Assignment 4

MAS365 Introduction to Numerical Analysis **Fall 2021**
Prof. Chang-Ock Lee **Due date: December 6 (Mon), 2021**

Note: Put your computer assignment in KLMS before the beginning of the class. If you did computer programming work, hand in your code and results in KLMS before the beginning of the class, too. For the plotting work, use MATLAB.

1. To find the integral
$$\int_0^{\pi/4} \cos^2 x \, dx \,,$$
we will use Gaussian quadrature. To get an accurate answer, there are two ways; one is use the higher order quadrature and another is use the composite Gaussian quadrature with, for example, three nodes. Discuss efficiency of the two approaches in terms of accuracy, cost, and ease of programming.

Nodes and weights of Gaussian quadrature

Number of nodes	Nodes	Weights
$n \le 5$	See material in the Wikipedia	
6	±.9324695142	.1713244924
	±.6612093865	.3607615730
	±.2386191861	.4679139346
7	±.9491079123	.1294849662
	±.7415311856	.2797053915
	±.4058451514	.3818300505
	0	.4179591837
8	±.9602898565	.1012285363
	±.7966664774	.2223810345
	±.5255324099	.3137066459
	±.1834346425	.3626837834

2. For $f(x) = \cos x$, find $f'(0.25)$ as accurately as possible using
$$f'(x) \approx \frac{f(x+h) - f(x)}{h} \,.$$

3. Solve the following equation by hand:
$$\begin{cases} y' = 1 + \dfrac{y}{x} \\ y(1) = 2 \end{cases}$$

4. Solve the above differential equation on the interval [1, 2] using the improved Euler's method with $h = 1/10$, $1/20$ and $1/40$. Discuss the reduction of errors.