EE326 Introduction to Information Theory and Coding, Homework #4

- Coverage: Lecture note 10
- Due date: November 11, 11:59 PM through KLMS
- Late submission: -50% for $<$ 12 hours, -100% for $>$12 hours
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Instruction:

- Use Matlab for the simulation. In case you never used Matlab before, it is fine to use Python or C.
- Write a report with detailed answers to each question. For the simulation results, include the screen captures.
- Submit your report together with all the source files (.m file for Matlab).

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1) Consider the $(7,4,3)$ Hamming code with parity check matrix H, for communication over binary symmetric channel with transition probability p. The transmitted codeword, the received vector, and the estimated codeword are denoted as $\mathbf{c}$, $\mathbf{r}$, and $\hat{\mathbf{c}}$, respectively. Note that $\hat{\mathbf{c}} = \mathbf{r}$ if $H\mathbf{r} = 0$, and $\hat{\mathbf{c}} = \mathbf{r} + \mathbf{e}_i$ if $H\mathbf{r} = \mathbf{h}_i$, where $\mathbf{h}_i$ is the ith column of H, $\mathbf{e}_i$ is a vector with a 1 in the i-th position and 0's elsewhere, and '+' represents the modulo-2 sum.
   a) (15 points) Develop a code that uniformly generates 10000 messages, sends the corresponding codewords through a binary symmetric channel with transition probability p, and estimates the transmitted codeword according to the decoding procedure.
   b) (10 points) From the simulation, compute the probability of undetected error $P_u^{(7,4,3)}(p)$ as follows:
   $$P_u^{(7,4,3)}(p) = \frac{\text{number of events s.t. } H\mathbf{r} = \mathbf{0}, \mathbf{r} \neq \mathbf{c}}{10000}.$$
   Plot for $P_u^{(7,4,3)}(p)$ for $p = 10^{-3} : 10^{-3} : 9 \cdot 10^{-3}, 10^{-2} : 10^{-2} : 9 \cdot 10^{-2}, 10^{-1} : 10^{-1} : 5 \cdot 10^{-1}$. Plot both axes in log scale.
   c) (10 points) From the simulation, compute the probability of detected and corrected error $P_{dc}^{(7,4,3)}(p)$ as follows:
   $$P_{dc}^{(7,4,3)}(p) = \frac{\text{number of events s.t. } H\mathbf{r} \neq \mathbf{0}, \hat{\mathbf{c}} = \mathbf{c}}{10000}.$$
   Plot $P_{dc}^{(7,4,3)}(p)$ for $p = 10^{-3} : 10^{-3} : 9 \cdot 10^{-3}, 10^{-2} : 10^{-2} : 9 \cdot 10^{-2}, 10^{-1} : 10^{-1} : 5 \cdot 10^{-1}$. Plot both axes in log scale.
   d) (10 points) From the simulation, compute the probability of detected but uncorrected error $P_{du}^{(7,4,3)}(p)$ as follows:
   $$P_{du}^{(7,4,3)}(p) = \frac{\text{number of events s.t. } H\mathbf{r} \neq \mathbf{0}, \hat{\mathbf{c}} \neq \mathbf{c}}{10000}.$$
   Plot $P_{du}^{(7,4,3)}(p)$ for $p = 10^{-3} : 10^{-3} : 9 \cdot 10^{-3}, 10^{-2} : 10^{-2} : 9 \cdot 10^{-2}, 10^{-1} : 10^{-1} : 5 \cdot 10^{-1}$. Plot both axes in log scale.
   e) (5 points) Plot the total probability of errors $P_t^{(7,4,3)}(p) = P_u^{(7,4,3)}(p) + P_{du}^{(7,4,3)}(p)$ for $p = 10^{-3} : 10^{-3} : 9 \cdot 10^{-3}, 10^{-2} : 10^{-2} : 9 \cdot 10^{-2}, 10^{-1} : 10^{-1} : 5 \cdot 10^{-1}$. Discuss the result.
2) (50 points) Repeat 1) for the (15, 11, 3) Hamming code, and denote the related error probabilities as $P_u^{(15,11,3)}(p)$, $P_{dc}^{(15,11,3)}(p)$, $P_{du}^{(15,11,3)}(p)$, and $P_t^{(15,11,3)}(p)$. Compare the corresponding probabilities for (7,4,3) code and (15,11,3) code and discuss the result.