is z, but it doesn't matter. Replace the variables in the expression $|y + z|$ with those values in either order, and calculate the result.

$$|-2 + 1| = |-1| = 1$$

If you're curious, try the other order for y and z to see that, because of the absolute value, it still works.

$$|1 + (-2)| = |1 - 2| = |-1| = 1$$

In either case the result is 1. The correct answer is (C).

Algebra Drill

Work these questions without your calculator using the skills you've learned so far. Answers and explanations can be found starting on page 425.

1 Mark for Review

For the inequality $3 > 9x - 3$, which of the following is a possible value of x?

(A) 0

(B) $\frac{2}{3}$

(C) 3

(D) 6

2 Mark for Review

For the equation $\sqrt{mx-5} = x+3$, the value of m is -3. Which of the following contains all possible solutions to the equation?

(A) $-3, 3$

(B) -2

(C) $-2, -7$

(D) $3, 6$

3 Mark for Review

The equation $\frac{ac}{20} = \frac{3d}{z}$ expresses the relationship among a, c, d, and z. Which of the following equations expresses z in terms of a, c and d?

(A) $z = \frac{60d}{ac}$

(B) $z = \frac{ac}{60d}$

(C) $z = \frac{3ac}{20}$

(D) $z = \frac{20ac}{3d}$

4 Mark for Review

A student spends $4\frac{1}{2}$ hours each day working on history and science assignments. It takes the student $\frac{1}{4}$ of an hour to complete a history assignment and $\frac{1}{2}$ of an hour to complete a science assignment. Which of the following equations represents the number of history assignments, h, and science assignments, s, the student can complete each day?

(A) $\left(h+\frac{1}{2}\right)\left(s+\frac{1}{4}\right) = \frac{9}{2}$

(B) $\left(\frac{1}{2}+\frac{1}{4}\right)(h+s) = \frac{9}{2}$

(C) $\frac{1}{4}h+\frac{1}{2}s = \frac{9}{2}$

(D) $\frac{1}{2}h+\frac{1}{4}s = \frac{9}{2}$

5 Mark for Review

If $x + 6 > 0$ and $1 - 2x > -1$, which of the following values of x is NOT a solution?

Ⓐ -6

Ⓑ -4

Ⓒ 0

Ⓓ $\frac{1}{2}$

6 Mark for Review

If $\frac{2x}{x^2+1} = \frac{2}{x+2}$, what is the value of x?

Ⓐ $-\frac{1}{4}$

Ⓑ $\frac{1}{2}$

Ⓒ 0

Ⓓ 2

7 Mark for Review

If the product of x and y is 76, and x is twice the square of y, which of the following pairs of equations could be used to determine the values of x and y?

Ⓐ $xy = 76$
$x = 2y^2$

Ⓑ $xy = 76$
$x = (2y)^2$

Ⓒ $x + y = 76$
$x = 4y^2$

Ⓓ $xy = 76$
$x = 2y$

8 Mark for Review

A tadpole that has just hatched from an egg weighs t grams. The equation $y = t(3)^w$ displays the weight y, in grams, of the tadpole w weeks after it hatches. If the tadpole reaches a weight of 9.72 grams at 5 weeks after hatching, what is the value of t?

9 Mark for Review

How many solutions exist to the equation $|x| = |2x - 1|$?

Ⓐ 0

Ⓑ 1

Ⓒ 2

Ⓓ 3

10 Mark for Review

The sum of three numbers, a, b, and c, is 400. One of the numbers, a, is 40 percent less than the sum of b and c. What is the value of $b + c$?

Ⓐ 40

Ⓑ 60

Ⓒ 150

Ⓓ 250

ALGEBRA DRILL ANSWERS AND EXPLANATIONS

1. **A** The question asks for a value in an inequality. To begin to isolate x, add 3 to both sides of the equation to get $6 > 9x$. Divide both sides of the equation by 9 to get $\frac{6}{9} > x$. Because 9 is positive, it is not necessary to flip the inequality sign. Reduce the fraction to get $\frac{2}{3} > x$. Only (A) is less than $\frac{2}{3}$. The correct answer is (A).

2. **B** The question asks for all possible solutions to an equation. Since the question gives the value of m, the first step is to plug that value into the original equation to get $\sqrt{-3x-5} = x+3$. Now square both sides of the equation to remove the square root: $\left(\sqrt{-3x-5)}\right)^2 = (x+3)^2$ or $-3x - 5 = x^2 + 6x + 9$. Now combine like terms. If you combine the terms on the right side of the equation, you can avoid having a negative x^2 term. The equation becomes $0 = x^2 + 9x + 14$. Factor the quadratic to find the roots: $0 = (x + 2)(x + 7)$. The possible solutions to the quadratic are -2 and -7. Don't forget to plug these numbers back into the original equation to check for extraneous solutions. Begin by checking $x = -2$. When you do this, you get $\sqrt{(-3)(-2)-5} = (-2) + 3$, or $\sqrt{6-5} = 1$, or $\sqrt{1} = 1$, which is true. Now, check $x = -7$. Set it up as $\sqrt{(-3)(-7)-5} = (-7) + 3$, and start simplifying to get $\sqrt{21-5} = -4$. You can technically stop simplifying here, as there is a negative number on the right-hand side of the equals sign. Remember, when taking a square root with a radical provided, it will yield the positive root only. So -7 cannot be part of the solution set. Be very careful of (C), which is a trap answer. The correct answer is (B).

3. **A** The question asks for an equation in terms of a specific variable. To begin to isolate z, cross-multiply to get $(20)(3d) = (ac)(z)$, which becomes $60d = (ac)(z)$. Divide both sides of the equation by ac to get $\frac{60d}{ac} = z$. The correct answer is (A).

4. **C** The question asks for an equation that represents a specific situation. Translate the information in bite-sized pieces and eliminate after each piece. All of the answer choices equal $\frac{9}{2}$, which represents the $4\frac{1}{2}$ hours each day the student spends on both types of assignment. The left side of the equation must add up to that total time. One piece of information says that it *takes the student* $\frac{1}{4}$ *of an hour to complete a history assignment*, and another piece says that h represents the number of history assignments. The total time spent on history assignments can be represented by $\frac{1}{4}h$. Eliminate (D) because it multiplies h by $\frac{1}{2}$ instead of by $\frac{1}{4}$. Choice (A) includes the term $\frac{1}{4}h$ after FOILing, but it also includes hs. There is no reason to multiply the number of history

assignments by the number of science assignments, so eliminate (A). Choice (B) includes the term $\frac{1}{2}h$ after FOILing, but h should only be multiplied by $\frac{1}{4}$. Eliminate (B). Choice (C) also correctly multiplies the time per science assignment by the number of science assignments to get $\frac{1}{2}s$. The correct answer is (C).

5. **A** The question asks for the value of x that is not a solution. Solve the first inequality by subtracting 6 from each side so that $x > -6$. You are looking for values that won't work for x, and x cannot equal -6. Therefore, the answer must be (A). Just to be sure, solve the next inequality by subtracting 1 from each side to get $-2x > -2$. Divide by -2, remembering to switch the sign because you are dividing by a negative number to get $x < 1$. The values in (B), (C), and (D) fit this requirement as well, so they are values for x and not the correct answer. The correct answer is (A).

6. **B** The question asks for the value of x. To solve this equation, use cross-multiplication to get $(2x)(x + 2) = (x^2 + 1)(2)$. Expand the equation to get $2x^2 + 4x = 2x^2 + 2$. Once you combine like terms, the result is $2x^2 - 2x^2 + 4x = 2$ or $4x = 2$. Solve for x by dividing both sides by 4 to get $x = \frac{1}{2}$. The correct answer is (B).

7. **A** The question asks for a pair of equations to represent the situation. Translate each statement, piece by piece. The first part tells you that "the product of x and y is 76." Since *product* means multiplication, the first equation must be $xy = 76$, so you can eliminate (C). The second part says that "x is twice the square of y," which translates to $x = 2y^2$, so eliminate (B) and (D), and (A) is the only choice left. Notice that only the y needs to be squared, which is why (B) is wrong. The second equation for (B) would be written as "the square of twice y," which is not what the question states. The correct answer is (A).

8. **0.04** The question asks for a value given a specific situation. The weight of the tadpole is increasing by a multiple over time, so write down the growth and decay formula. That formula is *final amount* = (*original amount*)(*multiplier*)$^{number\ of\ changes}$. The equation in the question already shows that 3 is the *multiplier*. The question states that *the tadpole reaches a weight of 9.72 grams at 5 weeks after hatching*, so 9.72 is the *final amount* and 5 is the *number of changes*. Plug in these values and solve for t. The equation becomes $9.72 = t(3)^5$. Start with the exponent on the right side of the equation to get $9.72 = t(243)$. Divide both sides of the equation by 243 to get $0.04 = t$. The correct answer is 0.04, which can also be entered in the fill-in box as .04 or a fractional equivalent.

9. **C** The question asks for the number of solutions to an equation. If $|x| = |2x - 1|$, either $x = 2x - 1$ or $-x = 2x - 1$. The solutions to these equations are 1 and $\frac{1}{3}$, respectively. However, the only thing you need to recognize is that the equation has two different solutions. Another option is making a quick graph on a calculator, which will show you that there are two solutions and what those solutions are. Either way, the correct answer is (C).

10. **D** The question asks for the value of $b + c$. This is a system of equations question in disguise. First, locate a piece of information in this question that you can work with. "The sum of three numbers, a, b, and c, is 400," seems very straightforward. Write the equation $a + b + c = 400$. Now the question tells you that "one of the numbers, a, is 40 percent less than the sum of b and c." Translate this piece by piece to get $a = (1 - 0.4)(b + c)$, or $a = 0.6(b + c)$. Distribute the 0.6 to get $a = 0.6b + 0.6c$. Arrange these variables so they line up with those in the first equation as $a - 0.6b - 0.6c = 0$. To solve for $b + c$, stack the equations and multiply the second equation by -1:

$$a + b + c = 400$$
$$-1(a - 0.6b - 0.6c) = 0(-1)$$

Now solve:

$$\begin{array}{r} a + b + c = 400 \\ \underline{-a + 0.6b + 0.6c = 0} \\ 1.6b + 1.6c = 400 \end{array}$$

Simplify by dividing both sides by 1.6 to get $b + c = 250$. The correct answer is (D).

Summary

- Don't "solve for x" or "solve for y" unless you absolutely have to. (Don't worry; your math teacher won't find out.) Instead, look for direct solutions to Digital SAT questions. Math section questions rarely require time-consuming computations or endless fiddling with big numbers. There's almost always a trick—if you can spot it.
- If a question contains an expression that can be factored, factor it. If it contains an expression that already has been factored, multiply it out.
- To solve systems of equations, simply add or subtract the equations. If you don't have the answer, look for multiples of your solutions. When the question asks for a single variable and addition and subtraction don't work, try to make something disappear. Multiply the equations to make the coefficient(s) of the variable(s) you don't want go to zero when the equations are added or subtracted.
- If a question asks for the number of solutions, the graphing calculator can help, but it's also useful to know the rules.
    - Two linear equations have ***no solution*** when they have the same coefficient on the variable but different constants. This makes the lines parallel.
    - Two linear equations have ***exactly one solution*** when a single (x, y) point is a solution to both equations. The two lines intersect once.
    - Two linear equations have ***infinitely many solutions*** when they have the same coefficient on the variable and the same constants. This makes them the same line.
- Some Digital SAT questions require algebraic manipulation. Use tricks when you can, but if you have to manipulate the equation, take your time and work carefully to avoid unnecessary mistakes. You don't get partial credit for getting the question mostly correct.
- When working with inequalities, don't forget to flip the sign when you multiply and divide by negative numbers.
- When asked to create an equation, start with the most straightforward piece of information. You can also use the equations in the answer choices to help you narrow down the possibilities for your equation. Eliminate any answers in which an equation doesn't match your equation.

- When a question asks for an extraneous solution, first solve the equation, and then plug the answers back into the equation. If the equation is not true when solved with the solution, then that solution is extraneous.

- When solving quadratic equations, you may need to use FOIL or factor to get the equation into the easiest form for the question task. Don't forget about the common equations that the test-writers use when writing questions about quadratics.

- To solve for the roots of a quadratic equation, set it equal to zero by moving all the terms to the left side of the equation, then factor and solve, or use the quadratic formula:

  $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

  When solving for the sum or product of the roots, you can also use these formulas:

  - sum of the roots: $-\frac{b}{a}$
  - product of the roots: $\frac{c}{a}$

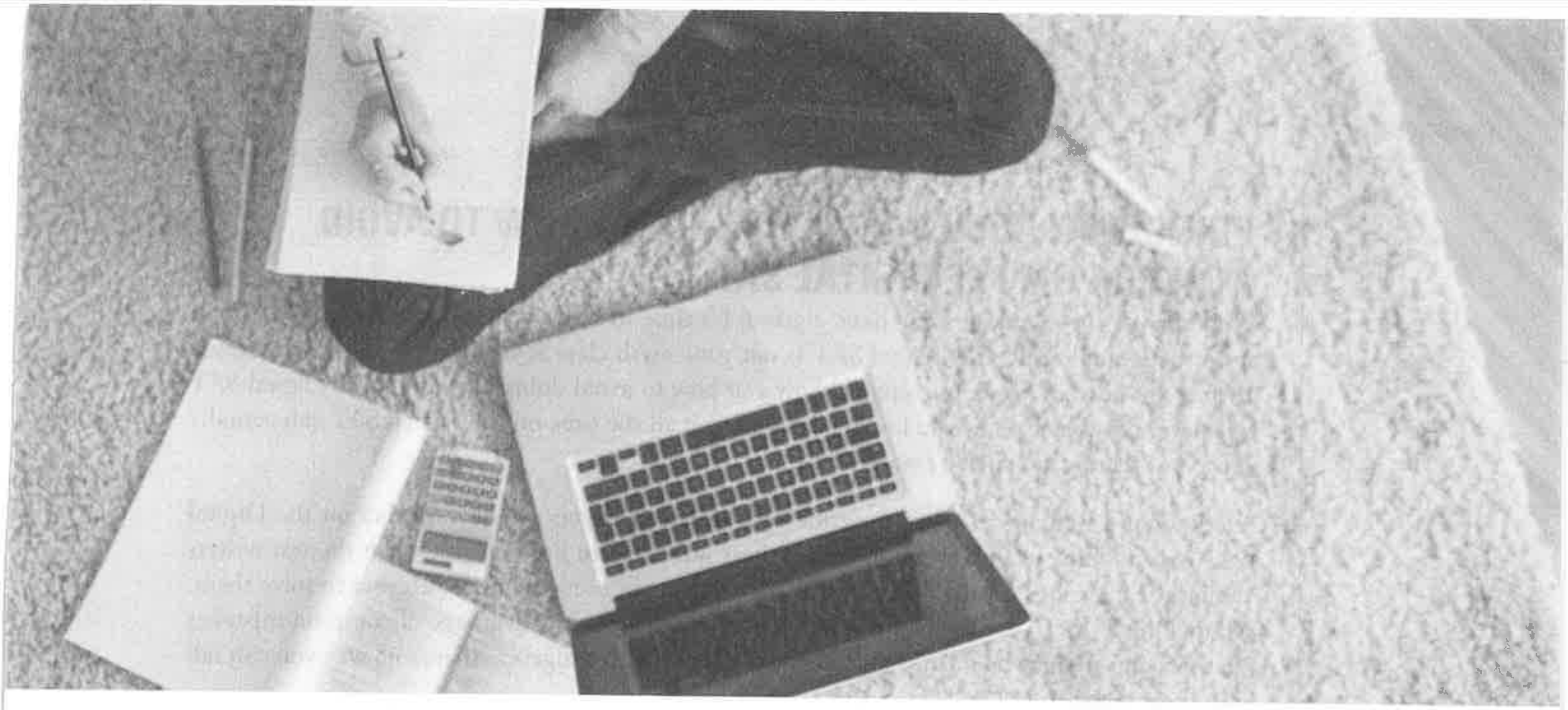
- The discriminant of a quadratic in the form $ax^2 + bx + c = 0$ is the value of $b^2 - 4ac$. If this value is positive, there are 2 real roots; if it is 0, there is 1 real root; if it is negative, there are no real roots. A graphing calculator can also help with questions about the number of solutions to a quadratic equation.

- When a question is about exponential growth or decay, use the following formula:

  $$\textit{final amount} = (\textit{original amount})(1 \pm \textit{rate})^{\text{number of changes}}$$

  If the change is by a multiple instead of a rate, use the same formula with *multiplier* inside the parentheses.

- The absolute value of a number is its distance from zero; distances are always positive. When working inside the | |, remember to consider both the positive and the negative values of the expression. Also remember that | | work like ( ); you need to complete all the operations inside the | | before you can make the value positive.



Chapter 22
Other Digital SAT Algebra Strategies

Now that you're familiar with the basics of algebra, it's time to learn how to avoid using algebra on the Digital SAT. Yes, you read that correctly. Algebra questions on the Digital SAT are filled with traps carefully laid by the test-writers, so you need to know how to work around them. This chapter gives you the strategies you need in order to turn tricky algebra questions into simple arithmetic.

PRINCETON REVIEW ALGEBRA—AKA HOW TO AVOID ALGEBRA ON THE DIGITAL SAT

Now that you've reviewed some basic algebra, it's time to find out when and how NOT to use it on the Digital SAT. The Digital SAT is not your math class at school, and all that matters is the correct answer. So we're going to show you how to avoid doing algebra on the Digital SAT whenever possible. Even if you love algebra, using it all the time on the Digital SAT can actually hurt your score, and we don't want that.

We know it's difficult to come to terms with this. But if you use only algebra on the Digital SAT, you're doing exactly what the test-writers want you to do. You see, when the test-writers design the questions on the Digital SAT, they expect the students to use algebra to solve them. Many Digital SAT problems have built-in traps meant to take advantage of common mistakes that students make when using algebra. But if you don't use algebra, there's no way you can fall into those traps.

Your Best Friend
Plugging In allows you to use your calculator on many of the algebra questions that show up on the Digital SAT.

Plus, when you avoid algebra, you add one other powerful tool to your tool belt: you can use the built-in calculator or your own! Calculators can help with algebra, but they're designed first and foremost for arithmetic. Our goal, then, is to turn algebra on the Digital SAT into arithmetic. We do that using techniques we call Plugging In and Plugging In the Answers (PITA).

PLUGGING IN THE ANSWERS (PITA)

Algebra uses letters to stand for numbers. You don't go to the grocery store to buy x eggs or y gallons of milk. Most people think about math in terms of numbers, not letters that stand for numbers.

You should think in terms of numbers on the Digital SAT as much as possible. On many Digital SAT algebra questions, even very difficult ones, you will be able to find the correct answer without using any algebra at all. You will do this by working backward from the answer choices instead of trying to solve the problem using your standard math-class methods.

Plugging In the Answers is a useful technique for solving word problems in which the answer choices are all numbers. Using this powerful technique can solve many algebra problems on the Digital SAT simply and quickly.

In algebra class at school, you solve word problems by using equations. Then, you check your solution by plugging in your answer to see if it works. Why not skip the equations entirely by simply checking the four possible solutions on the multiple-choice questions? One of these is the correct answer. You don't have to do any algebra, you will seldom have to try more than two choices, and you will never have to try all four. Use PITA for questions that ask for a specific amount.

Here's an example of using PITA instead of writing equations.

1 Mark for Review

On a certain assignment, student X takes 20 seconds per grammar question and 35 seconds per punctuation question. On the same assignment, student Y takes 30 seconds per grammar question and 55 seconds per punctuation question. It takes 310 seconds for student X to complete the assignment and 480 seconds for student Y to complete the assignment. If those are the only two types of questions on the assignment, how many punctuation questions are on the assignment?

(A) 5

(B) 6

(C) 7

(D) 8

Here's How to Crack It

The question asks for a value given a specific situation. Sure, you could make up some variables, translate words into equations, and use skills from the previous chapter to solve the system of equations, but that's what you would do in math class at school. On the Digital SAT, **there's a better way.** Notice that the question asks for a single, specific value: the number of punctuation questions on the assignment. Each answer choice also has a single, specific value. One of those values has to be the answer to the question, so start with the answers and plug them into the question.

Representation
Make sure you know what the numbers in the answer choices represent. Be sure to label them!

Before you plug in an answer, make sure you know what the answers represent. The question asks for the number of punctuation questions, so rewrite the answers on your scratch paper and label them "# of punct Qs" or something similar.

Now it's time to start working the steps of the problem. But first, notice that the answer choices are in numerically ascending order. The test-writers like to keep their questions organized, so they will always put the answers in order on this kind of question. You can use that to your advantage by starting with one of the middle answer choices. If it ends up being too big or small, you should be able to eliminate more than one answer. If the first answer you plug in happens to work, you can just stop there. Try (B) first.

Look at (B) and ask yourself, "If the number of punctuation questions is 6, what's the next thing I can figure out?" In this case, you could figure out how much time each student spends on punctuation questions. The question states that student X takes *35 seconds per punctuation question*, so student X will take (6)(35) = 210 seconds to finish the punctuation questions. It takes student X a total of 310 seconds to complete the assignment, so that leaves 310 – 210 = 100 seconds for the grammar questions. The question also states that *student X takes 20 seconds per grammar question*, so there are $\frac{100}{20} = 5$ grammar questions.

Next, do the same thing for student Y, who takes 55 seconds per punctuation question. If there are 6 punctuation questions, that's (6)(55) = 330 seconds. Student Y takes 480 seconds to complete the assignment, so that leaves 480 – 330 = 150 seconds for the grammar questions. The question states that *student Y takes 30 seconds per grammar question*, so there are $\frac{150}{30} = 5$ grammar questions.

Take a look at the numbers on your scratch paper. Did both students complete 6 punctuation questions? Yes, because that's the number you plugged in. Did both students complete 5 grammar questions? Yes, they did. Are you done? Yes, you are!

If you had started with a different number, somewhere along the way the math wouldn't have worked. You would end up with a fractional number of questions, or the two students wouldn't have the same number of each question type. When that happens, cross out the answer and try another one. In some cases, you'll be able to tell whether the number you tried was too big or too small, which helps you eliminate more answers and choose which one to try next. That's not necessary here because the first number you tried worked. The correct answer is (B).

Here are the steps for solving a problem using the PITA approach:

To solve a problem by plugging in the answers:

1. Rewrite the answer choices on your scratch paper and label them.
2. Starting with one of the middle answer choices, work the steps of the problem.
3. Look for something in the question that tells you what must happen for the answer to be correct.
4. When you find the correct answer, STOP.

You've seen how PITA gets you through a word problem quickly, efficiently, and accurately. But the technique also works on questions that aren't word problems but still ask for a specific value and have numbers in the answer choices. Here's an example.

2 Mark for Review

$$2x + y = 6$$
$$7x + 2y = 27$$

The system of equations above is satisfied by which of the following ordered pairs (x, y)?

Ⓐ $(-5, 4)$

Ⓑ $(4, -2)$

Ⓒ $(5, 4)$

Ⓓ $(5, -4)$

Which Way?
Sometimes, it's hard to tell if you need a larger number or a smaller number if the first answer you tried didn't work. Don't fret. Just pick a direction and go! Spend your time trying answers rather than worrying about going in the wrong direction.

Here's How to Crack It

The question asks for the coordinates of the point that satisfies the system of equations. When you feel the urge to do a whole lot of algebra, it is a good time to check whether it would be possible to just plug in the answers instead. In this case, trying your answer choices will be not only effective but also incredibly fast.

Start by rewriting the answer choices on your scratch paper and labeling them "(x, y)." It doesn't seem like you will be able to tell whether to move up or down this time, as the ordered pairs don't really have an ascending or descending order, but start in the middle anyway. Even if you end up trying three of the four, you will be saving time by plugging in the answers instead of solving.

Starting with (B) gives you 4 for x and –2 for y. Try that out in the first equation: $2(4) + (-2) = 6$. That matches the first equation, so this is a possibility. Try it out in the second equation: $7(4) + 2(-2) = 24$. That does not match the second equation, so you can eliminate (B).

Try out (C) next. If $x = 5$ and $y = 4$, then $2(5) + 4 = 14$, and you wanted it to be 6, so you can eliminate this answer choice as well.

Move on to (D). That would give you $2(5) + (-4) = 6$. So far so good! Try the second equation to see if this choice satisfies both: $7(5) + 2(-4) = 27$. This works, so the correct answer is (D).

You may recall that we covered questions like this in the last chapter. It is important to know how to solve these, in case a question like this comes up on a fill-in question. When you have answers available to you, though, don't be afraid to use them!

One last thing about PITA: here's how to determine whether you should use this approach to solve a problem.

Three ways to know that it's time for PITA:

1. There are numbers in the answer choices.
2. The question asks for a specific amount. Look for phrases like "the number of," "what was," or "how many."
3. You have the urge to write an algebraic equation to solve the problem.

Here's an example of using PITA instead of writing equations.

3 Mark for Review

A bakery sold exactly 85% of the cupcakes it baked on Tuesday. Which of the following could be the total number of cupcakes baked on Tuesday?

Ⓐ 130

Ⓑ 140

Ⓒ 145

Ⓓ 150

Here's How to Crack It

The question asks for a possible total number of cupcakes. Is your first reaction that there isn't nearly enough information here to start on this question? That makes it a great opportunity to plug in the answers! Start with one of the middle answer choices and test it out. Sometimes, even if you can't see how a question works ahead of time, it starts to make a lot more sense once you plug real numbers into it.

Choice (C) is 145, but 145 what? Read the question very carefully. The question asks for the total number of cupcakes baked on Tuesday, so rewrite the answer choices and label them "Total."

Next, work your way through the problem. If 145 is the total number of cupcakes baked on Tuesday, the number the bakery sold on Tuesday is 85% of 145, or 123.25. Have you ever bought 0.25 cupcakes at a bakery? It would be really weird for a bakery to sell fractions of cupcakes, so this answer could not be the total number baked on Tuesday.

In this particular question, it is hard to tell whether you should try bigger or smaller numbers next, but you have learned two things from your first attempt: you can get rid of (C), and the correct answer will be the one that gives you a whole number of cupcakes. Instead of spending time trying to predict which direction to go for the answer, just get to work on the other answer choices.

Try (B) next. If the bakery baked 140 cupcakes on Tuesday, it sold 85% of 140, or 119. Is there anything wrong with selling 119 cupcakes? No! Since the bakery sold only whole cupcakes, the correct answer is (B).

Here's another example of how PITA turns algebra into arithmetic.

4 Mark for Review

For what value of x is $|2x + 3| + 5 = 0$?

Ⓐ -4

Ⓑ 0

Ⓒ 4

Ⓓ There is no such value of x.

Here's How to Crack It

The question asks for the value of x. Although we covered it in the last chapter, solving algebraically on an absolute value question can be treacherous. There are so many ways to go wrong with those signs! Luckily, this absolute value question comes complete with answer choices, so we can simply plug in the answers to get a solution. As always, start by rewriting the answer choices on your scratch paper and labeling them. In this case, the label is simply "x."

Start with (C). When you put 4 in for x, you get $|2(4) + 3| + 5 = 0$, or $16 = 0$. This is clearly not true, so cross off (C) and move on to (B). If x is 0, then the original equation becomes $|2(0) + 3| + 5 = 0$ or $8 = 0$, so you can eliminate (B) as well. Next, try (A): $|2(-4) + 3| + 5 = 0$ could be rewritten as $|-8 + 3| + 5 = 0$, or $|-5| + 5 = 0$. As long as you remember that the absolute value of a number is always positive, it is clear that this gives you $5 + 5 = 0$. Since this is also clearly untrue, eliminate (A). Apparently, there is no such value of x! The correct answer is (D).

SOLVING RATIONAL EQUATIONS

A rational equation is basically an equation in which one (or more) of the terms is a fractional one. Rational equations look scary, but there are very simple ways of solving them. All in all, the test-writers can't get too messy here, so they will keep the math nice and tidy. Try the following question.

5 Mark for Review

$$x - 3 = \frac{5}{x - 3}$$

Which of the following is a possible value of $x - 3$ in the equation above?

Ⓐ $\sqrt{5}$

Ⓑ 5

Ⓒ $3 + \sqrt{5}$

Ⓓ 25

Watch Us Crack It
Watch the step-by-step video explanation of how to answer this question in your Student Tools.

Here's How to Crack It

The question asks for a possible value of $(x - 3)$ in an equation with fractions and binomials. You could use your skills with quadratics from the last chapter, but it's much simpler to use PITA! After all, there are numbers in the answers and the question asks for a specific value, so it sure looks like a PITA question. This approach will also avoid the issue of extraneous solutions.

Follow the PITA approach: start by rewriting the answers on your scratch paper and labeling them as "$x - 3$" and then start with one of the middle answer choices. The value in (C) looks complicated, so start with (B), 5. When $x - 3 = 5$, the equation becomes

$$5 = \frac{5}{5} \text{ or } 5 = 1$$

This is not true, so eliminate (B). Plugging in 25 would make the fraction too small, so ballpark out (D). Try the next simplest answer, (A). When $x - 3 = \sqrt{5}$, the equation becomes

$$\sqrt{5} = \frac{5}{\sqrt{5}}$$

Multiply both sides by $\sqrt{5}$ to get $5 = 5$, or use a calculator to find out that the two sides are equal. This makes the equation accurate, so $\sqrt{5}$ is a possible value of $x - 3$. The correct answer is (A).

PLUGGING IN YOUR OWN NUMBERS

Plugging In the Answers enables you to find the answer to questions whose answer choices are all numbers. What about questions that have answer choices containing variables? On these questions, you will usually be able to find the answer by plugging in your own numbers.

Plugging In is easy. It has three steps:

1. Pick numbers for the variables in the question.
2. Use your numbers to find an answer to the questions. Circle your answer.
3. Plug your number(s) for the variable(s) into the answer choices and eliminate choices that don't equal the answer you found in Step 2.

The Basics of Plugging In Your Own Numbers

This sort of Plugging In is simple to understand. Here's an example.

6 Mark for Review

Which of the following is equivalent to $\frac{3}{5} + \frac{y}{1-2y}$?

Ⓐ $\frac{3+y}{6-2y}$

Ⓑ $\frac{3-y}{5-10y}$

Ⓒ $\frac{2y}{5-10y}$

Ⓓ $\frac{3y}{5-10y}$

Here's How to Crack It

The question asks for an equivalent form of an expression. Rather than do complicated algebra, try plugging in. First, pick a number for y. Pick something easy to work with, like 2. On your scratch paper, write $y = 2$ so you won't forget. Then plug in 2 everywhere there's a y in the expression to get $\frac{3}{5}+\frac{2}{1-2(2)}$, which becomes $\frac{3}{5}+\frac{2}{1-4}$, then $\frac{3}{5}+\frac{2}{-3}$, and then $\frac{3}{5}-\frac{2}{3}$. Either find a common denominator and subtract the fractions by hand to get $\frac{9}{15}-\frac{10}{15}=-\frac{1}{15}$ or use a calculator to get the decimal equivalent of $-0.0\overline{6}$. In either form, this is your target value. Write it on the scratch paper and circle it.

Next, plug $y = 2$ into the answer choices and eliminate ones that do not equal the target value. Rewrite the answers one at a time with 2 in place of y.

A) $\frac{3+2}{6-2(2)}=\frac{5}{6-4}=\frac{5}{2}$ Not the target; eliminate!

B) $\frac{3-2}{5-10(2)}=\frac{1}{5-20}=\frac{1}{-15}=-\frac{1}{15}$ Matches the target; keep but check the rest just in case.

C) $\frac{2(2)}{5-10(2)}=\frac{4}{5-20}=\frac{4}{-15}=-\frac{4}{15}$ Not the target; eliminate!

D) $\frac{3(2)}{5-10(2)}=\frac{6}{5-20}=\frac{6}{-15}=-\frac{6}{15}$ Not the target; eliminate!

Only (B) matches the target value, so it's the correct answer.

Here's another example.

7 Mark for Review

During a special sale at a furniture store, Erica bought a floor lamp at a 10% discount. She paid a total of t dollars, which included the discounted price of the floor lamp and a 6% sales tax on the discounted price. In terms of t, what was the original price of the floor lamp?

Ⓐ $\frac{t}{0.96}$

Ⓑ $(0.9)(1.06)t$

Ⓒ $\frac{t}{(0.9)(1.06)}$

Ⓓ $0.96t$

Get Real

Trying to imagine how numbers behave in the abstract is a waste of time. So, if the question says that Tina is *x* years old, why not plug in your own age? That's real enough. You don't have to change your name to Tina.

Here's How to Crack It

The question asks for an expression to represent the situation. This could be a pretty tricky algebra question, but if you read the question carefully and plug in easy numbers, it will be a breeze.

Start at the beginning. When Erica bought that floor lamp on sale, what did you really wish you knew? It would be very helpful to start this problem knowing the original price of the floor lamp. So, start plugging in there. Plug in a number that you know how to take a percentage of, like 100. Write down "original = 100" and move on to the next step of the problem. Erica got a 10% discount, so take 10% of the original price. That means she got a \$10 discount, and the discounted price of her floor lamp was \$90. Write that down and move on to the sales tax. If you read carefully, it is clear that the sales tax is 6% of the discounted price. So, you need to take 6% of the \$90 discounted price, or \$5.40. To get her total, add the \$5.40 of tax to the \$90 for the discounted floor lamp, and you get \$95.40. This is where the careful reading comes in. The variable t in this question is supposed to be the total amount she paid, so make sure that you label this "t = \$95.40."

Next, read the last sentence of the question again to be sure you know which of the answers is your target answer. The question asks for the original price of the floor lamp, so circle the number you plugged in for the original price. Your target answer is 100.

On to the answer choices! When you put \$95.40 in for t in (A), you get 99.375. This is not your target answer, so you can eliminate (A). Choice (B) gives you 91.0116, so that will not work either. Plugging \$95.40 into (C) yields the target of 100, so hang on to it while you check (D) just in case. When you plug \$95.40 into (D), you get 91.584. Since that does not match your target, you can eliminate (D). The correct answer is (C).

Which Numbers?

Although you can plug in any number, you can make your life much easier by plugging in "good" numbers—numbers that are simple to work with or that make the problem easier to manipulate. Picking a small number, such as 2, will usually make finding the answer easier. If the question asks for a percentage, plug in 100. If the question has to do with minutes, try 30 or 120.

Except in special cases, you should avoid plugging in 0 and 1; these numbers have weird properties. Using them may allow you to eliminate only one or two choices at a time. You should also avoid plugging in any number that appears in the question or in any of the answer choices. Using those numbers could make more than one answer match your target. If more than one answer choice matches your target, plug in a new number and check those answer choices. You may have to plug in more than once to eliminate all three incorrect answers.

Be Good
"Good" numbers make a problem less confusing by simplifying the arithmetic. This is your chance to make the Digital SAT easier for you.

Many times you'll find that there is an advantage to picking a particular number, even a very large one, because it makes solving the problem more straightforward.

Here's an example.

8 Mark for Review

If 60 equally priced downloads cost x dollars, then how much do 9 downloads cost?

(A) $\frac{20}{3x}$

(B) $\frac{20x}{3}$

(C) $60x + 9$

(D) $\frac{3x}{20}$

Here's How to Crack It

The question asks for the cost of 9 downloads. Since the question is asking you to arrive at a number (how much 9 downloads cost) in terms of variable x, try plugging in. Should you plug in 2 for x? You could, but plugging in 120 would make the math easier. After all, if 60 downloads cost a total of $120, then each download costs $2. Write $x = 120$ on your scratch paper.

If each download costs $2, then 9 downloads cost $18. Write an 18 and circle it. You are looking for the answer choice that works out to 18 when you plug in $120 for x. Try each choice:

A) $\dfrac{20}{3(120)} \neq 18$

B) $\dfrac{20(120)}{3} \neq 18$

C) $60(120) + 9 \neq 18$

D) $\dfrac{3(120)}{20} = 18$

That last one matches the target answer, so the correct answer is (D).

Let's try another example.

Watch Us Crack It
Watch the step-by-step video explanation of how to answer this question in your Student Tools.

9 Mark for Review

A watch loses x minutes every y hours. At this rate, how many hours will the watch lose in one week?

Ⓐ $7xy$

Ⓑ $\dfrac{5y}{2x}$

Ⓒ $\dfrac{14y}{5x}$

Ⓓ $\dfrac{14x}{5y}$

Here's How to Crack It

The question asks for the number of hours lost in one week and gives information about minutes. This is an extremely difficult question for students who try to solve it using math-class algebra. You'll be able to find the answer easily, though, if you plug in carefully.

What numbers should you plug in? As always, you can plug in anything. However, if you think just a little bit before choosing the numbers, you can make the question easier to understand. There are three units of time—minutes, hours, and weeks—and that's a big part of the reason this question is hard to understand. If you choose units of time that are easy to think about, you'll make the question easier to handle.

Start by choosing a value for x, which represents the number of minutes that the watch loses. You might be tempted to choose $x = 60$ and that would make the math pretty easy. However, it's usually not a good idea to choose a conversion factor such as 60, the conversion factor between minutes and hours, for plugging in. When a question deals with time, 30 is usually a safer choice to avoid having multiple answers work the first time you plug in. So, write down $x = 30$.

Next, you need a number for y, which represents the number of hours. Again, you might be tempted to use $y = 24$, but that's the conversion factor between hours and days. Therefore, $y = 12$ is a safer choice. Write down $y = 12$.

Now, it's time to solve the problem to come up with a target. If the watch loses 30 minutes every 12 hours, then it loses 60 minutes every 24 hours. Put another way, the watch loses an hour each day. In one week, the watch will lose 7 hours. That's your target, so be sure to circle it.

Now, you just need to check the answer choices to see which one gives you 7 when $x = 30$ and $y = 12$.

A) $7xy = 7(30)(12)$ = Something too big! Cross it off.

B) $\frac{5y}{2x} = \frac{5(12)}{2(30)} = \frac{60}{60} = 1$. Also wrong.

C) $\frac{14y}{5x} = \frac{14(12)}{5(30)} = \frac{168}{150} = \frac{28}{25}$. Cross it off.

D) $\frac{14x}{5y} = \frac{14(30)}{5(12)} = \frac{420}{60} = 7$. Choose it!

The correct answer is (D).

MEANING IN CONTEXT

Some questions, instead of asking you to come up with an equation, just want you to recognize what a part of the equation stands for. These questions will look like algebra, with variables and equations, but they're often more about reading carefully, working in bite-sized pieces, and using your pencil.

First things first, though, you want to think about your POOD. Do these questions make sense to you, or are they a little confusing? Do you usually get them right quickly, or do they take up too much time? You can always use the Mark for Review tool to flag a meaning in context question to come back to later after you get some easier, quicker questions right.

If the question does fit your POOD, read carefully, write down labels for the parts of the equation, and use POE to get rid of any answer choices that don't match your labels.

Here's an example.

10 Mark for Review

There are m boxes of red marbles and n boxes of blue marbles in a crate, and the crate contains a total of 204 marbles. If this situation is represented by the equation $12m + 7n = 204$, what does the number 12 most likely represent?

Ⓐ The total number of red marbles

Ⓑ The total number of blue marbles

Ⓒ The number of red marbles in each box

Ⓓ The number of blue marbles in each box

Here's How to Crack It

The question asks for the meaning of a value in context. Start by reading the final question, which asks for the meaning of the number 12. Then label the parts of the equation with the information given. The question states that *there are m boxes of red marbles*, so label m as "# of boxes of red marbles." Similarly, label n as "# of boxes of blue marbles." The question also states that *the crate contains a total of 204 marbles*, so label 204 as "total # of marbles." The equation is now labeled as follows:

12(# of boxes of red marbles) + 7(# of boxes of blue marbles) = (total # of marbles)

Next, use Process of Elimination to get rid of answer choices that are not consistent with the labels. Since 12 is multiplied by the number of boxes of red marbles, it has something to do with the red marbles. Eliminate (B) and (D) because they refer to blue marbles. Since 12 is multiplied by the number of boxes of red marbles, it must represent a value per box, not a total value. Keep (C) because it is consistent with this information, and eliminate (A) because it refers to a total number. The correct answer is (C).

If labeling the equation isn't enough to eliminate three answers, try plugging in! This will help clarify what's going on in confusing equations or show the relationship between two parts of an equation.

Let's try a meaning in context question that uses plugging in.

11 Mark for Review

$$n = 1{,}273 - 4p$$

The equation above was used by the cafeteria in a large public high school to model the relationship between the number of slices of pizza, n, sold daily and the price of a slice of pizza, p, in dollars. What does the number 4 represent in this equation?

(A) For every \$4 the price of pizza decreases, the cafeteria sells 1 more slice of pizza.

(B) For every dollar the price of pizza decreases, the cafeteria sells 4 more slices of pizza.

(C) For every \$4 the price of pizza increases, the cafeteria sells 1 more slice of pizza.

(D) For every dollar the price of pizza increases, the cafeteria sells 4 more slices of pizza.

Here's How to Crack It

The question asks for the meaning of the number 4 in the context of the situation. First, read the question very carefully, and label the variables on your scratch paper. You know that p is the price of pizza, and n is the number of slices, so you can add that information to the equation. If you can, eliminate answer choices that don't make sense. But what if you can't eliminate anything, or you can eliminate only an answer choice or two?

Even with everything labeled, this equation is difficult to decode, so it's time to plug in! Try a few of your own numbers in the equation, and you will get a much better understanding of what is happening.

Try it out with $p = 2$. When you put 2 in for p, $n = 1{,}273 - 4(2)$ or 1,265.

So, when $p = 2$, $n = 1{,}265$. In other words, at \$2 a slice, the cafeteria sells 1,265 slices.

When $p = 3$, $n = 1{,}261$, so at \$3 a slice, the cafeteria sells 1,261 slices.

When $p = 4$, $n = 1{,}257$, so at \$4 a slice, the cafeteria sells 1,257 slices.

Now, use POE. First of all, is the cafeteria selling more pizza as the price goes up? No, as the price of pizza goes up, the cafeteria sells fewer slices of pizza. That means you can eliminate (C) and (D).

Choice (A) says that for every \$4 the price goes down, the cafeteria sells 1 more slice of pizza. Does your plugging in back that up? No. The cafeteria sells 8 more slices of pizza when the price drops from \$4 to \$2, so it will sell 16 more slices of pizza when the price drops by \$4, and (A) is no good.

Now take a look at (B). Does the cafeteria sell 4 more slices of pizza for every dollar the price drops? Yes! The correct answer is (B).

Here are the steps for solving meaning in context questions. The first three steps are often enough, but plug in when they aren't.

Meaning in Context

1. Read the question carefully. Make sure you know which part of the equation you are being asked to identify.
2. Use your scratch paper to rewrite the equation, replacing the parts of the equation you can identify with labels.
3. Eliminate any answer choices that clearly describe the wrong part of the equation, or go against what you have labeled.
4. Plug in! Use your own numbers to start seeing what is happening in the equation.
5. Use POE again, using the information you learned from plugging in real numbers, until you can get it down to one answer choice. Or get it down to as few choices as you can, and guess.

Let's look at a slightly different one now.

12 Mark for Review

$$7x + y = 133$$

Jeffrey has set a monthly budget for purchasing frozen blended mocha drinks from his local coffee shop. The equation above can be used to model the amount of his budget, y, in dollars, that remains after buying coffee for x days in a month. What does it mean that (19, 0) is a solution to this equation?

Ⓐ Jeffrey starts the month with a budget of $19.

Ⓑ Jeffrey spends $19 on coffee every day.

Ⓒ It takes 19 days for Jeffrey to drink 133 cups of coffee.

Ⓓ It takes 19 days for Jeffrey to run out of money in his budget for purchasing coffee.

Here's How to Crack It

The question asks about a point that is the solution to an equation in the context of the situation. Start by labeling the x and the y in the equation to keep track of what they stand for. Use your scratch paper to replace x with "days" and y with "budget." So 7 × days + budget = 133. Hmm, still not very clear, is it? One way to approach this is to plug in the point. If x = days = 19 when y = budget = 0, then Jeffrey will have no budget left after 19 days. This matches (D).

If you have trouble seeing this, you can use the answer choices to help you plug in. If (A) is true, the budget at the start of the month, when days = 0, is $19. Plug these values into the equation to see if it is true. Is 7 × 0 + 19 = 133? Not at all, so eliminate (A). If (B) is true, Jeffrey drinks a lot of coffee! Try some numbers and see if it works. For $x = 1$, the equation becomes $7(1) + y = 133$ or $y = 126$, and for $x = 2$, it is $7(2) + y = 133$ or $y = 119$. The difference in y, the budget remaining, is 126 – 119 = 7, so that's not $19 per day. Eliminate (B) so only (C) and (D) remain. These both have 19 for the number of days, and the point (19, 0) would indicate that 19 is the x-value, or days. If you saw that right away—great! That would allow you to skip right to testing (C) and (D).

For (C), you can plug in 19 for days in the equation to get 7 × 19 + budget = 133, or budget = 0. Does that tell you how many cups of coffee Jeffrey drank? You have no information about the cost of a single cup of coffee, so the answer can't be (C). It does tell you, however, that after 19 days, Jeffrey has no budget left. The correct answer is (D).

You might have noticed that the equation in this question could have been graphed in the xy-plane, which could help you visualize the situation. Some Digital SAT questions ask about the meaning of a piece of a linear graph, so keep the meaning in context approach in mind when you get to the Functions and Graphs chapter.

Other Digital SAT Algebra Strategies Drill

Work these algebra questions using Plugging In or PITA. Answers and explanations can be found starting on page 454.

1 Mark for Review

The length of a certain rectangle is twice the width. If the area of the rectangle is 128, what is the length of the rectangle?

(A) 4

(B) 8

(C) 16

(D) $21\frac{1}{3}$

2 Mark for Review

If $xy < 0$, which of the following must be true?

I. $x + y = 0$

II. $2y - 2x < 0$

III. $x^2 + y^2 > 0$

(A) I only

(B) III only

(C) I and III

(D) II and III

3 Mark for Review

If $\frac{\sqrt{x}}{2} = 2\sqrt{2}$, what is the value of x?

(A) 4

(B) 16

(C) $16\sqrt{2}$

(D) 32

4 Mark for Review

$$\sqrt{2x - k} = 3 - x$$

If $k = 3$, which of the following contains all possible solutions to the equation above?

(A) –2

(B) 2

(C) 2 and 6

(D) 6

5 Mark for Review

If Alex can fold 12 napkins in x minutes, how many napkins can he fold in y hours?

Ⓐ $\frac{720}{xy}$

Ⓑ $\frac{xy}{720}$

Ⓒ $\frac{720y}{x}$

Ⓓ $\frac{720x}{y}$

6 Mark for Review

Nails are sold in 8-ounce and 20-ounce boxes. If 50 boxes of nails were sold and the total weight of the nails sold was less than 600 ounces, what is the greatest possible number of 20-ounce boxes that could have been sold?

Ⓐ 16

Ⓑ 17

Ⓒ 25

Ⓓ 33

7 Mark for Review

In a certain school, 55% of the students sing in the choir. Of those who do not sing in the choir, 80% play in the band. If all students at the school participate in exactly one musical activity, what percentage of the students in the school participate in a musical activity other than singing in the choir or playing in the band?

Ⓐ 9%

Ⓑ 11%

Ⓒ 36%

Ⓓ 44%

8 Mark for Review

The equation $ax^2 - 12x + 12 = 0$ has exactly one solution for which of the following values of constant a?

Ⓐ -3

Ⓑ -2

Ⓒ 2

Ⓓ 3

9 Mark for Review

A gas station sells regular gasoline for $2.39 per gallon and premium gasoline for $2.79 per gallon. If the gas station sold a total of 550 gallons of both types of gasoline in one day for a total of $1,344.50, how many gallons of premium gasoline were sold?

(A) 25

(B) 75

(C) 175

(D) 475

10 Mark for Review

On a test, all right answers are worth the same number of points and all wrong answers are worth the same number of points. The equation $25C - 20W = 400$ represents the number of right answers, C, and wrong answers, W, a student had on a test when the student earned a total of 400 points. The point value of a right answer is how much more than the point value of a wrong answer?

(A) 20

(B) 25

(C) 45

(D) 80

OTHER DIGITAL SAT ALGEBRA STRATEGIES DRILL ANSWERS AND EXPLANATIONS

1. **C** The question asks for the length of the rectangle. This is a specific value, and there are numbers in the answer choices, so plug in the answers. If you start with (B), the length is 8, and the width is half that, or 4. Area is length × width. The area of this rectangle is 8 × 4, which is nowhere near 128. Eliminate (A) and (B), as both are too small. Try (C). If the length is 16, the width is 8. So, does 128 = 16 × 8? Yes, it does! Since this matches the information in the question, stop here. The correct answer is (C).

2. **B** The question asks which statements must be true. A question with unknown variables indicates a good place to plug in. You need numbers for x and y that will give you a negative product. Try $x = 1$ and $y = -2$. If you plug these into the statements in the Roman numerals, you find that (I) is false, but (II) and (III) are true. You can eliminate any answer choice that contains (I). This leaves (B) and (D). Now try different numbers to see if you can eliminate another choice. If you try $x = -1$ and $y = 2$, you find that (II) is false and (III) is still true. The correct answer is (B).

3. **D** The question asks for the value of x. This is a specific amount, and there are numbers in the answer choices, so plug in the answers, starting with (B). If $x = 16$, the left side of the equation is $\frac{\sqrt{16}}{2} = \frac{4}{2} = 2$. Does that equal $2\sqrt{2}$? No—it's too small. Choice (C) seems tough to work with, so try (D) next. If it is too big, (C) is your answer. For (D), $x = 32$, and the left side of the equation becomes $\frac{\sqrt{32}}{2} = \frac{\sqrt{16 \times 2}}{2} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$. It's a match. The correct answer is (D).

4. **B** The question asks for the possible solutions to an equation. Since the question asks for a specific value and the answers contain numbers in increasing order, plug in the answers. Write the answers on the scratch paper, label them as "x," and start with a number that appears more than once in the answers, such as 2. Plug $x = 2$ and $k = 3$ into the equation to get $\sqrt{2(2)-3} = 3-2$, which becomes $\sqrt{4-3} = 1$, then $\sqrt{1} = 1$, and finally $1 = 1$. This is true, so eliminate (A) and (D) because they do not include 2. Try 6 next, and plug $x = 6$ and $k = 3$ into the equation to get $\sqrt{2(6)-3} = 3-6$, which becomes $\sqrt{12-3} = -3$, then $\sqrt{9} = -3$, and finally $3 = -3$. (On the Digital SAT, the square root of a number is always positive.) This is not true, so eliminate (C). The correct answer is (B).

5. **C** The question asks for the number of napkins that can be folded in y hours. The two variables tell you this is a great place to plug in. Pick numbers that make the math easy. You can try $x = 30$ and $y = 2$. So in 2 hours there are 4 periods of 30 minutes each: 12 × 4 = 48. Alex can fold 48 napkins in 2 hours, so 48 is your target. Plug the values for x and y into the answer choices to see which one matches the target. Only (C) works, so the correct answer is (C).

6. **A** The question asks for the greatest possible number of 20-ounce boxes. Because this is a specific amount and there are numbers in the answer choices, this is a perfect opportunity to use PITA. Start with (C). If there are twenty-five 20-ounce boxes, then there are twenty-five 8-ounce boxes because a total of 50 boxes was purchased. In this case, the twenty-five 20-ounce boxes weigh 500 ounces, and the twenty-five 8-ounce boxes weigh 200 ounces; the total is 700 ounces. This is too big because the question says the total weight was less than 600. If (C) is too big, (D) must also be too big; eliminate both answers. If you try (B), the total weight is 604 ounces, which is still too big. Thus, the correct answer is (A).

7. **A** The question asks for a percentage. No specific values are given, so plug in. Since *percent* means "out of 100," plug in 100 for the total number of students to make the math easy. Next, work the steps of the question one at a time. The question states that *55% of the students sing in the choir*, so 55 of the 100 students sing in the choir. Read carefully: the next step refers to *those who do not sing in the choir*. If 55 sing in the choir, 100 – 55 = 45 do not sing in the choir. Take 80%, or $\frac{80}{100}$, of those 45 students to get $\frac{80}{100}(45) = 36$ students who play in the band. That leaves 100 – 55 – 36 = 9 students who participate in a musical activity other than choir or band. As a result of plugging in 100, there is no need to do math to convert to a percentage: 9 out of 100 is 9%. The correct answer is (A).

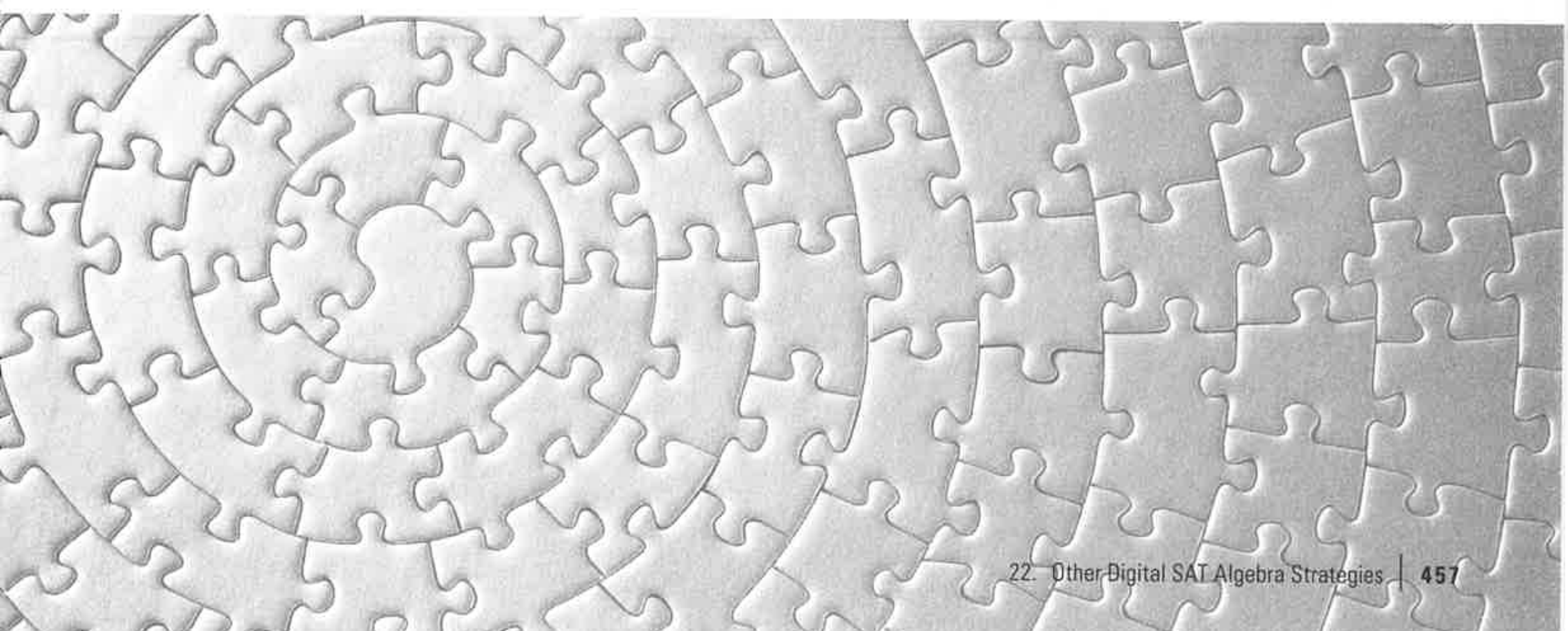
8. **D** The question asks for the value of a constant in a quadratic equation. One way to solve the question is to work with the discriminant, which you learned about in Chapter 21. However, the built-in calculator or a personal graphing calculator, combined with plugging in the answers, will get the question right faster. Start with (C), and plug in 2 for a. The equation becomes $2x^2 - 12x + 12 = 0$. Enter the left side of the equation into the built-in calculator. There are two dots where the graph intersects the x-axis, which means the quadratic has two solutions. Eliminate (C). It might not be obvious whether a bigger or smaller number is needed, but it won't take long to try other numbers. Try (B) and replace 2 with –2 in the equation. The calculator shows that the quadratic again has two solutions; eliminate (B). Try (D) and replace –2 with 3. The quadratic now has one solution at (2, 0). The correct answer is (D).

9. **B** The question asks for the number of gallons of premium gasoline that were sold. When asked for a specific value, try plugging in the answers. Label them as gallons of premium and start with the value in (B). If 75 gallons of premium were sold, the station would make 75($2.79) = $209.25 for those sales. A total of 550 gallons was sold, so the station would have sold 550 – 75 = 475 gallons of regular gasoline. The sales for the regular gasoline would be 475($2.39) = $1,135.25. The total sales for both types of gasoline would be $209.25 + $1,135.25 = $1,344.50. That matches the information in the question, so the correct answer is (B).

10. **C** The question asks for the relationship between two values in an equation. It will help to understand what the parts of the equation represent, so label parts of the equation and rewrite it. The question states that C represents the number of right answers, W represents the number of wrong answers, and 400 represents the total number of points. Rewrite the equation as 25(number of right answers) – 20(number of wrong answers) = (total number of points). Thus, 25 and –20 must represent the number of points earned for each right and wrong answer, respectively. To determine how much more a right answer is worth than a wrong answer, subtract the values to get $25 - (-20) = 45$. The correct answer is (C).

Summary

- When an algebra question asks for a specific amount and has numbers in the answer choices, plug each of the answer choices into the problem until you find one that works.
- If you start with one of the middle numbers, you may be able to cut your work. The answer choices will be in order, so if your number is too high or too low, you'll know what to eliminate.
- When the question has variables in the answer choices, you can often plug in your own amounts for the unknowns and do arithmetic instead of algebra.
- When you plug in, use "good" numbers—ones that are simple to work with and that make the problem easier to manipulate: 2, 5, 10, or 100 are generally easy numbers to use.
- On meaning in context questions, label the parts of the equation with information from the question, and eliminate answers that don't match the labels.
- Plugging In can also be used on meaning in context questions. If labeling the answers and using Process of Elimination isn't enough to answer the question, plug your own amounts into the equation so you can start to see what is going on.



Chapter 23
Functions and Graphs

The Digital SAT includes a large number of questions about functions, either by themselves or connected to a graph. Some of these questions can get quite complicated, but the same basic concepts apply no matter what. This chapter will give you the tools you need to work on questions about functions and understand how functions and graphs are connected.

CALL ON THE CALCULATOR

A complete understanding of a graphing calculator can make a significant difference in how long it takes to answer questions about the graphs of functions. Try solving the examples with the built-in calculator or your own to discover how powerful it can be. Some questions cannot be solved by the graphing calculator, however, so it's also important to understand the concepts covered in this chapter.

FUNCTION FUNDAMENTALS

Think of a **function** as just a machine for producing ordered pairs. You put in one number and the machine spits out another. Think of these as the input and the output. An input value, x, gets put into the function machine, f, and an output value, y, comes out of the function machine. The most common function is an $f(x)$ function. A simple way to keep functions straight is to remember that $f(x) = y$.

Let's look at a question.

1 Mark for Review

If $f(x) = x^3 - 4x + 8$, what is the value of $f(5)$?

Ⓐ 67

Ⓑ 97

Ⓒ 113

Ⓓ 147

What's This?

Anytime you see the notation $f(x)$, know that f isn't a variable; it's the name of the function. When you say it out loud, it's "f of x." Though $f(x)$ is the most common way to show that an equation is a mathematic function, any letter can be used. So you may see $g(x)$ or $h(d)$. Know that you're still dealing with a function.

Here's How to Crack It

The question asks for the value of $f(5)$ for the given function. Any time you see a number inside the parentheses, such as $f(5)$, plug in that number for x. The question is actually telling you to use Plugging In! Let's do it:

$$f(5) = 5^3 - 4(5) + 8$$
$$f(5) = 125 - 20 + 8$$
$$f(5) = 113$$

The correct answer is (C).

The previous question gave you a number to put into the function, which made it a Plugging In question. If the question gives you information about what comes out of the function and asks what should go in, it's a PITA question!

Here's an example of using PITA on a function question.

2 Mark for Review

For what value of x does $f(x) = 12$ if the function f is defined as $f(x) = \frac{1}{2}x + 2$?

(A) 5

(B) 8

(C) 20

(D) 24

Use PITA!
Don't forget that you can often plug in the answer choices on function questions! Noticing a pattern yet? Just a few easy tricks can unlock a lot of easy points.

Here's How to Crack It

The question asks for the input value of a function that results in a specific output value. The answer choices contain numbers in increasing order, and one of them must be the input that will result in an output of 12. Plug in the answers. Rewrite the answer choices on the scratch paper and label them "x." Next, start with one of the middle numbers. Try (B), 8. Plug $x = 8$ into the function to get $f(8) = \frac{1}{2}(8) + 2$, or $f(8) = 4 + 2$, and finally $f(8) = 6$. This does not match the output value of 12, so eliminate (B). The result was too small, so try a larger number and plug in $x = 20$. The function becomes $f(20) = \frac{1}{2}(20) + 2$, or $f(20) = 10 + 2$, and finally $f(20) = 12$. This matches the output value of 12, so stop here. The correct answer is (C).

In this case, it would have been possible to set the function equal to 12 and solve for x. However, solving will be difficult on more complicated problems, and it's easy to make mistakes with algebra. PITA is always a good idea when a question gives the output of a function and asks for the input.

Now that you know the basics of functions on the Digital SAT, try the next one.

3 Mark for Review

If $f(x) = x^2 + 2$, which of the following could be a value of $f(x)$?

Ⓐ -1

Ⓑ 0

Ⓒ 1

Ⓓ 2

Here's How to Crack It

The question asks for a possible value of the function. Therefore, the question is asking which of these values could be spit out of the $f(x)$ machine. Think about what is going in. No matter what you put in as a value for x, the value of x^2 has to be positive or zero. So, the lowest possible value of $x^2 + 2$ is 2. The correct answer is (D).

Note that you could also approach this question by plugging in the answers. If you plugged in 1 for $f(x)$, for instance, you would get $1 = x^2 + 2$, which becomes $x^2 = -1$, which is impossible.

Sometimes you'll get more complicated questions. As long as you know that when you put in x, your function will spit out another number, you'll be fine. Now try another one.

4 Mark for Review

What is the value of c if $g(x) = 4x^2 - 6$ and $g(x - c) = 4x^2 + 40x + 94$?

Ⓐ -40

Ⓑ -5

Ⓒ 5

Ⓓ 40

Here's How to Crack It

The question asks for the value of c. In function notation, the number inside the parentheses is the x-value that goes into the function. This question might seem complicated, but look beyond the math and use your Princeton Review Digital SAT knowledge. The question asks for a specific value and has numbers in the answers, so try PITA! Label the answer choices as "c" and start in the middle with (C), 5. Plug $c = 5$ into the question, and the value to put into the function becomes $x - 5$.

$$g(x - 5) = 4(x - 5)^2 - 6$$

FOIL the quadratic to get

$$g(x - 5) = 4(x^2 - 10x + 25) - 6$$

Distribute and combine like terms to get

$$g(x - 5) = 4x^2 - 40x + 100 - 6$$
$$g(x - 5) = 4x^2 - 40x + 94$$

Set the results equal to the quadratic given in the question.

$$4x^2 - 40x + 94 = 4x^2 + 40x + 94$$

This is not true, so eliminate (C). The middle term has the right coefficient but the wrong sign, so try (B), -5. The function becomes

$$g(x + 5) = 4(x + 5)^2 - 6$$

Carry out the same steps as before to get

$$g(x + 5) = 4(x^2 + 10x + 25) - 6$$
$$g(x + 5) = 4x^2 + 40x + 100 - 6$$
$$g(x + 5) = 4x^2 + 40x + 94$$

This is equal to the quadratic given in the question, so –5 is the value of c. The correct answer is (B).

Sometimes you may see a word problem that describes a function and then asks you to "build a function" that describes the real-world situation presented in the question. Take the following question, for example.

5 Mark for Review

Rock climbing routes are rated on a numbered scale with the highest number representing the most difficult route. Sally tried a range of shoe sizes on each of several routes of varying difficulty and found that when she wore smaller shoes, she could climb routes of greater difficulty. If D represents the difficulty rating of a route Sally successfully climbed and s represents the size of the shoes she wore on such a route, then which of the following could express D as a function of s?

Ⓐ $D(s) = s^2$

Ⓑ $D(s) = \sqrt{s}$

Ⓒ $D(s) = s - 3.5$

Ⓓ $D(s) = \frac{45}{s}$

Here's How to Crack It

The question asks for a function that best represents a situation. Start by thinking about the relationship described in the question: the smaller the shoes, the greater the difficulty. This is an inverse relationship. So, look for an inverse function. Only (D) expresses this type of relationship because a greater value of s leads to a smaller value of D.

If you aren't sure, try plugging in numbers to try it out. Plug in $s = 8$ and then $s = 10$ to see if the result for D is smaller when you use a larger shoe size. Since only (D) results in a smaller difficulty for a larger shoe size, the correct answer is (D).

One way the test-writers will make functions more complicated is to use percentages. Use your knowledge of function basics from this chapter and of percentages from earlier in this book to work through these questions.

Try this example.

6 Mark for Review

For every increase in the value of x by 1 in function z, the value of $z(x)$ increases by 25%. Which of the following functions defines z if $z(1) = 12.5$?

Ⓐ $z(x) = 10(1.25)^x$

Ⓑ $z(x) = 1.25(10)^x$

Ⓒ $z(x) = 0.75(10)^x$

Ⓓ $z(x) = 10(0.75)^x$

Here's How to Crack It

The question asks for the equation that defines a function. Recall that, in function notation, the number inside the parentheses is the x-value that goes into the function, or the input, and the value that comes out of the function is the y-value, or the output. The question provides a pair of input and output values by stating that $z(1) = 12.5$. Plug $x = 1$ into the answer choices and eliminate answers that do not result in $z(1) = 12.5$. These are the results:

A) $z(1) = 10(1.25)^1$	$z(1) = 10(1.25)$	$z(1) = 12.5$	Keep (A).
B) $z(1) = 1.25(10)^1$	$z(1) = 1.25(10)$	$z(1) = 12.5$	Keep (B).
C) $z(1) = 0.75(10)^1$	$z(1) = 0.75(10)$	$z(1) = 7.5$	Eliminate (C).
D) $z(1) = 10(0.75)^1$	$z(1) = 10(0.75)$	$z(1) = 7.5$	Eliminate (D).

Two answers worked, so try a different value. The question states that *for every increase in the value of x by 1 in function z, the value of z(x) increases by 25%*. Increase the input value by 1 to get $x = 1 + 1 = 2$. When $x = 1$, $z(x) = 12.5$, so when $x = 2$, $z(x)$ will increase by 25%. Take the value of $z(1)$, which is 12.5, and add 25% of it. *Percent* means out of 100, so translate 25% as $\frac{25}{100}$. Thus, $z(2) = 12.5 + \left(\frac{25}{100}\right)(12.5)$, or $z(2) = 15.625$. The other option is to use the built-in calculator, which automatically translates the % symbol and adds the word "of" to multiply the percent by the next number you enter. Either way, the target value for $z(2)$ is 15.625. Plug $x = 2$ into the remaining answer choices:

A) $z(2) = 10(1.25)^2$	$z(2) = 10(1.5625)$	$z(2) = 15.625$	Keep (A).
B) $z(2) = 1.25(10)^2$	$z(2) = 1.25(100)$	$z(2) = 125$	Eliminate (B).

Only one answer remains, so it must be correct. If the functions in the answer choices look familiar, that's because they are in the form of the growth and decay formula you learned about in Chapter 21: *final amount* = *original amount* $(1 \pm rate)^{number\ of\ changes}$. In correct answer (A), $z(x)$ is the *final amount*, 10 is the *original amount*, 0.25 is the *rate*, and x is the number of changes. But don't worry if you didn't catch that: plugging in values from the question was all you needed to do to get this one right. The correct answer is (A).

Functions and Tables

There is one more way the test-writers will test you on input and output values in a function, and that's by putting the values into a table. Stick to the basics of functions and look for chances to plug in.

Take a look at the following example.

7 Mark for Review

x	$f(x)$
1	8
2	17
3	26
4	35

Linear function f is represented in the table above, which shows four values of x and their corresponding values of $f(x)$. Which of the following functions accurately represents function f?

Ⓐ $f(x) = 8x$

Ⓑ $f(x) = 8x + 8$

Ⓒ $f(x) = 9x - 1$

Ⓓ $f(x) = 9x + 8$

Here's How to Crack It

The question asks for the function that represents values given in a table. Apply the function basics: x is the input, and $f(x)$, which is equivalent to y, is the output of the function. The table shows pairs of values for x and $f(x)$, and the correct function must work for every pair of values. Plug in values from the table and eliminate functions that don't work. Plug in $x = 1$ and $f(x) = 8$ to get the following results:

A) $8 = 8(1)$, or $8 = 8$ — True, so keep (A).
B) $8 = 8(1) + 8$, or $8 = 16$ — Not true; eliminate (B).
C) $8 = 9(1) - 1$, or $8 = 8$ — True, so keep (C).
D) $8 = 9(1) + 8$, or $8 = 17$ — Not true; eliminate (D).

Two answers worked for the first pair of input and output values, so try a second pair from the table. Plug $x = 2$ and $y = 17$ into the remaining answers to get these results:

A) $17 = 8(2)$, or $17 = 16$ — Not true; eliminate (A).
C) $17 = 9(2) - 1$, or $17 = 17$ — True, so keep (C).

The only answer that worked both times must be correct, so don't spend time trying the other two points. The correct answer is (C).

What's the Point?

Why did math folks come up with functions? To graph them, of course! When you put in a value for x, and your machine (or function) spits out another number, that's your y. You now have an ordered pair. Functions are just another way to express graphs. Knowing the connection between functions and graphs is useful, because you will most likely see questions involving graphs on the Digital SAT.

THE COORDINATE PLANE

A **coordinate plane,** or the ***xy*-plane,** is made up of two number lines that intersect at a right angle. The horizontal number line is called the ***x*-axis,** and the vertical number line is the ***y*-axis.**

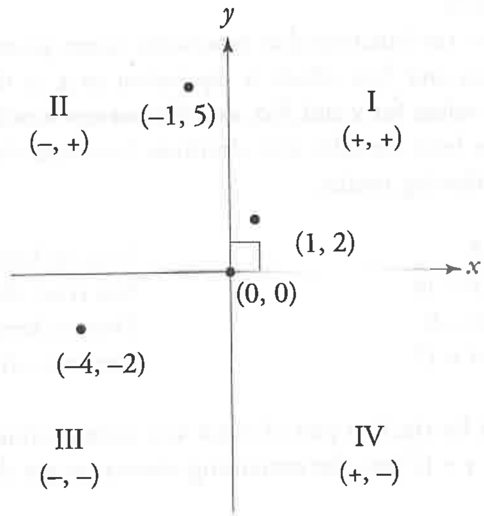
The four areas formed by the intersection of the axes are called **quadrants.** The location of any point can be described with a pair of numbers (x, y), just the way you would describe a point on a map: (0, 0) are the coordinates of the intersection of the two axes (also called the **origin**); (1, 2) are the coordinates of the point one space to the right and two spaces up; (–1, 5) are the coordinates of the point one space to the left and five spaces up; (–4, –2) are the coordinates of the point four spaces to the left and two spaces down. All of these points are located on the following figure.

Quadrants

A coordinate plane has four distinct areas known as quadrants. The quadrants are numbered counterclockwise, starting from the top right. They help determine generally whether x and y are positive or negative. Sometimes knowing what quadrant a point is in and what that means is all you need to find the answer.

Ways to Remember

Having trouble remembering that the x-coordinate comes before the y-coordinate in an ordered pair? Just remember the phrase "x before y, walk before you fly." The letter x also comes before y in the dictionary.



Some of the questions on the Digital SAT may require you to know certain properties of lines on the xy-plane. Let's talk about them.

POINTS ON A LINE

You may be asked if a point is on a line or on the graph of any other equation. Just plug the coordinates of the point into the equation of the line to determine if that point makes the equation a true statement.

8 Mark for Review

In the xy-plane, which of the following ordered pairs is a point on the line $y = 2x - 6$?

Ⓐ (6, 7)

Ⓑ (7, 7)

Ⓒ (7, 8)

Ⓓ (8, 7)

Here's How to Crack It

The question asks for a point that is on the given line. Plug in the answers, starting with (B). The (x, y) point is (7, 7), so plug in 7 for x and 7 for y. The equation becomes $7 = 2(7) - 6$ or $7 = 8$. This isn't true, so eliminate (B). The result was very close to a true statement, and the point in (C) has the same x-coordinate and a larger y-coordinate, so try that next. Because $8 = 2(7) - 6$, the correct answer is (C).

SLOPE

You always read a graph from left to right. As you read the graph, how much the line goes up or down is known as the slope. **Slope** is the rate of change of a line and is commonly known as "rise over run." It's denoted by the letter m. Essentially, it's the change in the y-coordinates over the change in the x-coordinates and can be found with the following formula:

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

This formula uses the points (x_1, y_1) and (x_2, y_2). For example, if you have the points (2, 3) and (7, 4), the slope of the line created by these points would be

$$m = \frac{(4 - 3)}{(7 - 2)}$$

So the slope of a line with points (2, 3) and (7, 4) would be $\frac{1}{5}$, which means that every time you go up 1 unit, you travel to the right 5 units.

EQUATIONS OF A LINE

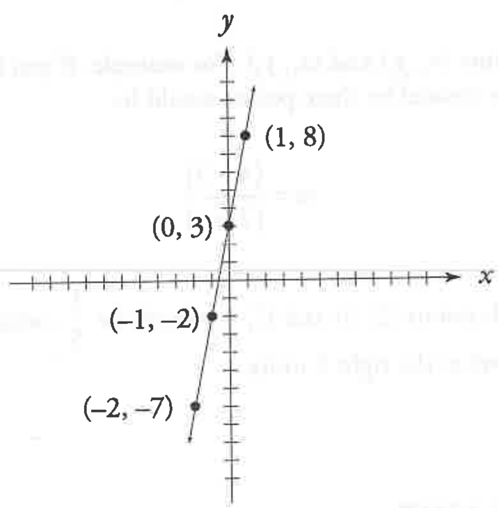
Slope-Intercept Form

The equation of a line can take multiple forms. The most common of these is known as the **slope-intercept form.** If you know the slope and the y-intercept, you can create the equation of a given line. A slope-intercept equation takes the form $y = mx + b$, where m is the slope and b is the ***y*-intercept** (the point where the function crosses the y-axis).

Let's say that you know that a certain line has a slope of 5 $\left(\text{which is the same as } \frac{5}{1}\right)$ and a y-intercept of 3. The equation of the line would be $y = 5x + 3$. You could graph this line simply by looking at this form of the equation. First, draw the y-intercept, (0, 3). Next, plug in a number for x and solve for y to get a coordinate pair of a point on the line. Then connect the point you just found with the y-intercept you already drew, and voilà, you have a line. If you want more points, you can create a table such as the following:

x	y
–2	–7
–1	–2
0	3
1	8

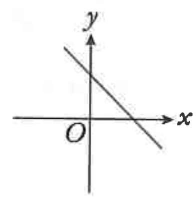
Take a look at the finished product:



One way the Digital SAT can test your understanding of lines is to show you a graph and ask you which equation describes that graph.

Here's an example.

9 Mark for Review



Which of the following could be the equation of the line represented in the graph above?

Ⓐ $y = -2x - 1$

Ⓑ $y = -2x + 4$

Ⓒ $y = 2x - 4$

Ⓓ $y = 2x + 4$

Here's How to Crack It

The question asks for the equation of a line based on the graph. Remember that the equation of a line is $y = mx + b$, where m is the slope and b is the y-intercept. Look at the graph and think about what the equation should look like. Since the line is sloping downward, it should have a negative slope, so you can eliminate (C) and (D). Next, since the line has a positive y-intercept, you can eliminate (A). The correct answer is (B).

Standard Form

Another way the equation of a line can be written is the **standard form** of $Ax + By = C$, where A, B, and C are constants and A and B do not equal zero. The test-writers will sometimes present equations in this form in the hopes that you will waste time putting it in slope-intercept form. If you know what to look for, the standard form can be just as useful as the slope-intercept form.

In standard form $Ax + By = C$:

- the slope of the line is $-\frac{A}{B}$
- the y-intercept of the line is $\frac{C}{B}$
- the x-intercept of the line is $\frac{C}{A}$

The equation $y = 5x + 3$ in the previous example would be $-5x + y = 3$ when written in the standard form. Using the information above, you can see that:

$$\text{slope} = -\left(\frac{-5}{1}\right) = 5$$

$$y\text{-intercept} = \frac{3}{1} = 3$$

$$x\text{-intercept} = \frac{3}{-5} = -\frac{3}{5}$$

The answers for the slope and the y-intercept were the same as when the slope-intercept form was used. Depending on the form of the equation in the question or in the answers, knowing these line equation facts can help save time on the test.

Let's look at how this may be tested.

10 Mark for Review

The graph of which of the following equations is parallel to the line with equation $y = -\frac{1}{3}x - \frac{1}{6}$?

Ⓐ $x - \frac{1}{3}y = 3$

Ⓑ $x - 3y = 2$

Ⓒ $x + 6y = 4$

Ⓓ $x + 3y = 5$

Here's How to Crack It

The question asks for the equation of a line that has a slope parallel to the slope of the line given. In the form $y = mx + b$, m represents the slope. The equation in the question is in that form, so the slope is $-\frac{1}{3}$. Parallel lines have equal slopes, so all you need to do now is find the answer choice that also has a slope of $-\frac{1}{3}$.

One way to do that would be to rewrite each answer in the $y = mx + b$ form. However, notice that the equations in the answer choices are in the $Ax + By = C$ form, and in that form the slope is equal to $-\frac{A}{B}$. Find the slope of each answer choice, and eliminate the ones that are not $-\frac{1}{3}$. The slope of the line in (A) is $-\frac{1}{-\frac{1}{3}} = 3$. This is not the correct slope, so eliminate (A). The slope of the line in (B) is $-\frac{1}{-3} = \frac{1}{3}$. This is also the wrong slope, so eliminate (B). The slope of the line in (C) is $-\frac{1}{6}$, which is also the wrong slope, so eliminate (C). The slope of the line in (D) is $-\frac{1}{3}$. This is the same slope as the line given in the question. The correct answer is (D).

PARALLEL AND PERPENDICULAR LINES

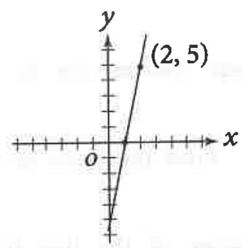
So now you know that **parallel lines** have the same slope. Whenever the Digital SAT brings up **perpendicular lines,** just remember that a perpendicular line has a slope that is the *negative reciprocal* of the other line's slope. For instance, if the slope of a line is 3, then the slope of a line perpendicular to it would be $-\frac{1}{3}$. Combine this with the skills you've already learned to work on a question about perpendicular lines.

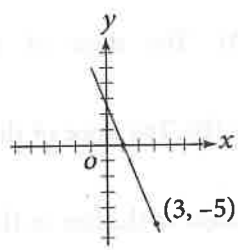
Parallel vs. Perpendicular
Parallel lines have the same slope and never intersect. Perpendicular lines have slopes that are negative reciprocals and intersect at a right angle.

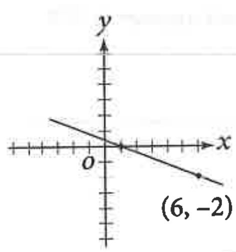
Here's an example.

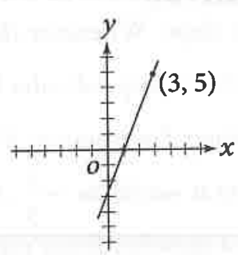
11 Mark for Review

Which of the following is the graph of a line perpendicular to the line defined by the equation $2x + 5y = 10$?

Ⓐ


Ⓑ


Ⓒ


Ⓓ


Here's How to Crack It

The question asks for a line perpendicular to the line $2x + 5y = 10$. Therefore, you need to find the slope of the line and then take the negative reciprocal to find the slope of the perpendicular line. You can convert the equation into the $y = mx + b$ format in order to find the slope, or simply remember that when an equation is presented in the form $Ax + By = C$, the slope is equal to $-\frac{A}{B}$. So the slope of this line is $-\frac{2}{5}$, and the slope of a perpendicular line would be $\frac{5}{2}$.

Look at the answer choices for one with a positive (upward) slope. Choices (B) and (C) slope downward, so eliminate them. Next, use points in the graph to find the slope of each answer. Eliminate (A); it has points at (1, 0) and (2, 5), for a slope of 5—too steep. The only remaining choice is (D), so the correct answer is (D).

TWO EQUATIONS WITH INFINITELY MANY SOLUTIONS

In the previous chapters on algebra, we discussed equations with one or multiple solutions. Now imagine an equation in which any value of x would create a viable solution to the equation.

$$x + 3 = x + 3$$

In this case, it is fairly obvious that any number you choose to put in for x will create a true equation. But what does it mean when two lines have infinitely many solutions? Let's look at an example.

12 Mark for Review

$$gx - hy = 78$$
$$4x + 3y = 13$$

In the system of equations above, g and h are constants. If the system has infinitely many solutions, what is the value of gh?

Ⓐ –432

Ⓑ –6

Ⓒ 6

Ⓓ 432

To Infinity...and Beyond!
When given two equations with infinitely many solutions, find a way to make them equal. The equations represent the same line.

Here's How to Crack It

The question asks for the value of gh, where g and h are coefficients in the system of equations. This question may have you scratching your head and moving on to the next question, but explore what you can do to solve this before you decide it's not worth your time. You may be surprised by how easy it is to solve a problem like this.

When they say that these equations have infinitely many solutions, what they are really saying is that these are the same equation, or that one equation is a multiple of the other equation. In other words, these two equations represent the same line. With that in mind, try to determine what needs to be done to make these equations equal. Since the right side of the equation is dealing with only a constant, first determine what you would need to do to make 13 equal to 78.

In this case, you need to multiply 13 by 6. Since you are working with equations, you need to do the same thing to both sides of the equation in order for the equation to remain equal.

$$6(4x + 3y) = 6 \times 13$$

$$24x + 18y = 78$$

Since both equations are now equal to 78, you can set them equal to each other, giving you this equation:

$$24x + 18y = gx - hy$$

You may know that when you have equations with the same variables on each side, the coefficients on those variables must be equal, so you can deduce that $g = 24$ and $h = -18$. (Be cautious when you evaluate this equation. The test-writers are being sneaky by using addition in one equation and subtraction in another.) Therefore, gh equals $24 \times -18 = -432$. The correct answer is (A).

TWO EQUATIONS WITH NO SOLUTIONS

You saw above that a system of equations can have infinitely many solutions. When solving equations, you likely assume, as most people do, that there will be at least one solution to the equation, but that is not always the case. Look at the following example.

$$3x - 6 = 3x + 7$$

If you solve this equation, you will find that $-6 = 7$. Since -6 can never equal 7, there is no value of x that can be put into this equation to make it true. In this case, the equation has no solutions.

What does it mean if two equations of lines have no solutions? Here's one to try.

13 Mark for Review

Which of the following accurately represents the set of solutions for the lines $6x + 12y = -24$ and $y = -\frac{1}{2}x + 2$?

Ⓐ (0, −4)

Ⓑ (0, 4)

Ⓒ There are no solutions.

Ⓓ There are infinitely many solutions.

There's Just No Solution
When given two equations with no solutions, find a way to compare slopes. The equations represent parallel lines.

Here's How to Crack It

The question asks for the solution to the system of equations. If the lines intersect, this will be the point of intersection. The answers in (C) and (D), though, suggest that the lines may be the same or parallel. Rather than plugging in the points in (A) and (B), look for a way to compare slopes. Start by putting the first line into $y = mx + b$ form: $12y = -6x - 24$. Divide the whole equation by 12, so $y = -\frac{1}{2}x - 2$. Since these lines have the same slope but different y-intercepts, the lines are parallel, and they will never intersect. Therefore, the correct answer is (C).

If two lines had different slopes, the lines would intersect at a single point such as (A) or (B). If the equations were identical, then they would be the same line and therefore have infinitely many solutions.

POINTS OF INTERSECTION

Earlier in this book you learned how to find the solution to a system of equations. There are several ways to do this, including stacking up the equations and adding or subtracting, setting them equal, or even plugging in the answers. The Digital SAT may also ask about the intersection of the graphs of two equations in the *xy*-plane, which is a similar idea.

Let's try one.

14 Mark for Review

In the *xy*-plane, which of the following is a point of intersection between the graphs of $y = x + 2$ and $y = x^2 + x - 2$?

Ⓐ (0, −2)

Ⓑ (0, 2)

Ⓒ (1, 0)

Ⓓ (2, 4)

Here's How to Crack It

Here's how you would apply PITA in a point of intersection question.

The question asks for the point of intersection for two equations. This is a point that is on the graphs of both equations. Therefore, the point would work if plugged into the equation of the line and the equation of the parabola.

So, use PITA by testing the answer choices: start with one of the answers in the middle and plug the point into each equation to see if it is true. The correct point of intersection will work in both functions. Try (C) in the first equation: does $0 = (1) + 2$? No. So, (C) is not the answer. Try (D) in the first equation: does $4 = (2) + 2$? Yes. So, try (D) in the second equation: does $4 = (2)2 + 2 - 2$? Yes. Since (2, 4) works in both equations, the correct answer is (D).

CALL ON THE CALCUATOR

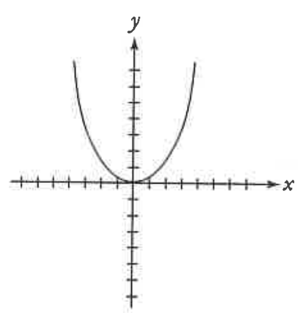
It's good to know the concepts and math behind the graphs of lines and the way they're tested on the Digital SAT. It's also good to know how to use the built-in graphing calculator or your own that you bring with you on test day. Go back to some of the previous examples and try them again using a graphing calculator (if you didn't use it the first time). You might be surprised by how easy some of them are to answer.

The rest of this chapter will deal with graphs of parabolas and circles instead of straight lines. The graphing calculator is extremely useful on many questions like this, and we'll show you how to take advantage of it.

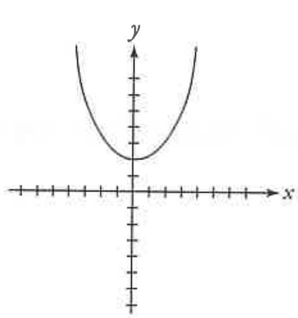
GRAPHING FUNCTIONS

One type of function question you might be asked is how the graph of a function would shift if you added a value to it.

Here is a quick guide for the graph of $f(x) = x^2$, as seen below:



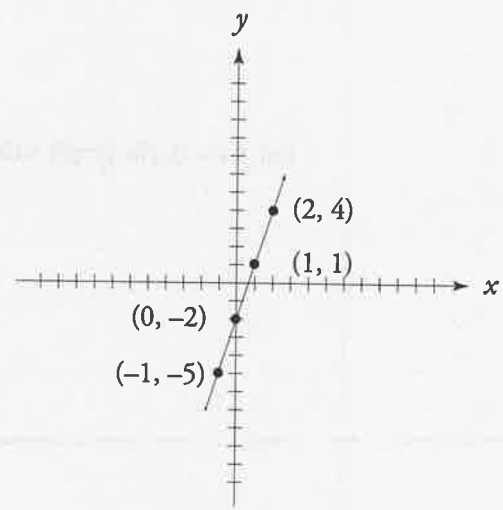
For $f(x) + c$, where c is a constant, the graph will shift up c units, as shown in the figure below:



$y = f(x)$

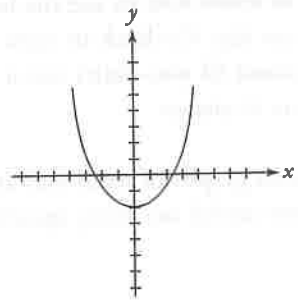
Sometimes, instead of seeing the typical $y = mx + b$ equation, or something similar, you'll see $f(x) = mx + b$. Look familiar? Graphs are just another way to show information from a function. Functions show information algebraically and graphs show functions geometrically (as pictured).

Here's an example. The function $f(x) = 3x - 2$ is shown graphically as the following:

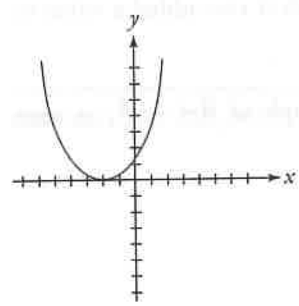


The reason the Digital SAT includes function questions is to test whether you can figure out the relationship between a function and its graph. To tackle these questions, you need to know that the independent variable, the x, is on the x-axis, and the dependent variable, the $f(x)$, is on the y-axis. For example, if you see a function of $f(x) = 7$, then you need to understand that this is a graph of a horizontal line where $y = 7$.

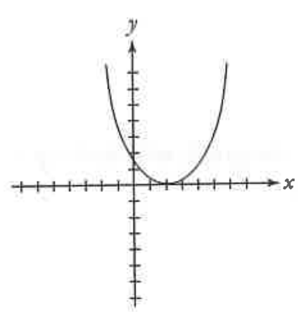
Conversely, $f(x) - c$ will shift the graph down by c units:



For $f(x + c)$, the graph will shift c units to the left:



For $f(x - c)$, the graph will shift to the right by c units:



You may have realized how easy these questions would become if you simply put them into your graphing calculator.

You can also plug in points to find the correct graph.

You have several options for dealing with questions like this. You can know the four rules for transforming graphs, you can plug in points to find the correct graph, or you can use a graphing calculator. The built-in calculator can graph the equation even if the test-writers give you an equation in an unusual form.

Try the next one using the built-in graphing calculator or your own.

15 Mark for Review

Function f can be graphed in the xy-plane by translating function g down 3 units. Function g is defined by $g(x) = -2x^3 + 4x^2 - x + 2$. What is the value of $f(1)$?

Watch Us Crack It
Watch the step-by-step video explanation of how to answer this question in your Student Tools.

Here's How to Crack It

The question asks for the value of a function. Start by graphing the equation of $g(x)$. In the built-in calculator, enter $g(x) = -2x^3 + 4x^2 - x + 2$ into the first entry field and you will see the graph in the graph display. The question asks about $f(1)$, so find the point where $x = 1$ on the graph of $g(x)$. You might need to scroll and zoom to see 1 on the x-axis, and use the mouse pointer to find the y-value when $x = 1$. The point is at (1, 3). The question states that the graph of function f is the graph of function g translated down 3 units. Count 3 units down from the point at (1, 3) in the graph of $g(x)$ to get to a point at (1, 0). At this point, $x = 1$ and $y = 0$. Since $f(x) = y$, the value of $f(1)$ is 0. The correct answer is 0.

EQUATIONS OF A PARABOLA

Standard Form

The Digital SAT will ask questions using three different forms of the equation for a parabola.

The standard form of a parabola equation is

$$y = ax^2 + bx + c$$

In the standard form of a parabola, the value of a tells whether a parabola opens upward or downward (if a is positive, the parabola opens upward, and if a is negative, the parabola opens downward).

Factored Form

We looked at equations for parabolas in Chapter 21 when we solved quadratics. The factored form of a quadratic equation reveals the roots of the parabola. These are also the solutions of x. Given a question about roots or solutions, it can be helpful to know the relationship between the equation and the graph of the parabola in the xy-plane.

The factored form of a parabola equation is

$$y = a(x - r_1)(x - r_2)$$

In the factored form, r_1 and r_2 are the roots or x-intercepts of the parabola.

For the next question, the equation is $y = x^2 - 4x - 12$. If you factored this, you'd get $y = (x + 2)(x - 6)$, and the roots would be at $x = -2$ and $x = 6$. You can see that those are the exact points on the graph that the parabola crosses the x-axis.

Vertex Form

The vertex form of a parabola equation is

$$y = a(x - h)^2 + k$$

In the vertex form, the point (h, k) is the vertex of the parabola.

In vertex form, the value of a still indicates which way the parabola opens. The most useful feature of vertex form is that it shows the vertex of the parabola, which is the minimum value of y when $a > 0$ or the maximum value of y when $a < 0$. When the equation is not in vertex form, you'll have to do some work to find the vertex.

Here's a typical question about the vertex of a parabola.

16 Mark for Review

The function g is defined by $g(x) = -x^2 + 4x + 12$. For what value of x does $g(x)$ reach its maximum?

Here's How to Crack It

The question asks for the value when a quadratic function reaches its maximum. A parabola reaches its minimum or maximum value at its vertex, so find the x-coordinate of the vertex. The simplest method is to enter the equation into the built-in calculator, then scroll and zoom as needed to find the vertex. The vertex is at (2, 16), so the value of the x-coordinate is 2.

There is also a way to find the x-coordinate using algebra. In vertex form, the vertex is at (h, k). When a quadratic is in standard form, $ax^2 + bx + c$, use the formula $h = -\frac{b}{2a}$ to find the x-coordinate of the vertex. The equation in this question is in standard form, so $a = -1$, $b = 4$, and $c = 12$. The formula becomes $h = -\frac{4}{2(-1)}$, or $h = -\frac{4}{-2}$, and then $h = 2$. You don't need this formula to answer this question, but if you needed to solve for the y-coordinate of the vertex, you could plug $x = 2$ back into the equation to get $-(2)^2 + 4(2) + 12 = -4 + 8 + 12 = 16$. This is the vertex of (2, 16) that you found using the calculator.

Using either method, the correct answer is 2.

In the chapter on algebra, we showed you how the Digital SAT asks questions about the number of solutions to a system of linear equations or a quadratic. Review that to know the rules and how to use the discriminant. However, the calculator provides additional options, and some of the other skills you've learned in this book apply to questions about graphing too.

Think about using a calculator and apply what you've learned so far to the following question.

17 Mark for Review

$$\frac{y}{3} = -\frac{7}{2}x$$
$$y = ax^2 + 12.25$$

In the system of equations above, a is a constant. If the system of equations has exactly one solution, what is the value of a?

Ⓐ -2.25

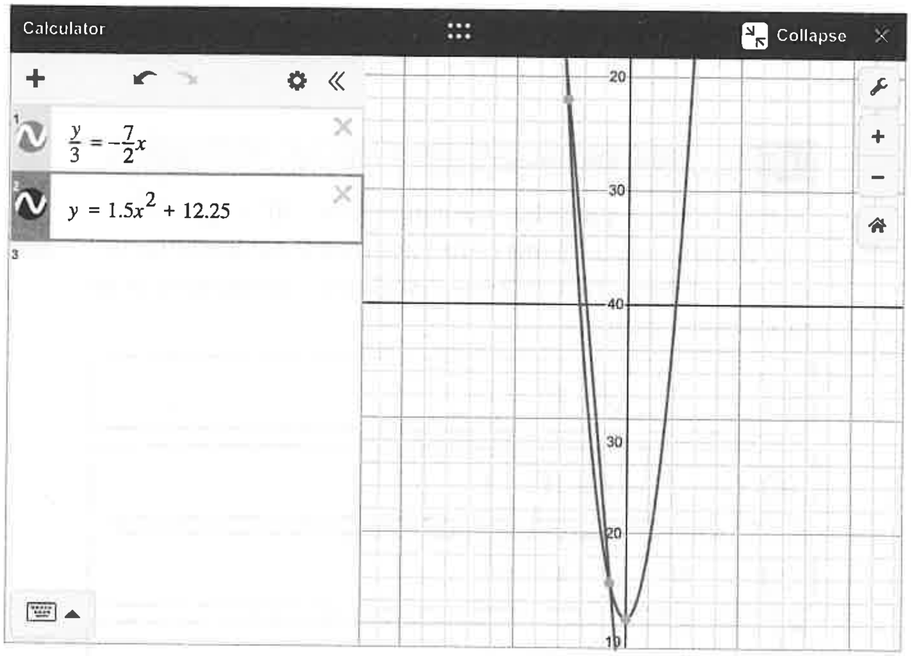
Ⓑ -1.5

Ⓒ 1.5

Ⓓ 2.25

Here's How to Crack It

The question asks for the value of a constant in a system of equations. The question asks for a specific value, and the answers contain numbers in increasing order, so plug in the answers. Yes, PITA can work on graphing questions, too! Start with an answer in the middle. Try (C), 1.5. Plug $a = 1.5$ into the second equation to get $y = (1.5)(x^2) + 12.25$. Next, enter both equations in the entry fields of the built-in graphing calculator and look at the results in the graph display. Zoom out enough to see both the parabola and the line clearly in the graph display (expanding the calculator window also helps). If you click one of the equations, several gray dots will appear on the graph. One shows the vertex of the parabola, and the other two show where the line intersects the parabola.



The question states that the *system of equations has exactly one solution*, not two, so eliminate (C). Next, try (D), 2.25. Keep the two equations you already entered and change 1.5 to 2.25 in the second equation. Click on one of the equations to see two gray dots: one is the vertex and the other is the single point where the line intersects the graph. The system now has exactly one solution, and the correct answer is (D).

EQUATION OF A CIRCLE

The Digital SAT will also ask questions about the equation of a circle in the *xy*-plane.

> The equation of a circle is
>
> $$(x-h)^2 + (y-k)^2 = r^2$$
>
> In the circle equation, the center of the circle is the point (h, k), and the radius of the circle is r.

Let's look at a question that tests the use of the circle equation.

18 Mark for Review

Circle P in the xy-plane has the equation $(x-4)^2+(y+1)^2 = 9$. Circle Q has the same radius as Circle P, but its center is 2 units to the left of that of Circle P. Which of the following is the equation of Circle Q?

Ⓐ $(x-4)^2+(y+1)^2=1$

Ⓑ $(x-2)^2+(y+1)^2=9$

Ⓒ $(x-6)^2+(y+1)^2=9$

Ⓓ $(x-4)^2+(y+3)^2=9$

Here's How to Crack It

The question asks for an equation that represents a graph. The equation of a circle in standard form is $(x-h)^2+(y-k)^2=r^2$, where (h, k) is the center and r is the radius. Since the equation of Circle P is $(x-4)^2+(y+1)^2=9$, the center is at $(4, -1)$ and $r^2=9$, so $r=3$. The question states that *Circle Q has the same radius as Circle P*, so the right side of the equation of Circle Q should also be 3^2, or 9; eliminate (A) because it has an incorrect value for r^2. The center of Circle Q is 2 units to the left of the center of Circle P. Two units to the left of $(4, -1)$ is $(2, -1)$, so the equation of Circle Q is $(x-2)^2+(y+1)^2=9$. The correct answer is (B).

On the last question, you could have graphed the original circle and the circles in the answers. That would be time-consuming, though, and it was easy to find the center given the equation in standard form. Some circle questions are better cracked using a graphing calculator, either the built-in calculator or your own. Try using a calculator to answer this question.

19 Mark for Review

What is the radius of a circle graphed in the xy-plane and given by the equation $3x^2 + 15x + 3y^2 - 3y = 28.5$?

Ⓐ 3

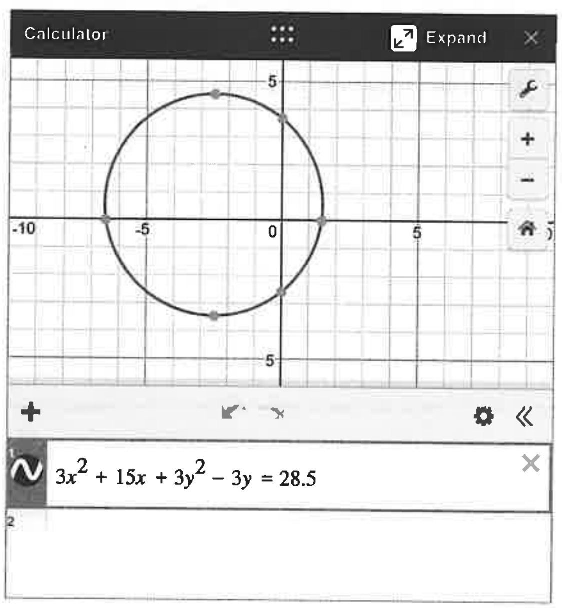
Ⓑ 4

Ⓒ $\sqrt{35}$

Ⓓ $\sqrt{48}$

Here's How to Crack It

The question asks for the radius of a circle given an equation for its graph. The equation is not given in standard form, so you cannot simply look at the equation and determine the radius. One approach is to graph the equation. Using the built-in calculator, enter the equation as given; the calculator will graph the circle correctly even though the equation isn't in standard form. Click in the entry field or on the circle to see several gray dots.



Click the dots at the maximum and minimum y-values to find the two ends of the diameter. The maximum y-value is at $(-2.5, 4.5)$, and the minimum y-value is at $(-2.5, -3.5)$. Because these points have the same x-coordinate, the distance between them is the diameter of the circle. Find the difference to get a diameter of $4.5 - (-3.5) = 8$. The radius of a circle is half the diameter, so the radius is 4.

The other way to solve for the radius is to convert the equation to standard form. The equation in standard form of a circle in the xy-plane is

$$(x - h)^2 + (y - k)^2 = r^2$$

where (h, k) is the center of the circle and r is the radius. The equation given in the question is not in standard form. To put it into standard form, complete the squares. Start by dividing the entire equation by 3 to remove the coefficients on the x^2 and y^2 terms. This leaves

$$x^2 + 5x + y^2 - y = 9.5$$

Next, take half the coefficient on the x term, or 2.5, square it to get 6.25, and add 6.25 to both sides. Now the equation is

$$x^2 + 5x + 6.25 + y^2 - y = 15.75$$

Do the same with the coefficient on the y term to get

$$x^2 + 5x + 6.25 + y^2 - y + 0.25 = 16$$

The standard form equation for this circle is thus

$$(x + 2.5)^2 + (y - 0.5)^2 = 16$$

The question didn't ask for the center of the circle, so focus on the radius. If $r^2 = 16$, then $r = 4$. Using a graphing calculator or completing the square, the correct answer is (B).

Got all of that? Now test your knowledge of functions and graphing with the following drills.

Functions and Graphs Drill

Use your new knowledge of functions, graphs, and coordinate geometry to complete the questions. Answers and explanations can be found starting on page 492.

1 Mark for Review

Function f is defined by $f(x) = x^2 - c$, where c is a constant. If $f(-2) = 6$, what is the value of c?

Ⓐ -10

Ⓑ -2

Ⓒ 0

Ⓓ 2

2 Mark for Review

A farming crew harvests 100 hectares a day on a wheat farm. If the crew maintains this rate, which of the following functions represents the number of hectares, h, the crew can harvest in d days?

Ⓐ $h(d) = d + 100$

Ⓑ $h(d) = d - 100$

Ⓒ $h(d) = 100d$

Ⓓ $h(d) = \frac{d}{100}$

3 Mark for Review

x	y
-3	-7
-1	-3
2	3

Based on the chart above, which of the following could express the relationship between x and y?

Ⓐ $y = x - 4$

Ⓑ $y = 2x - 1$

Ⓒ $y = 2x + 2$

Ⓓ $y = 3x - 3$

4 Mark for Review

Line l contains points (3, 2) and (4, 5). If line m is perpendicular to line l, which of the following could be the equation of line m?

Ⓐ $-5x + y = \frac{1}{3}$

Ⓑ $x + 3y = 15$

Ⓒ $x + 5y = 15$

Ⓓ $3x + y = 5$

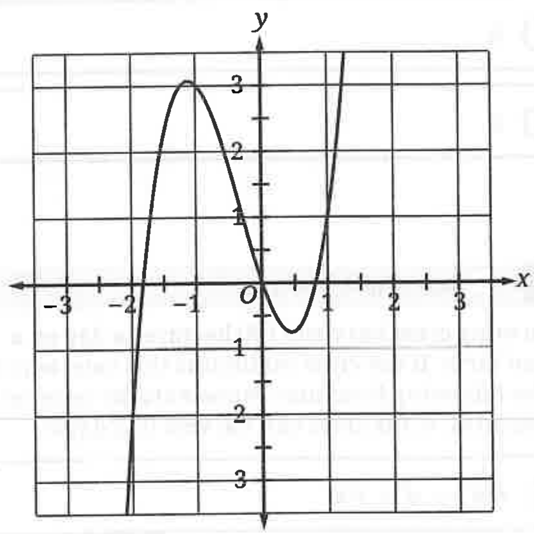
5 Mark for Review

If $f(x) = 2x^2 + 4$ for all real numbers x, what is the value of $f(3) + 3f(5)$?

6 Mark for Review

The variable c represents a positive constant in the exponential function $f(x) = \frac{1}{3}c^x$. What is the value of $f(3)$ if $f(4) = 27$?

7 Mark for Review



The graph of $y = g(x)$ in the xy-plane is shown above. The function g is defined by $g(x) = ax^3 + bx^2 - cx$, where a, b, and c are constants. For how many values of x does $g(x) = 0$?

Ⓐ Zero

Ⓑ One

Ⓒ Two

Ⓓ Three

8 Mark for Review

The acceleration of a ball rolling down a ramp can be estimated using the function $s(t) = 5t$, where s represents the speed of the ball and t represents time. Given this relationship, which of the following tables contains 4 values of t and the corresponding values of s?

Ⓐ

t	s
1	5
2	10
3	15
4	20

Ⓑ

t	s
1	5
2	10
3	30
4	120

Ⓒ

t	s
1	5
2	25
3	125
4	625

Ⓓ

t	s
5	1
10	2
15	3
20	4

9 Mark for Review

The graph of line l in the xy-plane passes through the points (2, 5) and (4, 11). The graph of line m has a slope of -2 and an x-intercept of 2. If point (x, y) is the point of intersection of lines l and m, what is the value of y?

Ⓐ $\frac{3}{5}$

Ⓑ $\frac{4}{5}$

Ⓒ 1

Ⓓ 2

10 Mark for Review

The movement of a roller coaster cart as it completes one ride can be modeled by the equation $y = -0.05(x - 55.5)^2 + 154$, which shows the cart's height above the ground y, in feet, x seconds after the ride starts, where $0 < x \leq 111$. Which of the following statements best describes the meaning of the vertex of the graph of the equation in the xy-plane?

Ⓐ The cart reached a maximum height of 154 feet above the ground.

Ⓑ The cart reached a maximum height of 55.5 feet above the ground.

Ⓒ The cart's final height was 154 feet above the ground.

Ⓓ The cart's final height was 55.5 feet above the ground.

FUNCTIONS AND GRAPHS DRILL ANSWERS AND EXPLANATIONS

1. **B** The question asks for the value of c. Start by plugging in what you know into the given function. If $f(x) = x^2 - c$, and $f(-2) = 6$, then plug in -2 for x in the function: $f(-2) = (-2)^2 - c$. Solve and replace $f(-2)$ with 6: $6 = 4 - c$; $2 = -c$; and $c = -2$. If you picked (A), you forgot that $(-2)^2$ is positive 4. The correct answer is (B).

2. **C** The question asks for a function that represents a specific situation. There are variables in the answer choices, and the question asks about the relationship between the number of hectares and the number of days, so plug in. Make $d = 2$. If the crew harvests 100 hectares in 1 day, it will harvest twice as many hectares, 200, in 2 days. Now plug $d = 2$ and $h(d) = 200$ into the answer choices and eliminate any that don't work. Choice (A) becomes $200 = 2 + 100$, or $200 = 102$. This is not true, so eliminate (A). Choice (B) becomes $200 = 2 - 100$, or $200 = -98$; eliminate (B). Choice (C) becomes $200 = 100(2)$, or $200 = 200$; this is true, so keep (C) but check (D) just in case. Choice (D) becomes $200 = \frac{2}{100}$; eliminate (D). The correct answer is (C).

3. **B** The question asks for the equation that best models pairs of values given in a table. Plug in the values from the chart. Use the pair $(-3, -7)$ from the top of the chart and eliminate answers that are not true. Choice (A) becomes $-7 = -3 - 4$, which is true. Keep it. Keep (B): $-7 = 2(-3) - 1$ is true. Get rid of (C), which becomes $-7 = 2(-3) + 2$: -7 does not equal -4. Get rid of (D): $-7 = 3(-3) - 3$, and -7 does not equal -12. Now use another pair just to test (A) and (B). Using $(-1, -3)$, (A) gives $-3 = -1 - 4$, which is not true, so eliminate it. The correct answer is (B).

4. **B** The question asks for the equation of line m. First, find the slope of line l by using the slope formula: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{4 - 3} = \frac{3}{1}$. A line perpendicular to line l must have a slope that is the negative reciprocal of l's slope. So, its slope should be $-\frac{1}{3}$. In the standard form of a line $Ax + By = C$, the slope is $-\frac{A}{B}$. Only (B) has a slope of $-\frac{1}{3}$. If you didn't remember the rule about the slope of perpendicular lines in standard form, you could have converted the answers to slope-intercept form and sketched out each of the lines to look for the answer that looked perpendicular to l. The correct answer is (B).

5. **184** The question asks for the value of expression that contains two functions. In function notation, the number inside the parentheses is the x-value that goes into the function, or the input, and the value that comes out of the function is the y-value, or the output. When there is a number in front of the function, the output value is multiplied by that number. Plug $x = 3$ into the function to get $f(3) = 2(3)^2 + 4$, then $f(3) = 2(9) + 4$. This becomes $f(3) = 18 + 4$, and then $f(3) = 22$. Plug $x = 5$ into the function to get $f(5) = 2(5)^2 + 4$, then $f(5) = 2(25) + 4$, then $f(5) = 50 + 4$, and then $f(5) = 54$. Multiply

this output by 3 to get (3)(54) = 162. Finally, add the values of $f(3)$ and $3f(5)$ to get 22 + 162 = 184. The correct answer is 184.

6. **9** The question asks for the value of a function. In function notation, $f(x) = y$. The number inside the parentheses is the x-value that goes into the function, or the input, and the value that comes out of the function is the y-value, or the output. According to the question, $f(4) = 27$, so when the input is 4, the output is 27. Plug in $f(x) = 27$ and $x = 4$, and solve for c. The function becomes $27 = \frac{1}{3}c^4$. Multiply both sides of the equation by 3 to get $81 = c^4$. Take the fourth root of both sides of the equation to get $c = 3$. With the built-in calculator, enter $81^{\frac{1}{4}}$ or use the $\boxed{\sqrt[n]{}}$ button under the Functions menu to take the fourth root of 81. The question asks for the value of $f(3)$, so plug $x = 3$ and $c = 3$ into the function, and solve for $f(3)$. The function becomes $f(3) = \frac{1}{3}(3^3)$, then $f(3) = \frac{1}{3}(27)$, which becomes $f(3) = 9$. The correct answer is 9.

7. **D** The question asks for the number of times the value of a function is 0. In function notation, $f(x) = y$. The number inside the parentheses is the x-value that goes into the function, or the input, and the value that comes out of the function is the y-value, or the output. When $g(x) = 0$, the y-value on the graph is 0, so this question is asking for the number of x-intercepts. Look at the graph to see that it crosses the x-axis 3 times, with points at approximately (–1.8, 0), (0, 0), and (0.8, 0). Thus, there are 3 times when $g(x) = 0$. The correct answer is (D).

8. **A** The question asks for correct values in a function. In function notation, the number inside the parentheses is the x-value that goes into the function, or the input, and the value that comes out of the function is the y-value, or the output. In this scenario, t is the input and s is the output. When given a function and asked for the table of values, plug values from the answer choices into the function and eliminate answers that don't work. Start with $t = 2$. Plug $t = 2$ into the function to get $s(2) = 5(2)$, or $s(2) = 10$. Eliminate (C) because it has an incorrect value of s when t is 2. Plug $t = 3$ into the function to get $s(3) = 5(3)$, or $s(3) = 15$. Eliminate (B) because it has an incorrect value of s when t is 3. Next, try a value for t that is in (D) but not (A). Plug $t = 5$ into the function to get $s(5) = 5(5)$, or $s(5) = 25$. Eliminate (D) because it has an incorrect value of s when t is 5. The correct answer is (A).

9. **D** The question asks for the value of y. First, find the slope of line l by using the slope formula: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{11-5}{4-2} = \frac{6}{2} = 3$. Plug this slope and one of the points on line l into the slope-intercept form $y = mx + b$ to solve for b, giving you the full equation of the line. If you use the point (2, 5), you get $5 = 3(2) + b$ or $5 = 6 + b$, so $b = -1$. Therefore, the equation for line l is $y = 3x - 1$. For line m, the slope is given as –2, and the x-intercept is 2. Be very careful not to jump to the conclusion that the equation of line m is $y = -2x + 2$. In the form $y = mx + b$, the b is the y-intercept, not the

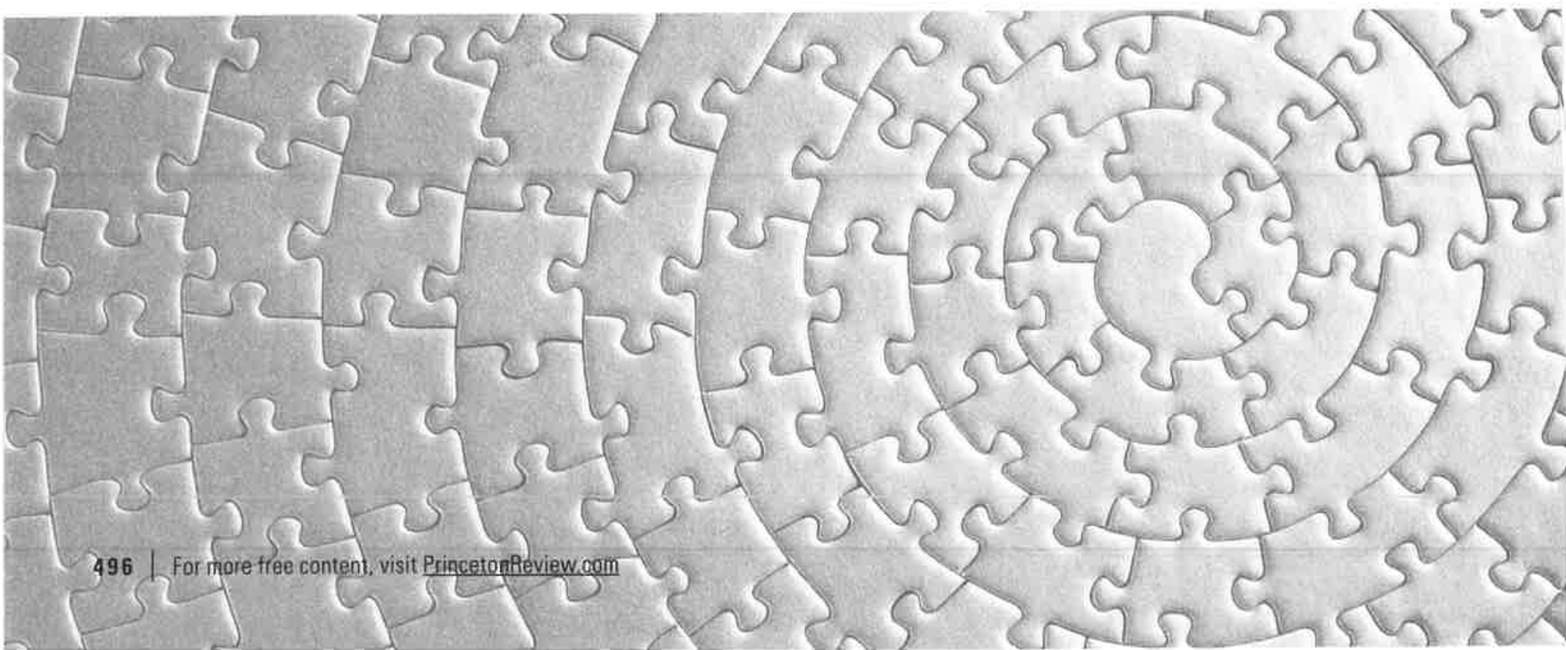
x-intercept. The x-intercept is where $y = 0$, so you know that (2, 0) is a point on line m. Use this point and the slope to find the equation of line m in the same way you did for line l: $0 = -2(2) + b$, so $b = 4$ and the equation is $y = -2x + 4$. Now set the x parts of the equations equal to find the point of intersection. If $3x - 1 = -2x + 4$, then $5x = 5$ and $x = 1$. Again, be careful! The question asked for the value of y. Plug $x = 1$ into one of the line equations to find y. For line l, the equation becomes $y = 3(1) - 1 = 3 - 1 = 2$. The correct answer is (D).

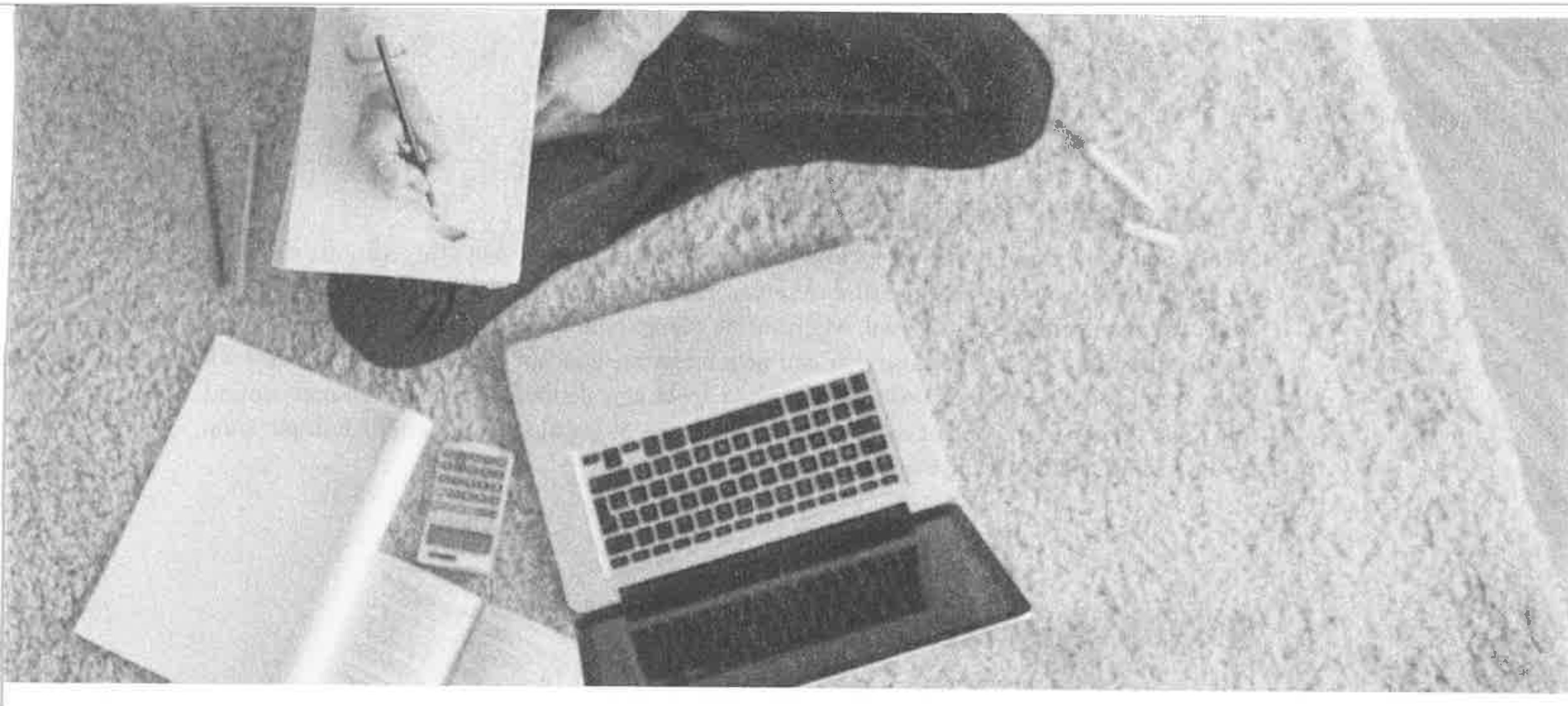
10. **A** The question asks about the graph of the data representing a certain situation. Start by reading the final question, which asks for the meaning of the vertex of the graph. The function is a quadratic in vertex form, $y = a(x - h)^2 + k$, in which the vertex is at (h, k). Since $a = -0.05$, the parabola opens downward, and the vertex of the parabola is at (55.5, 154). Entering the equation into a graphing calculator is a good way to visualize all of this information. Next, label parts of the equation given. The question states that y is the height above the ground, in feet, and x is the number of seconds after the ride starts. The greatest height is at the vertex, and the y-value of the vertex is 154, so it is the maximum height. The correct answer is (A).

Summary

- Given a function, you put an x-value in and get an $f(x)$ or y-value out.
- Look for ways to use Plugging In and PITA on function questions.
- For questions about the graphs of functions, remember that $f(x) = y$.
- If the graph contains a labeled point or the question gives you a point, plug it into the equations in the answers and eliminate any that aren't true.
- The equation of a line can take two forms. In either form, (x, y) is a point on the line.
  - In slope-intercept form, $y = mx + b$, the slope is m and the y-intercept is b.
  - In standard form, $Ax + By = C$, the slope is $-\frac{A}{B}$ and the y-intercept is $\frac{C}{B}$.
- Given two points on a line, (x_1, y_1) and (x_2, y_2), the slope is $\frac{(y_2 - y_1)}{(x_2 - x_1)}$.
- Two linear equations with infinitely many solutions represent the same line.
- Parallel lines have the same slopes and no points of intersection.
- Perpendicular lines have slopes that are negative reciprocals and intersect at a right angle.
- To find a point of intersection, plug the point into both equations to see if it works, or graph the lines on your calculator when it is allowed.
- The roots of a function, also known as solutions, zeroes, or x-intercepts, are the points where the graph crosses the x-axis and where $y = 0$.
- Graphs of functions can be moved up or down if a number is added to or subtracted from the function, respectively. They move left if a number is added inside the parentheses of the function or move right if a number is subtracted inside the parentheses.

- The standard form of a parabola is $y = ax^2 + bx + c$, where c is the y-intercept. If a is positive, the parabola opens up, and if a is negative, it opens down.
- The factored form of a parabola equation is $y = a(x - r_1)(x - r_2)$, where r_1 and r_2 are the roots or x-intercepts of the parabola.
- The vertex form of a parabola equation is $y = a(x - h)^2 + k$, where (h, k) is the vertex. To get a parabola in the standard form into vertex form, complete the square.
- The standard form of a circle equation is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius. To get a circle equation into the standard form, complete the square for both the x-terms and the y-terms.
- A graphing calculator can make an enormous difference on questions about graphs. Practice with the calculator you plan to use on test day—either the built-in calculator or your own—to learn all of the tricks and feel comfortable with using it during the test.





Chapter 24
Advanced Arithmetic

The Digital SAT includes questions about what the test-writers call Problem Solving and Data Analysis. Many of these questions test concepts you learned a few years ago, such as averages and proportions. More difficult questions will build on these basic concepts by requiring you to use charts and data or to combine multiple techniques. In this chapter, we will review the arithmetic and statistical concepts you'll need to know for the Digital SAT.

CALL ON THE CALCULATOR

While you work through the topics and examples in this chapter, don't forget to use the built-in calculator or your own. Make sure to review the Calculator Guide in your Student Tools so you can take advantage of this tool. Many of the examples in this chapter will show you how to do the question by hand because that can help make the concept clearer, but a calculator will help you do some questions very quickly while avoiding calculation errors. Even scary-sounding topics like margin of error come down to the basics of addition, subtraction, multiplication, and division, and that's what calculators do best!

RATIOS AND PROPORTIONS

A Ratio Is a Comparison

Many students get extremely nervous when they are asked to work with ratios. But there's no need to be nervous. A **ratio** is a comparison between the quantities of ingredients you have in a mixture, be it a class full of people or a bowl of cake batter. Ratios can be written to look like fractions—don't get them confused.

The ratio of x to y can be expressed in the following three ways:

1. $\frac{x}{y}$
2. the ratio of x to y
3. $x:y$

Ratios vs. Fractions
Keep in mind that a ratio compares part of something to another part. A fraction compares part of something to the whole thing.

Ratio: $\frac{\text{part}}{\text{part}}$

Fraction: $\frac{\text{part}}{\text{whole}}$

Part, Part, Whole

Ratios are a lot like fractions. In fact, anything you can do to a fraction (convert it to a decimal or percentage, reduce it, and so on), you can do to a ratio. The difference is that a fraction gives you a part (the top number) over a whole (the bottom number), while a ratio typically gives you two parts (boys to girls, cars to trucks, sugar to flour), and it is your job to come up with the whole. For example, if there is one cup of sugar for every two cups of flour in a recipe, that's three cups of stuff. The ratio of sugar to flour is 1:2. Add the parts to get the whole.

Ratio to Real

If a class contains 3 students and the ratio of boys to girls in that class is 2:1, how many boys and how many girls are there in the class? Of course, there are 2 boys and 1 girl.

Now, suppose a class contains 24 students and the ratio of boys to girls is still 2:1. How many boys and how many girls are there in the class? This is a little harder, but the answer is easy to find if you think about it. There are 16 boys and 8 girls.

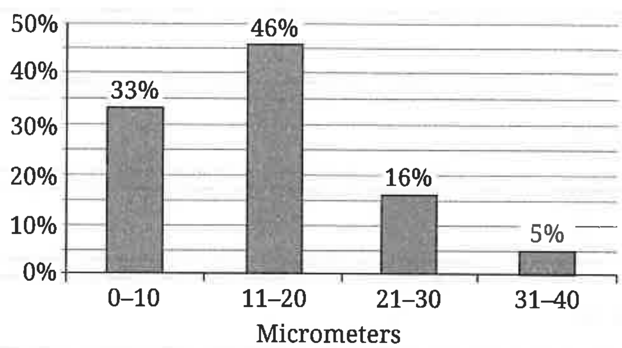
How did we get the answer? We added up the number of "parts" in the ratio (2 parts boys plus 1 part girls, or 3 parts all together) and divided it into the total number of students. In other words, we divided 24 by 3. This told us that the class contained 3 equal parts of 8 students each. From the given ratio (2:1), we knew that two of these parts consisted of boys and one of them consisted of girls.

The test-writers will often combine ratios with diagrams or data from charts and graphs. Don't let them intimidate you with these: just work in bite-sized pieces and write down the part-to-part relationships that you need to solve the question.

Try this example.

1 Mark for Review

A lapping slurry contains microbeads suspended in a solution and is used to polish a silicon wafer by abrasion of the surface. The distribution of the particle size, in micrometers, is shown below.



Which of the following is closest to the ratio of the number of 11–20 micrometer microbeads to the number of 31–40 micrometer microbeads?

Ⓐ 1:9

Ⓑ 2:1

Ⓒ 3:1

Ⓓ 9:1

Here's How to Crack It

The question asks for a ratio based on data from a graph. To find the ratio of 11–20 micrometer microbeads to 31–40 micrometer microbeads, read carefully, look up the right information in the graph, and set up the part-to-part relationship. The 11–20 micrometer microbeads make up 46% of the total, and the 31–40 micrometer microbeads make up 5% of the total. Plug in 100 for the total to get actual numbers of 46 and 5, respectively. That is a ratio of 46:5. Because the question is asking for the closest ratio, round the numbers to get a ratio of 45:5, which reduces to 9:1. Notice that (A) has the right numbers in the wrong order; always read carefully and use your scratch paper to avoid this mistake. The correct answer is (D).

Proportions Are Equal Ratios

Some Digital SAT math questions will contain two proportional, or equal, ratios from which one piece of information is missing.

Here's an example.

2 Mark for Review

If 2 packages contain a total of 12 doughnuts, how many doughnuts are there in 5 packages?

Ⓐ 24

Ⓑ 30

Ⓒ 36

Ⓓ 60

Here's How to Crack It

The question asks for the number of doughnuts in 5 packages. This question simply describes two equal ratios, one of which is missing a single piece of information. Here's the given information represented as two equal ratios:

$$\frac{2 \text{ (packages)}}{12 \text{ (doughnuts)}} = \frac{5 \text{ (packages)}}{x \text{ (doughnuts)}}$$

Because ratios can be written so they look like fractions, you can treat them exactly like fractions. To find the answer, all you have to do is solve for x. Start by cross-multiplying:

$$\frac{2}{12} \times \frac{5}{x}$$

$$\text{so, } 2x = 60$$

$$x = 30$$

The correct answer is (B).

Proportions: Advanced Principles

Many proportion questions will also involve unit conversion. Be sure to pay attention to the units and have the same units in both numerators and the same units in both denominators.

Let's look at an example.

3 Mark for Review

Gary is using a 3D printer to create a miniature version of himself. The scale of the miniature is 0.4 inches to 1 foot of Gary's actual height. If Gary is 5 feet and 9 inches tall, what will be the height of his 3D-printed miniature?
(12 inches = 1 foot)

Ⓐ 2.0 inches

Ⓑ 2.3 inches

Ⓒ 2.6 inches

Ⓓ 2.9 inches

Here's How to Crack It

The question asks for the height of the 3D miniature. The scale of the 3D printer is in inches and feet—0.4 inches on the miniature for every 1 foot in real life. Start by converting every measurement to inches. There are 12 inches in each foot, so the scale will be 0.4 inches = 12 inches in real life. Now convert Gary's height into inches. Begin by setting up a proportion to find out how many inches are in 5 feet.

$$\frac{12 \text{ inches}}{1 \text{ foot}} = \frac{x \text{ inches}}{5 \text{ feet}}$$

Cross-multiply to find that 5 feet equals 60 inches. Gary is 5 feet and 9 inches tall, so he is 60 + 9 = 69 inches tall. Now set up a proportion with the scale of the miniature and Gary's height in inches.

$$\frac{0.4 \text{ inches}}{12 \text{ inches}} = \frac{x \text{ inches}}{69 \text{ inches}}$$

Cross-multiply to get $12x = 27.6$, and then divide both sides by 12 to find that $x = 2.3$ inches. The correct answer is (B).

Now try a question that requires converting the units of both parts of a rate.

4 Mark for Review

A car is traveling on the highway at a speed of 65 miles per hour. Which of the following is the best approximation of the car's speed, in feet per second? (1 mile = 5,280 feet)

Ⓐ 95

Ⓑ 190

Ⓒ 5,720

Ⓓ 343,200

Here's How to Crack It

The question asks for a rate in different units. Take it one piece at a time to avoid getting confused. Start by converting miles to feet. The question states that *1 mile = 5,280 feet*, so set up a proportion to determine how many feet are in 65 miles, being sure to match up units. The proportion is $\frac{1\text{ mile}}{5{,}280\text{ feet}} = \frac{65\text{ miles}}{x\text{ feet}}$. Cross-multiply to get (5,280)(65) = x, or x = 343,200 feet per hour. This is answer choice (D), but you're only halfway done with the conversion. Eliminate trap answer (D).

Next, convert hours to seconds. There are 60 minutes in 1 hour and 60 seconds in 1 minute, so there are (60)(60) = 3,600 seconds in 1 hour. Thus, a rate of $\frac{343{,}200\text{ feet}}{1\text{ hour}}$ is the same as a rate of $\frac{343{,}200\text{ feet}}{3{,}600\text{ seconds}}$. Reduce the fraction to get a rate of $95.\overline{3}$ feet per second. The question asks for the *best approximation of the car's speed*, and the closest answer is 95. The correct answer is (A).

PERCENTAGES

Percentages Are Fractions

There should be nothing frightening about a percentage. It's just a convenient way of expressing a fraction with a denominator of 100.

Percent means "per 100" or "out of 100." If there are 100 questions on your math test and you answer 50 of them, you will have answered 50 out of 100, or $\frac{50}{100}$, or 50 percent. To think of it another way:

$$\frac{\text{part}}{\text{whole}} = \frac{x}{100} = x\text{ percent}$$

Memorize These Percentage-Decimal-Fraction Equivalents

These show up all the time, so go ahead and memorize them.

$0.01 = \frac{1}{100} = 1$ percent

$0.1 = \frac{1}{10} = 10$ percent

$0.2 = \frac{1}{5} = 20$ percent

$0.25 = \frac{1}{4} = 25$ percent

$0.5 = \frac{1}{2} = 50$ percent

$0.75 = \frac{3}{4} = 75$ percent

Converting Percentages to Fractions

To convert a percentage to a fraction, simply put the percentage over 100 and reduce:

$$80 \text{ percent} = \frac{80}{100} = \frac{8}{10} = \frac{4}{5}$$

Converting Fractions to Percentages

Because a percentage is just another way to express a fraction, you shouldn't be surprised to see how easy it is to convert a fraction to a percentage. To do so, simply use a calculator to divide the top of the fraction by the bottom of the fraction, and then multiply the result by 100. Here's an example:

Problem: Express $\frac{3}{4}$ as a percentage.

Solution: $\frac{3}{4} = 0.75 \times 100 = 75$ percent.

Converting fractions to percentages is easy with the built-in calculator or your own. In fact, a calculator is going to be the fastest and most accurate way to answer most questions about percentages.

Converting Percentages to Decimals

To convert a percentage to a decimal, simply move the decimal point *two places to the left*. For example, 25 percent can be expressed as the decimal 0.25; 50 percent is the same as 0.50 or 0.5; 100 percent is the same as 1.00 or 1.

Converting Decimals to Percentages

To convert a decimal to a percentage, just do the opposite of what you did in the preceding section. All you have to do is move the decimal point *two places to the right*. Thus, 0.5 = 50 percent; 0.375 = 37.5 percent; 2 = 200 percent.

The following drill will give you practice working with fractions, decimals, and percentages.

Call on the Calculator
Review the first two sections of the Digital SAT Calculator Guide in your Student Tools to remember how to work with fractions, decimals, and percentages using the calculator. If you're planning to use your own calculator, make sure you know how to use it to make these conversions.

FRACTIONS, DECIMALS, AND PERCENTS EXERCISE

Fill in the missing information in the following table. Answers can be found on page 532.

	Fraction	Decimal	Percent
	$\frac{1}{5}$	0.2	20%
1.	$\frac{1}{2}$		
2.		3.0	
3.			0.5%
4.	$\frac{1}{3}$		

Translation, Please!

Word problems can be translated into arithmetic symbols. Learning how to translate from English to math will help you immensely on the Digital SAT Math section. We covered some of this already, but as a review, here are some of the most common terms you will see in word problems and their math symbol equivalents:

Word	Symbol
is, are, costs	=
greater than, more than	+
fewer than, less than	–
of	× (multiply)
percent	÷ 100
what	*n* (variable)

Do You Speak Math?

Problem: What number is 5 more than 10 percent of 20?

Students often make careless errors on questions like this because they aren't sure how to translate the words they are reading into math. You won't make mistakes if you take the words slowly, one at a time, and translate each one into a mathematical symbol. Use the chart above to write this question in math. *What number* means "variable," so you can write that as n (or x or whatever letter works for you). *Is* means "equals," so now you have $n =$. Next you are given the number 5, so write that in your equation and you get $n = 5$. *More than* translates to +, and *10 percent* is $\frac{10}{100}$. That gives you $n = 5 + \frac{10}{100}$. Finally, *of 20* means multiply by 20, so now you have the equation:

$$n = 5 + \frac{10}{100}(20)$$

$$n = 5 + 2$$

$$n = 7$$

You will see the words *of, is, product, sum,* and *what* pop up a lot in math questions on the Digital SAT. Don't let these words fool you because they all translate into simple math functions. Memorize all of these terms and their math equivalents. It will save you time on the test and make your life with the Digital SAT much less unpleasant.

What Percent of What Percent of What?

On more challenging Digital SAT questions, you may be asked to determine the effect of a series of percentage increases or decreases. The key point to remember on such questions is that each successive increase or decrease is performed on the result of the previous one.

Here's an example.

5 Mark for Review

A business paid $300 to rent a piece of office equipment for one year. The rent was then increased by 10% each year thereafter. How much will the company pay for the first three years it rents the equipment?

Ⓐ $920

Ⓑ $960

Ⓒ $990

Ⓓ $993

Bite-Sized Pieces
Always handle percentage problems using Bite-Sized Pieces: one piece at a time.

Here's How to Crack It

The question asks for the cost of the equipment over three years. This question is a great place to use the Bite-Sized Pieces strategy. You know that the business paid $300 to rent the piece of office equipment for the first year. Then, you were told that the rent increases by 10 percent for each year thereafter. That's a sure sign that you're going to need the rent for the second year, so go ahead and calculate it. For the second year, the rent is $300 + \left(\frac{10}{100} \times 300\right) = 330$.

Now, the question tells you that the business rents the equipment for three years. So, you need to do the calculation one more time. At this point, you might want to set up a chart to help keep track of the information.

Year 1: $300

Year 2: $300 + \left(\frac{10}{100} \times 300\right) = \$330$

Year 3: $330 + \left(\frac{10}{100} \times 330\right) = \$363$

To find the answer, all you need to do is add up the costs for each of the three years.

Year 1:	$300
Year 2:	$330
Year 3:	$363
	$993

The correct answer is (D).

What Percent of What Percent of . . . Yikes!

Sometimes you may find successive percentage questions in which you aren't given actual numbers to work with. In such cases, you need to plug in some numbers.

Here's an example.

6 Mark for Review

A number is increased by 25 percent and then decreased by 20 percent. The result is what percent of the original number?

(A) 80

(B) 100

(C) 105

(D) 120

Plugging Away at Relationships
Questions dealing with percents, fractions, and other ways of expressing relationships among numbers are great chances to plug in!

Here's How to Crack It

The question asks for the result of a percent increase and a percent decrease on an original number. You aren't given a particular number to work with in this question—just "a number." Rather than trying to deal with the problem in the abstract, you should immediately plug in a number to work with. What number would be easiest to work with in a percentage question? Why, 100, of course.

1. 25 percent of 100 is 25, so 100 increased by 25 percent is 125.
2. Now you have to decrease 125 by 20 percent; 20 percent of 125 is 25, so 125 decreased by 20 percent is 100.
3. 100 (the result) is 100 percent of 100 (the number you plugged in). The correct answer is (B).

Remember, never try to solve a percentage problem by writing an equation if you can plug in numbers instead. Using Plugging In on percentage questions is faster, easier, and more accurate. Why work through long, arduous equations if you don't have to?

AVERAGES

What Is an Average?

The **average,** also called the **arithmetic mean,** of a set of n numbers is simply the sum of all the numbers divided by n. In other words, if you want to find the average of three numbers, add them up and divide by 3. For example, the average of 3, 7, and 8 is $\frac{(3+7+8)}{3}$, which equals $\frac{18}{3}$, or 6.

That was an easy example, but average questions on the Digital SAT won't always have clear solutions. That is, you won't always be given the information for averages in a way that is easy to work with. For that reason, use the formula $T = AN$, in which T is the *total*, A is the *average*, and N is the *number of things*. The total is the sum of all the numbers you're averaging, and the number of things is the number of elements you're averaging. Plug in the information you've been given, and then you can solve the equation for the quantity that you don't know.

Here's what the formula looks like using the simple average example we just gave you.

$$T = AN$$

$$3 + 7 + 8 = (A)(3)$$

$$18 = (A)(3)$$

$$A = 6$$

Here's another simple example:

Problem: If the mean of three test scores is 70, what is the total of all three test scores?

Solution: Just put the average (70) and the number of things (3 tests) into the formula to get $T = (70)(3)$. Then multiply to find the total, which is 210.

Total
When calculating averages and means, always find the total. It's the one piece of information that the Digital SAT loves to withhold.

Mark It!
Make sure you're using the formula $T = AN$ each time you see the word *mean* or *average* in a question.

Averages: Advanced Principles

To solve most difficult average questions, all you have to do is use the formula more than once. Most of the time you will use it to find the total of the number being averaged. Here's an example.

7 Mark for Review

Maria has taken four chemistry tests and has a mean score of 80. If she scores a 90 on her fifth chemistry test, what is her mean for these five tests?

Ⓐ 80

Ⓑ 81

Ⓒ 82

Ⓓ 84

Here's How to Crack It

The question asks for the mean score Maria received on all 5 tests. Start by writing out $T = AN$ and filling in what you know. You can put 80 in for the average and 4 in for the number of things to get $T = (80)(4)$. You can calculate that Maria has gotten 320 total points on her first four tests.

Now, since the question mentions another average, write the formula again and fill in the new information. This time, there are five tests, making the formula $T = (A)(5)$. The question asks for the average, so you also need to find the total. The total for all five tests is the total from the first four tests plus the score from the fifth test: $320 + 90 = 410$. Put that into the formula to get $410 = (A)(5)$ and divide to find the average: 82. The correct answer is (C).

Averages, and many other arithmetic topics, may be tested using charts and data. To find the numbers to average, look them up on the graphic provided and watch out for mismatched units.

8 Mark for Review

Charge No.	Battery Life
1	1:11
2	1:05
3	0:59
4	0:55
5	0:55
6	0:54
7	0:54

A toy drone is opened and charged to full battery life. The table above shows the duration of the battery life in hours and minutes between charges. What is the mean battery life for the first five charges?

(A) 55 minutes

(B) 58 minutes

(C) 1 hour and 1 minute

(D) 1 hour and 5 minutes

Here's How to Crack It

The question asks for the mean battery life for the first 5 charges. To find the average, add up the battery life values for the first 5 charges and divide by 5. Make sure that you convert the battery charge time for charges 1 and 2 into minutes before calculating: 1:11 = 60 + 11 = 71 minutes, and 1:05 = 60 + 5 = 65 minutes. The average is equal to $\frac{71+65+59+55+55}{5}=\frac{305}{5}=61$ minutes, which is equal to 1 hour and 1 minute. The correct answer is (C).

Call on the Calculator
The built-in calculator can give you an average at the click of a button. Read the section on the scientific calculator in the Digital SAT Calculator Guide in your Student Tools to review how to find that button in the Functions menu and how to use it to work with averages.

Don't forget that you can also plug in when answering average questions.

9 Mark for Review

The mean of a list of 5 numbers is n. When an additional number is added to the list, the mean of all 6 numbers is $n + 3$. Which of the following is the value, in terms of n, of the number added to the list?

Ⓐ $6n + 18$

Ⓑ $5n$

Ⓒ $n + 18$

Ⓓ $n + 6$

Here's How to Crack It

The question asks for the value of a number added to the list. There are variables in the answers, so plug in for the value of n, which is the average. If $n = 20$, then you can use the average formula to find the total of the five numbers on the list.

$$T = AN$$

$$T = (20)(5)$$

$$T = 100$$

A number is added and there is a new average, so it's time to write out the formula again. For this one, you know that there are 6 numbers and that their average is $n + 3 = 20 + 3 = 23$.

$$T = AN$$

$$T = (23)(6)$$

$$T = 138$$

Since the difference in the two totals was caused by the addition of the sixth number, the sixth number must be $138 - 100 = 38$. That's the target, so be sure to circle it. Now check the answer choices. Choice (A) becomes $6(20) + 18 = 120 + 18 = 138$. This does not match the target, so eliminate it. The value with (A) was much too large, so (B) will also be large. Try (C) and (D) next: (C) becomes $20 + 18 = 38$, and (D) becomes $20 + 6 = 26$. Only (C) matches the target, so the correct answer is (C).

On the Digital SAT, you'll also need to know four other statistical topics: ***median, mode, range,*** and ***standard deviation.*** These topics have pretty straightforward definitions. One way the Digital SAT will complicate the issue is by presenting the data in a chart or graph, making it harder to see the numbers you are working with.

WHAT IS A MEDIAN?

The **median** of a list of numbers is the number that is exactly in the middle of the list when the list is arranged from smallest to largest, as on a number line. For example, in the group 3, 6, 6, 6, 6, 7, 8, 9, 10, 10, 11, the median is 7. Five numbers come before 7 in the list, and 5 come after. Remember it this way: median sounds like *middle.*

Missing the Middle?
To find the median of a set containing an even number of items, take the average of the two middle numbers.

Let's see how this idea might be tested.

10 Mark for Review

	Milligrams of Gold				
	1	2	3	4	5
Limestone	0.45	0.58	0.55	0.42	0.41
Granite	0.94	0.87	0.82	0.55	0.73
Gneiss	0.38	0.60	0.37	0.40	0.34

Five samples of each of three different rock types were collected on a hiking trip in Colorado. Each sample was analyzed for its gold content. The milligrams of gold found in each sample are presented in the table above. How much larger is the median of the amount of gold in the granite samples than that of the limestone samples?

Ⓐ 0.00

Ⓑ 0.37

Ⓒ 0.45

Ⓓ 0.55

Here's How to Crack It

The question asks for a comparison of the medians of data for limestone and granite. Start by putting the gold weights for limestone in order to get

$$0.41, 0.42, 0.45, 0.55, 0.58$$

The median for limestone is the middle number: 0.45 mg.

Next, place the gold weights for granite in order to get

$$0.55, 0.73, 0.82, 0.87, 0.94$$

The median for granite is 0.82.

Therefore, the difference between the median amount of gold in the granite and limestone samples is $0.82 - 0.45 = 0.37$. The correct answer is (B).

WHAT IS A MODE?

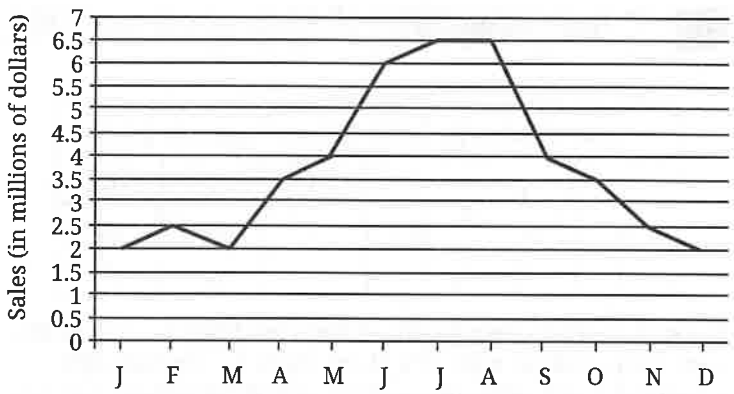
The **mode** of a group of numbers is the number in the list that appears most often. In the list 3, 4, 4, 5, 7, 7, 8, 8, 8, 9, 10, the mode is 8, because it appears three times while no other number in the group appears more than twice. Remember it this way: *mode* sounds like "most."

Because mode by itself is pretty easy to figure out, questions will usually combine mode with other concepts like mean, median, or range.

WHAT IS A RANGE?

The **range** of a list of numbers is the difference between the greatest number in the list and the least number in the list. For the list 4, 5, 5, 6, 7, 8, 9, 10, 20, the greatest number is 20 and the least is 4, so the range is $20 - 4 = 16$.

11 Mark for Review



The forecasted monthly sales of a type of sunscreen are presented in the figure above. Which of the following best describes the range of monthly sales, in millions of dollars, throughout the year shown?

Ⓐ 2.5

Ⓑ 3.5

Ⓒ 4.0

Ⓓ 4.5

Here's How to Crack It

The question asks for the range in monthly sales based on the graph. The range of a set of values is the difference between the greatest and the smallest value. The lowest monthly sales number for the sunscreen can be found where the line dips closest to the bottom of the graph. This happens in both January and March, when the forecasted sales are 2 million. Make sure to read the units carefully. The highest point is where the line goes closest to the top of the graph. This happens in July and August, when the forecasted monthly sales are 6.5 million. Therefore, the range is 6.5 million – 2 million = 4.5 million. The correct answer is (D).

The Digital SAT might even have a question that tests more than one of these statistical concepts at the same time. Take it one step at a time and use POE when you can.

12 Mark for Review

Precious Metals in Catalytic Converters, in grams					
1	2	2	3	4	6
6	6	9	9	10	10
11	13	14	14	15	17

The grams of precious metals in recycled catalytic converters were measured for a variety of automobiles. The data are presented in the table above. If the lowest data point, 1 gram, and highest data point, 17 grams, are removed from the set, which of the following quantities would change the most?

Ⓐ Mode

Ⓑ Mean

Ⓒ Range

Ⓓ Median

Here's How to Crack It

The question asks for the measure of the data that will change the most if two specific data points are removed. Start by evaluating the easier answer choices and save mean for last. The mode of the current list is 6, and removing 1 and 17 from the list won't change that. Eliminate (A). The range is the difference between the smallest number and the largest number on the list. Right now, the range is $17 - 1 = 16$. If those extremes are removed from the list, the new range is $15 - 2 = 13$, and the range changed by 3 units. Keep (C) for now. The median is the middle number in the list, or the average of the middle two numbers. Currently, both middle numbers are 9, so the median is 9. This won't change if 1 and 17 are removed, so eliminate (D). The mean of a list is not likely to change dramatically with the removal of the numbers at the extremes, so (C) is likely correct. To actually evaluate the mean, you need to add up all the numbers on the list and divide by the number of items in the list. For the current list, the total is 152 for the 18 items, so the average is $8.\overline{44}$. To find the new total if 1 and 17 are removed, don't re-add everything; just subtract 18 from the previous total. The new list will have only 16 items, so the new average is 8.375. This is only slightly different than the previous mean, so eliminate (B). The correct answer is (C).

WHAT IS A MARGIN OF ERROR?

Some questions will test **margin of error**, which is a range of percentages rather than numbers. A margin of error gives a range for random sampling errors in a survey or poll. It indicates how much the results might change if the poll were repeated or if the entire population were asked instead of a random sample. For example, if a survey shows that 70% of randomly sampled test-takers prefer the Digital SAT to the paper-and-pencil SAT, and there is a margin of error of ±5%, that means it is highly likely that between 65% and 75% of all test-takers prefer the Digital SAT.

Let's look at an example of how this can be tested.

13 Mark for Review

A summer beach volleyball league has 750 players in it. At the start of the season, 150 of the players are randomly chosen and polled on whether games will be played while it is raining or if the games should be canceled. The results of the poll show that 42 of the polled players would prefer to play in the rain. The margin of error on the poll is ±4%. What is the range of players in the entire league that would be expected to prefer to play volleyball in the rain rather than cancel the game?

Ⓐ 24–32

Ⓑ 39–48

Ⓒ 150–195

Ⓓ 180–240

Here's How to Crack It

The question asks for the range of players that would prefer to play in the rain. The first step is to determine the percent of polled players that wanted to play in the rain.

$$\frac{42}{150} = 0.28 \text{ or } 28\%$$

Now apply this percent to the entire population of the league. Since 28% of the polled players wanted to play in the rain, 28% of all players should want to play in the rain.

$$\frac{28}{100} \times 750 = 210$$

The only range that contains this value is (D), so that is the correct answer. To actually calculate the margin of error, add and subtract 4% to the actual percent of 28% to get a range of 24–32% of the total.

$$24\% \text{ of } 750 = 180$$

$$32\% \text{ of } 750 = 240$$

Therefore, the entire range is 180 to 240. The correct answer is (D).

WHAT IS A FREQUENCY TABLE?

Another way the Digital SAT tests statistical concepts is by using a **frequency table**. This is just what it sounds like: a table to show how frequently something happens. One column shows the numbers of something, like ages or scores on a test, and the other column shows how often that thing occurs. If you wanted to show, for example, how many meetings each member of a 15-person club attended, you could use the table below.

Meeting Attendance for this Cool Club I'm In

Meetings attended	Frequency
1	2
2	1
3	1
4	3
5	5
6	3

Frequency tables give you ways to find the mean, median, and mode of a list of numbers without needing to write out the whole list, so the test-writers will often combine those concepts with frequency tables in the same question.

Try out an example of this below.

14 Mark for Review

Number of Ice Cream Scoops for Customers at an Ice Cream Parlor

Number of Ice Cream Scoops	Frequency
8	1
5	2
4	2
3	7
2	6
1	6

The distribution of ice cream scoops for 24 customers at an ice cream parlor is displayed in the table above. Which of the following orders the median, mode, and range correctly?

Ⓐ mode < median < range

Ⓑ median < mode < range

Ⓒ median < range < mode

Ⓓ range < median < mode

Here's How to Crack It

The question asks for the correct order of the median, mode, and range of the data from least to greatest. Start with the mode, which is easy to determine from the table. Since the greatest number in the Frequency column is 7, the corresponding number of scoops is the mode. Thus, the mode is 3.

The range is also easy to determine from the table because the range of a list of numbers is the difference between the greatest number and the least number. The greatest number of ice cream scoops is 8 and the least number of ice cream scoops is 1, so the range is $8 - 1 = 7$. The mode of 3 is less than the range of 7. Eliminate (C) and (D) because they have the range as less than the mode.

Next, find the median, which is the middle number of an ordered list. You could write out all 24 numbers in this list and count up to the middle number(s), but there's a better way. When a list has an even number of terms, the median is the average of the two middle terms. Since $\frac{24}{2} = 12$, the median is the average of the 12th and 13th terms. To confirm this, notice that there are 12 – 1 = 11 terms to the left of the 12th term and 24 – 13 = 11 terms to the right of the 13th term. Now, use the frequency table to find the 12th and 13th terms. The table shows that 6 customers chose 1 scoop, and 6 customers chose 2 scoops, so the 12th term is 2. The next 7 customers chose 3 scoops, so the 13th term is 3. The average of 2 and 3 is $\frac{2+3}{2} = 2.5$. Thus, the median is 2.5. This is less than 3, so the median is less than the mode. Eliminate (A) because it has the mode as less than the median. The correct answer is (B).

WHAT IS STANDARD DEVIATION?

In real-world applications, **standard deviation** is a measure of how numbers are distributed around the mean, and the calculations can get complicated. But Digital SAT math is not the real world! Think of standard deviation as similar to range in that it shows the spread of a group of numbers. When the numbers are more spread out around the mean, the standard deviation is greater. When the numbers are clumped closer together around the mean, the standard deviation is smaller.

Take a look at the example on the next page of how a Digital SAT question might combine standard deviation with another statistical concept you already know.

15 Mark for Review

Data Set 1	Data Set 2
6	4
4	3
3	7
8	8
4	2
6	5
5	6

The table above shows two sets of data. Of the following statements comparing Data Set 1 to Data Set 2, which is true?

(A) The standard deviations are the same, and the medians are the same.

(B) The standard deviations are the same, and the medians are different.

(C) The standard deviations are different, and the medians are the same.

(D) The standard deviations are different, and the medians are different.

Here's How to Crack It

The question asks which statement is true about the medians and standard deviations of two data sets. Start by finding the median of each data set, recalling that the median of a group of numbers is the middle number when all values are arranged in order. Start by putting the lists in order. Data Set 1 is 3, 4, 4, 5, 6, 6, 8, and Data Set 2 is 2, 3, 4, 5, 6, 7, 8. Both Data Set 1 and Data Set 2 have 7 numbers, so the median will be the fourth number. The median of Data Set 1 is 5, and the median of Data Set 2 is 5. The medians are the same, so eliminate (B) and (D).

Standard deviation is a measure of the spread of a group of numbers. In Data Set 1, the numbers are clustered toward the middle. In Data Set 2, each number appears once and the numbers are evenly spread throughout the list. Thus, Data Set 1 has a smaller standard deviation than does Data Set 2. Eliminate (A), which says the standard deviations are the same. The correct answer is (C).

PROBABILITY

Probability is a mathematical expression of the likelihood of an event. The basis of probability is simple. The likelihood of any event is discussed in terms of all of the possible outcomes. To express the probability of a given event, *x*, you would count the number of possible outcomes, count the number of outcomes that give you what you want, and arrange them in a fraction, like this:

$$\text{Probability of } x = \frac{\text{number of outcomes that give you what you want}}{\text{total number of possible outcomes}}$$

Every probability is a fraction. The largest a probability can be is 1; a probability of 1 indicates total certainty. The smallest a probability can be is 0, meaning that it's something that cannot happen. Furthermore, you can find the probability that something WILL NOT happen by subtracting the probability that it WILL happen from 1. For example, if the meteorologist tells you that there is a 0.3 probability of rain today, then there must be a 0.7 probability that it won't rain, because $1 - 0.3 = 0.7$. Figuring out the probability of any single event is usually simple. When you flip a coin, for example, there are only two possible outcomes, heads and tails; the probability of getting heads is therefore 1 out of 2, or $\frac{1}{2}$. When you roll a die, there are six possible outcomes, 1 through 6; the odds of getting a 4 are therefore $\frac{1}{6}$. The odds of getting an even result when rolling a die are $\frac{1}{2}$ because there are 3 even results in 6 possible outcomes.

Here's an example of a probability question.

16 Mark for Review

A bag contains 7 blue marbles and 14 marbles that are not blue. If one marble is drawn at random from the bag, what is the probability that the marble is blue?

Ⓐ $\frac{1}{3}$

Ⓑ $\frac{1}{2}$

Ⓒ $\frac{2}{3}$

Ⓓ $\frac{3}{7}$

Here's How to Crack It

The question asks for the probability that a selected marble is blue. To make the probability, find the number of blue marbles and the total number of marbles. Here, there are 21 marbles in the bag, 7 of which are blue. The probability that a marble chosen at random would be blue is therefore $\frac{7}{21}$, or $\frac{1}{3}$. The correct answer is (A).

Let's look at a probability question based on a chart. Again, getting to the correct answer involves reading the chart carefully to find the right numbers to use.

17 Mark for Review

The table below shows the distribution of 300 high school students signing up for elective classes.

	Culinary Arts	World Literature	Computer Coding	Total
Freshmen	18	22	38	78
Sophomores	24	21	15	60
Juniors	30	9	18	57
Seniors	37	38	30	105
Total	109	90	101	300

If one of the students is randomly selected, what is the probability of selecting a student who is a junior taking culinary arts? (Express your answer as a decimal or fraction, not as a percent.)

See Chapter 26 for more about fill-in questions.

Here's How to Crack It

The question asks for a probability based on data in a table. Probability is defined as $\frac{\text{\# of outcomes that give you what you want}}{\text{total \# of possible outcomes}}$. Read the table carefully to find the numbers to make the probability. There are 300 total students, so that is the *total # of possible outcomes.* Of these 300 students, 30 are juniors taking culinary arts, so that is the *# of outcomes that give you what you want.* Therefore, the probability that a student chosen at random is a junior taking culinary arts is $\frac{30}{300}$. This answer cannot be entered into the fill-in box, which only accepts 5 characters when the answer is positive. All equivalent answers that fit will be accepted, so reduce the fraction or convert it to a decimal. The correct answer is $\frac{3}{30}$, $\frac{1}{10}$, 0.1, or another equivalent form.

RATES

Rate is a concept related to averages. Cars travel at an average speed. Work gets done at an average rate. Because the ideas are similar, the formulas you can use for rate problems are similar to the one for averages you learned about earlier. These formulas are $D = RT$ for distance and $W = RT$ for work.

Here's a simple example:

Problem: If a fisherman can tie 9 flies for fly fishing in an hour and a half, how long does it take him to tie one fly, in minutes?

Solution: First, convert the hour and a half to 90 minutes so your units are consistent. Then, fill in the formula with the work or amount done (9 flies) and the time (90 minutes).

$$W = RT$$

$$9 = (R)(90)$$

Divide 9 by 90 to get $R = \frac{1}{10}$, so the rate is one fly every 10 minutes.

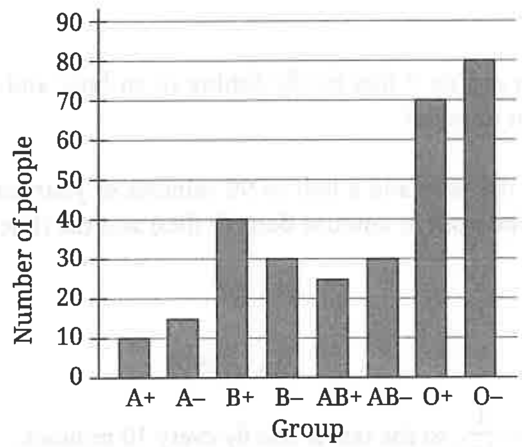
You have already seen that rate questions often involve proportions or unit conversion. As always, write down everything and label it carefully to avoid performing the wrong arithmetic operations.

IS THERE SCIENCE ON THE DIGITAL SAT?

No, there isn't, but the test-writers like to make it look like there is. Some questions will use scientific topics and other "real-world" contexts to make things look more complicated. Don't worry about memorizing data about rock types or knowing what a lapping slurry is; just focus on the math.

Take a look at this next example.

18 Mark for Review



A study about changes in blood type phenotypes after bone marrow transplants recorded the blood types of 300 people before they underwent a bone marrow transplant. How many more people in the study had blood type B– than blood type A+?

Here's How to Crack It

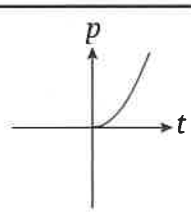
The question asks for the difference between two values based on a graph. Read the graph carefully and look up the necessary values. Find the bar on the graph for blood type B–, then look at the vertical axis to find the number of people with that blood type. The bar goes up to the line at 30, so 30 people have blood type B–. Do the same thing for A+ to see that 10 people have blood type A+. Translate *how many more* as subtraction to get 30 – 10 = 20. The correct answer is 20.

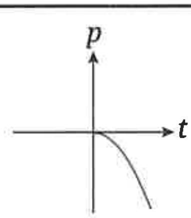
You may also be asked to graph the data presented in a table. Your knowledge of positive and negative relationships will help—you can eliminate things with the wrong relationship.

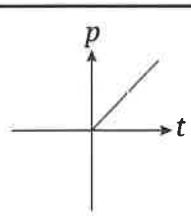
19 Mark for Review

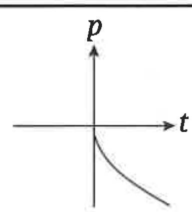
Temperature in °C (t)	Vapor Pressure in kPa (p)
10	4
20	9
30	37
40	66
50	100

A student conducting experiments in class noticed that the temperature of a given liquid affected the vapor pressure of the liquid, as shown in the table above. Which of the following graphs best represents the relationship between the temperature, t, and the vapor pressure, p, as indicated by the table?

Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

Here's How to Crack It

The question asks for the graph that shows the relationship between temperature and vapor pressure. Notice that the vapor pressure increases as the temperature increases. The line or curve of best fit will go up as you follow the graph from left to right, so eliminate (B) and (D). To determine if the correct graph is (A) or (C), try roughly plotting the data points, and then look at your graph. Notice that the vapor pressure does not increase by the same number for each 10-degree temperature increase. This is an exponential increase, not a linear increase. Therefore, the graph will be curved. Eliminate (C). The correct answer is (A).

Advanced Arithmetic Drill

Work these questions using the advanced arithmetic techniques covered in this chapter. Answers and explanations can be found starting on page 532.

1 Mark for Review

The volume of a water tank decreased by a total of 144 liters over the course of 9 days. What was the rate, in liters per day, at which the volume decreased?

(A) 16

(B) 135

(C) 153

(D) 1,296

2 Mark for Review

Number	Frequency
1	3
2	1
3	2
4	4

Which of the following lists of data is represented by the frequency table shown above?

(A) 1, 1, 2, 2, 2, 3, 4, 4, 4, 4

(B) 1, 1, 1, 2, 3, 3, 4, 4, 4, 4

(C) 1, 2, 2, 3, 3, 3, 4, 4, 4, 4

(D) 2, 3, 6, 16

3 Mark for Review

Steve ran a 12-mile race at an average speed of 8 miles per hour. If Adam ran the same race at an average speed of 6 miles per hour, how many minutes longer did Adam take to complete the race than did Steve?

(A) 12

(B) 16

(C) 24

(D) 30

4 Mark for Review

The amount of time that Amy walks is directly proportional to the distance that she walks. If she walks a distance of 2.5 miles in 50 minutes, how many miles will she walk in 2 hours?

(A) 4.5

(B) 5

(C) 6

(D) 6.5

5 Mark for Review

A total of 140,000 votes was cast for two candidates, Candidate A and Candidate B. If Candidate A won by a ratio of 4 to 3, how many votes were cast for Candidate B?

Ⓐ 30,000

Ⓑ 40,000

Ⓒ 60,000

Ⓓ 80,000

6 Mark for Review

A random sample of students at a local school was surveyed to determine the proportion of students with brown hair. Out of the 200 students sampled, 70 had brown hair. The margin of error of the survey was 6%. Which of the following is the most reasonable conclusion about the percentage of students with brown hair?

Ⓐ No less than 29% of the students have brown hair.

Ⓑ Between 29% and 41% of the students have brown hair.

Ⓒ Exactly 35% of the students have brown hair.

Ⓓ No more than 41% of the students have brown hair.

7 Mark for Review

29, 32, 31, 29, 24, 25, 26

The data set shown above contains 7 integer values. If an 8th integer between 20 and 50 is added to the data set, the new mean will be 2 more than the current mean. What is the value of the 8th integer added to the data set?

8 Mark for Review

Of all the houses in a certain neighborhood, 80% have garages. Of those houses with garages, 60% have two-car garages. If there are 56 houses with garages that are not two-car garages, how many houses are there in the neighborhood?

Ⓐ 93

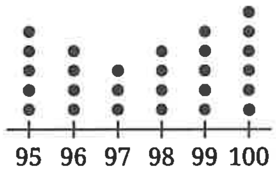
Ⓑ 117

Ⓒ 156

Ⓓ 175

9 Mark for Review

Data Set R



Data set R is represented by the dot plot above. Data set Q is created by subtracting 35 from each of the values in data set R. Which of the following statements correctly compares the standard deviations of data sets R and Q?

Ⓐ There is not enough information to compare these standard deviations.

Ⓑ The standard deviation of data set R is greater than the standard deviation of data set Q.

Ⓒ The standard deviation of data set R is less than the standard deviation of data set Q.

Ⓓ The standard deviation of data set R is equal to the standard deviation of data set Q.

10 Mark for Review

On Tuesday, a watchmaker made 4 more watches than he made on Monday. If he made 16% more watches on Tuesday than on Monday, how many watches did he make on Tuesday?

Ⓐ 20

Ⓑ 21

Ⓒ 25

Ⓓ 29

ANSWERS TO FRACTIONS, DECIMALS, AND PERCENTS EXERCISE

1.	$\frac{1}{2}$	0.5	50%
2.	$\frac{3}{1}$	3.0	300%
3.	$\frac{1}{200}$	0.005	0.5%
4.	$\frac{1}{3}$	$0.333\overline{3}$	$33\frac{1}{3}\%$

ADVANCED ARITHMETIC DRILL ANSWERS AND EXPLANATIONS

1. **A** The question asks for a rate. Begin by reading the question to find information about the rate. The question states that the volume *decreased by a total of 144 liters over the course of 9 days* and asks for the rate of decrease in *liters per day*. Divide the number of liters by the number of days to get $\frac{144 \text{ liters}}{9 \text{ days}} = \frac{16 \text{ liters}}{1 \text{ day}}$. The rate is 16 liters per day. The correct answer is (A).

2. **B** The question asks for the list of numbers that correctly represents a frequency table. A frequency table has two columns: the left-hand column contains the values, and the right-hand column contains the number of times each value occurs, or its frequency. Work in bite-sized pieces and eliminate answer choices that do not match the data. According to the table, the number 1 has a frequency of 3, so the number 1 should be in the list 3 times. Eliminate (A), (C), and (D) because they do not have the number 1 in the list 3 times. Choice (B) shows the correct frequency for each value. The correct answer is (B).

3. **D** The question asks for the difference in minutes between Steve's time and Adam's time. Use the formula $D = RT$ to calculate the time for each runner. Steve runs 12 miles at 8 miles per hour, so the formula becomes $12 = (8)(T)$. To find Steve's time, divide his distance by his rate, which means that he runs for $1\frac{1}{2}$ hours (or 1.5 if you're using your calculator). Adam runs the same 12 miles at 6 miles per hour, so the formula becomes $12 = (6)(T)$. This means that Adam runs for 2 hours. Adam takes one-half hour longer to complete the race, and one-half hour is 30 minutes. The correct answer is (D).

4. **C** The question asks for the distance Amy can walk in two hours if she walks at a given rate. Since you know the time that Amy walked and the distance she walked are directly proportional, you can set up a proportion to show her distance ÷ time. The time it took her to walk 2.5 miles is given in minutes and the requested time is in hours, so match the units in the proportion by putting 120(60 × 2) minutes in the second half of the ratio: $\frac{2.5}{50} = \frac{x}{120}$. To solve, cross-multiply, and you'll get $50x = 2.5 \times 120$; $50x = 300$; $x = 6$ miles. The correct answer is (C).

5. **C** The question asks for the number of votes Candidate B received. You know the ratio for the votes is 4 for Candidate A to every 3 for Candidate B, so the proportion of votes Candidate B received is 3 out of every 7 or $\frac{3}{7}$ of the total votes. There were 140,000 votes all together, so Candidate B received $\frac{3}{7}(140{,}000) = 60{,}000$ votes. The correct answer is (C).

6. **B** The question asks for a reasonable conclusion based on survey results and a margin of error. Work in bite-sized pieces and eliminate after each piece. Start by determining the percent of surveyed students with brown hair by dividing the number of students with brown hair by the total number of students and multiplying by 100: $\frac{70}{200} \times 100 = 35\%$. A margin of error expresses the amount of random sampling error in a survey's results. Eliminate (C) because it is only the result, 35%, not a range above and below the result. The margin of error is 6%, meaning that results within a range of 6% above and 6% below the estimate are reasonable. The study found that 35% of students have brown hair, so the reasonable results for this study are between 29% and 41%. Eliminate (A) because it only addresses the lower limit. Keep (B) because it addresses both the lower limit and the upper limit. Eliminate (D) because it only addresses the upper limit. The correct answer is (B).

7. **44** The question asks for a value given information about the mean of a data set. For averages, use the formula $T = AN$, in which T is the *Total*, A is the *Average*, and N is the *Number of things*. Start by finding the average of the seven integers given in the question. There are 7 values, so $N = 7$. Find the *Total* by adding the seven integers to get $T = 29 + 32 + 31 + 29 + 24 + 25 + 26 = 196$. The average formula becomes $196 = (A)(7)$. Divide both sides of the equation by 7 to get $A = 28$. The question states that, after an eighth integer is added to the data set, ***the new average will be 2 more than the current average***. Since the current average is 28, the new average will be $28 + 2 = 30$. There are now 8 integers in the data set, so plug $N = 8$ and $A = 30$ into the data set to get $T = (30)(8) = 240$. The question asks for the value of the eighth integer, so subtract the *Total* of the 7 integers from the *Total* of the 8 integers to get $240 - 196 = 44$ as the value of the eighth integer. The correct answer is 44.

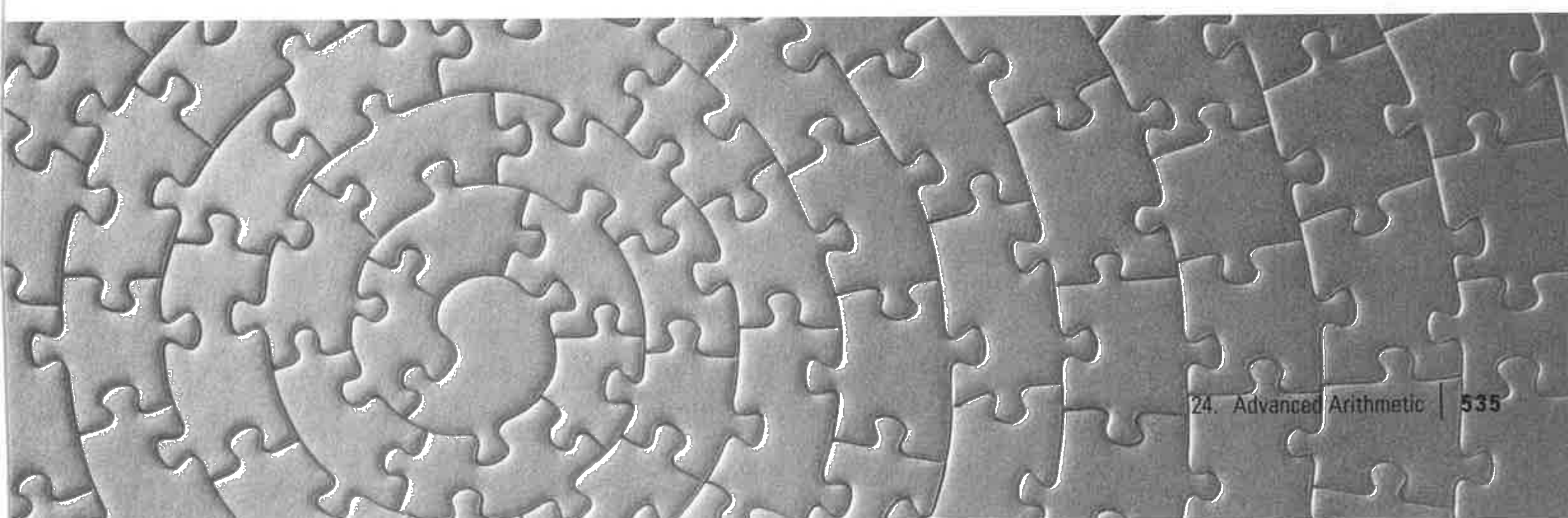
8. **D** The question asks for the number of houses in the neighborhood. Start by figuring out what percent of the houses do not have two-car garages. Since 60% of the houses with garages have two-car garages, 40% of the houses with garages do not have two-car garages. In other words, 40% of 80% of the houses do not have two-car garages. Translate that into math to get $\frac{40}{100} \times \frac{80}{100} = 0.32$, or 32% of the houses. The question states that 56 houses do not have two-car garages, which means 32% of the houses equals 56. Translating into math gives $\frac{32}{100} \times x = 56$. Solve for x, and you'll get 175. Another great option for questions that ask for something specific like number of houses is plugging in the answers. Either way, the correct answer is (D).

9. **D** The question asks for a comparison of the standard deviations of two data sets. Standard deviation is a measure of the spread of a group of numbers. A group of numbers close together has a small standard deviation, whereas a group of numbers spread out has a large standard deviation. Subtracting 35 from each number will change the values in the data set but will not change how they are distributed. The dot plot of data set Q will look the same as the dot plot of data set R except for the values along the bottom of the dot plot. Because the shape of the dot plot remains the same, so does the standard deviation, and the two data sets have the same standard deviation. The correct answer is (D).

10. **D** The question asks for the number of watches a watchmaker made on Tuesday. This is a specific amount, and there are numbers in the answer choices, so plug in the answers. Start with (B). If the watchmaker made 21 watches on Tuesday, then he must have made 17 watches on Monday. You know that he should have made 16% more watches on Tuesday than on Monday, so use the percent change formula $\left(\frac{\text{difference}}{\text{original}} \times 100\right)$ to see if you get 16%: $\frac{4}{17} \times 100 = 23.5\%$, which is too big. Eliminate (B). You want the 4 watches to be a smaller percent of the total, so you need a bigger total. Try a bigger answer choice, like (D). If he made 29 watches on Tuesday, then he made 25 watches on Monday. Now the percent change is $\frac{4}{25} = 0.16 = 16\%$, which is exactly what you want. The correct answer is (D).

Summary

- A ratio can be expressed as a fraction, but ratios are not fractions. A ratio compares parts to parts; a fraction compares a part to the whole.
- Set up proportions in the form $\frac{x_1}{y_1} = \frac{x_2}{y_2}$.
- A percentage is just a convenient way of expressing a fraction with a denominator of 100.
- To convert a percentage to a fraction, put the percentage over 100 and reduce.
- To convert a fraction to a percentage, use your calculator to divide the top of the fraction by the bottom of the fraction. Then multiply the result by 100.
- To convert a percentage to a decimal, move the decimal point two places to the left. To convert a decimal to a percentage, move the decimal point two places to the right.
- In questions that require you to find a series of percentage increases or decreases, remember that each successive increase or decrease is performed on the result of the previous one.
- If you need to find the percent increase or decrease, use % change = $\frac{\text{difference}}{\text{original}} \times 100$.
- To find the mean, or average, of several values, add up the values and divide the total by the number of values.
- Use the formula $T = AN$ to solve questions involving means, or averages. The key to most average questions is finding the total.
- The median of a group of numbers is the number that is exactly in the middle of the group when the group is arranged from smallest to largest, as on a number line. If there is an even number of numbers, the median is the average of the two middle numbers.
- The mode of a group of numbers is the number in the group that appears most often.
- The range of a group of numbers is the difference between the greatest number in the group and the least number.

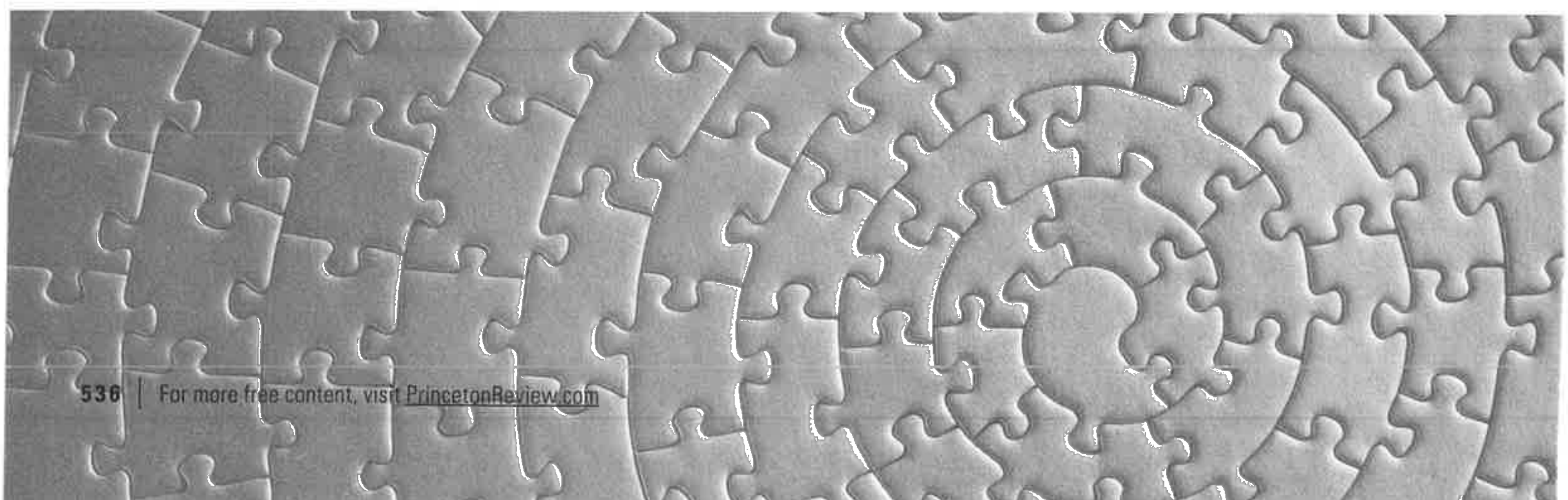


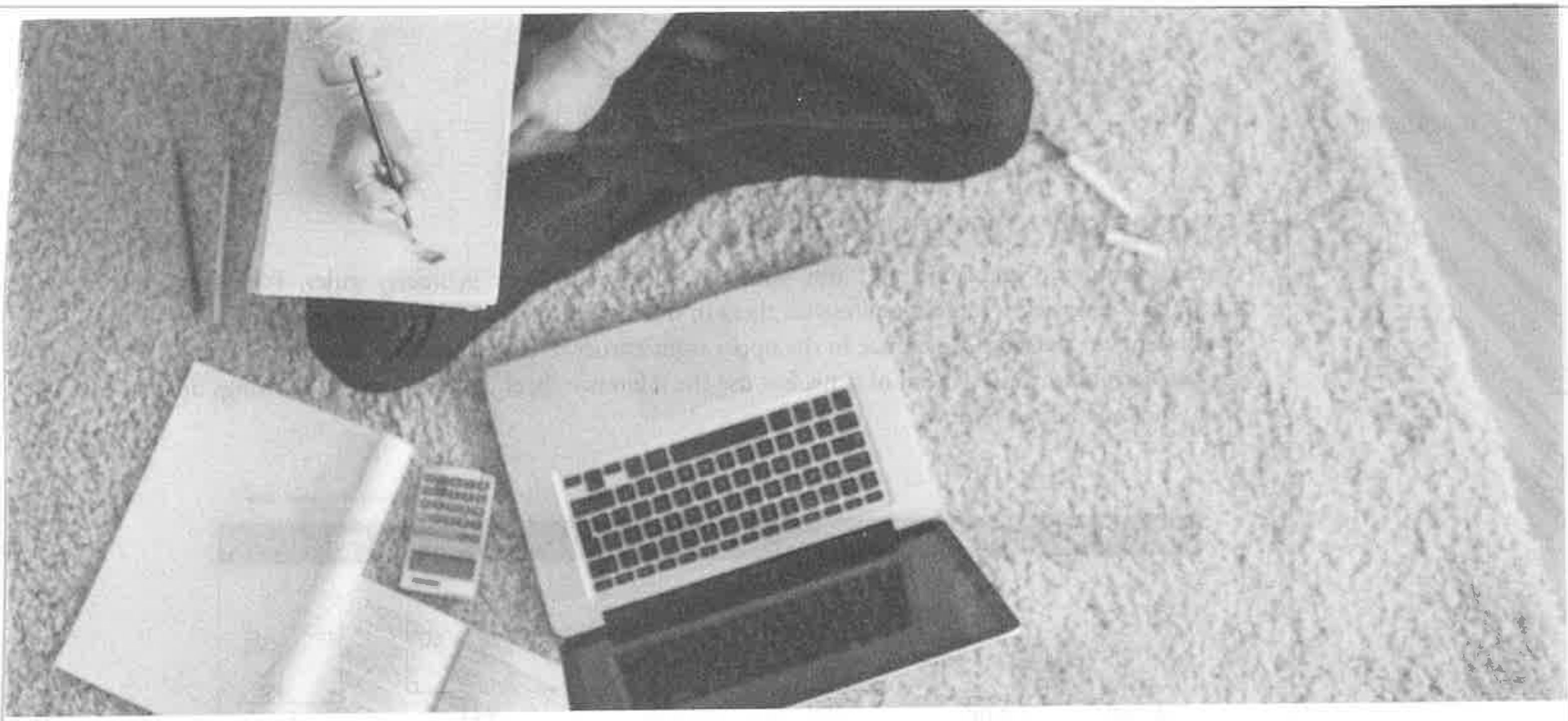
- Standard deviation is a measure of the spread, or distribution, of a group of numbers. More spread out means a large standard deviation, and more clustered together means a small standard deviation.

- Margin of error is the range above and below the value predicted by a survey within which the actual value is likely to be.

- Probability is expressed as a fraction:
    - $$\text{Probability} = \frac{\text{number of outcomes that give you what you want}}{\text{total number of possible outcomes}}$$

- On questions about rates, use the formulas $D = RT$ and $W = RT$. Be careful with the units—the Digital SAT will often require you to do a unit conversion such as minutes to hours or inches to feet.



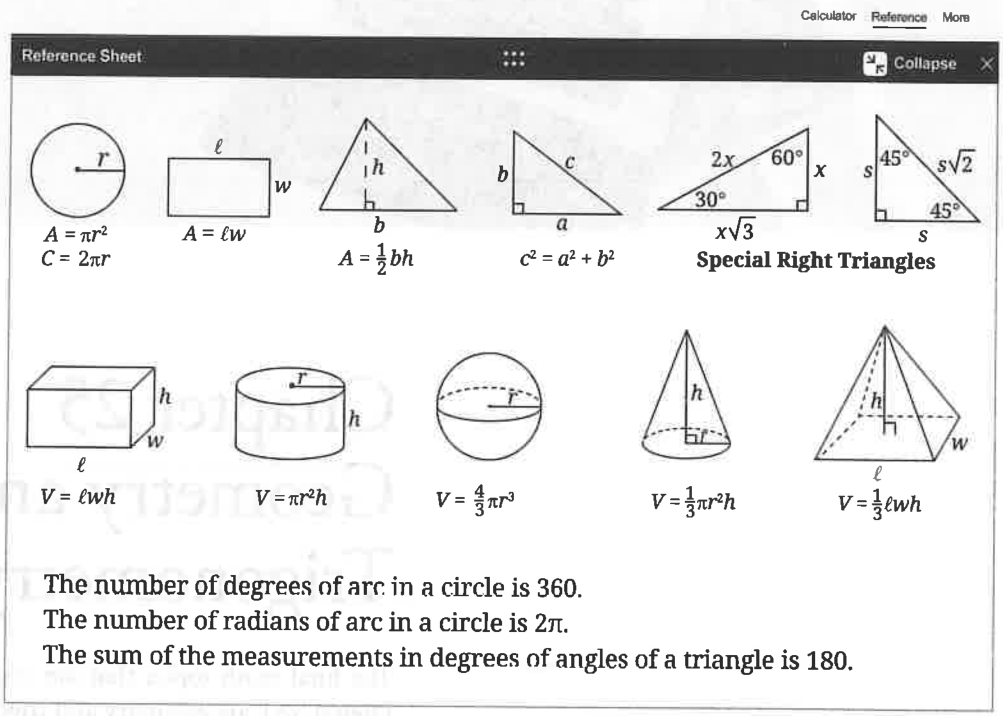


Chapter 25 Geometry and Trigonometry

The final math topics that are tested regularly on the Digital SAT are geometry and trigonometry. There will be 5–7 questions total on these topics, split between the two modules. These questions cover topics such as lines and angles, triangles, circles, and trig functions. This chapter covers each of those topics and more, and provides a step-by-step walkthrough for each type of question.

GEOMETRY ON THE DIGITAL SAT

Several geometry questions will test your knowledge of basic geometry rules, including formulas. Never fear! There is a reference sheet in the testing app that you can open at any time by clicking on the word Reference in the upper right corner of the screen. It saves time to know these facts and formulas ahead of time, but use the reference sheet to check or to look things up that you don't already know.



This reference sheet contains *some* of what you'll need to tackle geometry on the Digital SAT. In this chapter, we'll cover a basic approach for geometry questions and other information you'll need to know to handle geometry and trig questions on the Digital SAT.

Geometry: Basic Approach

For the handful of geometry questions that appear on the Digital SAT, we recommend the following step-by-step approach:

1. **Draw a figure** on your scratch paper. If the question contains the note "Figure not drawn to scale," draw the figure using information in the question.
2. **Label the figure** with any information given in the question. Sometimes you can plug in for parts of the figure as well.
3. **Write down formulas** that you might need for the question.
4. **Ballpark** if you're stuck or running short on time.

These four steps, combined with the techniques you've learned in the rest of this book and the geometry concepts this chapter will cover, will enable you to tackle any geometry question you might run across on the Digital SAT.

Before we dive in to the nitty-gritty, let's try a question using this approach.

1 Mark for Review

In triangle ABC, angle $B = 60°$ and AC is perpendicular to BC. If $AB = x$, what is the area of triangle ABC, in terms of x?

Ⓐ $\frac{x^2\sqrt{3}}{8}$

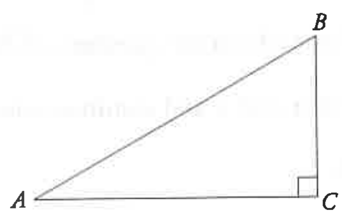
Ⓑ $\frac{x^2\sqrt{3}}{4}$

Ⓒ $\frac{x^2\sqrt{3}}{2}$

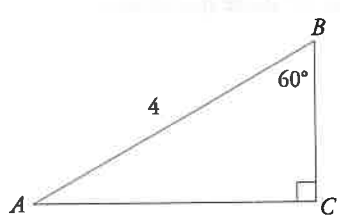
Ⓓ $x^2\sqrt{3}$

Here's How to Crack It

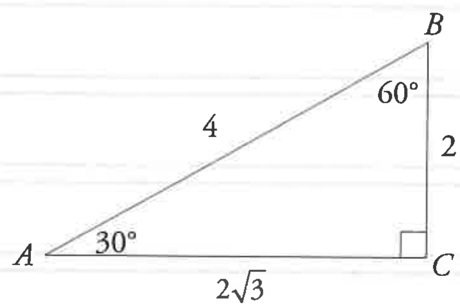
The question asks for the area of the triangle. Follow the steps outlined on the previous page. Start by drawing the figure. If AC is perpendicular to BC, then triangle ABC is a right triangle with the right angle at point C:



The next step is to label what you know. Angle $B = 60°$ can go right into the figure. Because $AB = x$, you can plug in for x; make $x = 4$. Label this information in the figure:



Next, figure out what other information you know. Because there are 180° in a triangle, angle $A = 180 - 90 - 60 = 30°$. This is a 30°-60°-90° special right triangle, which you are given information about in the reference sheet. Based on the figure given in the box, the hypotenuse is equal to $2x$. (Note that this is a different x from the one you plugged in for; the test-writers are trying to confuse you.) So, if the hypotenuse is 4, $x = \frac{4}{2} = 2$; this is the side opposite the 30° angle, BC. The remaining side, AC, is $x\sqrt{3}$, which is $2\sqrt{3}$. Label this information in your figure:



Now write down the formula you need. The question is asking for the area, so use the area of a triangle formula from the reference sheet: $A = \frac{1}{2}bh$. Fill in what you know. Because this is a right triangle, you can use the two legs of the triangle as the base and the height. Make $b = 2\sqrt{3}$ and $h = 2$ in the equation and solve: $A = \frac{1}{2}\left(2\sqrt{3}\right)(2) = 2\sqrt{3}$. This is your target; circle it. Now plug in $x = 4$ (that's the x from the question, NOT the x from the information in the reference sheet!) into each answer choice and eliminate what doesn't equal $2\sqrt{3}$. Only (A) works, so the correct answer is (A).

Now that we've covered how to approach geometry questions, let's look more closely at some of the geometry concepts you'll need for these questions.

LINES AND ANGLES

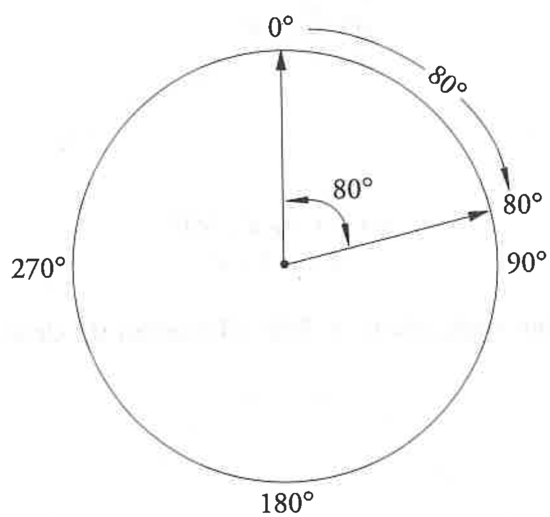
Here are the basic rules you need to know for questions about lines and angles on the Digital SAT.

1. **A circle contains 360 degrees.**

   Every circle contains 360 degrees. Each degree is $\frac{1}{360}$ of the total distance around the outside of the circle. It doesn't matter whether the circle is large or small; it still has exactly 360 degrees.

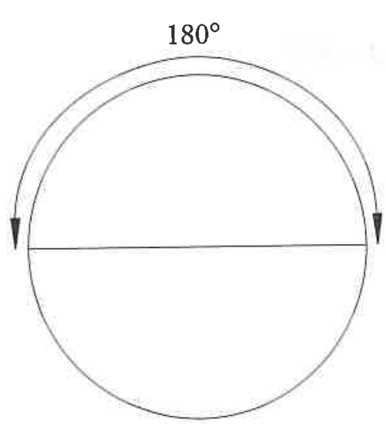
2. **When you think about angles, remember circles.**

   An angle is formed when two line segments extend from a common point. If you think of the point as the center of a circle, the measure of the angle is the number of degrees enclosed by the lines when they pass through the edge of the circle. Once again, the size of the circle doesn't matter; neither does the length of the lines. Refer to the following figure.

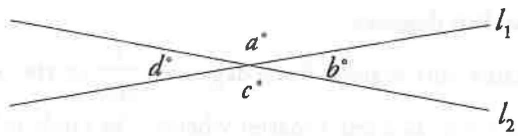


3. **A line is a 180° angle.**

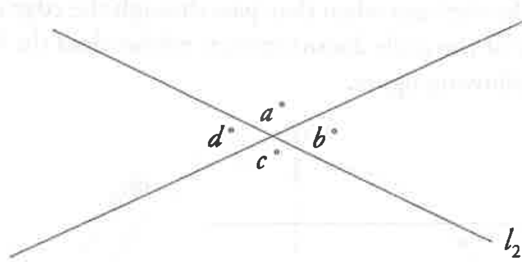
   You probably don't think of a line as an angle, but it is one. Think of it as a flat angle. The following drawings should help:



4. **When two lines intersect, four angles are formed.**
The following figure should make this clear. The four angles are indicated by letters.

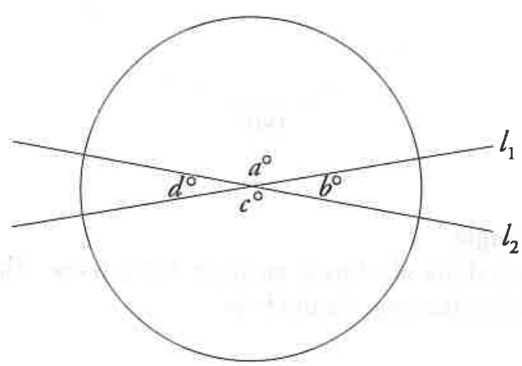


5. **When two lines intersect, the angles opposite each other will have the same measures.**
Such angles are called **vertical angles.** In the following figure, angles a and c are equal; so are angles b and d.



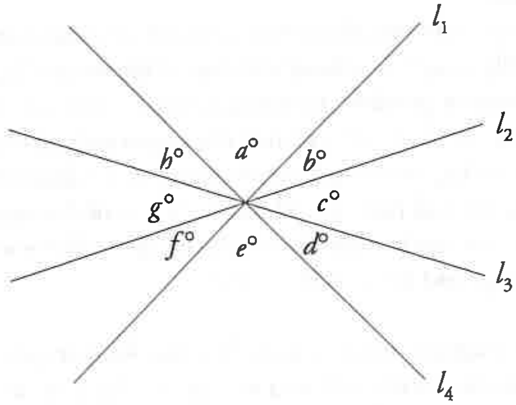
$$a + b + c + d = 360°$$
$$a = c, b = d$$

The measures of these four angles add up to 360°. (Remember the circle.)



$$a + b + c + d = 360°$$

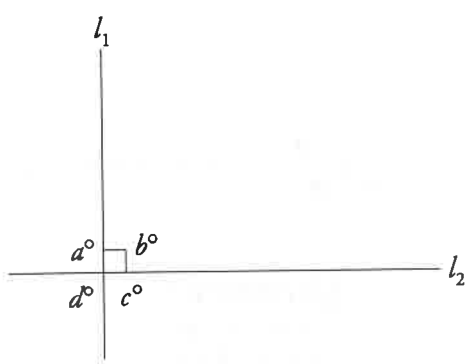
It doesn't matter how many lines you intersect through a single point. The total measure of all the angles formed will still be 360°.



$$a + b + c + d + e + f + g + h = 360°$$
$$a = e,\ b = f,\ c = g,\ d = h$$

6. **If two lines are perpendicular to each other, each of the four angles formed is 90°.**
A 90° angle is called a **right angle.**

> **Perpendicular:**
> Meeting at right (90°) angles



Angles *a*, *b*, *c*, and *d* all equal 90°.

> **Flip and Negate**
> If two lines are perpendicular, then their slopes are negative reciprocals; i.e., if l_1 has a slope of 2 and l_2 is perpendicular to l_1, then l_2 must have a slope of $-\frac{1}{2}$.

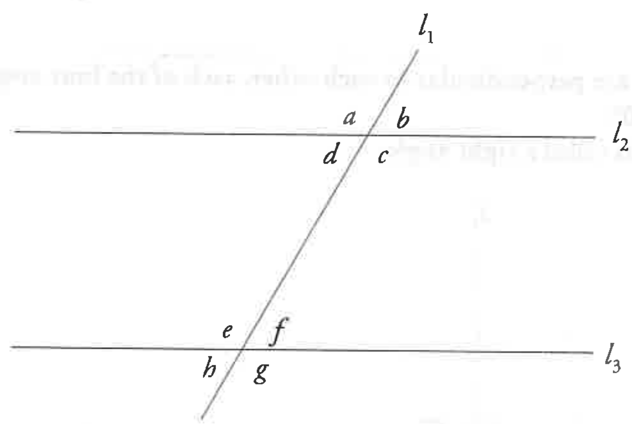
The little box at the intersection of the two lines is the symbol for a right angle. If the lines are not perpendicular to each other, then none of the angles will be right angles. Don't assume that an angle is a right angle unless you are specifically told that it is a right angle, either in the question or with the right angle symbol.

Parallel Lines
Parallel lines have the same slope.

7. **When two parallel lines are cut by a third line, all of the small angles are equal, all of the big angles are equal, and the sum of any big angle and any small angle is 180°.**

Parallel lines are two lines that never intersect, and the rules about parallel lines are usually taught in school with lots of big words. But we like to avoid big words whenever possible. Simply put, when a line cuts through two parallel lines, two kinds of angles are created: big angles and small angles. You can tell which angles are big and which are small just by looking at them. All the big angles look equal, and they are. The same is true of the small angles. Lastly, any big angle plus any small angle always equals 180°. (The test-writers like rules about angles that add up to 180° or 360°.)

In any geometry question, never assume that two lines are parallel unless the question or figure specifically tells you so. The two lines in the following figure are parallel. Angle *a* is a big angle, and it has the same measure as angles *c*, *e*, and *g*, which are also big angles. Angle *b* is a small angle, and it has the same measure as angles *d*, *f*, and *h*, which are also small angles.

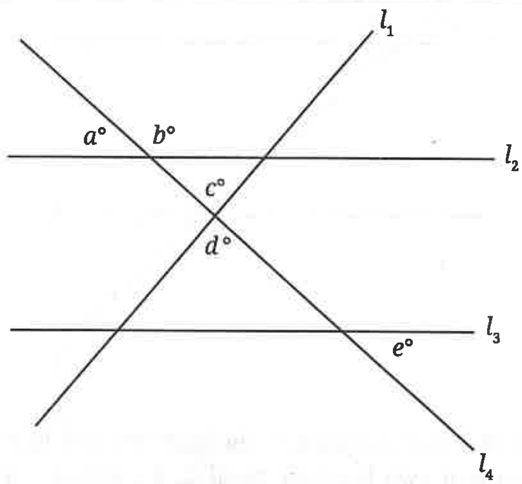


l_2 is parallel to l_3
$a = c = e = g$
$b = d = f = h$

You should be able to see that the degree measures of angles *a*, *b*, *c*, and *d* add up to 360°. So do those of angles *e*, *f*, *g*, and *h*. If you have trouble seeing it, draw a circle around the angles. What is the degree measure of a circle? Also, the sum of any small angle (such as *d*) and any big angle (such as *g*) is 180°.

Let's see how these concepts might be tested on the Digital SAT.

2 Mark for Review



Note: Figure not drawn to scale.

In the figure above, l_2 is parallel to l_3. If $b = 130$, what is the value of e?

Ⓐ 40

Ⓑ 50

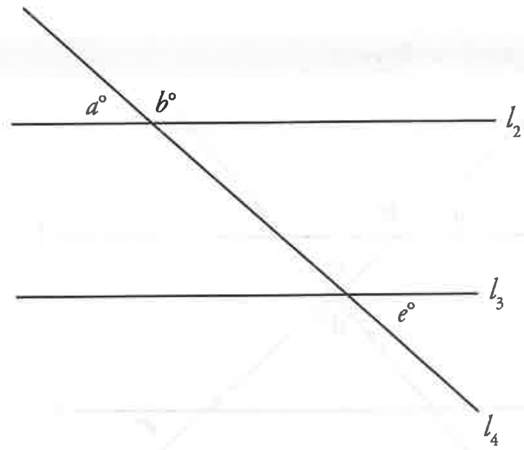
Ⓒ 90

Ⓓ 130

Here's How to Crack It

The question asks for the value of an angle on a figure. Use the Geometry Basic Approach. Start by redrawing the figure on your scratch paper. This figure has a lot going on, so only draw the parts you need and add more later if necessary. Angle e is between lines 3 and 4, and line 2 is parallel to line 3. That all seems important, so include it in your figure.

The drawing should look something like this:

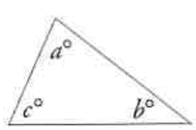


Next, label the figure with information given in the question, and label angle b as 130°. When a geometry question states that two lines are parallel, it's almost certainly testing the rules about angles. Recall that when two parallel lines are cut by a third line, two kinds of angles are created: big and small. All of the small angles are equal to each other, all of the big angles are equal to each other, and any small angle + any big angle = 180°. Angle b is a big angle and angle e is a small angle, so $b + e = 180$. Plug in the measure of angle b to get $130 + e = 180$. Subtract 130 from both sides of the equation to get $e = 50$. The correct answer is (B).

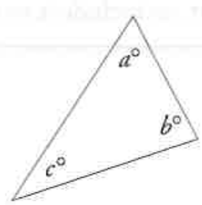
TRIANGLES

Here are some basic triangle rules you'll need to know for the Digital SAT.

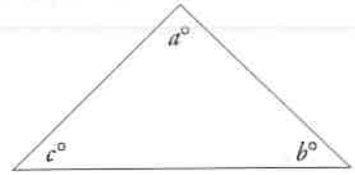
1. **Every triangle contains 180°.**
   The word *triangle* means "three angles," and every triangle contains three interior angles. The measure of these three angles always adds up to exactly 180°. You don't need to know why this is true or how to prove it. You just need to know it. And we mean ***know*** it.



$a + b + c = 180°$

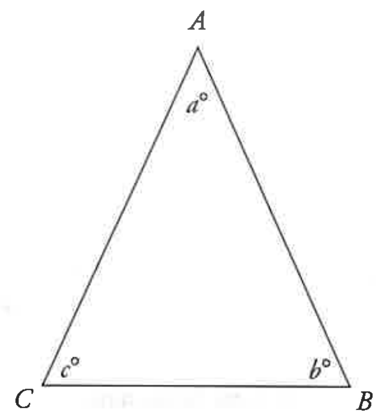


$a + b + c = 180°$



$a + b + c = 180°$

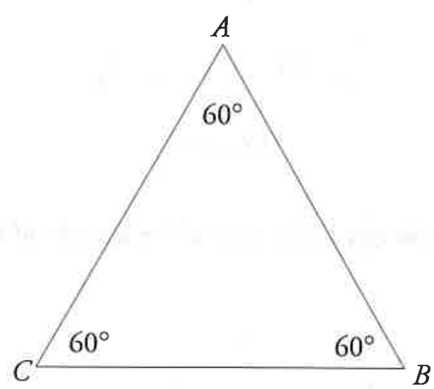
2. **An isosceles triangle is one in which two of the sides are equal in length.** The angles opposite those equal sides are also equal because angles opposite equal sides are also equal.



$AB = AC \quad AB \neq BC$
$c = b \quad c \neq a$

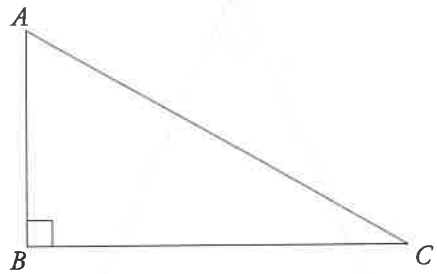
3. **An equilateral triangle is one in which all three sides are equal in length.** Because the angles opposite equal sides are also equal, all three angles in an equilateral triangle are equal too. (Their measures are always 60° each.)

Equilateral Triangles
An equilateral triangle is also isosceles.



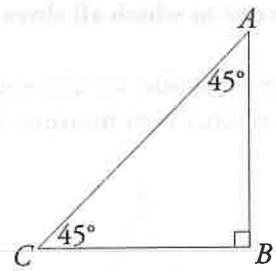
$AB = BC = AC$

4. **A right triangle is a triangle in which one of the angles is a right angle (90°).**
The longest side of a right triangle, which is always opposite the 90° angle, is called the **hypotenuse.** The other two sides are called **legs.**



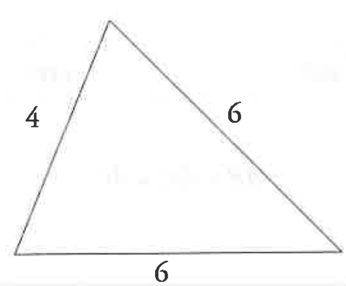
AC is the hypotenuse.

Some right triangles are also **isosceles.** The angles in an isosceles right triangle always measure 45°, 45°, and 90°.



$AB = BC$

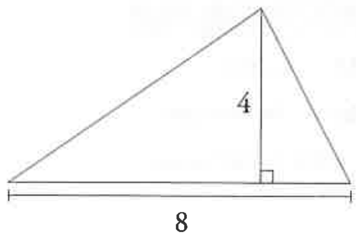
5. **The perimeter of a triangle is the sum of the lengths of its sides.**



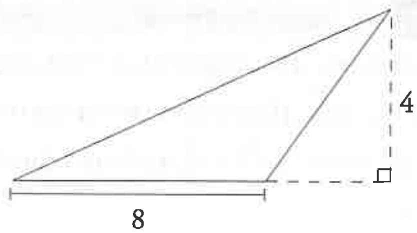
perimeter = 4 + 6 + 6 = 16

6. The area of a triangle is $\frac{1}{2}$ (base × height).

In or Out
The height can be found with a line dropped inside or outside the triangle—just as long as it's perpendicular to the base.



area = $\frac{1}{2}(8 \times 4) = 16$

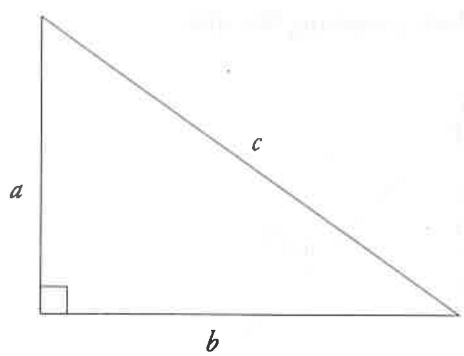


area = $\frac{1}{2}(8 \times 4) = 16$

Pythagorean Theorem

The **Pythagorean Theorem** states that in a right triangle, the square of the hypotenuse equals the sum of the squares of the other two sides. As mentioned earlier, the hypotenuse is the longest side of a right triangle; it's the side opposite the right angle. The square of the hypotenuse is its length squared. Applying the Pythagorean Theorem to the following drawing, we find that $a^2 + b^2 = c^2$.

Pythagorean Theorem
$a^2 + b^2 = c^2$, where c is the hypotenuse of a right triangle. Learn it; love it.



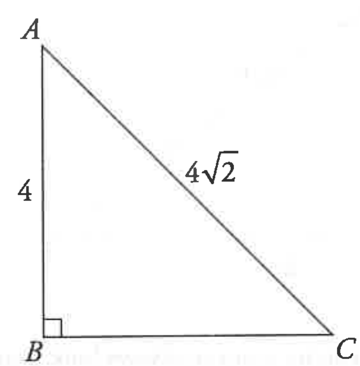
If you forget the Pythagorean Theorem, you can always look it up on the reference sheet.

3 Mark for Review

In triangle ABC, angle B is a right angle, $AB = 4$, and $AC = 4\sqrt{2}$. If the perimeter of the triangle can be written in the form $4\left(\sqrt{2} + a\right)$, and a is a constant, what is the value of a?

Here's How to Crack It

The question asks for the value of a constant given information about a triangle. Use the Geometry Basic Approach. Start by drawing a right triangle on your scratch paper and put the right angle symbol at B. Next, label the figure with information from the question. Label side AB as 4 and side AC as $4\sqrt{2}$. Notice that side AC is opposite the right angle, so it is the hypotenuse. The drawing should look something like this:



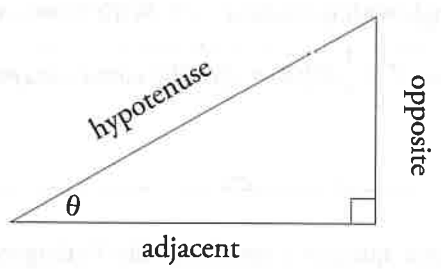
To find the length of the third side, use the Pythagorean Theorem: $a^2 + b^2 = c^2$. Plug in the known values to get $4^2 + b^2 = \left(4\sqrt{2}\right)^2$. Square the numbers to get $16 + b^2 = 32$, then subtract 16 from both sides of the equation to get $b^2 = 16$. Take the square root of both sides of the equation to get $b = 4$. Label side BC as 4. This happens to be an isosceles right triangle, which is one of the triangles on the reference sheet. If you catch that, you might be able to find the length of the third side faster, but using the Pythagorean Theorem works well, too.

The perimeter of a geometric shape is the sum of the lengths of the sides, so $P = 4 + 4 + 4\sqrt{2}$, or $P = 8 + 4\sqrt{2}$. Set this equal to the different form of the perimeter given in the question to get $8 + 4\sqrt{2} = 4\left(\sqrt{2} + a\right)$. Distribute on the right side of the equation to get $8 + 4\sqrt{2} = 4\sqrt{2} + 4a$. Subtract $4\sqrt{2}$ from both sides of the equation to get $8 = 4a$. Divide both sides of the equation by 4 to get $2 = a$. The correct answer is 2.

TRIGONOMETRY

SOHCAHTOA

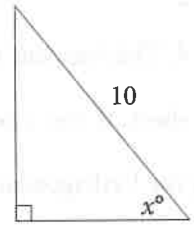
Trigonometry will appear on the Digital SAT Math section. But fear not! Many trigonometry questions you will see mostly require you to know the basic definitions of the three main trigonometric functions. **SOHCAHTOA** is a way to remember the three functions.



$$\text{sine } \theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \text{cosine } \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \text{tangent } \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Check out this next example.

4 Mark for Review



In the triangle above, $\sin x = 0.8$ and $\cos x = 0.6$. What is the area of the triangle?

Ⓐ 0.48

Ⓑ 4.8

Ⓒ 24

Ⓓ 48

Here's How to Crack It

The question asks for the area of the triangle. Use the definitions of sine and cosine to find the two legs of the triangle. Sine is $\frac{\text{opposite}}{\text{hypotenuse}}$, so if $\sin x = 0.8$, then $0.8 = \frac{\text{opposite}}{10}$. Multiply both sides by 10 and you find the side opposite the angle with measure $x°$ is 8. Similarly, cosine is $\frac{\text{adjacent}}{\text{hypotenuse}}$, so if $\cos x = 0.6$, then $0.6 = \frac{\text{adjacent}}{10}$. Multiply both sides by 10 to determine that the side adjacent to the angle with measure $x°$ is 6. With those two sides, find the area. The formula for area is $A = \frac{1}{2}bh$, so $A = \frac{1}{2}(6)(8) = 24$. The correct answer is (C).

The test-writers love to ask you questions involving the Pythagorean Theorem along with SOHCAHTOA. See the following question.

5 Mark for Review

In triangle ABC, AC is perpendicular to BC and $\cos(B) = \frac{12}{13}$.

What is the value of $\tan(B)$?

Ⓐ $\frac{5}{13}$

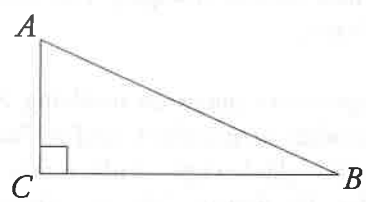
Ⓑ $\frac{5}{12}$

Ⓒ $\frac{12}{13}$

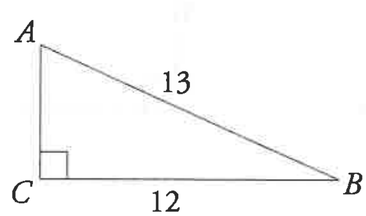
Ⓓ $\frac{12}{5}$

Here's How to Crack It

The question asks for the value of the tangent of an angle. Use the Geometry Basic Approach: start by drawing triangle ABC.



Next, label what you can. You don't know the actual side lengths, but because $\cos(B) = \frac{12}{13}$, you do know the relationship between the side adjacent to angle B and the hypotenuse. You can plug in for this relationship: make BC (the side adjacent to the angle) 12 and AB (the hypotenuse) 13:



You need to find tan (B), which means you need $\frac{\text{opposite}}{\text{adjacent}}$. You already know the adjacent side is 12, but you still need the side opposite, AC. Use the Pythagorean Theorem to find the missing side:

$$a^2 + b^2 = c^2$$
$$12^2 + b^2 = 13^2$$
$$144 + b^2 = 169$$
$$b^2 = 25$$
$$b = 5$$

Therefore, $AC = 5$, and tan $(B) = \frac{5}{12}$, so the correct answer is (B).

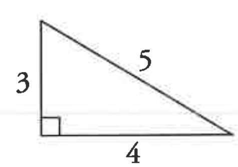
Your Friend the Rectangle
Be on the lookout for questions in which the application of the Pythagorean Theorem is not obvious. For example, every rectangle contains two right triangles. That means that if you know the length and width of the rectangle, you also know the length of the diagonal, which is the hypotenuse of both triangles.

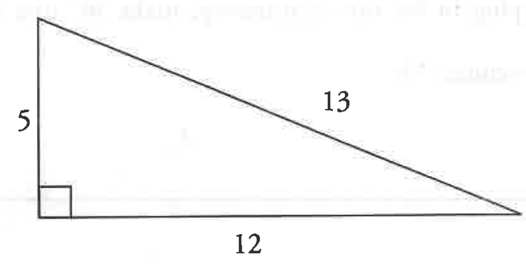
Special Right Triangles

Both of the previous questions you worked also used special right triangles. While in the last question we used the Pythagorean Theorem to find the missing side, if you memorize these special triangles, you can avoid using the Pythagorean Theorem in a lot of cases.

When it comes to geometry questions involving right triangles, the Digital SAT is often fairly predictable, as questions tend to focus on certain relationships. In these questions, the triangles have particular ratios. There are two different types of special right triangles. The first involves the ratio of sides, and the second involves the ratio of angles.

The most common special right triangles with side ratios are known as **Pythagorean triples.** Here are the test-writers' favorites:



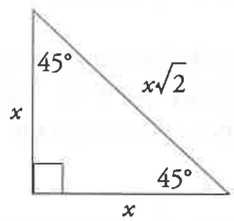


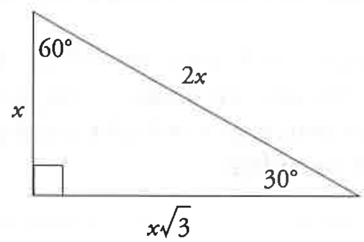
Relax; It's Just a Ratio
A 3-4-5 triangle may be hiding, disguised as 6-8-10 or 18-24-30. It's all the same ratio, though, so be on the lookout.

If you memorize these two sets of Pythagorean triples (3-4-5 and 5-12-13), you'll often be able to find the answer without using the Pythagorean Theorem. If you're given a right triangle with a side of 3 and a hypotenuse of 5, you know right away that the other side has to be 4. Likewise, if you see a right triangle with sides of 5 and 12, you know the hypotenuse must be 13.

The test-writers also like to use right triangles with sides that are simply multiples of the common Pythagorean triples. For example, you might see a 6-8-10 or a 10-24-26 triangle. These sides are simply the sides of the 3-4-5 and 5-12-13 triangles multiplied by 2.

There are two types of special right triangles that have a specific ratio of angles. They are the **30°-60°-90° triangle** and the **45°-45°-90° triangle.** The sides of these triangles always have the same fixed ratio to each other. The ratios are as follows:





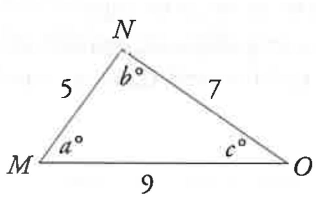
Don't Forget the Reference Sheet!
The relationships of the sides of special right triangles can be found in the reference sheet in the testing app, so you don't necessarily need to memorize them. However, you should be able to recognize them.

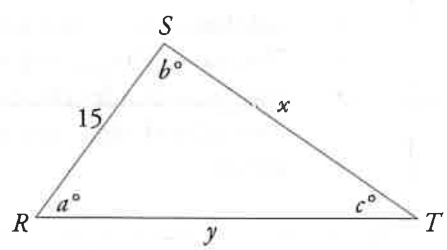
Let's talk about a 45°-45°-90° triangle first. Did you notice that this is also an isosceles right triangle? The legs will always be the same length. And the hypotenuse will always be the length of one leg times $\sqrt{2}$. Its ratio of side to side to hypotenuse is always $1:1:\sqrt{2}$. For example, if you have a 45°-45°-90° triangle with a leg length of 3, then the second leg length will also be 3 and the hypotenuse will be $3\sqrt{2}$.

Now let's talk about a 30°-60°-90° triangle. The ratio of shorter leg to longer leg to hypotenuse is always $1:\sqrt{3}:2$. For example, if the shorter leg of a 30°-60°-90° triangle is 5, then the longer leg would be $5\sqrt{3}$ and the hypotenuse would be 10.

Similar and Congruent Triangles

Similar triangles have the same shape, but they are not necessarily the same size. Having the same shape means that the angles of the triangles are identical and that the corresponding sides have the same ratio. Look at the following two similar triangles:





These two triangles both have the same set of angles, but they aren't the same size. Whenever this is true, the sides of one triangle are proportional to those of the other. Notice that sides NO and ST are both opposite the angle that is $a°$. These are called corresponding sides, because they correspond to the same angle. So the lengths of $\overline{NO}$ and $\overline{ST}$ are proportional to each other.

In order to figure out the lengths of the other sides, set up a proportion: $\frac{MN}{RS} = \frac{NO}{ST}$. Now fill in the information that you know: $\frac{5}{15} = \frac{7}{x}$. Cross-multiply and you find that $x = 21$. You could also figure out the length of y: $\frac{NO}{ST} = \frac{MO}{RT}$. Therefore, $\frac{7}{21} = \frac{9}{y}$, and $y = 27$.

Whenever you have to deal with sides of similar triangles, just set up a proportion.

Once in a while, a Digital SAT question will ask about what information is necessary in determining that two triangles are similar. These questions usually focus on angles, but it's important to know how both angles and sides can play a role. For two triangles to be similar, you need one of these relationships:

Two triangles are <u>similar</u> when at least **one** of the following is true:

- All three angles of the triangles are congruent (AAA).
- Pairs of sides of the triangles are in proportion, and the angle in between those sides is congruent (SAS).
- All three sides of one triangle are in proportion to the corresponding three sides of the other triangle (SSS).

In the previous example, there was enough information about triangles *MNO* and *RST* to use the AAA rule but not the SAS rule or the SSS rule.

Similar or Congruent?
All congruent triangles are similar.

Not all similar triangles are congruent.

AAA proves that triangles are similar but not necessarily congruent.

You might also be asked about the information needed to prove that two triangles are congruent. Here are the rules for that:

Two triangles are <u>congruent</u> when at least **one** of the following is true:

- All three sides are equal (SSS).
- Two pairs of angles and the side between them are equal (ASA).
- Two pairs of sides and the angle between them are equal (SAS).
- Two pairs of angles and a side that *isn't* between them are equal (AAS).

That's a lot, isn't it? The good news is that these topics aren't tested very often, so if they aren't in your POOD, you'll be okay. Here's a question that will help you review these concepts.

6 Mark for Review

In triangles PQR and WXY, angles Q and X each measure $101°$ and angles R and Y each measure $37°$. This information is sufficient to prove which of the following?

I. Triangles PQR and WXY are similar.
II. Triangles PQR and WXY are congruent.

Ⓐ Neither I nor II

Ⓑ I only

Ⓒ II only

Ⓓ I and II

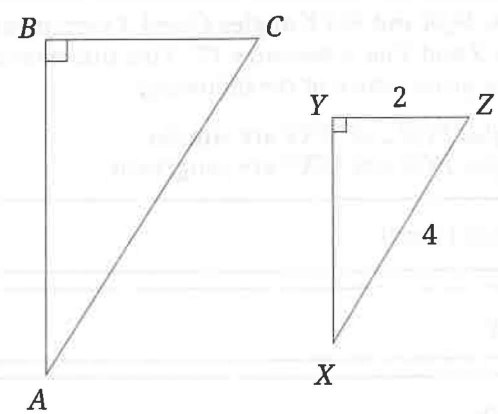
Here's How to Crack It

The question asks what can be proven about two triangles using the given information. Eliminate (C) immediately because it is impossible for a triangle to be congruent without also being similar. The information in the question is about angles only, and at least one pair of corresponding sides must be the same length for triangles to be congruent. There is not sufficient information to determine whether the triangles are congruent; eliminate (D). Since all triangles have 180 degrees and the triangles have two equal angles, the third angles must also equal each other. All three angles are congruent, which matches the AAA rule for similar triangles, so the information is sufficient to prove that the triangles are similar; eliminate (A). The correct answer is (B).

Similar Triangles and Trig

Finally, there's a special relationship between similar triangles and trigonometry. Side lengths in similar triangles are proportional, and the trigonometric functions give the proportions of the sides of a triangle. Therefore, if two triangles are similar, the corresponding trigonometric functions are equal! Let's look at how this might work in a question.

7 Mark for Review



In the figure above, triangle ABC is similar to triangle XYZ. What is the value of cos(A)?

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\sqrt{3}$

(D) 2

Here's How to Crack It

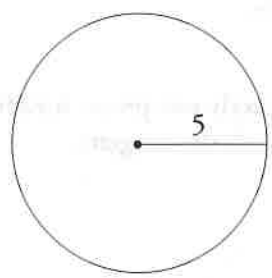
The question asks for the value of cos (A) but gives measurements on triangle XYZ. Because the two triangles are similar, the value of corresponding trigonometric functions will be equal. Therefore, cos (A) = cos (X). The value of cos X is $\frac{\text{adjacent}}{\text{hypotenuse}}$ or $\frac{XY}{XZ}$. You could use the Pythagorean Theorem to find XY, but it's easier to use the special right triangle discussed earlier. Because the hypotenuse is twice one of the legs, you know this is a 30°-60°-90° triangle. YZ is the shortest side (x), so XY is $x\sqrt{3}$ or $2\sqrt{3}$. Therefore, cos (X) = $\frac{2\sqrt{3}}{4}$, which reduces to $\frac{\sqrt{3}}{2}$. Because cos (X) = cos (A), cos (A) also equals $\frac{\sqrt{3}}{2}$. The correct answer is (B).

CIRCLES

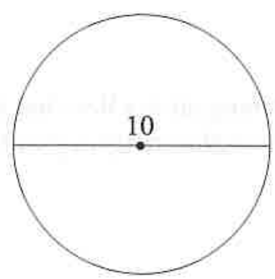
Here are the rules you'll need in order to tackle circle questions on the Digital SAT.

1. **The circumference of a circle is $2\pi r$ or πd, where r is the radius of the circle and d is the diameter.**
   This information is in the reference sheet, so don't stress over memorizing these formulas. You will always be able to open the reference sheet in the testing app if you forget them. Just keep in mind that the diameter is always twice the length of the radius (and that the radius is half the diameter).

> **A Few Formulas**
> Area = πr^2
> Circumference = $2\pi r$ or πd
> Diameter = $2r$



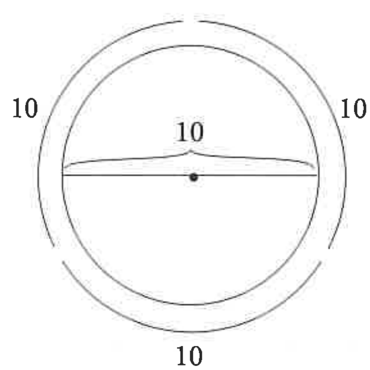
circumference = $2 \times \pi \times 5 = 10\pi$



circumference = 10π

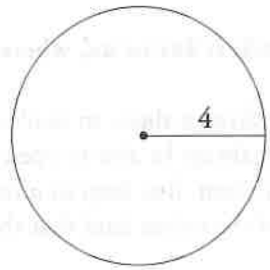
In math class you probably learned that $\pi = 3.14$ (or even 3.14159). On the Digital SAT, $\pi = 3^+$ (a little more than 3) is a good enough approximation. Even with a calculator, using $\pi = 3$ will give you all the information you need to solve difficult Digital SAT multiple-choice geometry questions.

> **Leave That π Alone!**
> Most of the time, you won't multiply π out in circle questions. Because the answer choices will usually be in terms of π (6π instead of 18.849...), you can save yourself some trouble by leaving your work in terms of π.



circumference = about 30

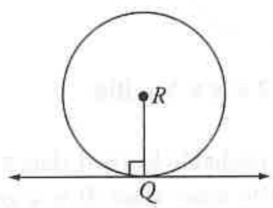
2. **The area of a circle is πr^2, where r is the radius of the circle.**



area = $\pi(4)^2 = 16\pi$

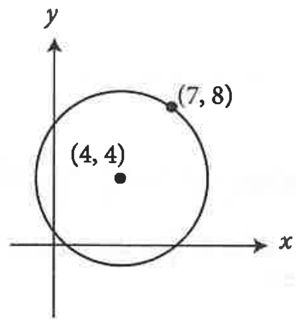
3. **A tangent is a line that touches a circle at exactly one point. A radius drawn from that tangent point forms a 90° angle with the tangent.**

Circles Have Names?
If a question refers to Circle R, it means that the center of the circle is point *R*.



Let's see how these rules can show up on the Digital SAT.

8 Mark for Review



The circle defined by the equation $(x - 4)^2 + (y - 4)^2 = 25$ has its center at point (4, 4) and includes point (7, 8) on the circle. This is shown in the figure above. What is the area of the circle shown?

Ⓐ 5π

Ⓑ 10π

Ⓒ 16π

Ⓓ 25π

Here's How to Crack It

The question asks for the area of the circle, so write down the formula for area of a circle: $A = \pi r^2$. That means you need to determine the radius of the circle. If you remember the circle formula from Chapter 23, you simply need to recall that $r^2 = 25$ and multiply by π to find the area. If not, you can find the distance between (4, 4) and (7, 8) by drawing a right triangle. The triangle is a 3-4-5 right triangle, so the distance between (4, 4) and (7, 8) (and thus the radius) is 5. If the radius is 5, then the area is $\pi(5)^2$, or 25π. The correct answer is (D).

Converting Degrees to Radians

Some geometry questions will ask you to convert an angle measurement from degrees to radians. While this may sound scary, doing this conversion only requires that you remember that $180° = \pi$ radians. Use this relationship, which is included on the reference sheet, to set up a proportion and convert the units.

9 Mark for Review

An angle in a circle measures $\frac{\pi}{6}$ radians. What is the measure of the angle in degrees?

Here's How to Crack It

The question asks for the measure of an angle in degrees. Use the relationship between radians and degrees to set up a proportion. If $180° = \pi$ radians, the proportion will look like this:

$$\frac{180°}{\pi \text{ radians}} = \frac{x°}{\frac{\pi}{6} \text{ radians}}$$

Cross-multiply to get $(\pi)(x) = (180)\left(\frac{\pi}{6}\right)$. Simplify the right side of the equation to get $\pi x = 30\pi$, then divide both sides of the equation by π to get $x = 30$. The correct answer is 30.

$\boxed{30}$

Arcs and Sectors

Many circle questions on the Digital SAT will not ask about the whole circle. Rather, you'll be asked about arcs or sectors. Both arcs and sectors are portions of a circle: arcs are portions of the circumference, and sectors are portions of the area. Luckily, both arcs and sectors have the same relationship with the circle, based on the central angle (the angle at the center of the circle that creates the arc or sector):

$$\frac{\text{part}}{\text{whole}} = \frac{\text{central angle}}{360°} = \frac{\text{arc length}}{2\pi r} = \frac{\text{sector area}}{\pi r^2}$$

Note that these relationships are all proportions. Arcs and sectors are proportional to the circumference and area, respectively, as the central angle is to 360°.

Questions sometimes refer to "minor" or "major" arcs or sectors. A minor arc or sector is one that has a central angle of less than 180°, whereas a major arc or sector has a central angle greater than 180° (in other words, it goes the long way around the circle). Let's see how arcs and sectors might show up in a question.

10 Mark for Review

Points A and B lie on circle O (not shown). $AO = 3$ and angle $O = 120°$. What is the area of minor sector AOB?

Ⓐ $\frac{\pi}{3}$

Ⓑ π

Ⓒ 3π

Ⓓ 9π

Here's How to Crack It

The question asks for the area of minor sector AOB in circle O. Because O is the name of the circle, it's also the center of the circle, so AO is the radius. Angle O is the central angle of sector AOB, so you have all the pieces you need to find the sector. Put them into a proportion:

$$\frac{120°}{360°} = \frac{x}{\pi(3)^2}$$

Cross-multiply to get $360x = 1{,}080\pi$ (remember to not multiply out π). Divide both sides by 360, and you get $x = 3\pi$. The correct answer is (C).

Relationship Between Arc and Angle in Radians

Sometimes you'll be asked for an arc length, but you'll be given the angle in radians instead of degrees. Fear not! Rather than making the question more complicated, the test-writers have actually given you a gift! All you need to do is memorize this formula:

$$s = r\theta$$

In this formula, s is the arc length, r is the radius, and θ is the central angle in radians. If you know this formula, these questions will be a snap!

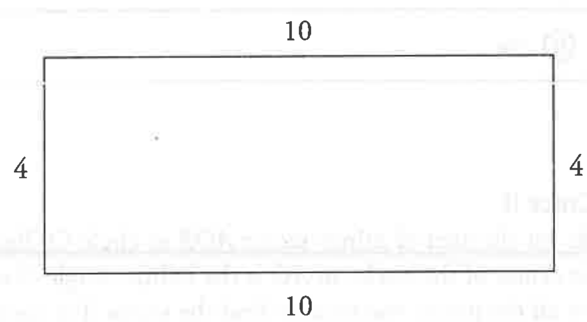
When an arc measure is given in degrees, it's even simpler: the arc is the same number of degrees as the angle that defines it.

Little Boxes
Here's a progression of quadrilaterals from least specific to most specific:
quadrilateral is any 4-sided figure
↓
parallelogram is a quadrilateral in which opposite sides are parallel
↓
rectangle is a parallelogram in which all angles = 90°
↓
square is a rectangle in which all sides are equal

RECTANGLES AND SQUARES

Here are some rules you'll need to know about rectangles and squares:

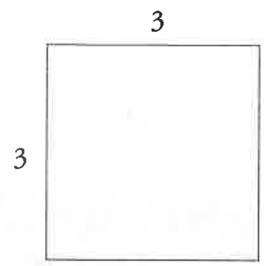
1. **The perimeter of a rectangle is the sum of the lengths of its sides.** Just add them up.



perimeter = 10 + 4 + 10 + 4 = 28

2. **The area of a rectangle is length × width.**
The area of the preceding rectangle, therefore, is 10 × 4, or 40.

3. **A square is a rectangle whose four sides are all equal in length.**
The perimeter of a square, therefore, is four times the length of any side. The area is the length of any side squared.



perimeter = 4(3) = 12
area = $3^2 = 9$

4. **In rectangles and squares all angles are 90° angles.**
It can't be a square or a rectangle unless all angles are 90°.

Let's check out an example.

11 Mark for Review

If the perimeter of a square is 28, what is the length of the diagonal of the square?

Ⓐ $2\sqrt{14}$

Ⓑ $7\sqrt{2}$

Ⓒ $7\sqrt{3}$

Ⓓ 14

Here's How to Crack It

The question asks for the diagonal of a square based on its perimeter. The perimeter of a square is $4s$. So, $28 = 4s$. Divide by 4 to find $s = 7$. The diagonal of a square divides the square into two 45°-45°-90° triangles, with sides in the ratio of $x:x:x\sqrt{2}$. If the side is 7, the diagonal is $7\sqrt{2}$. The correct answer is (B).

VOLUME

Volume questions on the Digital SAT can seem intimidating at times. The test-writers love to give you questions featuring unusual shapes such as pyramids and spheres. Luckily, the reference sheet in the testing app contains all the formulas you will ever need for volume questions on the Digital SAT. Simply apply the Basic Approach for geometry using the given formulas and you'll be in good shape (pun entirely intended)!

Let's look at an example.

12 Mark for Review

A sphere has a volume of 36π. What is the surface area of the sphere? (The surface area of a sphere is given by the formula $A = 4\pi r^2$.)

Ⓐ 3π

Ⓑ 9π

Ⓒ 27π

Ⓓ 36π

Here's How to Crack It

The question asks for the surface area of a sphere given its volume. Start by writing down the formula for volume of a sphere from the reference sheet: $V = \frac{4}{3}\pi r^3$. Put what you know into the equation: $36\pi = \frac{4}{3}\pi r^3$. From this you can solve for r. Divide both sides by π to get $36 = \frac{4}{3}r^3$. Multiply both sides by 3 to clear the fraction: $36(3) = 4r^3$. Note we left 36 as 36, because the next step is to divide both sides by 4, and 36 divided by 4 is 9, so $9(3) = r^3$ or $27 = r^3$. Take the cube root of both sides to get $r = 3$. Now that you have the radius, use the formula provided to find the surface area: $A = 4\pi(3)^2$, which comes out to 36π. The correct answer is (D).

BALLPARKING

You may be thinking, "Wait a second, isn't there an easier way?" By now, you should know that of course there is, and we're going to show you. On many Digital SAT geometry questions, you won't have to calculate an exact answer. Instead, you can estimate an answer choice. We call this **Ballparking,** a strategy mentioned earlier in this book.

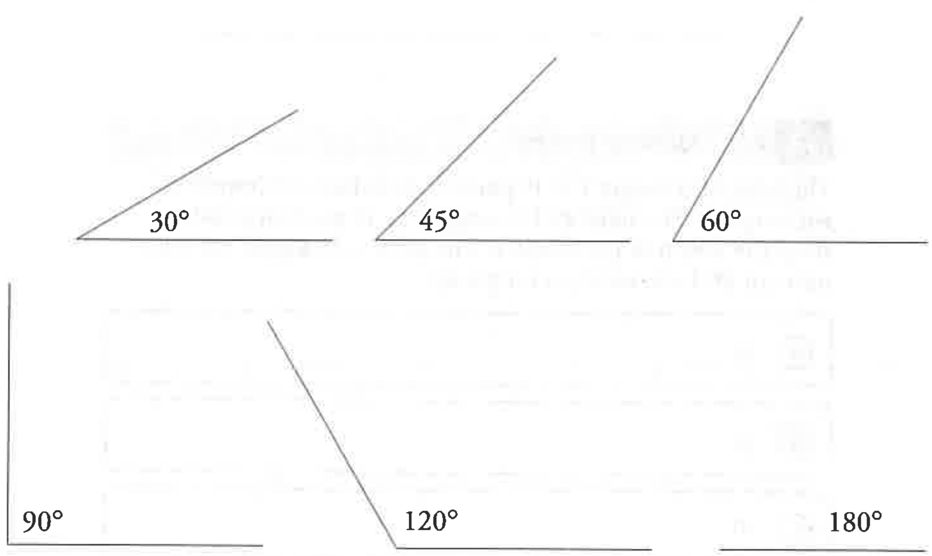
Rocket Science?
The SAT is a college admissions test, not an exercise in precision. Because approximately 33 of its 44 Math questions are multiple-choice, you can afford to approximate numbers like π, $\sqrt{2}$, and $\sqrt{3}$ (3+, 1.4, and 1.7+, respectively).

Ballparking is extremely useful on Digital SAT geometry questions. At the very least, it will help you avoid careless mistakes by immediately eliminating answers that could not possibly be correct. On many questions, Ballparking will allow you to find the answer without even working out the problem at all.

For example, on many Digital SAT geometry questions, you will be presented with a drawing in which some information is given and you will be asked to find some of the information that is missing. In most such questions, you're expected to apply some formula or perform some calculation, often an algebraic one. But look at the drawing and make a rough estimate of the answer (based on the given information) before you try to work it out. You might be able to eliminate an answer choice or two or even narrow it down to one.

The basic principles you just learned (such as the number of degrees in a triangle and the fact that $\pi \approx 3$) will be enormously helpful to you in Ballparking on the Digital SAT.

Even though many geometric figures are marked with a note that they are not drawn to scale, you will also find it very helpful if you have a good sense of how large certain common angles are. Study the following examples.



How High Is the Ceiling?

If your friend stood next to a wall in your living room and asked you how high the ceiling was, what would you do? Would you get out your trigonometry textbook and try to triangulate using the shadow cast by your pal? Of course not. You'd look at your friend and think something like this: "Dave's about 6 feet tall. The ceiling's a couple of feet higher than he is. It must be about 8 feet high."

Your Ballpark answer wouldn't be exact, but it would be close. If someone later claimed that the ceiling in the living room was 15 feet high, you'd be able to tell her with confidence that she was mistaken.

The Correct Choice
Remember that the Digital SAT is a mostly multiple-choice test. This means that you don't always have to come up with an answer; you just have to identify the correct one from among the four choices provided.

You'll be able to do the same thing on the Digital SAT. If line segment A has a length of π and line segment B is exactly half as long, then the length of line segment B is a little more than 1.5. All such questions are ideal for Ballparking.

PLUGGING IN

As you learned already, Plugging In is a powerful technique for solving Digital SAT algebra questions. It is also very useful on geometry problems. For some questions, you will be able to plug in values for missing information and then use the results either to find the answer directly or to eliminate answers that cannot be correct.

Here's an example.

13 Mark for Review

The base of triangle T is 40 percent less than the length of rectangle R. The height of triangle T is 50 percent greater than the width of rectangle R. The area of triangle T is what percent of the area of rectangle R?

Ⓐ 10

Ⓑ 45

Ⓒ 90

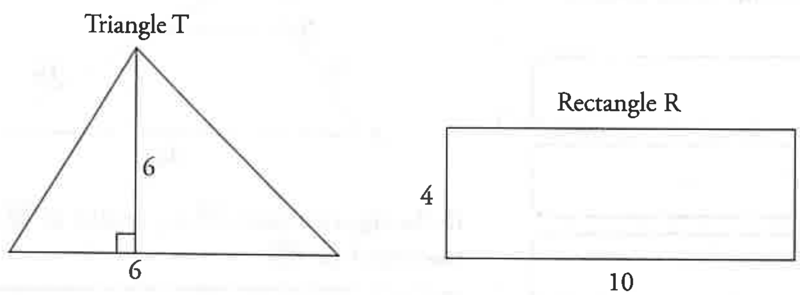
Ⓓ 110

Watch Us Crack It
Watch the step-by-step video explanation of how to answer this question in your Student Tools.

Here's How to Crack It

The question asks for the relationship between the areas of triangle T and rectangle R. This is a challenging question. Don't worry—you'll still be able to find the right answer by sketching and plugging in.

When plugging in, always use numbers that are easy to work with. Say the length of the rectangle is 10; that means that the base of the triangle, which is 40 percent smaller, is 6. If you plug 4 in for the width of rectangle R, the height of triangle T is 6. You should come up with two sketches that look like this:



T has an area of $\frac{1}{2}bh$, or 18. R has an area of 40. Now set up the translation: $18 = \frac{x}{100}(40)$, where x represents what percent the triangle is of the rectangle.

Solve for x and you get 45. The correct answer is (B).

Drill 1: Geometry

Answers and explanations can be found starting on page 574.

1 Mark for Review

If a rectangular swimming pool has a volume of 16,500 cubic feet, a uniform depth of 10 feet, and a length of 75 feet, what is the width of the pool, in feet?

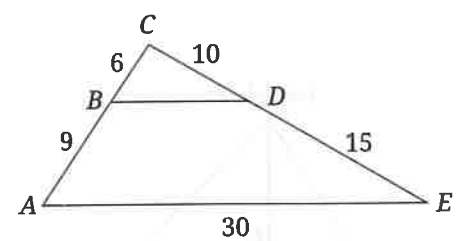
Ⓐ 22

Ⓑ 26

Ⓒ 32

Ⓓ 110

2 Mark for Review



In the figure above, $\overline{AE}$ is parallel to $\overline{BD}$. what is the length of $\overline{BD}$?

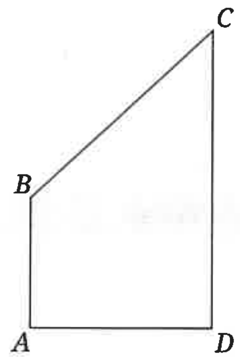
Ⓐ 8

Ⓑ 9

Ⓒ 12

Ⓓ 15

3 Mark for Review



In the quadrilateral above, the ratio of side length $\overline{AD}$ to $\overline{BC}$ is 5 to 7. If the length of $\overline{AD}$ is decreased by 1, how must the length of $\overline{BC}$ change to maintain this ratio?

(A) It must decrease by 1 unit.

(B) It must increase by 1 unit.

(C) It must decrease by 1.4 units.

(D) It must increase by 1.4 units.

4 Mark for Review

Cube X has a side length 22 times the side length of cube Y. How many times greater is the volume of cube X than that of cube Y?

5 Mark for Review

In right triangle ABC, angle A is a right angle, $AB = x$, and $BC = y$. The value of AC can be represented by which of the following expressions?

(A) $\sqrt{y - x}$

(B) $\sqrt{y^2 - x^2}$

(C) $\sqrt{x^2 + y^2}$

(D) $y - x$

6 Mark for Review

A pyramid has a height of 4 centimeters (cm) and a regular hexagonal base with an area of 15 cm^2. If the pyramid has a mass of 21.129 grams (g), what is the density of the pyramid in $\frac{g}{cm^3}$?

(A) 1.06

(B) 1.18

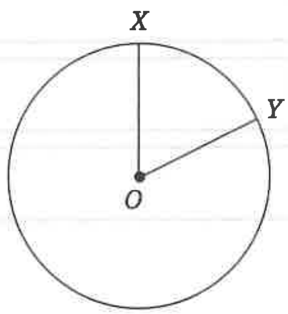
(C) 2.09

(D) 6.51

Drill 2: Trigonometry

Answers and explanations can be found on page 576.

1 Mark for Review



In the figure above, circle O has a radius of 8, and angle XOY measures $\frac{5}{16}\pi$ radians. What is the measure of minor arc XY?

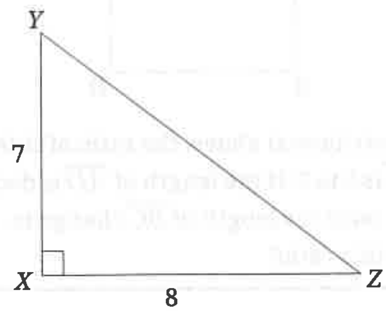
Ⓐ $\frac{5}{16}\pi$

Ⓑ $\frac{5}{2}\pi$

Ⓒ 5π

Ⓓ 16π

2 Mark for Review



What is the value of $\tan(Z)$?

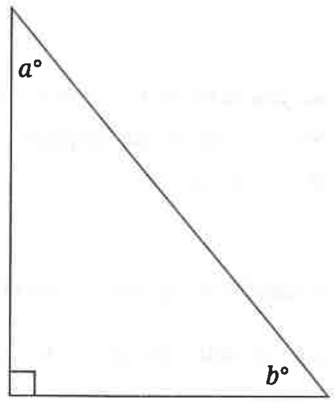
Ⓐ $\frac{7\sqrt{115}}{115}$

Ⓑ $\frac{8\sqrt{115}}{115}$

Ⓒ $\frac{7}{8}$

Ⓓ $\frac{8}{7}$

3 Mark for Review



In the figure above, $\sin a = x$. What is the value of $\cos b$?

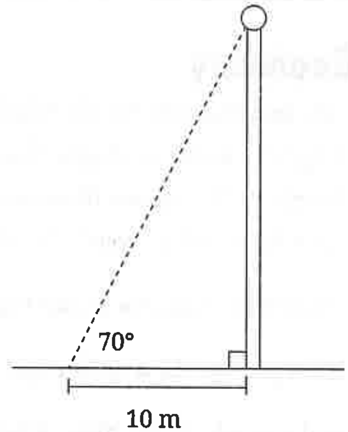
Ⓐ x

Ⓑ $\frac{1}{x}$

Ⓒ $|1-x|$

Ⓓ $\frac{90-x}{90}$

4 Mark for Review



Martin wants to know how tall a certain flagpole is. Martin walks 10 meters from the flagpole, lies on the ground, and measures an angle of 70° from the ground to the base of the ball at the top of the flagpole. Approximately how tall, in meters, is the flagpole from the ground to the base of the ball at the top of the flagpole?

Ⓐ 3 meters

Ⓑ 9 meters

Ⓒ 27 meters

Ⓓ 29 meters

DRILL 1 AND DRILL 2 ANSWERS AND EXPLANATIONS

Drill 1: Geometry

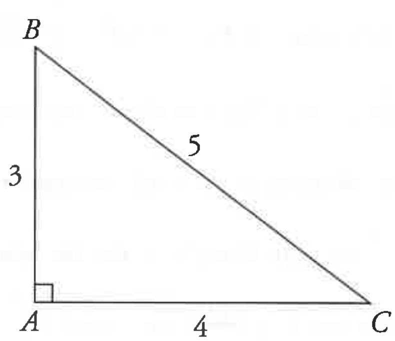
1. **A** The question asks for the width of the pool. For this question, you need to know that volume equals *length* × *width* × *height*. You know that the volume is 16,500, the depth (or height) is 10, and the length is 75. Just put those numbers into the formula: $16{,}500 = 75 \times w \times 10$. Use your calculator to solve for w, which equals 22. The correct answer is (A).

2. **C** The question asks for a value in a geometric figure. Use the Geometry Basic Approach. Start by redrawing the figure on the scratch paper. Split the figure into two triangles to make things clearer: triangle ACE and triangle BCD. Next, label the figure with the given information. The only new information is that $\overline{AE}$ is parallel to $\overline{BD}$. Use the knowledge that parallel lines cut by a third line create big and small angles, and label angles A and B as congruent and angles D and E as congruent. Triangles ACE and BCD have the same angle C, which means they have three congruent angles and are similar triangles. The sides of similar triangles have an equivalent ratio, so set up a proportion. First, add CD to DE to get the length of CE: $10 + 15 = 25$. Now use the ratio of CD to CE to find the ratio of BD to AE. The proportion is $\frac{10}{25} = \frac{x}{30}$. Cross-multiply to get $(25)(x) = (10)(30)$, which becomes $25x = 300$. Divide both sides of the equation by 25 to get $x = 12$. The correct answer is (C).

3. **C** The question asks for the change in a value given a proportion. Use the Geometry Basic Approach. Start by redrawing the figure on the scratch paper. Next, label the figure with information from the question. Since no specific numbers are given, only a ratio, plug in. Label AD as 5 and BC as 7. Draw the figure a second time and label it based on the change described in the question. Since AD is decreased by 1, label AD on the second figure as 4. Since AD decreased and the ratio stays the same, BC must also decrease. Eliminate (B) and (D) because they would both make BC longer. To find the new length of BC, set up a proportion for $\frac{AD}{BC}: \frac{5}{7} = \frac{4}{x}$. Cross-multiply to get $(7)(4) = (5)(x)$, which becomes $28 = 5x$. Divide both sides of the equation by 5 to get $x = 5.6$. Since the original length of BC was 7, the change is $7 - 5.6 = 1.4$. Eliminate (A) because it gives the wrong value for the decrease. The correct answer is (C).

4. **10648** The question asks for the relationship between the volumes of two geometric figures. Use the Geometry Basic Approach. Draw two cubes—or at least two squares to represent one face of each cube—on the scratch paper. Next, label the figure with the given information. No specific values are given, so plug in and label the side length of cube Y as 2. Since the side length of cube X is 22 times that of cube Y, label the side length of cube X as 44. The reference sheet doesn't give the formula for the volume of a cube, but it does give the volume of a rectangular solid: $V = lwh$. All three sides of a cube are the same length, so the formula becomes $V = s^3$. Plug in the two side lengths to determine the volume of each cube. The volume of cube Y becomes $V = 2^3$, or $V = 8$. The volume of cube X becomes $V = 44^3$, or $V = 85{,}184$. To answer the final question, divide the volume of cube X by the volume of cube Y to get $\frac{85{,}184}{8} = 10{,}648$. Leave out the comma when entering the answer in the fill-in box. The correct answer is 10648.

5. **B** The question asks for an expression that represents a value in a geometric figure. Use the Geometry Basic Approach. Start by drawing a triangle on the scratch paper with a right angle at A. Next, label the figure with the information given. No specific values are given, so plug in. To make things easier, use one of the Pythagorean triples and make it a 3-4-5 right triangle. The drawing should look something like this:



Write down $x = 3$ and $y = 5$ to keep track of which variable has which value. The question asks for AC, which is 4. This is the target value; write it down and circle it. Now plug $x = 3$ and $y = 5$ into the answer choices and eliminate any that do not match the target value. Choice (A) becomes $\sqrt{5-3} = \sqrt{2}$. This does not match the target value of 4, so eliminate (A). Choice (B) becomes $\sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16} = 4$. This matches the target, so keep (B), but check the remaining answers just in case. Choice (C) becomes $\sqrt{3^2 + 5^2} = \sqrt{9+25} = \sqrt{36} = 6$; eliminate (C). Choice (D) becomes $5 - 3 = 2$; eliminate (D). The correct answer is (B).

6. **A** The question asks for the density of a geometric figure. Start by determining the volume of the pyramid. Look up the formula for the volume of a right regular pyramid in the reference sheet and write it down: $V = \frac{1}{3}lwh$. The base of a pyramid is the length times the width—or the area of the base—and the area of the base is given as 15 cm^2, so plug in 15 cm^2 for *lw*. Also plug in 4 cm for *h*. The formula becomes $V = \frac{1}{3}(15\text{ cm}^2)(4\text{ cm})$, or V = 20 cm^3. Next, use the units to figure out how to solve for the density. Since density is in $\frac{\text{g}}{\text{cm}^3}$, the mass is in g, and the volume is in cm^3, density must be mass divided by volume. This happens to be the formula for density, which can be written as $D = \frac{m}{V}$. Plug in m = 21.129 and V = 20 to get $D = \frac{21.129}{20}$, or D = 1.05645. This rounds to 1.06, which is an answer choice. The correct answer is (A).

Drill 2: Trigonometry

1. **B** The question asks for the measure of minor arc *XY*. Since the question gives you the measure of the central angle in radians, you can use the formula $s = r\theta$ to find the arc length: $s = (8)\left(\frac{5}{16}\pi\right) = \frac{40}{16}\pi$, which reduces to $\frac{5}{2}\pi$. The correct answer is (B).
2. **C** The question asks for the tangent of an angle, which is defined as $\frac{\text{opposite}}{\text{adjacent}}$. The side opposite angle Z is 7, and the side adjacent to this angle Z is 8, so tan(Z) = $\frac{7}{8}$. The correct answer is (C).
3. **A** The question asks for the value of cos b. You can plug in when you're dealing with a geometry question with unknowns. When you're plugging in for a right triangle, use one of the special right triangles to make your life easier. Use a 3-4-5 right triangle. Make the side opposite a 3, the side adjacent to a 4, and the hypotenuse 5. Because sine is $\frac{\text{opposite}}{\text{hypotenuse}}$, $\sin a = \frac{3}{5}$, so $x = \frac{3}{5}$. Cosine is $\frac{\text{adjacent}}{\text{hypotenuse}}$, so $\cos b = \frac{3}{5}$. This is your target; circle it. Make $x = \frac{3}{5}$ in each answer choice and look for the answer that equals $\frac{3}{5}$. Only (A) works, so the correct answer is (A).
4. **C** The question asks how tall the flagpole is. Use SOHCAHTOA and your calculator to find the height of the flagpole. From the 70° angle, you know the adjacent side of the triangle, and you want to find the opposite side, so you need to use tangent. Tangent = $\frac{\text{opposite}}{\text{adjacent}}$, so tan 70° = $\frac{x}{10\text{m}}$, where x is the height of the flagpole up to the ball. Isolate x by multiplying both sides by 10: 10 tan 70° = x. Use your calculator to find that 10 tan 70° = 27.47, which is closest to 27. The correct answer is (C).

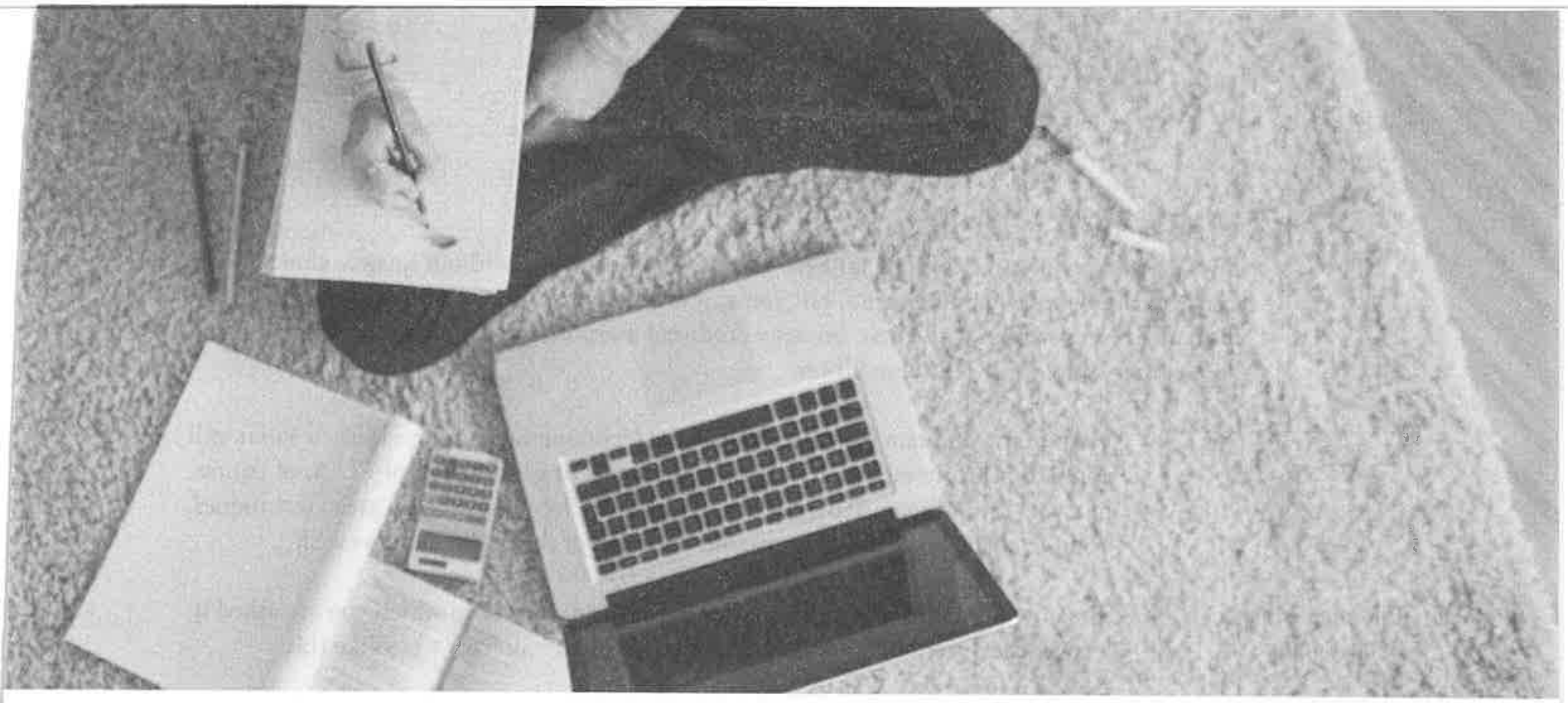
Summary

- **Degrees and angles**
  - A circle contains 360°.
  - When you think about angles, remember circles.
  - A line is a 180° angle.
  - When two lines intersect, four angles are formed; the sum of their measures is 360°.
  - When two parallel lines are cut by a third line, the small angles are equal, the big angles are equal, and the sum of a big angle and a small angle is 180°.

- **Triangles**
  - Every triangle contains 180°.
  - An isosceles triangle is one in which two of the sides are equal in length, and the two angles opposite the equal sides are equal in measure.
  - An equilateral triangle is one in which all three sides are equal in length, and all three angles are equal in measure (60°).
  - The area of a triangle is $\frac{1}{2}bh$.
  - The height of a triangle must form a right angle with the base.
  - The Pythagorean Theorem states that in a right triangle, the square of the hypotenuse equals the sum of the squares of the two legs. Remember the test-writers' favorite Pythagorean triples (3-4-5 and 5-12-13).
  - Remember the other special right triangles: 45°-45°-90° and 30°-60°-90°.
  - Similar triangles have the same angles, and their lengths are in proportion.
  - Congruent triangles have the same angles and their side lengths are exactly the same.

  - For trigonometry questions, remember SOHCAHTOA:
    - $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
    - $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
    - $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

- **Circles**
  - The circumference of a circle is $2\pi r$ or πd, where r is the radius of the circle and d is the diameter.
  - The area of a circle is πr^2, where r is the radius of the circle.
  - A tangent touches a circle at one point; any radius that touches that tangent forms a 90° angle with the tangent.
  - Arcs are proportional to the circumference based on the central angle: $\frac{\text{central angle}}{360°} = \frac{\text{arc length}}{2\pi r}$.
  - Sectors are proportional to the area based on the central angle: $\frac{\text{central angle}}{360°} = \frac{\text{sector area}}{\pi r^2}$.
  - If the central angle is given in radians, the measure of the arc is given by $s = r\theta$.
- **Rectangles and squares**
  - The perimeter of a rectangle is the sum of the lengths of its sides.
  - The area of a rectangle is *length* × *width*.
  - A square is a rectangle whose four sides are all equal in length.
  - The volume of a rectangular solid is *length* × *width* × *height*. The formulas to compute the volumes of other three-dimensional figures are supplied in the reference sheet that can be opened at any time in the testing app.
- When you encounter a geometry question on the Digital SAT, ballpark the answer before trying to work it out.
- Always draw the figure on your scratch paper.
- When information is missing from a diagram, ballpark or plug in.



Chapter 26
Fill-Ins

On the Digital SAT, approximately 11 of the 44 Math questions will require you to produce your own answer. Although the format of these questions is different from that of the multiple-choice questions, the mathematical concepts tested aren't all that different. In this chapter, we'll show you how to apply what you have learned in the previous chapters to answering fill-in questions.

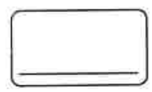
WHAT IS A FILL-IN?

Both Math modules on the Digital SAT have several questions without answer choices. The exact breakdown per module varies, but you can expect to see about 11 total questions in this format. The test-writers call these Student-Produced Response questions, but we're going to keep things simple and call them fill-ins.

Different Format, Same Content
Fill-in questions test the same math topics as multiple-choice questions:
- Algebra
- Advanced Math
- Problem Solving and Data Analysis
- Geometry and Trig

Despite the different format, many of the techniques that you've learned so far still apply to fill-in questions. You can't use Process of Elimination or PITA, of course, but you can still use Plugging In, Bite-Sized Pieces, and other great techniques. Your calculator will still help you out on many of these questions, as well.

The only difficulty with fill-ins is getting used to the way in which you are asked to answer the question. For each fill-in question, you will have a box like this:



To enter your answer, click inside the box and start typing. The numbers you enter will automatically appear left to right, and the testing app will show a preview of your answer so you can make sure it looks right.

THE INSTRUCTIONS

The fill-in instructions will appear on the left side of the screen for every fill-in question. There are buttons in the middle of the screen that look like this:

You can use the buttons to make the instructions on the left and the question on the right take up more or less of the screen. The instructions and examples in the testing app look like those on the next page.

Student-produced response questions

- If you find **more than one correct answer**, enter only one answer.
- You can enter up to 5 characters for a **positive** answer and up to 6 characters (including the negative sign) for a **negative** answer.
- If your answer is a **fraction** that doesn't fit in the provided space, enter the decimal equivalent.
- If your answer is a **decimal** that doesn't fit in the provided space, enter it by truncating or rounding at the fourth digit.
- If your answer is a **mixed number** (such as $3\frac{1}{2}$), enter it as an improper fraction (7/2) or its decimal equivalent (3.5).
- Don't enter **symbols** such as a percent sign, comma, or dollar sign.

Examples

Answer	Acceptable ways to enter answer	Unacceptable: will NOT receive credit
3.5	3.5 3.50 7/2	31/2 3 1/2
$\frac{2}{3}$	2/3 .6666 .6667 0.666 0.667	0.66 .66 0.67 .67
$-\frac{1}{3}$	–1/3 –.3333 –0.333	–.33 –0.33

Fill the Space
Know what you can and can't enter into the fill-in box:

- 5 characters for a positive answer
- 6 characters for a negative answer
- Don't enter extra zeros if the answer is short
- Do enter as much of a long decimal as will fit
- Don't enter a fraction that doesn't fit
- Do enter reduced fractions that fit

You don't want to have to spend time rereading the instructions every time, so make sure you know them well. Here is all the information you need to know about entering a fill-in answer:

1. There is space to enter 5 characters if the answer is positive and 6 characters—including the negative sign—if the answer is negative.
2. You do not need to type the comma for numbers longer than three digits, such as 4,200. In fact, the testing app will not allow it.
3. The testing app also will not allow symbols such as %, $, or π. Square roots, units, and variables cannot be entered.
4. You can enter your answer as either a fraction or a decimal. For example, .5, 0.5, and 1/2 are all acceptable answers. Use the forward slash for fractions.
5. If your answer is a fraction, it must fit within the space. Do not try to enter something like $\frac{200}{500}$ as a fraction: either reduce it or convert it to a decimal. Entering 20/50, 2/5, .4, and 0.4 would all count as the correct answer.
6. Fractions do not need to be in the lowest reduced form. As long as it fits, it's fine.
7. You cannot fill in mixed numbers. Convert all mixed numbers to improper fractions or decimals. If your answer is $2\frac{1}{2}$, you must convert it to 5/2 or 2.5. If you enter 21/2, the testing app will read your answer as $\frac{21}{2}$.
8. If your answer is a decimal that will not fit in the space provided, either enter as many digits as will fit or round the last digit. The fraction $-\frac{2}{3}$ can be entered in decimal form as –0.666, –0.667, –.6666, or –.6667.
9. Some questions will have more than one right answer. Any correct answer you enter will count as correct; do not try to enter multiple answers.

FILL-INS: A TEST DRIVE

To get a feel for this format, let's work through two examples. As you will see, fill-in questions are just regular Digital SAT Math questions.

1 Mark for Review

If $a + 2 = 6$ and $b + 3 = 21$, what is the value of $\frac{b}{a}$?

Here's How to Crack It

The question asks for the value of $\frac{b}{a}$. You need to solve the first equation for a and the second equation for b. Start with the first equation, and solve for a. By subtracting 2 from both sides of the equation, you should see that $a = 4$.

Now move to the second equation, and solve for b. By subtracting 3 from both sides of the second equation, you should see that $b = 18$.

The question asked you to find the value of $\frac{b}{a}$. That's easy. The value of b is 18, and the value of a is 4. Therefore, the value of $\frac{b}{a}$ is $\frac{18}{4}$.

That's an odd-looking fraction. How in the world do you fill it in? Ask yourself this question: "Does $\frac{18}{4}$ fit?" Yes! Fill in $\frac{18}{4}$.

18/4

Your math teacher wouldn't like it, but the scoring computer will. You shouldn't waste time reducing $\frac{18}{4}$ to a prettier fraction or converting it to a decimal. Spend that time on another question instead. The fewer steps you take, the less likely you will be to make a careless mistake.

2 Mark for Review

The radius of circle O is 212 times the radius of circle P. If the area of circle O is t times the area of circle P, what is the value of t?

Here's How to Crack It

The question asks for the relationship between the areas of two geometric figures. It doesn't matter that this is a fill-in instead of a multiple-choice question: you still start with the Geometry Basic Approach. Draw two circles on your scratch paper. Next, label the figure. Mark the center of one circle as O and the center of the other as P, and draw in the radius of each circle. The question doesn't give you any numbers for the radius or area of circle P, so plug in. Make the radius 2 and label that on the figure. Finally, write down formulas. The area of a circle is given by $A = \pi r^2$. Plug in $r = 2$ to get $A = \pi(2)^2$, or $A = 4\pi$ for circle P.

The question states that *the radius of circle O is 212 times the radius of circle P*, so multiply 2 by 212 to get $r = (2)(212) = 424$. Label the radius of circle O as 424. Plug $r = 424$ into the area formula to get $A = \pi(424)^2$, or $A = 179{,}776\pi$. To solve for t, divide the area of circle O by the area of circle P to get $t = \frac{179{,}776\pi}{4\pi}$, or $t = 44{,}944$. This is a big number! However, it's still only 5 characters long, so it will fit in the fill-in box. The fill-in box doesn't accept commas, so don't worry about that. The correct answer is 44944.

44944

MORE POOD

The fill-in questions are mixed in with the multiple-choice questions, and both math modules have an approximate order of difficulty. More important than the question order is your Personal Order of Difficulty (POOD), a strategy that encourages you to focus on the questions you know how to answer first. Don't spend too much time on a question you are unsure about, no matter which format it is.

Keep in mind, of course, that many of the math techniques that you've learned are still very effective on fill-in questions. The Geometry Basic Approach and Plugging In both worked well on the previous question. If you're able to plug in or take an educated guess, go ahead and fill in that answer. As always, there's no penalty for getting it wrong.

Here's another fill-in question that you can answer by using a technique you've learned before.

3 Mark for Review

Town A has 2,200 residents. The mean age of the residents of Town A is 34. Town B has 3,680 residents with a mean age of 40. What is the mean age of the residents of Town A and Town B combined?

Here's How to Crack It

The question asks for a mean, or average. Work the question in bite-sized pieces and start with Town A. For averages, use the formula $T = AN$, in which T is the *Total*, A is the *Average*, and N is the *Number of things*. The question states that *Town A has 2,200 residents*, so that is the *Number of things*. The question also states that the *mean age of the residents of Town A is 34*, so that is the *Average*. Plug these numbers into the average formula to get $T = (34)(2{,}200)$, or $T = 74{,}800$. Do the same thing for Town B: the *Number of things* is 3,680 residents, and the *Average* is the mean age of 40, so the formula becomes $T = (40)(3{,}680)$, or $T = 147{,}200$.

Next, add the two totals to get $74{,}800 + 147{,}200 = 222{,}000$. This is the *Total* for the two towns combined. The *Number of things* for the two towns combined is $2{,}200 + 3{,}680 = 5{,}880$ residents. Use the average formula one more time to get $222{,}000 = (A)(5{,}880)$. Divide both sides of the equation by 5,880 to get $37.7551020408 = A$.

There clearly isn't room to enter this answer in the fill-in box, so either cut it off or round when you run out of room. You can enter 37.75 or 37.76 and get the question right. Don't round too much, though: if you enter 37.8, you'll get the question wrong. Enter the full five characters to get credit for a positive answer. The correct answer is 37.75 or 37.76.

37.75 **or** 37.76

Careless Mistakes

On fill-in questions, you obviously can't use POE to get rid of bad answer choices, and Plugging In the Answers won't work either. In order to earn points on fill-in questions, you're going to have to find the answer yourself, as well as be extremely careful when you enter your answers in the fill-in box. If you need to, double-check your work to make sure you have solved correctly. If you suspect that the question is a difficult one and you get an answer too easily, you may have made a careless mistake or fallen into a trap.

Try the example below with this in mind.

4 Mark for Review

A teacher is grading two assignments that each had to be a specific length: research papers and short stories. Each research paper has 5 more pages than each short story. How many pages are in a research paper if 7 research papers and 5 short stories have a total of 275 pages?

Here's How to Crack It

The question asks for the number of pages in a research paper given other information about two assignments. Use another skill from earlier in this book and translate English to math in bite-sized pieces. The question states that *each research paper has 5 more pages than each short story.* Let r represent the number of pages in a research paper. The word *has* translates to =. The phrase *5 more than* translates to 5 +. Finally, let s represent the number of pages in a short story. The sentence, therefore, translates to $r = 5 + s$. Do the same thing with the information that *7 research papers and 5 short stories have a total of 275 pages.* Use r and s again for the number of pages in a research paper and a short story, respectively. Translate ***and*** as + and ***have a total of*** as =, and the sentence translates to $7r + 5s = 275$. You now have two equations with the same two variables:

$$r = 5 + s$$

$$7r + 5s = 275$$

Substitute $5 + s$ for r in the second equation to get

$$7(5 + s) + 5s = 275$$

Distribute the 7.

$$35 + 7s + 5s = 275$$

Combine like terms on the left side, then subtract 35 from both sides.

$$12s = 240$$

Isolate s.

$$s = 20$$

It's tempting to fill in 20 and call it a day, but always read the final question! The question asks for the number of pages in a research paper, not in a short story. Plug 20 for s into the first equation to solve for r.

$$r = 5 + 20 = 25$$

Thus, $r = 25$, so fill in that value. The correct answer is 25.

25

MULTIPLE CORRECT ANSWERS

As you've already seen, some fill-in questions will have more than one possible correct answer. It won't matter which correct answer you enter as long as it really is correct. This happens frequently when the answer is a fraction or a decimal. It can also happen when there is more than one solution to an equation.

Let's look at one of those.

5 Mark for Review

What is one possible solution to the equation $|a + 3| = 7$?

Here's How to Crack It

The question asks for a possible solution to an equation with an absolute value. With an absolute value, the value inside the absolute value bars can be either positive or negative. Set $a + 3$ equal to both 7 and -7, and solve both equations. When $a + 3 = 7$, subtract 3 from both sides of the equation to get $a = 4$. When $a + 3 = -7$, subtract 3 from both sides of the equation to get $a = -10$. Enter either 4 or -10 and you'll get the question right.

In this case, that was more work than you needed to do. The question asked for *one possible solution*, so you could have stopped after finding one value. However, questions about absolute value might ask for a specific solution—either the positive solution or the negative solution—so always read the final question (RTFQ) to make sure you don't enter a value that isn't correct.

4 **or** –10

Fill-In Drill

Answers and explanations can be found starting on page 591.

1 Mark for Review

If $a^b = 4$, and $3b = 2$, what is the value of a?

2 Mark for Review

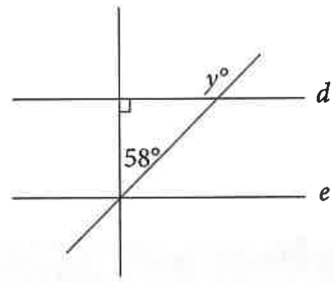
If $4x + 2y = 24$ and $\frac{7y}{2x} = 7$, what is the value of x?

3 Mark for Review

$$n = 12(2)^{\frac{t}{3}}$$

The number of mice in a certain colony is shown by the formula above, such that n is the number of mice and t is the time, in months, since the start of the colony. If 2 years have passed since the start of the colony, how many mice does the colony contain now?

4 Mark for Review



In the figure above, if d is parallel to e, what is the value of y?

5 Mark for Review

The function g is defined by $g(x) = -(x - 3)(x + 11)$. For what value of x is the value of $g(x)$ at its maximum?

6 Mark for Review

If line m is defined by the equation $-3x = -2y - 12$ and line n is parallel to line m, what is the slope of line n?

7 Mark for Review

If Alexandra pays $56.65 for a table, and this amount includes a tax of 3% on the price of the table, what is the amount, in dollars, that she pays in tax?

8 Mark for Review

In triangle ABC, where angle A is a right angle, $\sin(C)$ is $\frac{13}{85}$. What is the value of $\tan(B)$?

9 Mark for Review

The kinetic energy (KE) of a ball in motion is given by the equation $KE = \frac{1}{2}mv^2$, where m is the mass of the ball in kilograms (kg) and v is the velocity in meters per second $\left(\frac{\text{m}}{\text{s}}\right)$. A ball with a mass of 5 kg and a kinetic energy of 18.225 kg$\left(\frac{\text{m}^2}{\text{s}^2}\right)$ is to be rolled along the ground. What is the velocity of the ball in meters per second, assuming there is no friction?

10 Mark for Review

$$x(mx + 42) + 18 = 0$$

If the equation above has exactly two real solutions and m is an integer constant, what is the greatest possible value of m?

FILL-IN DRILL ANSWERS AND EXPLANATIONS

1. **8** The question asks for the value of a. Using $3b = 2$, solve for b by dividing both sides by 3 to get $b = \frac{2}{3}$. That means $a^{\frac{2}{3}} = 4$. Fractional exponents tell you to use the denominator as the root and use the numerator as a regular exponent. So, $\sqrt[3]{a^2} = 4$. First, cube both sides to find $a^2 = 4^3 = 64$. Next, take the square root of both sides to find $a = 8$. The correct answer is 8.

2. **3** The question asks for the value of x in a system of equations. To solve for one variable, find a way to make the other variable disappear when stacking and adding the equations. Start by dividing both sides of the first equation by 2 to get $2x + y = 12$. Multiply both sides of the second equation by $2x$ to get $7y = 14x$, and then subtract $7y$ from both sides of the equation to get $0 = 14x - 7y$. Now divide both sides of the equation by 7 to get $0 = 2x - y$. The y-terms in the two equations now have the same coefficient with opposite signs, so stack and add the equations.

$$\begin{array}{r} 2x + y = 12 \\ + \underline{2x - y = 0} \\ 4x \quad\ = 12 \end{array}$$

Divide both sides of the resulting equation by 4 to get $x = 3$. The correct answer is 3.

3. **3072** The question asks for a value in an equation and gives conflicting units. The question states that t is in months and asks about years. There are 12 months in 1 year, so there are $(12)(2) = 24$ months in 2 years. Thus, $t = 24$. Plug this value into the given equation to get $n = 12(2)^{\frac{24}{3}}$, which becomes $n = 12(2)^8$. Use a calculator to get $n = 12(256)$, or $n = 3{,}072$. Leave out the comma when entering the answer in the fill-in box. The correct answer is 3072.

4. **148** The question asks for the measure of an angle on a figure. Use the Geometry Basic Approach. Start by redrawing the figure on the scratch paper, including the labels. There is a triangle in the middle of the figure, and all triangles contain 180°. Right angles are 90°, so the third angle measures 180° – 90° – 58° = 32°. That angle and angle y make up a straight line, and there are 180° in a line, so $32° + y° = 180°$. Subtract 32° from both sides of the equation to get $y° = 148°$. The fill-in box does not allow degree signs, so leave it out. The correct answer is 148.

5. **–4** The question asks for the value when a quadratic function reaches its maximum. A parabola reaches its minimum or maximum value at its vertex, so find the x-coordinate of the vertex. One method is to enter the equation into a graphing calculator, then scroll and zoom as needed to find the vertex. The vertex is at (–4, 49), so the value of the x-coordinate is –4. Another way to solve is to recognize that the vertex of a parabola is on the axis of symmetry, which is the midpoint between the two x-intercepts. Find the x-intercepts by setting each factor equal to 0 and solving. When $x - 3 = 0$, $x = 3$. When

$x + 11 = 0$, $x = -11$. The x-coordinate of the midpoint is the average of the two x-intercepts, which is $\frac{3+(-11)}{2} = \frac{-8}{2} = -4$. Using either method, the correct answer is –4.

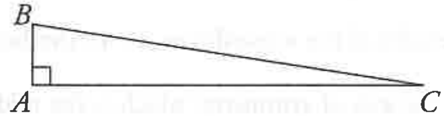
6. $\frac{3}{2}$ **or 1.5**

The question asks for the slope of a line. The question states that line m and line n are parallel, which means they have the same slope but different y-intercepts. The question gives the equation of line m, so find the slope of that line. Rearrange the equation so it is in slope-intercept form, $y = mx + b$, in which m is the slope and b is the y-intercept. Add $2y$ to both sides of the equation to get $-3x + 2y = -12$, then add $3x$ to both sides of the equation to get $2y = 3x - 12$. Finally, divide both sides of the equation by 2 to get $y = \frac{3}{2}x - 6$. The slope of line m is $\frac{3}{2}$, so the slope of line n is also $\frac{3}{2}$. The answer can be entered as a fraction or in decimal form, which is 1.5. The correct answer is $\frac{3}{2}$ or 1.5.

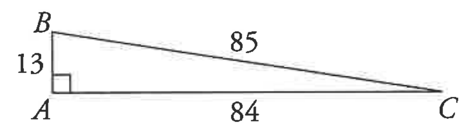
7. **1.65** The question asks for a value given a specific situation. Translate the information in bite-sized pieces. The question states that *Alexandra pays $56.65 for a table,* which includes *a tax of 3% on the price of the table.* The unknown is the price of the table before tax, so make that x. *Percent* means out of 100, so translate 3% as $\frac{3}{100}$. The tax is added to the cost before tax, so add those two values and set them equal to the total cost of $56.65. The equation becomes $x + \left(\frac{3}{100}\right)x = \$56.65$. Simplify the left side of the equation to get $x + 0.03x = \$56.65$, and then $1.03x = \$56.65$. Divide both sides of the equation by 1.03 to get $x = \$55.00$. This is the cost before tax. The question asks for the amount of tax, *in dollars*, so subtract the cost before tax from the final cost to get $\$56.65 - \$55.00 = \$1.65$. The fill-in box does not allow dollar signs, so leave it out. The correct answer is 1.65.

8. $\frac{84}{13}$ **or 6.461 or 6.462**

The question asks for the value of a trigonometric function. Use the Geometry Basic Approach. Begin by drawing a right triangle. Next, label the vertices and label the right angle as angle A. The drawing should look something like this:



Next, write out SOHCAHTOA to remember the trig functions. The SOH part defines the sine as $\frac{\text{opposite}}{\text{hypotenuse}}$, and the question states that $\sin(C) = \frac{13}{85}$. Label the side opposite angle C, which is $\overline{AB}$, as 13 and the hypotenuse, which is $\overline{BC}$, as 85. To find the length of the third side, use the Pythagorean Theorem: $a^2 + b^2 = c^2$. Plug in the known values to get $13^2 + b^2 = 85^2$. Square the numbers to get $169 + b^2 = 7{,}225$, then subtract 169 from both sides of the equation to get $b^2 = 7{,}056$. Take the square root of both sides of the equation to get $b = 84$. With all three side lengths labeled, the drawing looks like this:



To find $\tan(B)$, use SOHCAHTOA again. The TOA part defines the tangent as $\frac{\text{opposite}}{\text{adjacent}}$. The side opposite angle B is 84, and the side adjacent to angle B is 13, so $\tan(B) = \frac{84}{13}$. The answer can also be entered in the fill-in box as a decimal. In this case, $\frac{84}{13} = 6.\overline{461538}$, which is too long. Either stop when there's no more room and enter 6.461 or round the last digit and enter 6.462. The correct answer is $\frac{84}{13}$ or equivalent forms.

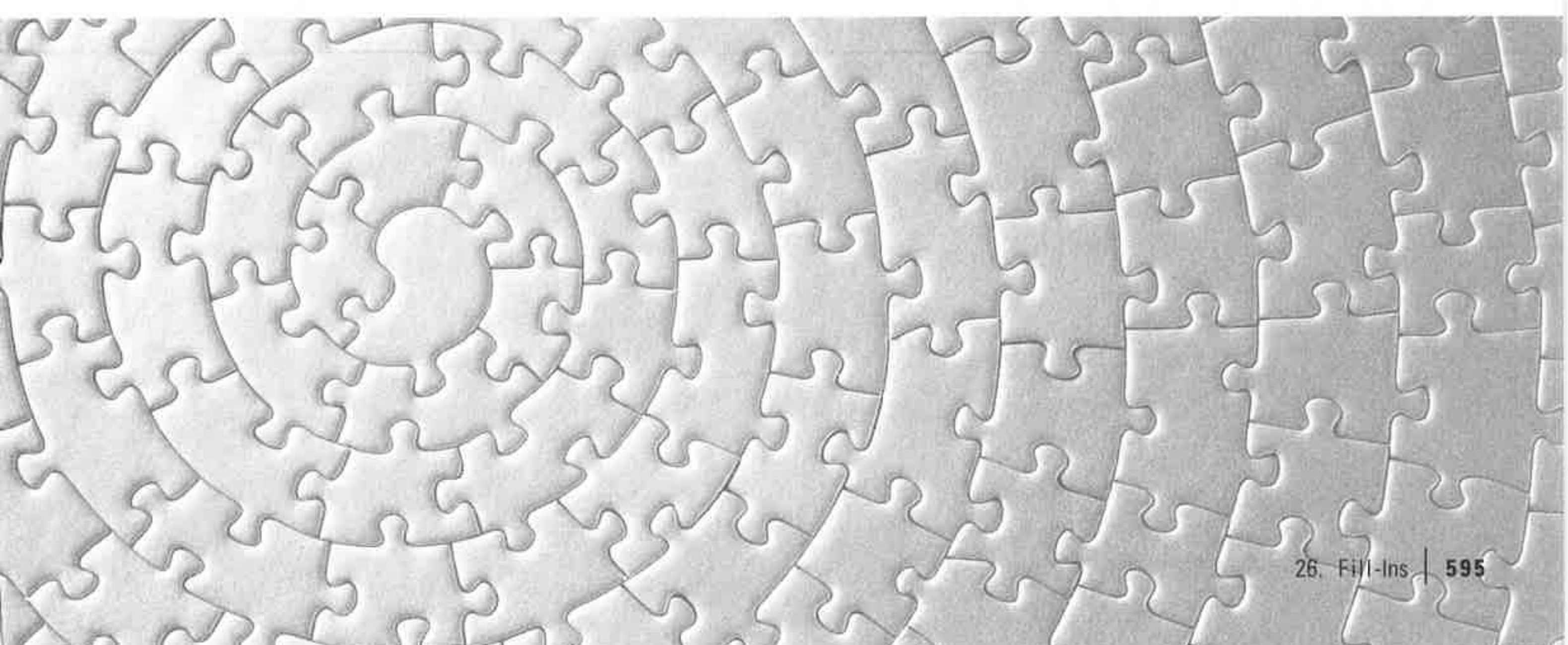
9. **2.7** The question asks for a value given an equation. Plug in the values given in the question and solve for the other value. The question states that the ball has a mass of 5 kg, and that m represents mass, so plug in $m = 5$. The question also states that the kinetic energy is 18.225, and that KE represents kinetic energy, so plug in $KE = 18.225$. The equation becomes $18.225 = \frac{1}{2}(5)(v)^2$. Multiply both sides of the equation by 2 to get $36.45 = (5)(v)^2$. Divide both sides of the equation by 5 to get $7.29 = v^2$, then take the square root of both sides of the equation to get $2.7 = v$. The correct answer is 2.7.

10. **24** The question asks for the value of a constant in a quadratic equation. Start by distributing the x on the left side of the equation to get $mx^2 + 42x + 18 = 0$. The question states that the system *has exactly two real solutions*. To determine the number of solutions to a quadratic, use the discriminant. The discriminant is the part of the quadratic formula under the square root sign, and it can be written as $D = b^2 - 4ac$. When the discriminant is positive, the quadratic has exactly two real solutions; when the discriminant is 0, the quadratic has exactly one real solution; and when the discriminant is negative, the quadratic has no real solutions. Since this quadratic has exactly two real solutions, the discriminant must be positive.

The quadratic is now in standard form, $ax^2 + bx + c = 0$, so $a = m$, $b = 42$, and $c = 18$. Plug these into the discriminant formula to get $D = 42^2 - 4(m)(18)$, which becomes $D = 1{,}764 - 72m$. In order for D to be positive, the result must be greater than 0, so write the inequality $1{,}764 - 72m > 0$. Add $72m$ to both sides of the inequality to get $1{,}764 > 72m$, and then divide both sides of the inequality by 72 to get $24.5 > m$. The question states that m is an integer, so the greatest possible value of m is 24. The correct answer is 24.

Summary

- Both of the Math modules on the Digital SAT contain several questions without answer choices. The test-writers call these questions "student-produced responses." We call them fill-ins because you have to fill in your own answer.
- Despite their format, fill-ins are really just like other Math questions on the Digital SAT, and many of the same techniques that you have learned still apply.
- The fill-in questions and multiple-choice questions are mixed together in a loose order of difficulty. Use your knowledge of your own strengths and weaknesses to decide which ones to tackle first and which ones, if any, to skip.
- The fill-in format increases the likelihood of careless errors. Know the instructions and check your work carefully.
- Just like the rest of the Digital SAT, there is no guessing penalty for fill-ins, so you should always fill in an answer, even if it's a guess.
- Enter only one answer even if the question has multiple possible answers. It doesn't matter which answer you enter, as long as it's one of the possible answers.
- Enter up to 5 characters when the answer is positive. Enter up to 6 characters, including the negative sign, when the answer is negative.
- The characters that can be entered are the digits 0–9, the negative sign, the forward slash (/) for fractions, and the decimal point. Special characters such as % or π cannot be entered.
- If the answer to a fill-in question contains a fraction or decimal, you can enter the answer in either form. Use whichever form is easier and less likely to cause mistakes.
- If your answer is a fraction that doesn't fit in the space, either reduce the fraction or convert it to a decimal.



- If a fraction fits in the space, you don't have to reduce the fraction before entering it.
- Do not enter mixed numbers. Convert mixed numbers to fractions or decimals before entering your answer.
- If your answer is a long or repeating decimal, fill up all of the space. Either keep entering digits until the space is full or round the last digit that will fit.