PART 1

The Digital SAT Math

Introduction to the Digital SAT Math

The College Board has made the SAT digital. The digitalized SAT will be administered in schools or test centers with a proctor present starting in **spring 2024** if you plan to take the SAT in the U.S. The digital SAT still scores on a 1600 point-scale. (Reading and Writing section score: 200-800 and Math section score: 200-800)

The Digital SAT Math test formats (Adaptive Test)

Math section has two adaptive test design.

- **Module 1**: 22 questions (including 2 unscored questions- research purposes only) Students are given a broad mix of easy, medium, and hard questions.
  -Time: 35 min, the use of calculator is permitted.

- **Module 2**: 22 questions (including 2 unscored questions- research purposes only) Students are given a targeted mix of questions of varying difficulties based on their performance in module 1.
  -Time: 35 min, the use of calculator is permitted.

Important! – After completing the first-stage module on math section, **the test application routes students to one of two possible second-stage modules of questions**. One whose questions are, on average, of lower difficulty than those in the first module and the other whose questions are, on average, of higher difficulty than those in the first module.
And you will get the score based on the route and your performance.

The Digital SAT Math test formats (Nonadaptive Test)

Math section has two nonadaptive test design.

- **Module 1**: 27 questions (including 2 unscored questions- research purposes only) Students are given a broad mix of easy, medium, and hard questions.
  -Time: 43 min, the use of calculator is permitted.

- **Module 2**: 27 questions (including 2 unscored questions- research purposes only) Same difficulty level as module 1.
  -Time: 43 min, the use of calculator is permitted.

- Any student who wants to take the digital SAT on a paper braille test in an approved accommodation will need to be approved by College Board.

Math section Content domains.

- **Algebra** (13-15 questions, about 35%)
    - linear equations (functions) with one or two variables, System of linear equations, and linear inequalities one or two variables. Meaning of numbers or variables in the context.

- **Advanced Math** (13-15 questions, about 35%)
    - Equivalent equations, non-linear equations in one variable and systems of equations in two variables, non-linear functions. Solving rational equations and graphs of rational and polynomial functions.

- **Problem-solving and Data Analysis** (5-7 questions, about 15%)

- Ratios, rates, proportional relationship, units. Percentages, one or two variable data: distributions and measures of center and spread, Scatterplots. Probability and conditional probability, Inference from sample statistics and margin of error.

- **Geometry and Trigonometry** (5-7 questions, about 15%)

- Area and volume. Lines, angles, and triangles, right triangles, trigonometry, Circles.

How to calculate your scores in this book

- Use the answer keys on the end of each test to find the number of questions in module 1 and module 2 that you answered correctly.

- Add the number of correct answers you got on both module 1 and module 2. That will be your raw scores. Remember, there are no guess penalties. Even though the real College Board digital SAT counts only 20 questions only for scoring (2 unscored questions for research purposes), **we count all 22 questions to get your score in this book.**

- Look for the expected Math score (200-800) in the Score Conversion table provided below the answer keys using your calculated raw scores.

Test-taking Strategies

- **Know the test basics**: 2 modules. 22 questions each module. 35 minutes. Remembering the importance of doing well in the first stage module in order to route the higher difficulty level in module 2. No guessing penalties.

- **Get familiar with Digital Testing Tools**: Cross out answer choices in multiple choice questions. Display or hide a countdown timer. Access the Math section's reference sheet during testing. Flag questions within a given test module to return to. Access a display informing you of how many questions in each module you've either flagged or left unanswered and allowing you to jump to any questions within a module. Use of the built-in Desmos Graphing Calculator or your own calculator on the entire math sections.

- **Know the two test questions formats**: About 75% of questions are multiple choice and the rest are SPR format (Students-Produced Response)

- **Get Familiar with the Various Question Types**: Study the SAT Math point notes summarized in the beginning of this book first and do all Practice tests available on both College Board web site (see below) and 15 full-length practice tests provided in this book.

- 4 free official full-length digital College Board SAT tests are located at https://satsuite.collegeboard.org/digital/digital-practice-preparation/practice-tests/linear

PART 2

Digital SAT Math Points Note

Algebra

(13-15 questions, about 35%)

Topics: linear equations (functions) with one or two variables, System of linear equations, and Linear inequalities one or two variables. Meaning of numbers or variables in the context.

- **Linear equations**: Standard form $ax + by = c$
  Slope-intercept form $y = mx + b$,
  Point-slope form $y - y_1 = m(x - x_1)$.

(Practice problems for algebra)

1) In the XY-plane, line l is perpendicular to line $2x - 3y = 1$. If the line l passes through a point (2, 1), which of the following is an equation of line l?

   A) $3x + 2y = 8$
   B) $2x - 3y = -8$
   C) $y = -\frac{2}{3}x + 4$
   D) $y = -\frac{2}{3}x + \frac{7}{3}$

x	y
-2	-4
1/3	3
1	5
3	11

2) From the table above, four pairs of XY-coordinate points in a line are shown. If a line $ax + by = -4$, where $a \text{ and } b \text{ are constants}$, represents the relationship in the line, what is the value of a?

3) A construction contractor uses the function h defined by $h(x) = 5{,}000 + 250x$, where x is the area of floor in square feet to estimate the cost of labor, in dollars, to build a wooden floor in a certain area. If the contractor gives the estimate of labor cost to build a wooden floor of a house in that area is \$15,000, what is the area of the wooden floor, in square feet, of the house?

CONTINUE

- **The system of equations**:
  (Ways to solve system of equations)
  1) Linear combination method (match the coefficients of one variable and eliminate it)
  2) Substitution method (isolate one variable and substitute it into the other equation)

Standard form: $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$ Point-slope form: $\begin{cases} y = m_1x + b_1 \\ y = m_2x + b_2 \end{cases}$

Standard form	Types of solutions	Point-slope form
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	No solution	$m_1 = m_2\ and\ b_1 \neq b_2$
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Infinitely many solutions	$m_1 = m_2\ and\ b_1 = b_2$
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	One solution	$m_1 \neq m_2$

$$4u + 5v = a$$
$$-12u - \frac{v}{b} = 2$$

4) In the given system of equation above, $a\ and\ b$ are constants. If the system has infinitely many solutions, what is the absolute value of $a - b$?

5) Adrian plans to work out every morning. He runs at 8 miles per hour and swims at 4 miles per hour. His goal is to practice both exercises at least a total of 15 miles in no more than 2 hours a day. If he spends k hours in running and m hours in swimming, which of the following system of inequalities represent Adrian's goal?

A) $\begin{cases} k + m \leq 2 \\ \frac{8}{k} + \frac{4}{m} \geq 15 \end{cases}$

B) $\begin{cases} k + m \leq 2 \\ \frac{k}{8} + \frac{m}{4} \geq 15 \end{cases}$

C) $\begin{cases} 8k + 4m \geq 15 \\ k + m \geq 2 \end{cases}$

D) $\begin{cases} 8k + 4m \geq 15 \\ k + m \leq 2 \end{cases}$

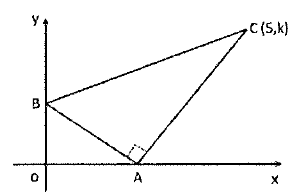
6) Janet purchased a blouse and a purse at a local store. The store offered a special discount on certain items. She spent a total of \$215.20 for both items. If the store offered no tax on the blouse and 10% sales tax on the purse she purchased and the sum of the prices before tax was \$198, what was the price, in dollars, of the purse?

- Properties of parallel and perpendicular lines (in the point-slope forms)

Parallel lines	$m_1 = m_2\ and\ b_1 \neq b_2$
Perpendicular lines	$m_1 = -\frac{1}{m_2}$ or $m_1 \cdot m_2 = -1$

- Distance, midpoint, and slope formula between two points $A(x_1, y_1)\ and\ B(x_2, y_2)$

Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ Slope $= \frac{y_2-y_1}{x_2-x_1}$



7) In the XY-plane above, the coordinates of points B and C are (0, 2) and (5, k), respectively. If $\overline{AB}$ is perpendicular to $\overline{AC}$ and the slope of $\overline{AB}$ is $-\frac{3}{5}$, what is the value of k?

8) The distance between two points $A(2, a)\ and\ B(-1, b)$ is 5. What is the value of $\frac{1}{2}|b - a|$?

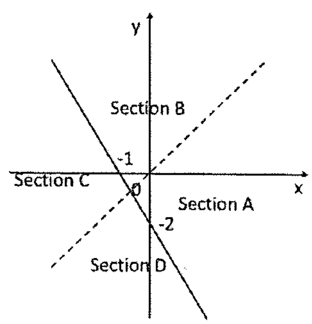
CONTINUE

9) The end points of a line segment AB are A(2, 12) and B(-4, -9), respectively. If points M is on $\overline{AB}$ such that $AM:MB = 1:2$, what are the coordinates of point M?

A) $(0,8)$
B) $(-1,4)$
C) $(0,5)$
D) $(-1,2)$

10) Which of the following equations represent a line parallel to the graph of the equation $\frac{1}{5}x + \frac{1}{3}y = -2$?

A) $5x + 3y = 1$
B) $5x - 3y = -4$
C) $3x - 5y = 9$
D) $3x + 5y - 2 = 0$



$$\begin{cases} y - x > 0 \\ -2x - y \le 2 \end{cases}$$

11) A system of inequalities and a graph are shown in the XY-plane above, which section of the graph could represent all of the solutions to the system?

A) Section A B) Section B C) Section C D) Section D

12) Elliott scored 85, 89, 95, and 80 on his exams before his last exam. If all exams weigh equally, which of the following inequalities could get him all the possible scores of the fifth exam, m that he would result in a mean score on all five exams at least 90?

A) $85 + 89 + 95 + 80 + m \le 450$
B) $85 + 89 + 95 + 80 + m \ge 360$
C) $\frac{85+89+95+80+m}{5} \le 90$
D) $85 + 89 + 95 + 80 \ge 450 - m$

CONTINUE

Answers and explanations for Practice problems

(Algebra)

1) A)

We can write an equation of the line l as $3x + 2y = a$ because it is perpendicular to $2x - 3y = 1$. And plug the point (2, 1) into the equation. Then $6 + 2 = a$. So, $a = 8$. finally, the equation of the line l is $3x + 2y = 8$.

2) 6

Plug any two points in the table into the given equation $ax + by = -4$. We get $-2a - 4b = -4$ when plug the point (-2, -4) and $a + 5b = -4$ when plug the point (1, 5). Solve the system of equations. Then you get $a = 6$ and $b = -2$.

3) 40

In the given equation $h(x) = 5{,}000 + 250x$, plug 15,000 into $h(x)$ and solve for x.
$15{,}000 = 5{,}000 + 250x$, then you get $x = 40$. So, the area of the wooden floor is 40 square feet.

4) $\frac{11}{15}$

If the system of equations has infinitely many solutions, it means two equations are identical.
Multiply the first equation by -3. Then you get $-12u - 15v = -3a$. Compare it with the second equation. You get $-15 = -\frac{1}{b}$ and $-3a = 2$. Therefore $a = -\frac{2}{3}$ and $b = \frac{1}{15}$. So, $|a - b| = \left|\left(-\frac{2}{3}\right) - \frac{1}{15}\right| = \left|-\frac{11}{15}\right| = \frac{11}{15}$

5) D)

K is the number of hours for running and m is the number of hours for swimming. So, we can set up $k + m \leq 2$ because his goal is no more than 2 hours of both exercise a day. And distance = rate x time. So, we can set up another inequality equation $8k + 4m \geq 15$ since he wants to practice at lease a total of 15 miles for both exercises.

6) 172

Let x is the price of the browse and y is the price of the purse she purchased. The sum of the prices before tax is $x + y = 198$. And she paid a total of $215.20 including 10% sales tax on the purse only. So, we can set up $x + 1.1y = 215.20$. now, solve the system of equations. You get $x = 26 \; and \; y = 172$.

7) $\frac{25}{9}$

Let the coordinates of the point A be $(x, 0)$. Since the slope of $\overline{AB}$ is $-\frac{3}{5}$, we can set up $-\frac{3}{5} = \frac{2-0}{0-x}$.
So, $x = \frac{10}{3}$. The slope of $\overline{AC}$ is $\frac{5}{3}$ because $\overline{AB}$ is perpendicular to $\overline{AC}$. So, we can set up $\frac{5}{3} = \frac{k-0}{5-\frac{10}{3}}$.
Solve for k. $k = \frac{25}{9}$.

8) 2

Use the distance formula between two points A and B. $5 = \sqrt{(-1-2)^2 + (b-a)^2}$. Square the equation and you get $16 = (b-a)^2$. Put square root on both sides of the equation. You get $|b - a| = 4$. Therefore, $\frac{1}{2}|b - a| = \frac{1}{2} \cdot 4 = 2$.

CONTINUE

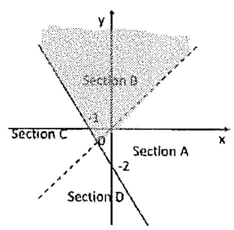
9) C)
Use the formula to determine the point M partitioning AB into the ratio $a\colon b$. Then, you get

$$M(x,y)=\left(\frac{bx_1+ax_2}{a+b},\frac{by_1+ay_2}{a+b}\right)$$

Then, $M(x,y)=\left(\frac{2\cdot 2+1\cdot -4}{1+2},\frac{2\cdot 12+1\cdot -9}{1+2}\right)=(0,5)$

10) D)
From the given equation $\frac{1}{5}x+\frac{1}{3}y=-2$, the slope of the line is $-\frac{3}{5}$. So, look for one which has the same slope. Then only D) has the value of slope is $-\frac{3}{5}$.

11) B)



Change the form of the equations into slope intercept form. $y > x\ and\ y \geq -2x-2$. When you graph both inequalities in the XY-plane shown above, you get the section B as the solutions to the system. Or you can use a point in each section and use the process of elimination. For instance, when you plug (0, 2) in the section B, the point satisfies both inequalities. So, section B is the solution to the system.

12) D)
We can set up an equation for the mean of the scores.
$\frac{85+89+95+80+m}{5}\geq 90$. And we can change it by multiplying 5 on both sides of the inequality and subtract m on both sides. You get $85+89+95+80 \geq 450-m$.

CONTINUE

Advanced Math

(13-15 questions, about 35%)

Topics: Equivalent equations, non-linear equations in one variable and systems of equations in two variables, non-linear functions. Solving rational equations and graphs of rational and polynomial functions.

- **Equivalent equations**: The expressions on both sides of the given equation are always identical for all values of the given variables. It means that the equation is true for all values of the variable.

  (Practice problems for Advanced math)

  1) $x^3 + 2x^2 - 4x + 1 = (x + b)(x^2 + cx - 1)$
     In the given equation above, b and c are constants. If the equation is true for all values of x, what is the value of $b + c$?

     A) 2
     B) 3
     C) 1
     D) -2

  2) Which of the following is equivalent to $3b^2 + 18a^4 - 5b^2$?

     A) $2(3a - b)(3a + b)$
     B) $2(3a - b^2)(3a + b^2)$
     C) $3(2a^2 - b)(2a^2 + b)$
     D) $2(3a^2 - b)(3a^2 + b)$

- **Quadratic Equation**:

  i. Standard form $\boldsymbol{y = ax^2 + bx + c}$.
     Vertex (h, k) : $h = -\frac{b}{2a}$. And substitute h into the equation to find the value of k.
     Axis of symmetry: $x = h$. (In SAT math, very useful information to find the unknown Y values or unknown x-intercept.)
     Shape: $a > 0$: opens upwards. $a < 0$: opens downwards.
     This form shows the y-intercept: the value of c.

  ii. Vertex form $\boldsymbol{y = a(x - h)^2 + k}$.
     Vertex (h, k). when $a > 0$, the graph has a minimum value of k at $x = h$.
     When $a < 0$, the graph has a maximum value of k at $x = h$.
     This form shows the maximum/minimum values of the function.

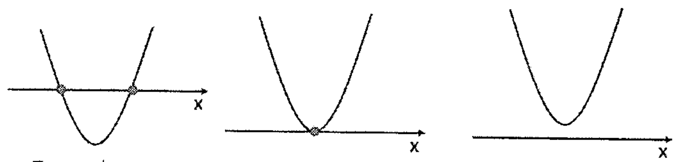
  iii. Zeros form $\boldsymbol{y = a(x - x_1)(x - x_2)}$.
     Vertex (h, k): $h = \frac{x_1+x_2}{2}$ (the mid-point of x_1 *and* x_2) And substitute h into the equation to find the value of k.
     This form shows two x-intercepts (zeros) in the equation.

- How to find solutions of quadratic equations (x-intercepts or zeros)

  i. Factor if possible.
  $a(x - x_1)(x - x_2) = 0$. Then, $x = x_1$ or $x = x_2$.

  ii. Use the quadratic formula.
  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Note: **D(Discriminant)**$= b^2 - 4ac$.
The value of D will tell you how many real solutions the equation will have.



Two real roots	One real root	No real root
$D > 0$	$D = 0$	$D < 0$

  iii. Complete the square.
  $x^2 + bx + c = 0$. Add $\left(\frac{b}{2}\right)^2$ on both sides of the equation. And factor it.

- Sum or Product of two roots of a quadratic equation in standard form (known as Fundamental theorem of algebra)

$$y = ax^2 + bx + c$$
$$\text{Sum} = -\frac{b}{a} \quad \text{Product} = \frac{c}{a}$$

(Quadratic Equations/Functions Practice Problems)

3) What is the value of m if $20x^2 + mx - 21 = (5x + a)(bx + 7)$, where $a, b, and\ m\ are\ constant$?

A) -13
B) 13
C) 23
D) -23

4) $f(x) = kx^2 + 2$, where k is a constant. what is the same value of $f(3)$?

A) $f(1)$
B) $f(4)$
C) $f(-4)$
D) $f(-3)$

CONTINUE

5) A ball is launched straight upward from a cliff (10m above the ground). The motion of a ball could be described as $h(t) = -4.9t^2 + 24t + 10$, where t is the time, in second, the ball is in the air and h is the height, in meters, of the ball after it was launched. How long will it take for a ball to reach its peak?

A) 2.45 sec
B) 24 sec
C) 10 sec
D) 4.9 sec

6) In the system of equations $\begin{cases} y = ax^2 + b \\ y = 1 \end{cases}$, for which of the following values of a and b does the system have two solutions?

A) $a = 2, b = 1$
B) $a = -1, b = 5$
C) $a = 2, b = 5$
D) $a = -2, b = 0$

7) In the system of equations $\begin{cases} f(x) = -x^2 + 2x + 6 \\ h(x) = k \end{cases}$, $where\ k\ is\ a\ constant.$ $f(x) \leq h(x)$ for all real values of x. What is the minimum value of k?

8) In the system of equations $\begin{cases} y = -(x + 2)^2 \\ y = m \end{cases}$, where m is a constant. If the system has two points of intersections in the XY-plane and the distance between two points is 12, what is the value of m?

A) -36
B) 36
C) -64
D) 64

CONTINUE

9) In a quadratic equation $2x^2 + 6x - 8 = 0$, Let a and b are two different roots. What is the value of $(a-1)(b-1)$?

10) The graph of $y = x^2 - 4x + k$ intersects the line $y = -2$ at one point. What is the value of k?

11) In a quadratic equation $h(t) = -16t^2 + 128t$. The function $h(t)$ represents the height, in feet, of an object t seconds after it is thrown upwards from the ground with an initial speed 128 feet per second. How long will the object stay above 192 ft from the ground?

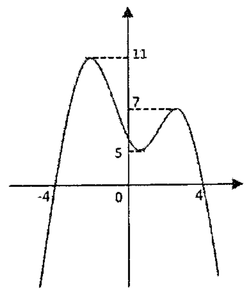
A) 2 sec
B) 4 sec
C) 6 sec
D) 8 sec

12) In the equation $\frac{x^2}{4} - 3x + k^2 = 0$, where k is a constant. If the equation has exactly one solution, what could be the value of k?

A) 3
B) 2
C) 1
D) 0

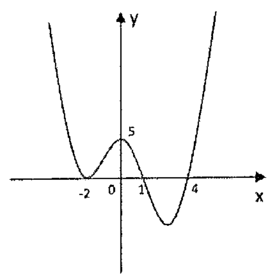
CONTINUE

- Graphs of Polynomial and rational functions



13) The graph of a polynomial function $y = f(x)$ is shown above. If a horizontal line $g(x) = k$(not shown), where k is a constant, meets four times with $y = f(x)$, which of the following could be the value of k?

A) 7
B) 8
C) 6
D) 4



14) Which of the following could be the equation of the graph shown above?

A) $y = \frac{5}{16}(x-2)^2(x+1)(x+4)$
B) $y = -\frac{5}{16}(x+2)^2(x-1)(x-4)$
C) $y = \frac{5}{16}(x+2)^2(x-1)(x-4)$
D) $y = 5(x+2)^2(x-1)(x-4)$

$$\frac{k^2}{\sqrt{k^2-x^2}} = \frac{x^2}{\sqrt{k^2-x^2}} + 29$$

15) In the equation shown above, k is a constant. Which of the following is one of the solutions to the equation?

A) $-k$
B) $k^2 - 29^2$
C) $-\sqrt{k^2-29^2}$
D) $\sqrt{29^2-k^2}$

CONTINUE

Answers and explanations for Practice problems

(Advanced math)

1) A)
If you multiply out the right side of the equation, you get $x^3 + (c + b)x^2 + (bc - 1)x - b$.
now, compare it with the left side of the equation $x^3 + 2x^2 - 4x + 1$. Then $b = -1$. and $2 = c - 1$.
So, $c = 3$. Therefore, $b + c = 2$.

2) D)
From the given equation $3b^2 + 18a^4 - 5b^2$, simplify it. $18a^4 - 2b^2 = 2(9a^4 - b^2)$
$= 2(3a^2 - b)(3a^2 + b)$.

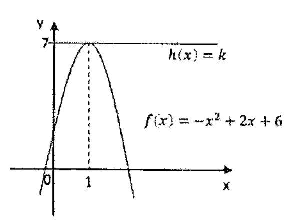
3) C)
Expand the right side of the equation. $5bx^2 + (35 + ab)x + 7a$. Compare it to the left side of the Equation. Then you get $a = -3$ *and* $b = 4$. And $m = 35 + (-3)(4) = 23$.

4) D)
The vertex of the parabola is located at (0, 2). By the axis of symmetry, we know that $f(-3)$ is same as $f(3)$.

5) A)
The ball will reach its peak at the vertex of the parabola. $t = -\frac{b}{2a} = -\frac{24}{2(-4.9)} = 2.45\ sec.$

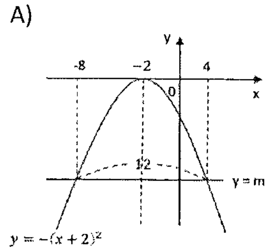
6) B)
Set two equations equal to each other. $1 = ax^2 + b$. Now, put it in the standard form of quadratic equation. $ax^2 + b - 1 = 0$. In order to have two solutions, the discriminant must be greater than zero.
$D = b^2 - 4ac = 0 - 4(a)(b - 1) > 0$. Then you get $4a - 4ab > 0$. Use the answer choices to find the answer.
Only B) $4(-1) - 4(-1)(5) > 0$.

7) 7
Graph both equations first. The x-coordinate of the vertex of $f(x) = -x^2 + 2x + 6$ is $-\frac{b}{2a} = -\frac{2}{2(-1)} = 1$. and $y = f(1) = -1 + 2 + 6 = 7$. So, the vertex is (1, 7).



so, the minimum value of k to satisfy $f(x) \le h(x)$ is 7.

8) A)



In the graph on the left, plug x=4 into the first equation. Then, $y = -36$.
So, the value of m is -36.

9) 0

Expand $(a-1)(b-1) = ab - a - b + 1 = ab - (a+b) + 1$.

From the fundamental theorem of algebra, $a + b = \frac{-6}{2} = -3$. And $ab = \frac{-8}{2} = -4$.

Therefore, $ab - (a+b) + 1 = (-4) - (-3) + 1 = 0$.

10) 2

Set two equations equal to each other to find the intersections. $x^2 - 4x + k = -2$. Now, put it into the standard form. $x^2 - 4x + k + 2 = 0$. For this quadratic equation to have one solution, the discriminant must be equal to zero. $D = b^2 - 4ac = (-4)^2 - 4 \cdot 1 \cdot (k+2) = 0$. Therefore, $k = 2$.

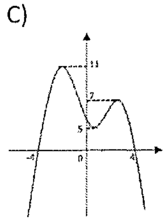
11) B)

Substitute 192 into the h(t) and solve for t. $192 = -16t^2 + 128t$. And $16t^2 - 128t + 192 = 0$. $t^2 - 8t + 12 = 0$. Then, $t = 2\ or\ 6$. It means that the object will be above 192ft from the ground between 2 and 6 seconds. Thus, it will stay above 192ft for 6-2= 4 sec.

12) A)

For a quadratic equation to have one solution, the discriminant must be zero. $D = b^2 - 4ac = (-3)^2 - 4\left(\frac{1}{4}\right)(k^2) = 0$. thus, $k = \pm 3$.

13) C)



$y = f(x)$ is shown and $g(x) = k$ is a horizontal line. For two equations to have four intersections in the graph, the value of k must be $5 < k < 7$ from the graph.

14) C)

From the given polynomial function graph, the function has zeros -2(double roots), 1, and 4. So we can set up $y = a(x+2)^2(x-1)(x-4)$. Now, plug the point (0, 5) into the equation to find the value of a. Then we get $a = \frac{5}{16}$.

15) C)

Multiply $\sqrt{k^2 - x^2}$ on both sides of the equation. $k^2 = x^2 + 29\sqrt{k^2 - x^2}$. It becomes $(k^2 - x^2) - 29\sqrt{k^2 - x^2} = 0$. And factor it. $\sqrt{k^2 - x^2}\left(\sqrt{k^2 - x^2} - 29\right) = 0$. Thus, $\sqrt{k^2 - x^2} = 29$. ($\sqrt{k^2 - x^2} \neq 0$ from the domain). Now, square both sides and solve for x. $x = \pm\sqrt{k^2 - 29^2}$.

Problem-Solving and Data analysis

(5-7 questions, about 15%)

Topics: Ratios, rates, proportional relationship, units. Percentages, one or two variable data: distributions and measures of center and spread, Scatterplots. Probability and conditional probability, Inference from sample statistics and margin of error.

- **Exponential growth/decay**

$p(t) = A(1+r)^t$, where r is % in decimal and t is the time unit. (Exponential growth)
$p(t) = A(1-r)^t$, where r is % in decimal and t is the time unit (Exponential decay)

(Exponential growth/decay Practice Problems)

1) A certain animal population in South Africa is about 2,000 currently. The scientists expect the population will continue to decay 5% every year due to the environmental issues.
$P(t) = 2{,}000\ a^t$, where t in in years. If this exponential model, P(t) represents the population t years from now, what is the value of a?

A) 1.05
B) 0.95
C) 5
D) 0.05

2) Elliott measured the temperature of a tea placed in his room with a constant temperature of 75 degrees Fahrenheit. The temperature of tea was 180°F at 7:00 a.m. and 120°F at 7:10 a.m. Assume that the temperature of tea continues to decrease close to the room temperature. Which of the following best models the temperature T(m), in degrees Fahrenheit, of the tea m minutes after it was placed in his room at 7:00 a.m.?

A) $T(m) = 180(0.67)^{\frac{m}{10}}$
B) $T(m) = 180(1.67)^{\frac{m}{10}}$
C) $T(m) = 75 + 105(0.43)^m$
D) $T(m) = 75 + 105(0.43)^{\frac{m}{10}}$

3) A spacecraft launches from a launching pad from the ground. The spacecraft ascends from the ground to an altitude of 100,000 ft at a constant rate of 1,000 feet per minute. What type of function best represents the relationship between the altitude of the spacecraft and time?

A) Decreasing exponential
B) Decreasing linear
C) Increasing exponential
D) Increasing linear

CONTINUE

- **Percent increase/decrease**

Percent increase = $\frac{Increase}{Original\ amount} \times 100\ (\%)$

Percent decrease = $\frac{decrease}{Original\ amount} \times 100\ (\%)$

- **Ratio / Proportions**

A ratio is a comparison of two numbers in the same unit.
A proportion is an equation with a ratio on each side. It is a statement that two ratios are equal.

(Practice problems with related topics)

4) If $5a + 3b$ is equal to 300 percent of 6b, what is the value of $\frac{a}{b}$?

5) The price of a certain product was first decreased by 20% and then the new price was increased by 20%. Which of the following is true about the price change from the initial price?

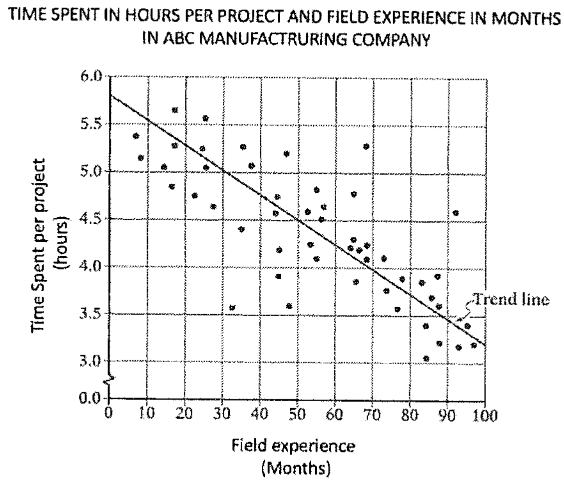
A) The price stayed the same.
B) The price went up by 4 percent.
C) The price went down by 4 percent.
D) Not enough information to compute.

6) On an engineer's blueprint, 1 inch represents 2 feet in real dimensions. If a heating oven is represented on the blueprint by a rectangle that has sides of lengths 4 inches and 5 inches, what is the actual area of the oven, in square feet?

A) 40
B) 80
C) 120
D) 160

7) County A has two school districts (1st grade to 12th grade). The first school district has an area of 110 square miles and a population density of 90 students per square miles. And the second district has an area of 70 square miles and a population density of 120 students per square miles. What is the student population density, the number of students per square miles, for all of County A? (Answer into the nearest integer.)

- Interpreting data / relationships in scatterplot, table, and equations in Statistics



(Types of questions could be asked in this context from the scatterplot above)

8) According to the scatterplot, a line of best fit is drawn in the graph above. Which of the following equations best represents the line of best fit?

A) $y = \frac{1}{40}x - 5.75$
B) $y = -\frac{1}{40}x + 4.5$
C) $y = -40x + 5.75$
D) $y = -\frac{1}{40}x + 5.75$

9) For how many data shown was the number of hours spent per project less than the number of hours predicted by the line of best fit if the data were chosen over 80-month field experience?

A) 5
B) 6
C) 7
D) 8

10) Which of the following best interpret the slope of the line of best fit in this context?

A) An engineer in the company would spend an hour less on each project if the person experienced every 40 projects.
B) An engineer in the company would spend an hour less on each project if the person had every 40 months field experience.
C) An engineer in the company could finish one more project if the person had more than 40-month field experience.
D) An engineer in the company would spend an hour less on each project if the person had every 40 hours field experience.

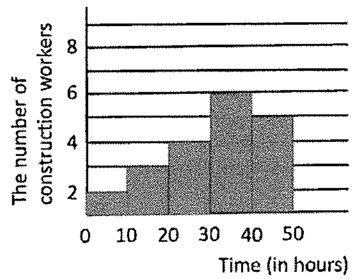
CONTINUE

Pet distribution in ABC High school students' household

	Less than 10 kg	10-15 kg	Greater than 15 kg	Total
DOG	14	20	48	82
CAT	9	7	22	38
Total	23	27	70	120

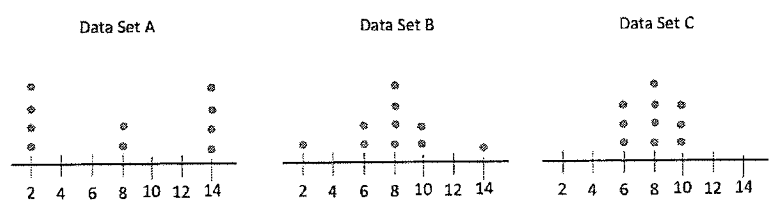
11) The table above summarizes the distribution of Pets weight, in kilograms, in ABC High school students' household. What is the probability of selecting a cat, given that the pet's weight is greater than 15kg?

A) $\frac{24}{35}$
B) $\frac{11}{35}$
C) $\frac{11}{60}$
D) $\frac{2}{5}$



12) In the histogram shown summarizes the distribution of time, in hours, worked by 20 construction workers last week. The first bar represents the number of workers who worked at least 0 hours and less than 10 hours and the second bar represents the number of workers who worked at least 10 hours and less than 20 hours and so on. Which of the following could be the median and mean amount of time worked, in hours, for 20 construction workers assuming only whole numbers of working hour considered?

A) Median = 25, Mean = 25
B) Median = 32, Mean = 27
C) Median = 33, Mean = 35
D) Median = 27, Mean = 33



13) The dot plots show the distribution of math quiz scores in 3 classes of 10 students each class. Which of the following is the correct order about the standard deviations?

A) $A < B < C$ B) $A < C < B$ C) $C < B < A$ D) $B < C < A$

CONTINUE

- **Strategy in survey problems**

1) If the subjects in the sample of a study were selected at random from the entire population, the result can be generalized to the entire population.
2) If the subjects in the sample were randomly assigned to the treatments, it may be appropriate to make conclusions about cause and effect.

	Subjects selected at random	Subjects NOT selected at random
Subjects randomly assigned to treatments	• Results can be generalized to the entire population • Conclusions about cause and effect can be appropriately be drawn	• Results CANNOT be generalized to the entire population • Conclusions about cause and effect can be appropriately be drawn
Subjects NOT randomly assigned to treatments	• Results can be generalized to the entire population • Conclusions about cause and effect should NOT be drawn	• Results CANNOT be generalized to the entire population • Conclusions about cause and effect should NOT be drawn

(Practice problems in data analysis and statistics)

14) A sample of 50 sixth-grade students were randomly selected from a certain elementary school. The 50 students completed a survey regarding to morning meditation before the first class starts, and 45 students replied that the meditation in the morning was helpful to focus in class. Which of the following is the largest population to which the results of the survey can be applied?

A) All students at the same school
B) The 50 students who were surveyed
C) All sixth-grade students in the county in which the school is located
D) All sixth-grade students at the same school

15) A community college offered a Japan tour over the summer to students who would take Japanese course in the fall. The students who visited Japan over the summer through the program did better in the course than students who didn't visit Japan over the summer. Based on the results, which of the following is the most appropriate conclusion?

A) Visiting foreign countries over the summer will cause an improvement for any student who takes the same foreign language course.
B) Visiting Japan over the summer will cause an improvement for any student who takes Japanese language course.
C) Visiting Japan over the summer was the cause of the improvement for the students at the specific community college.
D) No conclusion about the cause can be made regarding students who visited Japan over the summer and their performance in the Japanese course because students who visited Japan were volunteered.

16) A data set of 20 different numbers has a mean of 55 and a median of 55 as well. A new data set was created by adding 10 to each number in the original set of data that is greater than the median and subtracting 10 to each number in the original set of data that is less than the median. Which of the following does not have the same value in both the original and the new data set?

A) Median
B) Mean
C) Standard Deviation
D) Sum of the data

Sample	Percent in favor of new menu	Margin of error
A	78%	1.5%
B	68%	4.3%

17) The results of two random samples of survey for a new item in a local ice-cream shop are shown above. The samples were selected from the same population and the margins of error were calculated using the same method. Which of the following is the most proper reason that the margin of error for sample A is less than that of sample B?

A) Sample A had a larger sample size.
B) Sample B had a larger sample size.
C) Sample A had a higher percentage of favorable response.
D) Sample A could be more knowledgeable in new item.

18) In which of the following tables is the relationship between the values of x and their corresponding y-values non-linear?

A)

x	1	2	3	4
y	4	6	8	10

B)

x	1	2	3	4
y	-12	-6	0	6

C)

x	1	2	3	4
y	2	4	8	16

A)

x	1	2	3	4
y	4.5	6.5	8.5	10.5

CONTINUE

Answers and explanations for Practice problems

(Problem-Solving and Data analysis)

1) B)
Exponential decay problem. $A = P(1 - \%\ in\ decimal)^t$. Thus, $a = 1 - 0.05 = 0.95$.

2) D)
It's important to notice that the temp won't decrease below the room temperature. So, eliminate A) and B). Now, plug $t = 10$ into the answer choices to see which one comes up with 120. Only D) gives the correct value.

3) D)
The spacecraft ascends at a constant rate of 1,000 ft/min. it means that the altitude is linearly increasing according to the time.

4) 3
First translate the words into mathematical expression. $5a + 3b = 3 \cdot 6b$. Simplify it. $5a = 15b$. $a = 3b$. Therefore, $\frac{a}{b} = 3$.

5) C)
Let's say the original price is 100. It becomes 80 after decreasing 20%. And then it was increased by 20%. So, $80 + 80 \cdot 0.2 = 96$. Thus, the price was changed 4% down from the initial price.

6) B)
First convert the blueprint dimensions into the real dimensions. So, it is 8 ft x 10 ft. therefore, the actual area of the oven is 80 square feet.

7) 102
Total number of students of the two school districts in the county A
= $110\ miles^2 \times \frac{90\ students}{miles^2} + 70\ miles^{\ 2} \times \frac{120\ students}{miles^2} = 18300\ students$. Now, the student population density is obtained by dividing it by the total area of two school districts.
The student population density $= \frac{18300\ students}{180\ miles^2} = 101.667 \sim 102\ students\ per\ square\ miles.$

8) D)
Check the y-intercept. Its around 5.75. Eliminate A) and B). Pick any two obvious points to find the slope. Use (50, 4.5) and (70, 4). $m = \frac{4.5-4}{50-70} = -\frac{1}{40}$.

9) A)
Locate data over 80-month field experience from the graph. Now, how many data are there below the line of best fit. You will see there are 5 data.

10) B)
Check the units of x and y in the graph shown. Then you can see the meaning of the slope of the line of best fit. It means that any engineer in the company would spend an hour less on each project if the person had every 40 months field experience.

11) B)
The probability $= \frac{favorite\ outcomes}{total\ outcomes\ within\ the\ constraint} = \frac{22}{70} = \frac{11}{35}$.

12) B)
First median should be the average of 10th and 11th data in order. From the histogram, you know that the median is located between 30-40. Eliminate A) and D). Now, calculate possible mean of time worked. The minimum possible mean $= \frac{0\times2+10\times3+20\times4+30\times6+40\times5}{20} = 24.5$. and the maximum possible mean $= \frac{9\times2+19\times3+29\times4+39\times6+49\times5}{20} = 33.5$. therefore, the mean could be any number between 24.5 and 33.5, inclusive.

13) C)
The standard deviation will be bigger if the more data are located far from the median of the data. Thus, we can figure that $C < B < A$.

14) D)
The survey was conducted to 50 sixth-grade students in a certain elementary school. We can definitely apply the result more than the students who were surveyed. Eliminate B). Even if students are in the same school, we could get way different result for the younger graders. So, eliminate A). there could be a lot of different circumstances and location factors even if they are all sixth-graders in the entire county. So, it's not appropriate to extend the result in that broad range. Eliminate C).

15) D)
We cannot reach any meaningful conclusion in this survey because the participants in the Japan tour were not selected at random. It was actually volunteered. It means that any result from this survey could be biased. In addition, students who visited the Japan could be really serious and lead to great motivation to study Japanese culture and language. Therefore, we can't conclude that the Japan tour over the summer itself directly caused their improvement in their language course grade.

16) C)
The changes were made by adding 10 to each number that is greater than the median and subtracting 10 to each number that is less than the mean. We know that the median stays same. Eliminate A). and the Mean won't change its value either because the total sum and the number of the data didn't change. Eliminate B) and D). The standard deviation will be greater than the original set because the data were further away from the median value after changes.

17) A)
The margin of error solely depends on the sample size. The larger sample size, the smaller margin of error the survey will get as a result.

18) C)
Check which table values provides non-linear values in y. if the data has a common difference, it means it is a linear relationship. Only C) doesn't have a common difference.

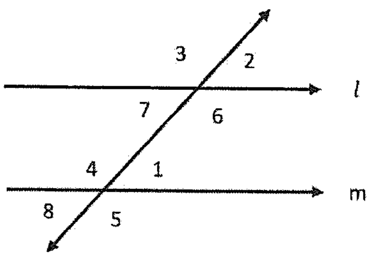
CONTINUE

Geometry and Trigonometry

(5-7 questions, about 15%)

Topics: Area and volume. Lines, angles, and triangles, right triangles, trigonometry, Circles.

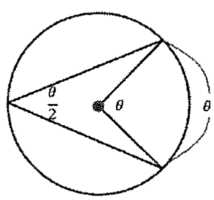
- Angles associated with parallel lines



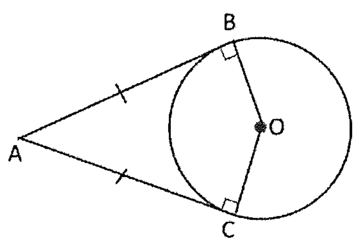
when $l \parallel m$,

- Corresponding angles are congruent: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$, $\angle 5 \cong \angle 6, \angle 7 \cong \angle 8$
- Alternate interior angles are congruent: $\angle 1 \cong \angle 7, \angle 4 \cong \angle 6$
- Same side interior angles are supplementary: $\angle 1 + \angle 6 = 180°, \angle 4 + \angle 7 = 180°$
- Alternate exterior angles are congruent: $\angle 3 \cong \angle 5, \angle 2 \cong \angle 8$
- Vertical angles are congruent: $\angle 1 \cong \angle 8, \angle 4 \cong \angle 5$

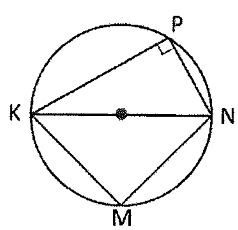
- Angles associated with circles



- Measure of arc is the same as the central angle
- The measure of inscribed angle is half the measure of the same intercepted arc



- The lengths of two tangent segments from a point outside of a circle are congruent ($AB = AC$)
- the segments form a right angle with the radius drawn to the point of tangency ($OB \perp AB, OC \perp AC$)

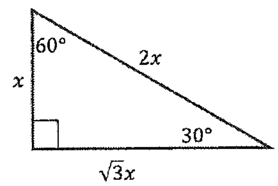


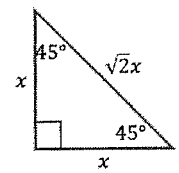
- Any inscribed angle drawn from the endpoints of diameter ($\angle KPN$ *or* $\angle$KMN are right angles)
- If a quadrilateral is inscribed in a circle, its opposite angles are supplementary. ($\angle KPN + \angle KMN = 180°$)

- Angles in polygons

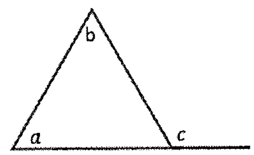
  i. Sum of interior angles in any polygon = $(n-2) \cdot 180°$, where n is the number of sides.
  ii. Each interior angle of a regular polygon = $\frac{(n-2) \cdot 180°}{n}$, where n is the number of sides.
  iii. Sum of exterior angles of any polygon = $360°$
  iv. Each exterior angle of a regular polygon = $\frac{360°}{n}$, where n is the number of sides.

- Angles associated with triangles

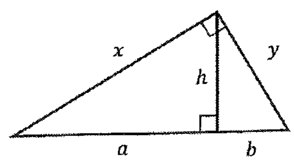




3 – 4 – 5
5 – 12 – 13
7 – 24 – 25
8 – 15 – 17



- One exterior angle is same as the sum of two remote interior angles
  $\angle a + \angle b = \angle c$



$x = \sqrt{a \cdot (a+b)}$

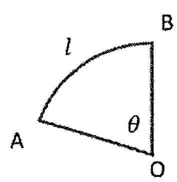
$y = \sqrt{b \cdot (a+b)}$

$h = \sqrt{a \cdot b}$

- Circle equation

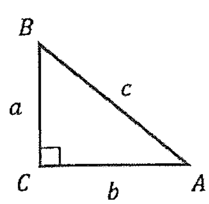
$(x-h)^2 + (y-k)^2 = r^2$, where (h,k) is the coordinates of the center of the circle and r is the radius.

- Area of a sector and Length of an arc



- Area of a sector = $\frac{\theta}{360°} \cdot \pi r^2$, where θ is the central angle in degrees
- Length of an arc = $\frac{\theta}{360°} \cdot 2\pi r$, where θ is the central angle in degrees

- Trigonometry



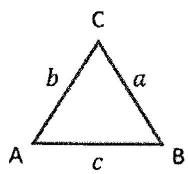
- sin A = $\frac{a}{c}$, cos A = $\frac{b}{c}$, tan A = $\frac{a}{b}$ (Soh-Cah-Toa)
- sin A = cos B (co-functions)

$Note: A = 90° - B$ or $A + B = 90°$ (complementary angles)

- How to convert angles in different mode (radians to degrees or degrees to radians)
  - Radians to degrees : multiply by $\frac{180°}{\pi}$
  - Degrees to radians : multiply by $\frac{\pi}{180°}$

CONTINUE

- Isosceles triangle property and triangle inequalities



Isosceles Triangle Property

- If $\angle A \cong \angle B$, then $a = b$
- If $a = b$, then $\angle A \cong \angle B$

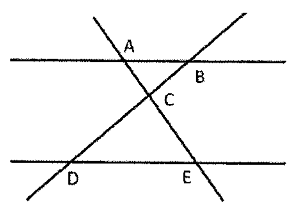
Triangle inequalities

- $|a - b| < c < a + b$

(Practice Problems in Geometry and Trigonometry)

1) In a right triangle, the tangent of one of the two acute angle is $\frac{1}{3}$. Which of the following is the sine of the other angle?

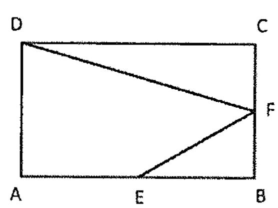
A) $\frac{1}{\sqrt{10}}$ B) $\sqrt{10}$ C) $\frac{\sqrt{10}}{3}$ D) $\frac{3}{\sqrt{10}}$



Note: Figure not drawn to scale

2) In the figure above, $\Delta\ ABC$ is similar to ΔEDC. Which of the following must be true?

A) $\overline{AB} \perp \overline{AE}$ B) $\overline{BD} \perp \overline{DE}$ C) $\overline{AB} \parallel \overline{DE}$ D) $\overline{BC} \perp \overline{CE}$

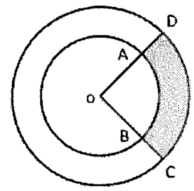


Note: Figure not drawn to scale

3) In the rectangle ABCD above, Points E and F are midpoints of the sides AB and BC, respectively. If $\tan \angle FDC = \frac{1}{2}$, what is the value of $\sin\angle BEF$?

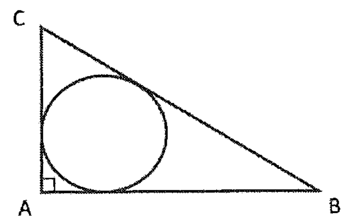
A) $\frac{1}{2}$
B) $\frac{1}{\sqrt{2}}$
C) $\frac{\sqrt{3}}{2}$
D) $\sqrt{2}$

CONTINUE



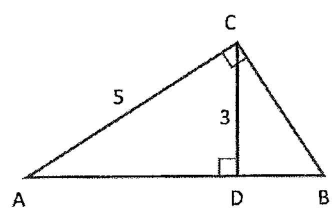
4) Two concentric circles with a center O are drawn above. The area of the shaded region is 16π. If the measure of angle AOB is $\frac{2\pi}{9}$ and $OA:AD = 3:2$, what is the length of a minor arc CD?

A) $\frac{10\pi}{9}$ B) $\frac{10\pi}{3}$ C) $\frac{9\pi}{5}$ D) 3π



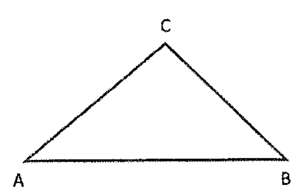
5) In the right triangle ABC above, $AC = 7, BC = 25$. What is the length of the radius of the inscribed circle?

A) 2 B) 3 C) 4 D) 5



6) In the right triangle ABC above, what is the value of $cos\angle CBD$?

A) $\frac{3}{4}$ B) $\frac{3}{5}$ C) $\frac{3}{5}$ D) $\frac{5}{3}$



Note: Figure not drawn to scale

7) In the triangle ABC above, $\sin\angle A = cos\angle B$. Which of the following is correct for triangle ABC?

A) Acute triangle
B) Obtuse triangle
C) Right triangle
D) Isosceles triangle

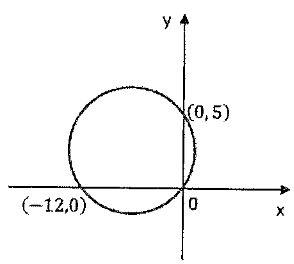
CONTINUE

8) The number of radians in a 540-degree angle can be written as $k\pi$, where k is a constant. What is the value of k?

9) A circle in the XY-plane has equation $(x-1)^2+(y+2)^2=9$. Which of the following points lie in the interior of the circle?

A) $(2,-1)$
B) $(-3,2)$
C) $(2,4)$
D) $(-2,0)$

10) The graph of $2x^2+x+2y^2+y=\frac{1}{4}$ in the XY-plane is a circle. What is the length of the radius of the circle?



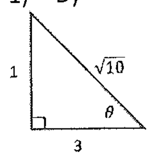
11) The graph of a circle is drawn in the XY-plane above. If the circle intersects at three points as shown, what is the length of the radius of the circle?

CONTINUE

Answers and explanations for Practice problems

(Geometry and Trigonometry)

1) D)

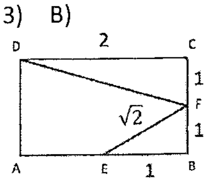


Draw a right triangle and put the dimensions for all sides. Now, sine of the other angle $= \frac{3}{\sqrt{10}}$

2) C)

Since two triangles are similar, the corresponding angles are congruent. So, $\angle A \cong \angle E$ *and* $\angle B \cong \angle D$. If two lines are cut by transversal and alternate interior angles are congruent, then two lines must be parallel.

3) B)

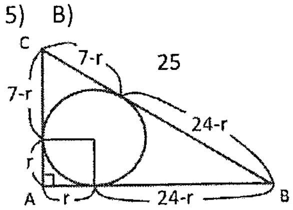


we can put numbers in each segment based on information. tan $\angle FDC = \frac{1}{2}$ and E and F are midpoints of sides AB and BC, respectively. Thus, $sin\angle BEF = \frac{1}{\sqrt{2}}$.

4) B)

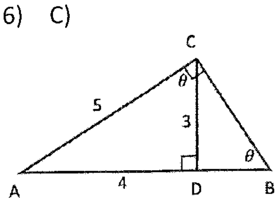
From the ratio given, $OA = 3x, AD = 2x, and\ OD = 5x$. We can set up an equation for the area of shaded region. $16\pi = (\pi(5x)^2 - \pi(3x)^2) \cdot \frac{\frac{2\pi}{9}}{2\pi}$. Then, $16 = \frac{1}{9}(25x^2 - 9x^2)$. And $x = 3$. Now, the length of a minor arc CD = $2\pi(OD) \cdot \frac{\frac{2\pi}{9}}{2\pi}$. And substitute OD = 15. Then, $CD = 2\pi(15) \cdot \frac{1}{9} = \frac{10\pi}{3}$.

5) B)



Use Pythagorean triple 7-24-25 to find AB. And you can break down the lengths of each segment as shown in the figure. Then you can set up an equation $(7 - r) + (24 - r) = 25$. And $r = 3$.

6) C)



From the drawing on the left, $\angle CBD \cong \angle ACD$ using similarity. Thus, $cos\angle CBD = cos\angle ACD = \frac{3}{5}$.

CONTINUE

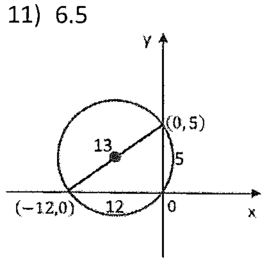
7) C)
Based on co-functions, if $\sin\angle A = cos\angle B$, then we know that $\angle A + \angle B = 90°$. Since the sum of interior angles of a triangle is 180°, $\angle C = 90°$. Therefore, we know that the triangle is a right triangle.

8) 3
To convert the angle from degrees to radians, multiply it by $\frac{\pi}{180°}$.
$540° \times \frac{\pi}{180°} = 3\pi\ rad$. Thus, the value of k is 3.

9) A)
Plug those points (x, y) in the answer choices into the left side of the circle equation and find which one is less than 9. When substitute A) (2, -1) into the equation, you get $(2-1)^2 + (-1+2)^2 = 2 < 9$.

10) $\frac{1}{2}$
Divide the equation $2x^2 + x + 2y^2 + y = \frac{1}{4}$ by 2 and complete the square of the equation.
$x^2 + \frac{1}{2}x + y^2 + \frac{1}{2}y = \frac{1}{8}$. And $x^2 + \frac{1}{2}x + \frac{1}{16} + y^2 + \frac{1}{2}y + \frac{1}{16} = \frac{1}{8} + \frac{1}{16} + \frac{1}{16}$. It becomes
$\left(x + \frac{1}{4}\right)^2 + \left(y + \frac{1}{4}\right)^2 = \frac{1}{4}$. Therefore, $r^2 = \frac{1}{4}$. And $r = \frac{1}{2}$.

11) 6.5



When draw a segment from (0, 5) to (-12, 0), then you can make a right triangle as shown in the figure. It means that the segment added in the figure is the diameter of the circle. Therefore, the length of the radius would be $\frac{1}{2}(13) = 6.5$.

CONTINUE

PART 3

Practice Tests