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COLUMBIA ACADEMY

enrichment beyond the classroom

Written by Jaehoon Song (Lecturer)

1. **Circle Equations** (10 points)
   Circle A in the xy-plane has the equation $(x + 5)^2 + (y - 5)^2 = 4$. Circle B has the same center as circle A. The radius of circle B is two times the radius of circle A. The equation defining circle B in the xy-plane is $(x + 5)^2 + (y - 5)^2 = k$, where k is a constant. What is the value of k?

   Answer:

   □

2. **Circle Diameter** (10 points)
   What is the diameter of the circle in the xy-plane with equation $(x - 5)^2 + (y - 3)^2 = 16$?

   (A) 4

   (B) 8

   (C) 16

   (D) 32

   Answer:

   □

3. **Arc and Angle Measure** (10 points)
   Point O is the center of a circle. The measure (central angle) of arc RS on this circle is $100°$. What is the measure, in degrees, of its associated angle (major-minor relationship) ROS?

   Answer:

   □

4. **Circle Radius** (10 points)
   The equation $(x + 6)^2 + (y + 3)^2 = 121$ defines a circle in the xy-plane. What is the radius of the circle?

   Answer:

   □

5. **Tangent Line Slope** (10 points)

A circle in the xy-plane has its center at $(-4, -6)$. Line k is tangent to this circle at the point $(-7, -7)$. What is the slope of line k?

(A) -3

(B) $-\frac{1}{3}$

(C) $\frac{1}{3}$

(D) 3

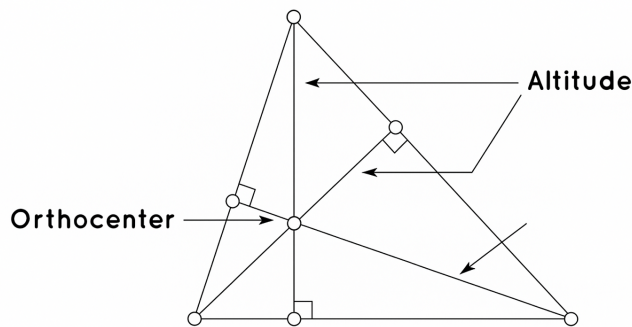**Answer**:

☐

6. **Triangle Area** (10 points)

Figure 1: reference attached

In triangle ABC, the measure of angle B is $90°$ and $\overline{BD}$ is an **altitude** of the triangle. The length of $\overline{AB}$ is 15 and the length of $\overline{AC}$ is 23 greater than the length of $\overline{AB}$. What is the value of $\frac{BC}{BD}$?

(A) $\frac{15}{38}$

(B) $\frac{15}{23}$

(C) $\frac{23}{15}$

(D) $\frac{38}{15}$

Answer:

☐

7. **Triangle Angle** (10 points)

In $\triangle XYZ$, the measure of $\angle X$ is $24°$ and the measure of $\angle Y$ is $98°$. What is the measure of $\angle Z$?

(A) $58°$

(B) $74°$

(C) $122°$

(D) $212°$

Answer:

☐

8. **Tree Height** (10 points)

Two nearby trees are perpendicular to the ground, which is flat. One of these trees is 10 feet tall and has a shadow that is 5 feet long. At the same time, the shadow of the other tree is 2 feet long. How tall, in feet, is the other tree?

(A) 3

(B) 4

(C) 8
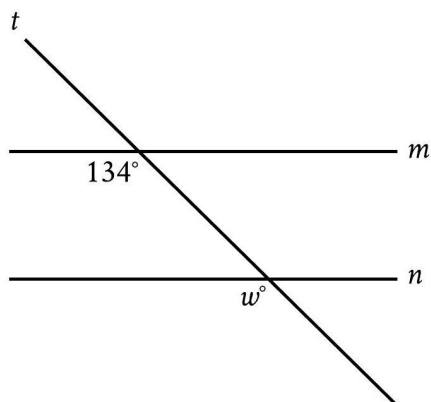
(D) 27

Answer:

☐

9. **Parallel Lines** (10 points)

Figure 2: reference attached

In the figure, line m is parallel to line n. What is the value of w?

(A) 13

(B) 34

(C) 66

(D) 134

Answer:

☐

10. **Triangle Congruence** (10 points)

In triangles ABC and DEF, angles B and E each have measure $27°$ and angles C and F each have measure $41°$. Which additional piece of information is sufficient to determine whether triangle ABC is congruent to triangle DEF?

(A) The measure of angle A

(B) The length of side AB

(C) The lengths of sides BC and EF

(D) No additional information is necessary

Answer:

☐

11. **Triangle Similarity** (10 points)

In triangles LMN and RST, angles L and R each have measure $60°$, $LN = 10$, and $RT = 30$. Which additional piece of information is sufficient to prove that triangle LMN is similar to triangle RST?

(A) $MN = 7$ and $ST = 7$

(B) $MN = 7$ and $ST = 21$

(C) The measures of angles M and S are $70°$ and $60°$, respectively.

(D) The measures of angles M and T are $70°$ and $50°$, respectively.

Answer:

☐

12. **Triangle Ratio** (10 points)
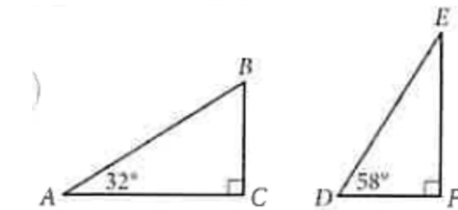
Figure 3: reference attached

Triangles ABC and DEF are shown above. Which of the following is equal to the ratio $\frac{BC}{AB}$?

(A) $\frac{DE}{DF}$

(B) $\frac{DF}{DE}$

(C) $\frac{DF}{EF}$

(D) $\frac{EF}{DE}$
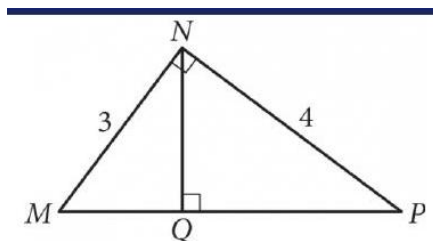
Answer:

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13. **Circle Length** (10 points)

Figure 4: reference attached

In the figure above, what is the length of $\overline{NQ}$?

(A) 2.2

(B) 2.3

(C) 2.4

(D) 2.5

Answer:

☐

14. **Triangle Angle** (10 points)

Triangle XYZ is similar to triangle RST such that X, Y, and Z correspond to R, S, and T, respectively. The measure of $\angle Z$ is $20°$ and $2XY = RS$. What is the measure of $\angle T$?

(A) $2°$

(B) $10°$

(C) $20°$

(D) $40°$

Answer:
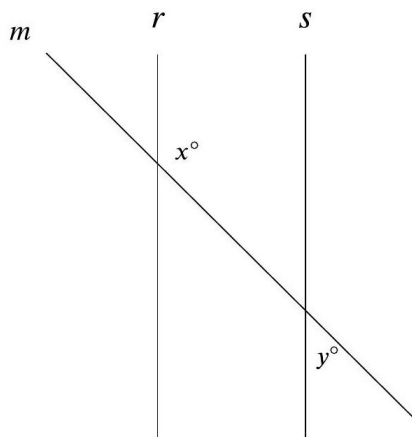
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15. **Parallel Lines** (10 points)

Figure 5: reference attached

Note: Figure not drawn to scale.

In the figure shown, lines r and s are parallel, and line m intersects both lines. If $y < 65$, which of the following must be true?

(A) $x < 115$

(B) $x > 115$

(C) $x + y < 180$

(D) $x + y > 180$

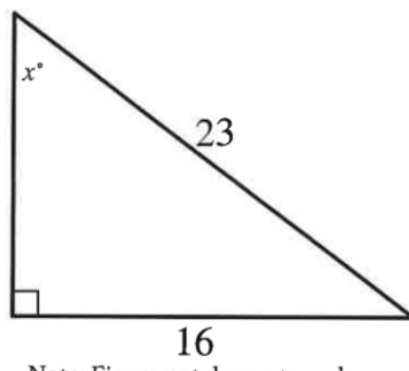**Answer**:

16. **Trigonometry** (10 points)

Figure 6: reference attached

In the triangle shown, what is the value of $\sin x^\circ$?
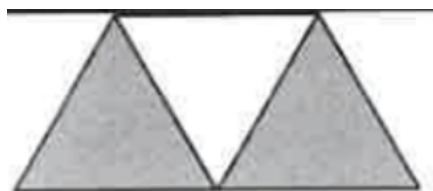
Answer:

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17. **Logo Area** (10 points)

Figure 7: reference attached

A graphic designer is creating a logo for a company. The logo is shown in the figure above. The logo is in the shape of a trapezoid and consists of three congruent equilateral triangles. If the perimeter of the logo is 20 centimeters, what is the combined area of the shaded regions, in square centimeters, of the logo?

(A) $2\sqrt{3}$
(B) $4\sqrt{3}$
(C) $8\sqrt{3}$
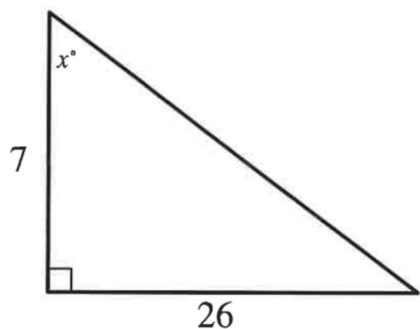(D) 16

Answer:

☐

18. **Triangle Tangent** (10 points)

Figure 8: reference attached

In the triangle shown, what is the value of $\tan x°$?

(A) $\frac{1}{26}$

(B) $\frac{19}{26}$

(C) $\frac{26}{7}$

(D) $\frac{33}{7}$

Answer:

☐

19. **Triangle Height** (10 points)
The perimeter of an equilateral triangle is 624 centimeters. The height of this triangle is $k\sqrt{3}$ centimeters, where k is a constant. What is the value of k?

Answer:

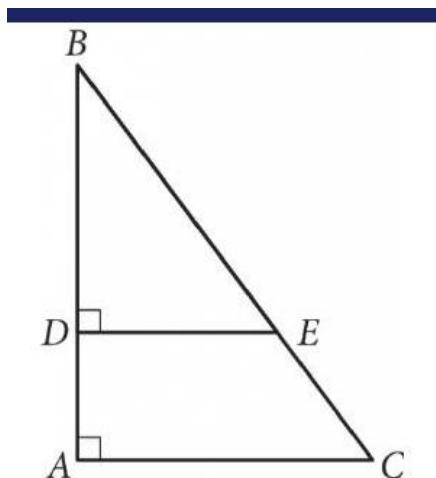☐

20. **Triangle Length** (10 points)

Figure 9: reference attached

In the figure above, $\tan B = \frac{3}{4}$. If $BC = 15$ and $DA = 4$, what is the length of $\overline{DE}$?

Answer:

□