

Advanced Math

(13-15 questions, about 35%)

Topics: Equivalent equations, non-linear equations in one variable and systems of equations in two variables, non-linear functions. Solving rational equations and graphs of rational and polynomial functions.

- **Equivalent equations:** The expressions on both sides of the given equation are always identical for all values of the given variables. It means that the equation is true for all values of the variable.

(Practice problems for Advanced math)

1) $x^3 + 2x^2 - 4x + 1 = (x + b)(x^2 + cx - 1)$

In the given equation above, b and c are constants. If the equation is true for all values of x , what is the value of $b + c$?

- A) 2
- B) 3
- C) 1
- D) -2

2) Which of the following is equivalent to $3b^2 + 18a^4 - 5b^2$?

- A) $2(3a - b)(3a + b)$
- B) $2(3a - b^2)(3a + b^2)$
- C) $3(2a^2 - b)(2a^2 + b)$
- D) $2(3a^2 - b)(3a^2 + b)$

- **Quadratic Equation:**

i. Standard form $y = ax^2 + bx + c$.

Vertex (h, k) : $h = -\frac{b}{2a}$. And substitute h into the equation to find the value of k .

Axis of symmetry: $x = h$. (In SAT math, very useful information to find the unknown Y values or unknown x -intercept.)

Shape: $a > 0$: opens upwards. $a < 0$: opens downwards.

This form shows the y -intercept: the value of c .

ii. Vertex form $y = a(x - h)^2 + k$.

Vertex (h, k) . when $a > 0$, the graph has a minimum value of k at $x = h$.

When $a < 0$, the graph has a maximum value of k at $x = h$.

This form shows the maximum/minimum values of the function.

iii. Zeros form $y = a(x - x_1)(x - x_2)$.

Vertex (h, k) : $h = \frac{x_1 + x_2}{2}$ (the mid-point of x_1 and x_2) And substitute h into the equation to find the value of k .

This form shows two x -intercepts (zeros) in the equation.

- How to find solutions of quadratic equations (x-intercepts or zeros)

- Factor if possible.

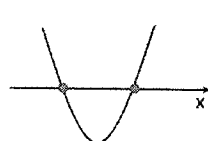
$$a(x - x_1)(x - x_2) = 0. \text{ Then, } x = x_1 \text{ or } x = x_2.$$

- Use the quadratic formula.

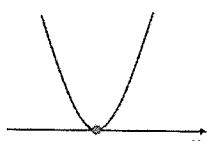
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Note: $D(\text{Discriminant}) = b^2 - 4ac$.

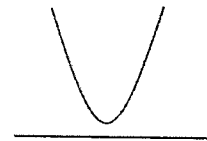
The value of D will tell you how many real solutions the equation will have.



Two real roots
 $D > 0$



One real root
 $D = 0$



No real root
 $D < 0$

- Complete the square.

$$x^2 + bx + c = 0. \text{ Add } \left(\frac{b}{2}\right)^2 \text{ on both sides of the equation. And factor it.}$$

- Sum or Product of two roots of a quadratic equation in standard form (known as Fundamental theorem of algebra)

$$y = ax^2 + bx + c$$

$$\text{Sum} = -\frac{b}{a} \quad \text{Product} = \frac{c}{a}$$

(Quadratic Equations/Functions Practice Problems)

- 3) What is the value of m if $20x^2 + mx - 21 = (5x + a)(bx + 7)$, where a, b , and m are constant?

- 13
- 13
- 23
- 23

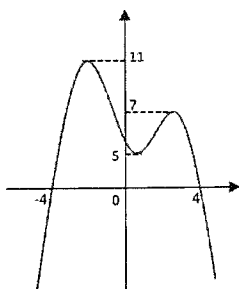
- 4) $f(x) = kx^2 + 2$, where k is a constant. what is the same value of $f(3)$?

- $f(1)$
- $f(4)$
- $f(-4)$
- $f(-3)$

- 5) A ball is launched straight upward from a cliff (10m above the ground). The motion of a ball could be described as $h(t) = -4.9t^2 + 24t + 10$, where t is the time, in second, the ball is in the air and h is the height, in meters, of the ball after it was launched. How long will it take for a ball to reach its peak?
- A) 2.45 sec
B) 24 sec
C) 10 sec
D) 4.9 sec
- 6) In the system of equations $\begin{cases} y = ax^2 + b \\ y = 1 \end{cases}$, for which of the following values of a and b does the system have two solutions?
- A) $a = 2, b = 1$
B) $a = -1, b = 5$
C) $a = 2, b = 5$
D) $a = -2, b = 0$
- 7) In the system of equations $\begin{cases} f(x) = -x^2 + 2x + 6 \\ h(x) = k \end{cases}$, where k is a constant. $f(x) \leq h(x)$ for all real values of x . What is the minimum value of k ?
- 8) In the system of equations $\begin{cases} y = -(x + 2)^2 \\ y = m \end{cases}$, where m is a constant. If the system has two points of intersections in the XY-plane and the distance between two points is 12, what is the value of m ?
- A) -36
B) 36
C) -64
D) 64

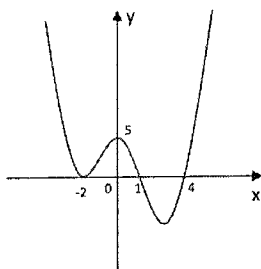
- 9) In a quadratic equation $2x^2 + 6x - 8 = 0$, Let a and b are two different roots. What is the value of $(a - 1)(b - 1)$?
- 10) The graph of $y = x^2 - 4x + k$ intersects the line $y = -2$ at one point. What is the value of k ?
- 11) In a quadratic equation $h(t) = -16t^2 + 128t$. The function $h(t)$ represents the height, in feet, of an object t seconds after it is thrown upwards from the ground with an initial speed 128 feet per second. How long will the object stay above 192 ft from the ground?
- A) 2 sec
B) 4 sec
C) 6 sec
D) 8 sec
- 12) In the equation $\frac{x^2}{4} - 3x + k^2 = 0$, where k is a constant. If the equation has exactly one solution, what could be the value of k ?
- A) 3
B) 2
C) 1
D) 0

- Graphs of Polynomial and rational functions



- 13) The graph of a polynomial function $y = f(x)$ is shown above. If a horizontal line $g(x) = k$ (not shown), where k is a constant, meets four times with $y = f(x)$, which of the following could be the value of k ?

- A) 7
- B) 8
- C) 6
- D) 4



- 14) Which of the following could be the equation of the graph shown above?

- A) $y = \frac{5}{16}(x - 2)^2(x + 1)(x + 4)$
- B) $y = -\frac{5}{16}(x + 2)^2(x - 1)(x - 4)$
- C) $y = \frac{5}{16}(x + 2)^2(x - 1)(x - 4)$
- D) $y = 5(x + 2)^2(x - 1)(x - 4)$

$$\frac{k^2}{\sqrt{k^2 - x^2}} = \frac{x^2}{\sqrt{k^2 - x^2}} + 29$$

- 15) In the equation shown above, k is a constant. Which of the following is one of the solutions to the equation?

- A) $-k$
- B) $k^2 - 29^2$
- C) $-\sqrt{k^2 - 29^2}$
- D) $\sqrt{29^2 - k^2}$