Assignment 2

MAS365 Introduction to Numerical Analysis **Fall 2021**
Prof. Chang-Ock Lee **Due date: Oct. 13 (Wed), 2021**

Note: Put your homework in KLMS before the beginning of the class. If you did computer programming work, hand in your code and results in KLMS before the beginning of the class, too. For the plotting work, use MATLAB.

1. Use Müller's method to find all zeros of $f(x) = 8x^3 - 12x^2 - 50x + 75$. Use your own stop criterion.

2. Rearrange the following expressions so as to avoid loss of significance. Then verify that the rearranged formulae really avoid the loss of significance.

   (a) $y - \sqrt{y^2 - 1}$ for y large

   (b) $1 - \cos^3\theta$ for θ near 0 Hint: Use half-angle formulas.

   (c) $z^2 - 200z + 10001$ for z near 100

3. Find the Cholesky decomposition of the following matrix

$$\begin{bmatrix} 6 & 2 & 1 & -1 & 1 \\ 2 & 4 & 1 & 0 & 0 \\ 1 & 1 & 4 & -1 & 0 \\ -1 & 0 & -1 & 3 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

   Compare the results with those using MATLAB commands for the Cholesky decomposition.

4. The $n \times n$ Hilbert matrix $H^{(n)}$ defined by

$$H_{ij}^{(n)} = \frac{1}{i + j - 1}, \quad 1 \le i, j \le n,$$

   is an ill-conditioned matrix.

(a) Solve the linear system

$$H^{(4)} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

using five digit rounding arithmetic, and compare the actual error. You may do the five digit rounding arithmetic by hand or using the commands `digits` and `vpa` in MATLAB. You can use MATLAB to find the solution and consider it the true solution.

(b) You may find the exact inverse of $H^{(4)}$ in MATLAB. What is the condition number $\kappa_\infty(H^{(4)})$?