Problem-Solving and Data analysis

(5-7 questions, about 15%)

Topics: Ratios, rates, proportional relationship, units. Percentages, one or two variable data: distributions and measures of center and spread, Scatterplots. Probability and conditional probability, Inference from sample statistics and margin of error.

- **Exponential growth/decay**

$p(t) = A(1 + r)^t$, where r is % in decimal and t is the time unit. (Exponential growth)
$p(t) = A(1 - r)^t$, where r is % in decimal and t is the time unit (Exponential decay)

(Exponential growth/decay Practice Problems)

1) A certain animal population in South Africa is about 2,000 currently. The scientists expect the population will continue to decay 5% every year due to the environmental issues.
$P(t) = 2{,}000\ a^t$, where t in in years. If this exponential model, P(t) represents the population t years from now, what is the value of a?

A) 1.05
B) 0.95
C) 5
D) 0.05

2) Elliott measured the temperature of a tea placed in his room with a constant temperature of 75 degrees Fahrenheit. The temperature of tea was 180°F at 7:00 a.m. and 120°F at 7:10 a.m. Assume that the temperature of tea continues to decrease close to the room temperature. Which of the following best models the temperature T(m), in degrees Fahrenheit, of the tea m minutes after it was placed in his room at 7:00 a.m.?

A) $T(m) = 180(0.67)^{\frac{m}{10}}$
B) $T(m) = 180(1.67)^{\frac{m}{10}}$
C) $T(m) = 75 + 105(0.43)^m$
D) $T(m) = 75 + 105(0.43)^{\frac{m}{10}}$

3) A spacecraft launches from a launching pad from the ground. The spacecraft ascends from the ground to an altitude of 100,000 ft at a constant rate of 1,000 feet per minute. What type of function best represents the relationship between the altitude of the spacecraft and time?

A) Decreasing exponential
B) Decreasing linear
C) Increasing exponential
D) Increasing linear

CONTINUE

- **Percent increase/decrease**

Percent increase = $\frac{Increase}{Original\ amount} \times 100\ (\%)$

Percent decrease = $\frac{decrease}{Original\ amount} \times 100\ (\%)$

- **Ratio / Proportions**

A ratio is a comparison of two numbers in the same unit.
A proportion is an equation with a ratio on each side. It is a statement that two ratios are equal.

(Practice problems with related topics)

4) If $5a + 3b$ is equal to 300 percent of 6b, what is the value of $\frac{a}{b}$?

5) The price of a certain product was first decreased by 20% and then the new price was increased by 20%. Which of the following is true about the price change from the initial price?

A) The price stayed the same.
B) The price went up by 4 percent.
C) The price went down by 4 percent.
D) Not enough information to compute.

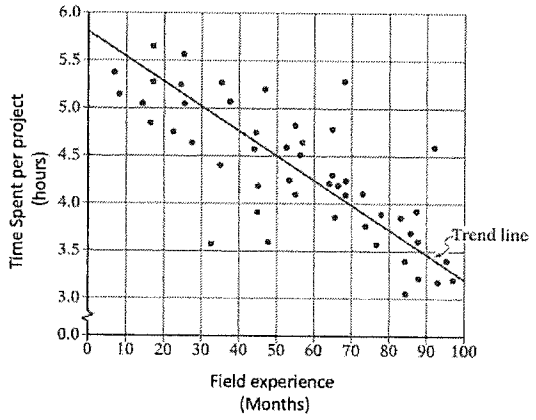
6) On an engineer's blueprint, 1 inch represents 2 feet in real dimensions. If a heating oven is represented on the blueprint by a rectangle that has sides of lengths 4 inches and 5 inches, what is the actual area of the oven, in square feet?

A) 40
B) 80
C) 120
D) 160

7) County A has two school districts (1st grade to 12th grade). The first school district has an area of 110 square miles and a population density of 90 students per square miles. And the second district has an area of 70 square miles and a population density of 120 students per square miles. What is the student population density, the number of students per square miles, for all of County A? (Answer into the nearest integer.)

- Interpreting data / relationships in scatterplot, table, and equations in Statistics

TIME SPENT IN HOURS PER PROJECT AND FIELD EXPERIENCE IN MONTHS
IN ABC MANUFACTRURING COMPANY



(Types of questions could be asked in this context from the scatterplot above)

8) According to the scatterplot, a line of best fit is drawn in the graph above. Which of the following equations best represents the line of best fit?

A) $y = \frac{1}{40}x - 5.75$
B) $y = -\frac{1}{40}x + 4.5$
C) $y = -40x + 5.75$
D) $y = -\frac{1}{40}x + 5.75$

9) For how many data shown was the number of hours spent per project less than the number of hours predicted by the line of best fit if the data were chosen over 80-month field experience?

A) 5
B) 6
C) 7
D) 8

10) Which of the following best interpret the slope of the line of best fit in this context?

A) An engineer in the company would spend an hour less on each project if the person experienced every 40 projects.
B) An engineer in the company would spend an hour less on each project if the person had every 40 months field experience.
C) An engineer in the company could finish one more project if the person had more than 40-month field experience.
D) An engineer in the company would spend an hour less on each project if the person had every 40 hours field experience.

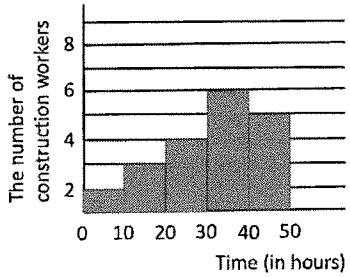
CONTINUE

Pet distribution in ABC High school students' household

	Less than 10 kg	10-15 kg	Greater than 15 kg	Total
DOG	14	20	48	82
CAT	9	7	22	38
Total	23	27	70	120

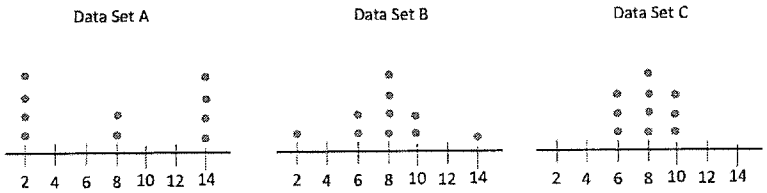
11) The table above summarizes the distribution of Pets weight, in kilograms, in ABC High school students' household. What is the probability of selecting a cat, given that the pet's weight is greater than 15kg?

A) $\frac{24}{35}$
B) $\frac{11}{35}$
C) $\frac{11}{60}$
D) $\frac{2}{5}$



12) In the histogram shown summarizes the distribution of time, in hours, worked by 20 construction workers last week. The first bar represents the number of workers who worked at least 0 hours and less than 10 hours and the second bar represents the number of workers who worked at least 10 hours and less than 20 hours and so on. Which of the following could be the median and mean amount of time worked, in hours, for 20 construction workers assuming only whole numbers of working hour considered?

A) Median = 25, Mean = 25
B) Median = 32, Mean = 27
C) Median = 33, Mean = 35
D) Median = 27, Mean = 33



13) The dot plots show the distribution of math quiz scores in 3 classes of 10 students each class. Which of the following is the correct order about the standard deviations?

A) $A < B < C$ B) $A < C < B$ C) $C < B < A$ D) $B < C < A$

- **Strategy in survey problems**

1) If the subjects in the sample of a study were selected at random from the entire population, the result can be generalized to the entire population.
2) If the subjects in the sample were randomly assigned to the treatments, it may be appropriate to make conclusions about cause and effect.

	Subjects selected at random	Subjects NOT selected at random
Subjects randomly assigned to treatments	• Results can be generalized to the entire population • Conclusions about cause and effect can be appropriately be drawn	• Results CANNOT be generalized to the entire population • Conclusions about cause and effect can be appropriately be drawn
Subjects NOT randomly assigned to treatments	• Results can be generalized to the entire population • Conclusions about cause and effect should NOT be drawn	• Results CANNOT be generalized to the entire population • Conclusions about cause and effect should NOT be drawn

(Practice problems in data analysis and statistics)

14) A sample of 50 sixth-grade students were randomly selected from a certain elementary school. The 50 students completed a survey regarding to morning meditation before the first class starts, and 45 students replied that the meditation in the morning was helpful to focus in class. Which of the following is the largest population to which the results of the survey can be applied?

A) All students at the same school
B) The 50 students who were surveyed
C) All sixth-grade students in the county in which the school is located
D) All sixth-grade students at the same school

15) A community college offered a Japan tour over the summer to students who would take Japanese course in the fall. The students who visited Japan over the summer through the program did better in the course than students who didn't visit Japan over the summer. Based on the results, which of the following is the most appropriate conclusion?

A) Visiting foreign countries over the summer will cause an improvement for any student who takes the same foreign language course.
B) Visiting Japan over the summer will cause an improvement for any student who takes Japanese language course.
C) Visiting Japan over the summer was the cause of the improvement for the students at the specific community college.
D) No conclusion about the cause can be made regarding students who visited Japan over the summer and their performance in the Japanese course because students who visited Japan were volunteered.

CONTINUE

16) A data set of 20 different numbers has a mean of 55 and a median of 55 as well. A new data set was created by adding 10 to each number in the original set of data that is greater than the median and subtracting 10 to each number in the original set of data that is less than the median. Which of the following does not have the same value in both the original and the new data set?

A) Median
B) Mean
C) Standard Deviation
D) Sum of the data

Sample	Percent in favor of new menu	Margin of error
A	78%	1.5%
B	68%	4.3%

17) The results of two random samples of survey for a new item in a local ice-cream shop are shown above. The samples were selected from the same population and the margins of error were calculated using the same method. Which of the following is the most proper reason that the margin of error for sample A is less than that of sample B?

A) Sample A had a larger sample size.
B) Sample B had a larger sample size.
C) Sample A had a higher percentage of favorable response.
D) Sample A could be more knowledgeable in new item.

18) In which of the following tables is the relationship between the values of x and their corresponding y-values non-linear?

A)

x	1	2	3	4
y	4	6	8	10

B)

x	1	2	3	4
y	-12	-6	0	6

C)

x	1	2	3	4
y	2	4	8	16

A)

x	1	2	3	4
y	4.5	6.5	8.5	10.5

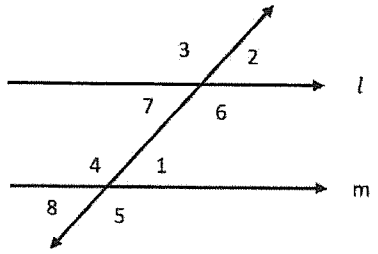
CONTINUE

Geometry and Trigonometry

(5-7 questions, about 15%)

Topics: Area and volume. Lines, angles, and triangles, right triangles, trigonometry, Circles.

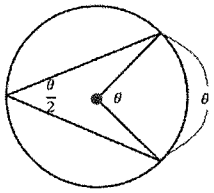
- Angles associated with parallel lines



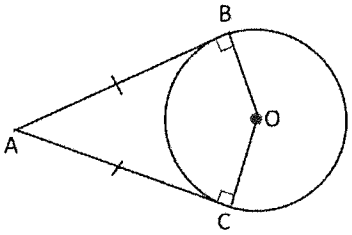
when $l \parallel$ m,

- Corresponding angles are congruent: $\angle 1 \cong \angle 2,\ \angle 3 \cong \angle 4,\ \angle 5 \cong \angle 6, \angle 7 \cong \angle 8$
- Alternate interior angles are congruent: $\angle 1 \cong \angle 7, \angle 4 \cong \angle 6$
- Same side interior angles are supplementary: $\angle 1 + \angle 6 = 180°, \angle 4 + \angle 7 = 180°$
- Alternate exterior angles are congruent: $\angle 3 \cong \angle 5, \angle 2 \cong \angle 8$
- Vertical angles are congruent: $\angle 1 \cong \angle 8, \angle 4 \cong \angle 5$

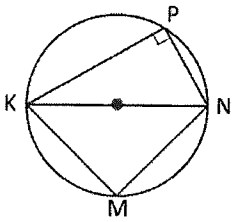
- Angles associated with circles



- Measure of arc is the same as the central angle
- The measure of inscribed angle is half the measure of the same intercepted arc



- The lengths of two tangent segments from a point outside of a circle are congruent ($AB = AC$)
- the segments form a right angle with the radius drawn to the point of tangency ($OB \perp AB, OC \perp AC$)



- Any inscribed angle drawn from the endpoints of diameter ($\angle KPN$ *or* $\angle$KMN are right angles)
- If a quadrilateral is inscribed in a circle, its opposite angles are supplementary. ($\angle KPN + \angle KMN = 180°$)

- Angles in polygons

i. Sum of interior angles in any polygon = $(n-2) \cdot 180°$, where n is the number of sides.

ii. Each interior angle of a regular polygon = $\frac{(n-2)\cdot 180°}{n}$, where n is the number of sides.

iii. Sum of exterior angles of any polygon = $360°$

iv. Each exterior angle of a regular polygon = $\frac{360°}{n}$, where n is the number of sides.

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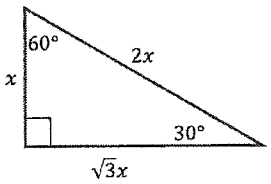
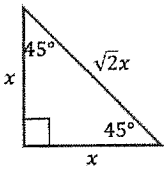
- Angles associated with triangles



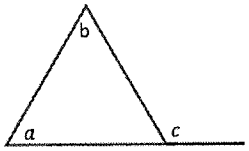


3 – 4 – 5

5 – 12 – 13

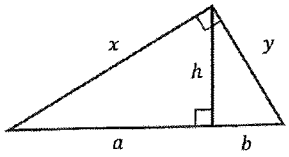
7 – 24 – 25

8 – 15 – 17



- One exterior angle is same as the sum of two remote interior angles

$\angle a + \angle b = \angle c$



$x = \sqrt{a \cdot (a+b)}$

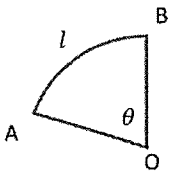
$y = \sqrt{b \cdot (a+b)}$

$h = \sqrt{a \cdot b}$

- Circle equation

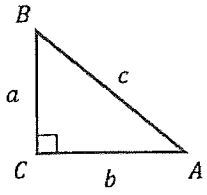
$(x-h)^2 + (y-k)^2 = r^2$, where (h, k) is the coordinates of the center of the circle and r is the radius.

- Area of a sector and Length of an arc



- Area of a sector = $\frac{\theta}{360°} \cdot \pi r^2$, where θ is the central angle in degrees
- Length of an arc = $\frac{\theta}{360°} \cdot 2\pi r$, where θ is the central angle in degrees

- Trigonometry



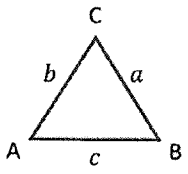
- $\sin A = \frac{a}{c}, \cos A = \frac{b}{c}, \tan A = \frac{a}{b}$ (Soh-Cah-Toa)
- sin A = cos B (co-functions)

$Note: A = 90° - B$ or $A + B = 90°$ (complementary angles)

- How to convert angles in different mode (radians to degrees or degrees to radians)
  - Radians to degrees : multiply by $\frac{180°}{\pi}$
  - Degrees to radians : multiply by $\frac{\pi}{180°}$

CONTINUE

- Isosceles triangle property and triangle inequalities



Isosceles Triangle Property

- If $\angle A \cong \angle B$, then $a = b$
- If $a = b$, then $\angle A \cong \angle B$

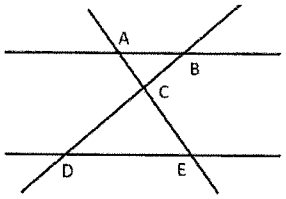
Triangle inequalities

- $|a - b| < c < a + b$

(Practice Problems in Geometry and Trigonometry)

1) In a right triangle, the tangent of one of the two acute angle is $\frac{1}{3}$. Which of the following is the sine of the other angle?

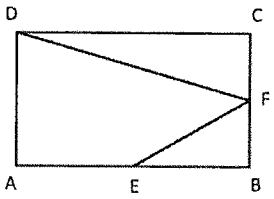
A) $\frac{1}{\sqrt{10}}$ B) $\sqrt{10}$ C) $\frac{\sqrt{10}}{3}$ D) $\frac{3}{\sqrt{10}}$



Note: Figure not drawn to scale

2) In the figure above, $\Delta\, ABC$ is similar to ΔEDC. Which of the following must be true?

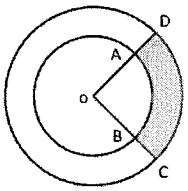
A) $\overline{AB} \perp \overline{AE}$ B) $\overline{BD} \perp \overline{DE}$ C) $\overline{AB} \parallel \overline{DE}$ D) $\overline{BC} \perp \overline{CE}$



Note: Figure not drawn to scale

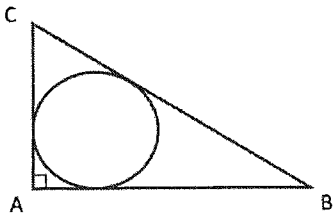
3) In the rectangle ABCD above, Points E and F are midpoints of the sides AB and BC, respectively. If $\tan \angle FDC = \frac{1}{2}$, what is the value of $sin \angle BEF$?

A) $\frac{1}{2}$
B) $\frac{1}{\sqrt{2}}$
C) $\frac{\sqrt{3}}{2}$
D) $\sqrt{2}$



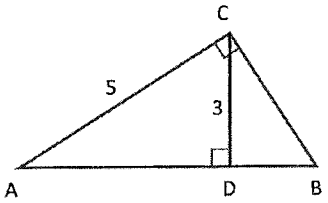
4) Two concentric circles with a center O are drawn above. The area of the shaded region is 16π. If the measure of angle AOB is $\frac{2\pi}{9}$ and $OA:AD = 3:2$, what is the length of a minor arc CD?

A) $\frac{10\pi}{9}$ B) $\frac{10\pi}{3}$ C) $\frac{9\pi}{5}$ D) 3π



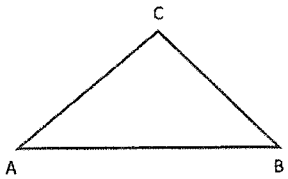
5) In the right triangle ABC above, $AC = 7, BC = 25$. What is the length of the radius of the inscribed circle?

A) 2 B) 3 C) 4 D) 5



6) In the right triangle ABC above, what is the value of $cos\angle CBD$?

A) $\frac{3}{4}$ B) $\frac{3}{5}$ C) $\frac{3}{5}$ D) $\frac{5}{3}$



Note: Figure not drawn to scale

7) In the triangle ABC above, $\sin\angle A = cos\angle B$. Which of the following is correct for triangle ABC?

A) Acute triangle
B) Obtuse triangle
C) Right triangle
D) Isosceles triangle

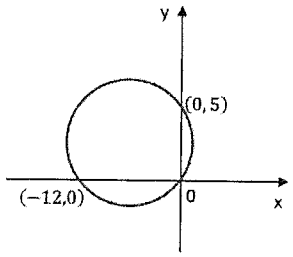
CONTINUE

8) The number of radians in a 540-degree angle can be written as $k\pi$, where k is a constant. What is the value of k?

9) A circle in the XY-plane has equation $(x-1)^2+(y+2)^2=9$. Which of the following points lie in the interior of the circle?

A) $(2,-1)$
B) $(-3,2)$
C) $(2,4)$
D) $(-2,0)$

10) The graph of $2x^2+x+2y^2+y=\frac{1}{4}$ in the XY-plane is a circle. What is the length of the radius of the circle?



11) The graph of a circle is drawn in the XY-plane above. If the circle intersects at three points as shown, what is the length of the radius of the circle?