Physics Higher Level Internal Assessment

How does the salinity of a water solution affect the speed at which light propagates through it?

An investigation into electromagnetic waves

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Introduction

Optics, the study of the creation and behaviour of light and its branches in optical geometry and physical optics, has been long researched by scientists and philosophers of science since 700BC [Glasses History, 2022]. Modern optics, now finding its uses within the fields of technology, science, and environmental studies, makes up a large portion of contemporary, quotidian life; it allows for modern communications, electronics, and digital cameras to operate. Its magnitude of uses within specialty fields as well as my personal interest within technology and technological instruments lead me to want to investigate deeper into the nature of light, as well as the different aspects that can affect its propagation.

Background information

Light, or broadly speaking electromagnetic waves in general, propagate through the universe, with magnetic and electric fields as its medium. Or in other words, light travels through transverse oscillations within the two fields [Nasa, 2022]. Its propagation occurs at a constant velocity ($c = 3.00 \times 10^8 ms^{-1}$) no matter the wavelength of the electromagnetic wave, so long as the medium remains constant. [Physics Classroom, n.d.].

This propagation of electromagnetic waves can happen through most mediums, even if it were solid (Gamma rays are radiation hazards as they can easily penetrate through the barriers on the human body [EPA, 2022]), by way of absorption and reemission of the wave energy [Physics Classroom, n.d.]. According to the same source, when atoms encounter an electromagnetic wave, the electrons absorb energy, causing them to vibrate. After a short period, the vibrating electrons create oscillations within the electric and magnetic fields, forming a new electromagnetic wave with the same frequency as the original. This process is repeated through regions of space between atoms in a medium. However, because there is a time delay between the absorption of the energy and the reemission of the wave, the net propagation speed of the wave as a whole becomes less than ***c*** when it travels into a medium that is not a complete vacuum. The absorbance coefficient of the atoms within the medium is called the optical density of that medium and is what dictates the speed at which electromagnetic waves actually travel through it [BYJU, 2022]. The refractive index, a measure of optical density, is formally defined within the equation:

$$n = \frac{c}{v}$$

Snell's law

When a wave travels from one medium to another, with different optical densities, depending on the change in velocity between the two mediums, the wave may bend towards or away from the normal. This phenomenon is called *refraction.* Given an initial medium with refractive index n_1 and a subsequent medium with refractive index n_2, if $n_1 < n_2$, the wave would refract outwards, and vice versa. As the ratio $\frac{n_1}{n_2}$ is constant for all wavelengths, the proportion to which any given light bends relative to the incident ray is always constant. This relationship, titled Snell's law after the Dutch mathematician Willebrord Snellious can be written as such:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

Factors that affect the refractive index

Decisively, one of the factors that can influence the optical density of a medium, in the case of a solution, is its density, for solutions, this translates to their concentration. As the concentration increases, the rate at which the electromagnetic wave encounters and therefore is absorbed and reemitted increases, causing the net wave speed to decrease and therefore, the refractive index decreases. In a sucrose concentration, according to [Adnan & Hammadi, 2017], the relationship between concentration and refractive index is proportional. For the same reason, temperature also influences the refractive index of a medium; as the temperature increases and the particles in the medium become more energetic and consequently less dense/viscous, the refractive index decreases [UTSC, n.d.]. Furthermore, due to chemical and ionic make-ups, different materials may have different absorbances, different ions or molecule may be able to absorb electromagnetic waves more easily, causing them to be able to absorb and reemit at different rates. It is for these reasons that the investigation was carried out strictly using only solutions involving regular Table salt within a temperature-controlled environment set at 22 °C.

Investigation overview

Independent and dependent variables

		Range of values (g/L)
Independent:	Concentration of salt in saline solution	0, 50, 100, 150, 250, 300

The range of values of the independent variable was decided based on data gathered from preliminary experimentation as well as research of the saturation level of a saline solution. The saturation concentration of the saline solution used in this experiment was found to be at a maximum of 300g/L.

Dependent: | Refractive Index of solution

The angle of incidence as well as angle of refraction was measured for each value of concentration of saline. Using these values, a plot of angle of incidence vs angle of refraction was drawn to compute both the refractive index (using the reciprocal of the mean gradient) as well as the uncertainty in the gradient (taken as half the gradient of the minimum line of best fit subtracted from the maximum line of best fit)

Hypothesis

Due to the nature of how electromagnetic waves propagate, I believe that the relationship between the concentration of a saline solution and the speed of light within the medium will be inversely proportional; As concentration increases, it is expected that density will also increase proportionally, leading to a higher likelihood of light rays encountering particles and getting absorbed. This increased probability will lead to a proportional increase in the net time delay of the light way when it propagates through the medium, therefore reducing the speed of light.

Safety and environmental considerations

There are minimal safety considerations to be taken for the method, all equipment used contain minimal sharp edges and do not require any safety equipment. However, it should be noted that the eye is sensitive to laser radiation and exposure to strong lasers can lead to visual loss [Verret, 2015]. Furthermore, the wasteful disposal of saline solutions (especially in cases where it is irrigated into rivers and streams) can lead to the reduction of growth of plants and crops as most are sensitive to salinity [Shrivastava & Kumar, 2014]. All saline solutions used within this experiment will henceforth be repurposed for other experiments.

Methodology

1. 1L of Saline solutions were prepared using distilled water and table salt at concentrations: 0, 50g, 100g, 150g, 250g, 300g
2. The saline solution was poured into a cubic glass tank of side lengths 200mm (191mm inside length) and placed on top of a piece of A4 paper, the midpoint of 2 opposite edges were marked on the paper and a normal drawn to the edge of the cube onto the paper, this was repeated for the opposite edge
3. A laser was shown directly at the intersection between the normal and the edge at different angles (10 °, 15°, 20°, 25°, 30° from the normal) and the point at which the light ray escaped the tank on the opposite edge was marked
4. The x distance between the normal and the mark where the light escaped was measured with a ruler
5. The angle of refraction was calculated using trigonometry, where the angle could be given as $\theta_1 = \tan^{-1}(\frac{x}{19.1})$ where x is the x displacement from the normal of the light ray.
6. As the refractive index of air ≈ 1, a plot of the sine of the angle of incidence against the sine of the angle of refraction was graphed, taking the reciprocal of the gradient as the refractive index of the solution
7. Step 3-6 was repeated for all concentrations of saline solutions
8. A plot of the refractive index against concentration was created

Extract of raw data (see appendix for all experimental values)

concentration (g/L)	angle of incident (± 0.5°)	angle of incident (rad)	x-displacement (cm) (±0.05)	Angle of refraction (rad)	Angle of refraction uncertainty
0	30	0.52	7.8	0.39	0.003
0	25	0.44	6.4	0.32	0.003
0	20	0.35	5.1	0.26	0.003
0	15	0.26	3.8	0.20	0.003
0	10	0.17	2.6	0.14	0.003

Uncertainty calculation

As the saline solution was produced by dissolving different masses of table salt into 1L of distilled water, the uncertainty formula used to propagate uncertainties was

$$\frac{\delta y}{y}\{for\, y = bc\} = \frac{\delta b}{b} + \frac{\delta c}{c}$$

Where δb = uncertainty of mass of table salt = 0.005g, δc = uncertainty of distilled water volume = 50mL

As the angles of incident were measured using an analogue protractor with a smallest increment of 1°, an uncertainty of 0.5° was used. However, to propagate this uncertainty to the sine of the angle (in radians), a standard deviation of an expected range of measurements was used. Using half of the sum of the maximum and minimum possible values.

$$\delta \sin \theta_0 = \frac{\left[\sin\left(\theta_0 + \frac{0.5\pi}{180}\right) - \sin\left(\theta_0 - \frac{0.5\pi}{180}\right)\right]}{2}$$

Where $\theta_0 =$ angle of incidence. For the angle of refraction, as $\theta_1 = \tan^{-1}(\frac{x}{19.1})$ was used to compute the angle of refraction, the uncertainty must first be propagated from the uncertainties of the measurements of the side length of the glass box and the x-displacement of the light

$$\delta\theta_1 = \delta \tan^{-1}(\frac{x}{19.1}) = \frac{\left[\tan^{-1}\left(\frac{x + 0.05}{19.05}\right) - \tan^{-1}\left(\frac{x - 0.05}{19.15}\right)\right]}{2}$$

Where $\theta_0 =$ angle of refraction. These uncertainty values were then used to propagate $\sin(\theta_r)$:

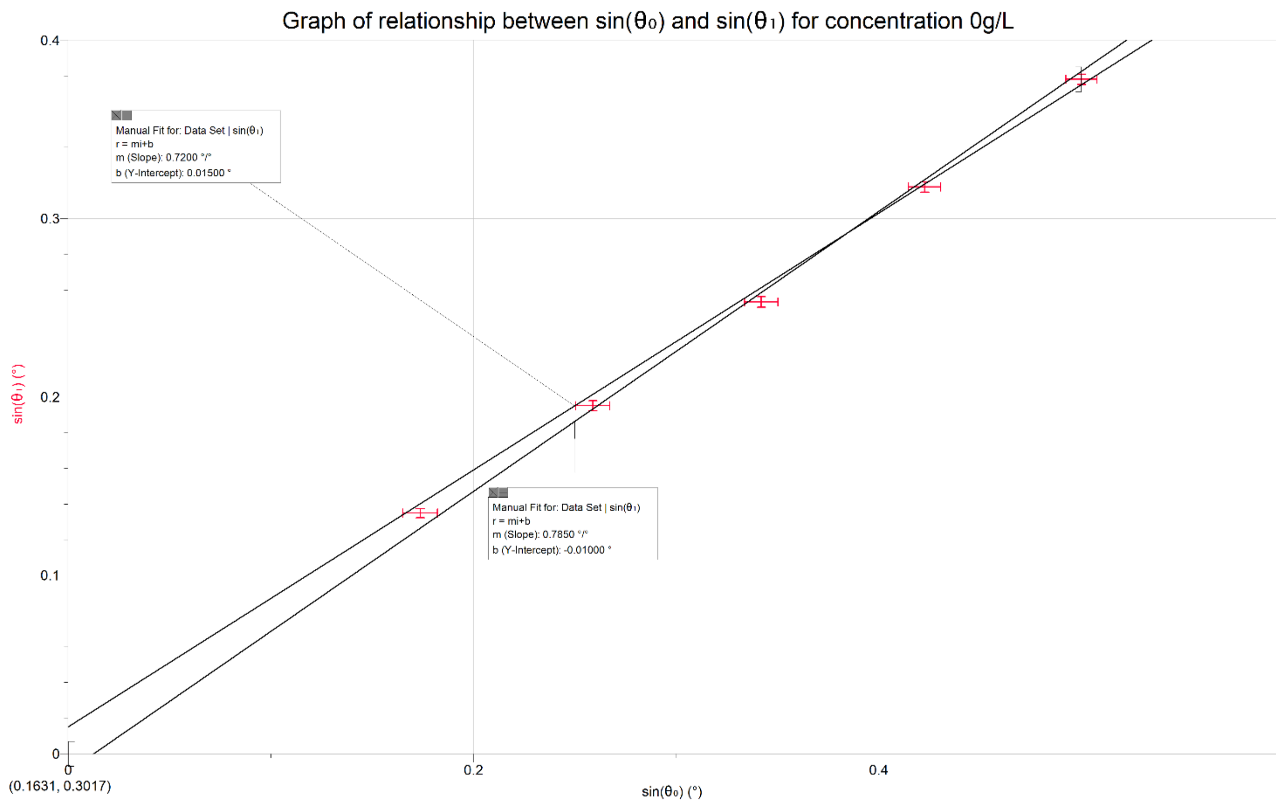
$$\delta \sin \theta_1 = \frac{[\sin(\theta + \delta\theta_1) - \sin(\theta - \delta\theta_1)]}{2}$$

Below is the table for $\sin \theta_r$ and $\sin \theta_i$ for concentration 0g/L as well as their respective standard deviations which were used to plot the uncertainties onto the graph of $\sin \theta_r$ against $\sin \theta_r$.

$\sin \theta_0$	$\sin \theta_1$	Standard deviation $\sin \theta_r$	Standard deviation $\sin \theta_r$
0.00	0.00	0.000	0.000
0.50	0.38	0.008	0.003
0.42	0.32	0.008	0.003
0.34	0.26	0.008	0.003
0.26	0.20	0.008	0.003
0.17	0.13	0.009	0.003

A maximum and minimum line of best fit was plotted on the graph to compute an average slope of best fit $(\frac{max\ slope - min\ slope}{2})$ as well as the uncertainty in the relationship $\left(\frac{\max slope + \min slope}{2}\right)$. Following, the reciprocal of the average slope of best fit was taken as the refractive index and the uncertainty was then propagated again using the formula of uncertainty propagation for division

$$\frac{\delta y}{y}\left\{for\ y = \frac{b}{c}\right\} = \frac{\delta b}{b} + \frac{\delta c}{c}$$

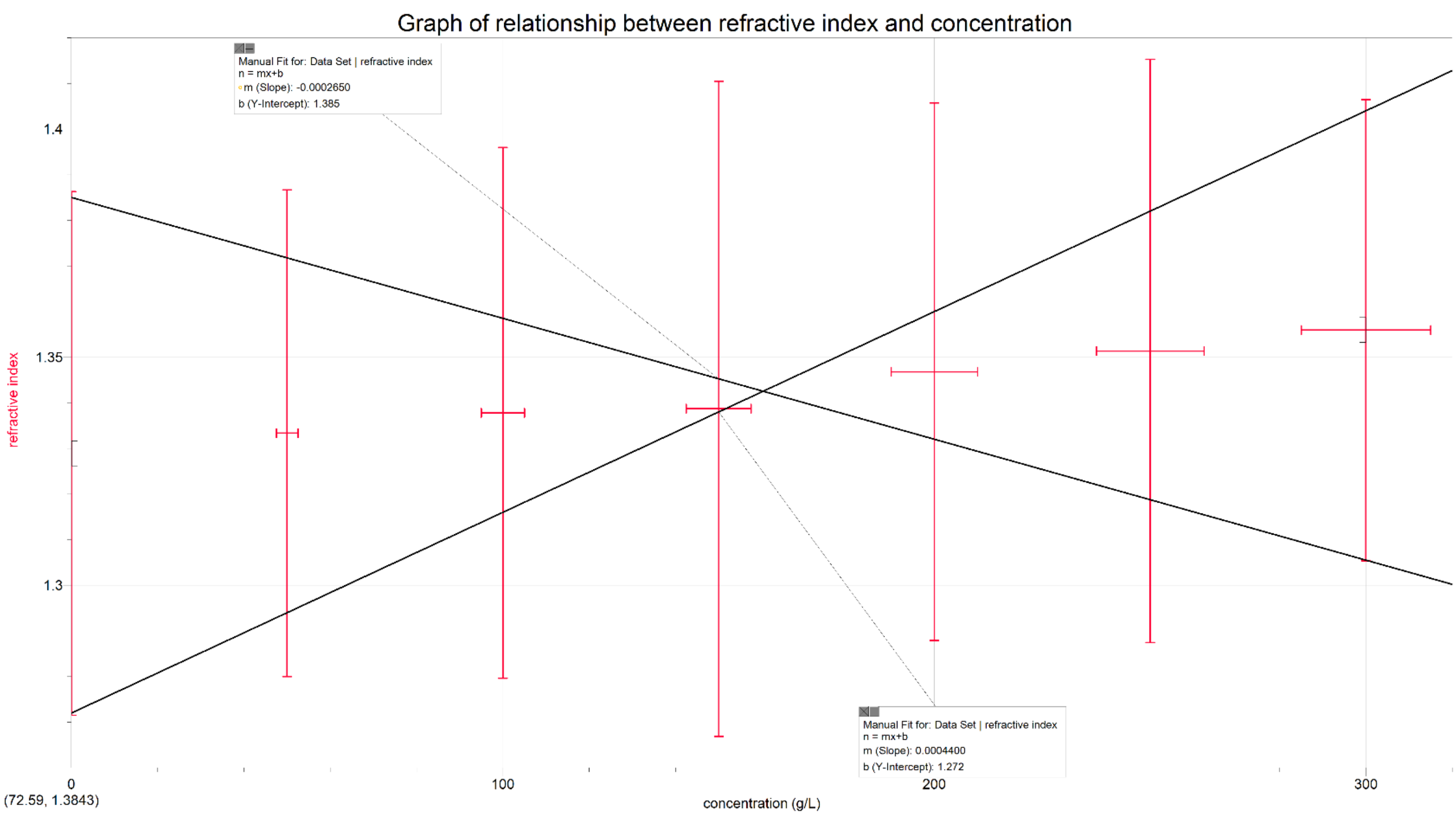


As $\delta b = 0$, it can be inferred that $\frac{\delta y}{y} = \frac{\delta c}{c}$. Therefore, the percentage uncertainty of the refractive index is equal to the percentage uncertainty of its reciprocal (the slope of the graph)

concentration (g/L)	Absolute Uncertainty (concentration)	Refractive index	Absolute uncertainty refractive index
0	0	1.33	0.00

This process was repeated for all respective measured values of each concentration of saline to plot a graph of refractive index against concentration of saline solution

concentration (g/L)	1/Refractive index	Uncertainty (concentration)	Absolute Uncertainty (refractive index)	Refractive index	Percentage uncertainty (refractive index)	Absolute uncertainty refractive index
0	0.753	0	0.033	1.33	0.04	0.06
50	0.750	3	0.030	1.33	0.04	0.05
100	0.748	5	0.033	1.34	0.04	0.06
150	0.746	8	0.040	1.34	0.05	0.07
200	0.743	10	0.033	1.35	0.04	0.06
250	0.740	13	0.035	1.35	0.05	0.06
300	0.738	15	0.028	1.36	0.04	0.05



Data analysis

Plotting a maximum and minimum line of best fit, as well as averaging their slopes, the average slope of the line of best fit that correlates concentration and refractive index = 0.0000875 ± 0.00035250. Thus, the results show a positive correlation between the concentration of a saline solution and its refractive index. It should be noted, however that these results have a percentage uncertainty of 403%, evidenced by the large error bars on the graph. Furthermore, despite the maximum slope of the graph indicating a positive gradient of 0.00044, the minimum line of best fit has a negative gradient of -0,000256. Even more so, as evidenced by the graph, due to the large error bars, as well as a relative flatness of the correlation, the error bars have large overlap with each other throughout the data points, with there even being overlap between the error bars at concentration = 0g/L and those at concentration = 300g/L. All these quantitative and qualitative arguments point towards generally inconclusive data. However, it must also be noted the general trend off the error bars themselves. There is a general upwards trend of the error bars, despite their size and overlap. For example, there is an increase between the lowest edge of the error bar at concentration 0g/L (refractive index = 1.27) and the lowest edge of the error bar at concentration 300g/L (refractive index = 1.31). This relationship is also mirrored at the highest edge. This general upwards is also shown through the positive sign of the average slope of the line of best fit, which shows that, although ultimately definite, there is a positive correlation between the independent and dependent variables that is shown by the results gathered.

Conclusion

In conclusion, the results show a general linear correlation between concentration and refractive index. These indications are also in line with [Adnan & Hammadi, 2017] which aimed to investigate the variation of refractive index against concentration of different solutions (NaCl included) and also found that the refractive index of a saline solution peaked at approximately 1.365. This agreement further confirms the reliability of the data gathered as well as the conclusions drawn.

As the refractive index can be defined as $n = \frac{c}{v}$ where c is the speed of light in a vacuum and v is the speed of light in the refractive medium, it can thus be inferred that — because c is a constant — refractive index is inversely proportional to the speed of light inside the vacuum. Therefore, the results also indicate an inverse relationship between the concentration and the speed of light propagation within the medium, which is in line with the hypothesis provided. These results further serve to confirm the concept of lights propagation by the absorbance and reemission of waves, as solutions that are more concentrated, and thus denser, would have more particles that absorb the lights waves, leading to an increase in the total time dilation that occurs when a light wave propagates through the solution.

These results serve usefulness especially in the field of deep-sea optical imaging, where the speed of light at different depths of the ocean (and therefore different degrees of salinity), serves particular importance in the accurate visualisation of the deep-sea floor. It can thus be used to correct the image produced for varying speeds of light at different levels of concentration to produce more accurate images and representations of the habitat.

However, it should be noted that these results are limited to saline solutions, more specifically solutions of water and table salt, and can only be used as a basis for further investigation of refractive index in other types of solutions, (e.g., sucrose)

Evaluation of strengths and limitations

The strengths of this investigation include:

1. A wide range of concentrations were investigated (from 0 to the maximum saturation percentage of the solution)
2. The Temperature, atmospheric pressure, and volume of the solution was kept consistent by conducting the experiment in the same room to ensure the concentration and therefore density is precise
3. The same wavelength of light was used throughout the investigation to isolate the effects of concentration on the refractive index, as the wavelength of light can also affect its propagation speed, following the equation $s = \lambda f$
4. A range of angles of incident and angles of refraction were measured to plot the relationship between them, this was used to draw maximum and minimum lines of best fit, confirming the relationship between the two values and increasing the reliability and accuracy of the investigation

Limitations:

Limitation	**Significance of Limitation**	**Effect on Results**	**Improvement**	**Why improvement would reduce error**
Measurements of x displacement were taken with a 20cm ruler	Due to the lack of precision of the instrument, there was a limit as to how certain the measurement taken was, leading to speculation about the true value of the measurement	Error bars on each graph were large, which influenced the magnitude of the uncertainty of the final graph	Use a phone camera in conjunction with the inbuilt image analysis functionality of logger pro to make measurements	Allow the measurement of x displacement to be made with a higher level of precision
Angles of incidence were measured using common protractors	Due to the lack of precision of the instrument, there was a limit as to how certain the measurement taken was, leading to speculation about the true value of the measurement	Error bars on each graph were large, which influenced the magnitude of the uncertainty of the final graph	A jig could have been created for each desired value of desired angle of incidence	ensure that the desired angle of incidence is actually measured
Repetitions for each respective angle of incidence and refraction were not taken	Reliability of each angle of refraction measured is not ensured, therefore its true value is not certain	Taking more trials could have eliminated anomalous data that is otherwise not clear	For each angle of incidence, complete 3 trials of data collection to average the results	Further reduce the uncertainty of the angle of refraction and allow for the highlighting of anomalous data
Table salt was used instead of pure NaCl	Whilst not highly significant, the usage of table salt means that the saline solution produced in this investigation was not completely pure, meaning the relationship shown by the results cannot be completely trusted for all saline solutions	Part of the results may be skewed due to interactions between the light rays and floating or dissolved impurities within the table salt	Use NaCl to produce the saline solution	The solution would then be pure.

Further research

Investigations could be conducted on the effect of concentration on refractive index for other aqueous solutions, including mixtures, sucrose and heterogeneous solutions such as with water and oil. This may help in furthering understanding of light behaviour at the boundaries between mediums as well as how Brownian motion of levitating particles within fluids may affect its propagation.

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Appendix

Appendix 1 (Table of raw data)

concentration (g/L)	Other θ_0(rad)	θ_0(rad)	θ_0 (rad)	x displacement (cm)	θ_1 (rad)
0	60	30	0.524	7.6	0.379
0	65	25	0.436	6.3	0.319
0	70	20	0.349	5.0	0.256
0	75	15	0.262	3.8	0.196
0	80	10	0.175	2.6	0.135
50	60	30	0.524	7.8	0.388
50	65	25	0.436	6.5	0.328
50	70	20	0.349	5.2	0.266
50	75	15	0.262	4.0	0.206
50	80	10	0.175	2.7	0.140
100	60	30	0.524	7.8	0.388
100	65	25	0.436	6.4	0.323
100	70	20	0.349	5.1	0.261
100	75	15	0.262	3.8	0.196
100	80	10	0.175	2.6	0.135
150	60	30	0.524	7.8	0.388
150	65	25	0.436	6.6	0.333
150	70	20	0.349	5.2	0.266
150	75	15	0.262	4.0	0.206
150	80	10	0.175	2.7	0.140
200	60	30	0.524	7.8	0.388
200	65	25	0.436	6.5	0.328
200	70	20	0.349	5.2	0.266
200	75	15	0.262	4.0	0.206
200	80	10	0.175	2.7	0.140
250	60	30	0.524	7.3	0.365
250	65	25	0.436	6.1	0.309
250	70	20	0.349	4.8	0.246
250	75	15	0.262	3.7	0.191
250	80	10	0.175	2.3	0.120
300	60	30	0.524	7.4	0.370
300	65	25	0.436	6.0	0.304
300	70	20	0.349	4.7	0.241
300	75	15	0.262	3.6	0.186
300	80	10	0.175	2.3	0.120

Appendix 2 (Standard deviation tables for concentrations)

Concentration 0g/L			
Sin(θ_0) (rad)	sin(θ_1) (rad)	Standard deviation θ_0	Standard deviation θ_1
0.000	0.000	0.000	0.000
0.500	0.370	0.008	0.003
0.423	0.313	0.008	0.003
0.342	0.253	0.008	0.003
0.259	0.195	0.008	0.003
0.174	0.135	0.009	0.003

Concentration 50g/L			
Sin(θ_0) (rad)	sin(θ_1) (rad)	Standard deviation θ_0	Standard deviation θ_1
0.000	0.000	0.000	0.000
0.500	0.378	0.008	0.003
0.423	0.322	0.008	0.003
0.342	0.263	0.008	0.003
0.259	0.205	0.008	0.003
0.174	0.140	0.009	0.003

Concentration 100g/L			
Sin(θ_0) (rad)	sin(θ_1) (rad)	Standard deviation θ_0	Standard deviation θ_1
0.000	0.000	0.000	0.000
0.500	0.378	0.008	0.003
0.423	0.318	0.008	0.003
0.342	0.258	0.008	0.003
0.259	0.195	0.008	0.003
0.174	0.135	0.009	0.003

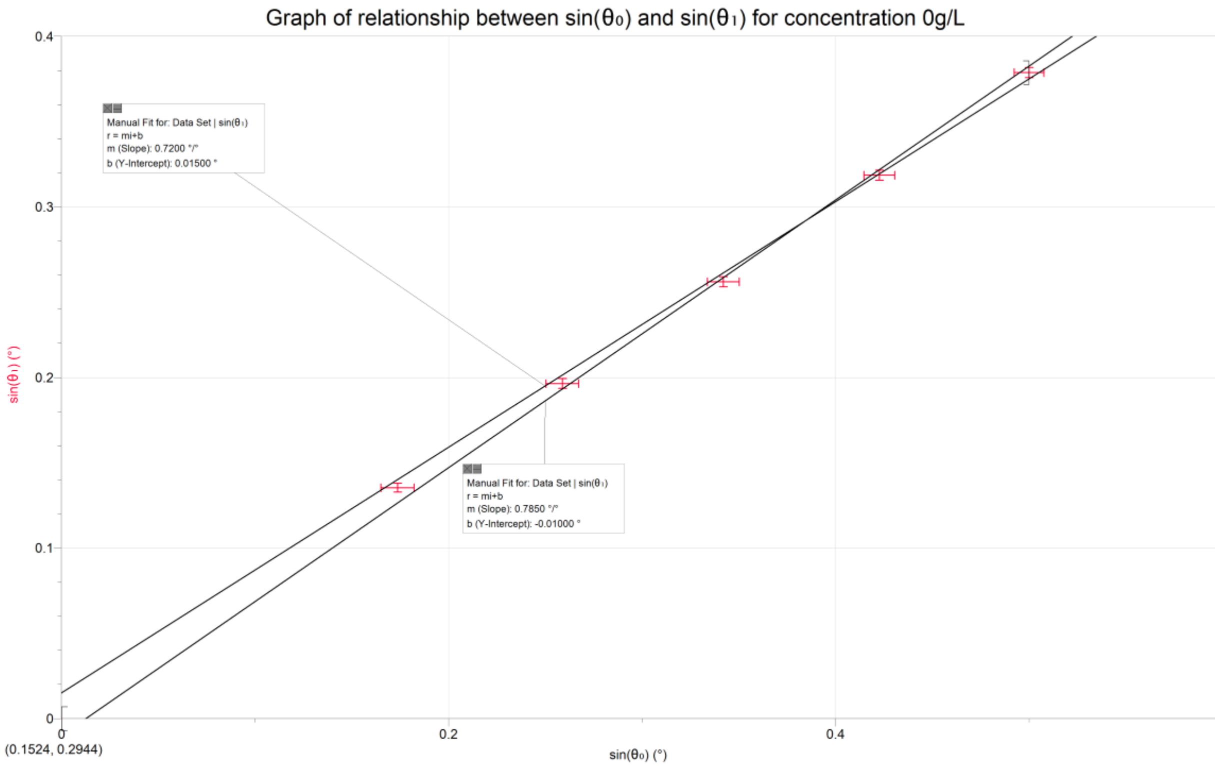
Concentration 150g/L			
Sin(θ_0) (rad)	sin(θ_1) (rad)	Standard deviation θ_0	Standard deviation θ_1
0.000	0.000	0.000	0.000
0.500	0.378	0.008	0.003
0.423	0.327	0.008	0.003
0.342	0.263	0.008	0.003
0.259	0.205	0.008	0.003
0.174	0.140	0.009	0.003

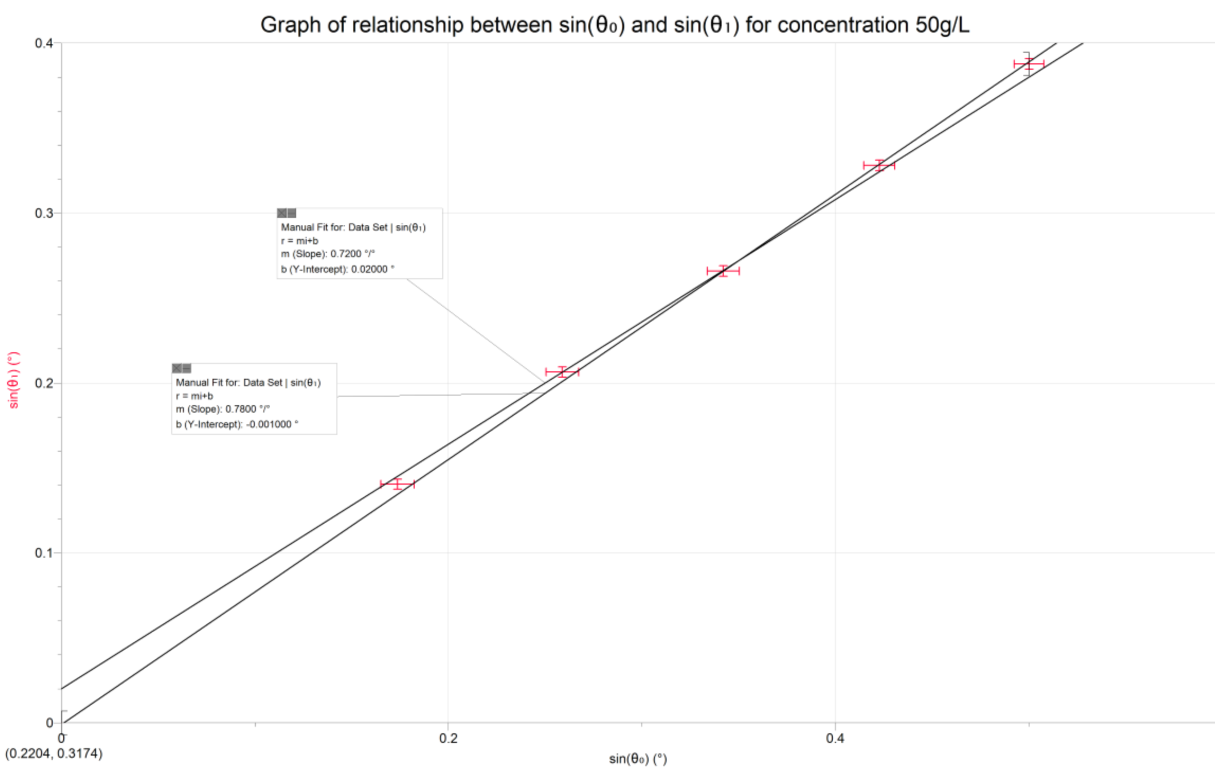
Concentration 200g/L			
Sin(θ_0) (rad)	sin(θ_1) (rad)	Standard deviation θ_0	Standard deviation θ_1
0.000	0.000	0.000	0.000
0.500	0.378	0.015	0.003
0.423	0.322	0.008	0.003
0.342	0.263	0.008	0.003
0.259	0.205	0.008	0.003
0.174	0.140	0.009	0.003

Concentration 250g/L			
Sin(θ_0) (rad)	sin(θ_1) (rad)	Standard deviation θ_0	Standard deviation θ_1
0.000	0.000	0.000	0.000
0.500	0.357	0.008	0.003
0.423	0.304	0.008	0.003
0.342	0.244	0.008	0.003
0.259	0.190	0.008	0.003
0.174	0.120	0.009	0.003

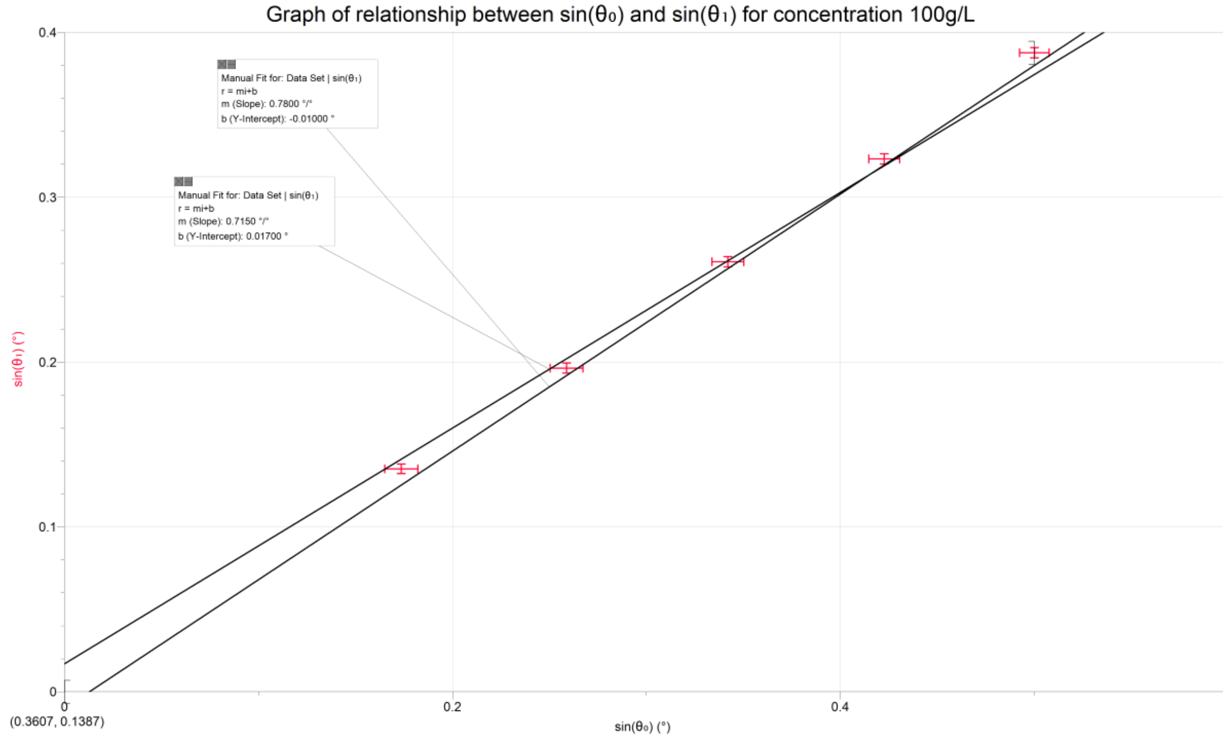
Concentration 250g/L			
Sin(θ_0) (rad)	sin(θ_1) (rad)	Standard deviation θ_0	Standard deviation θ_1
0.000	0.000	0.000	0.000
0.500	0.361	0.008	0.003
0.423	0.300	0.008	0.003
0.342	0.239	0.008	0.003
0.259	0.185	0.008	0.003
0.174	0.120	0.009	0.003

Appendix 3 (graphs of $\sin\theta_0$ vs $\sin\theta_1$)

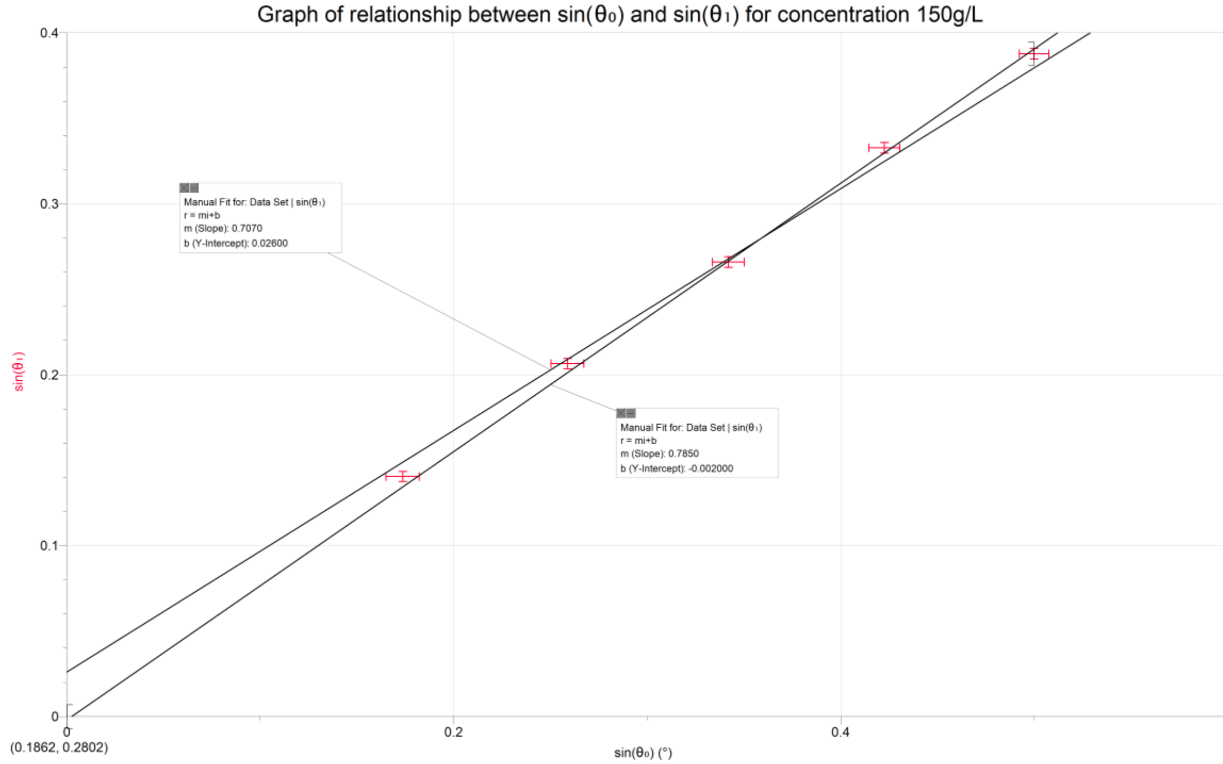


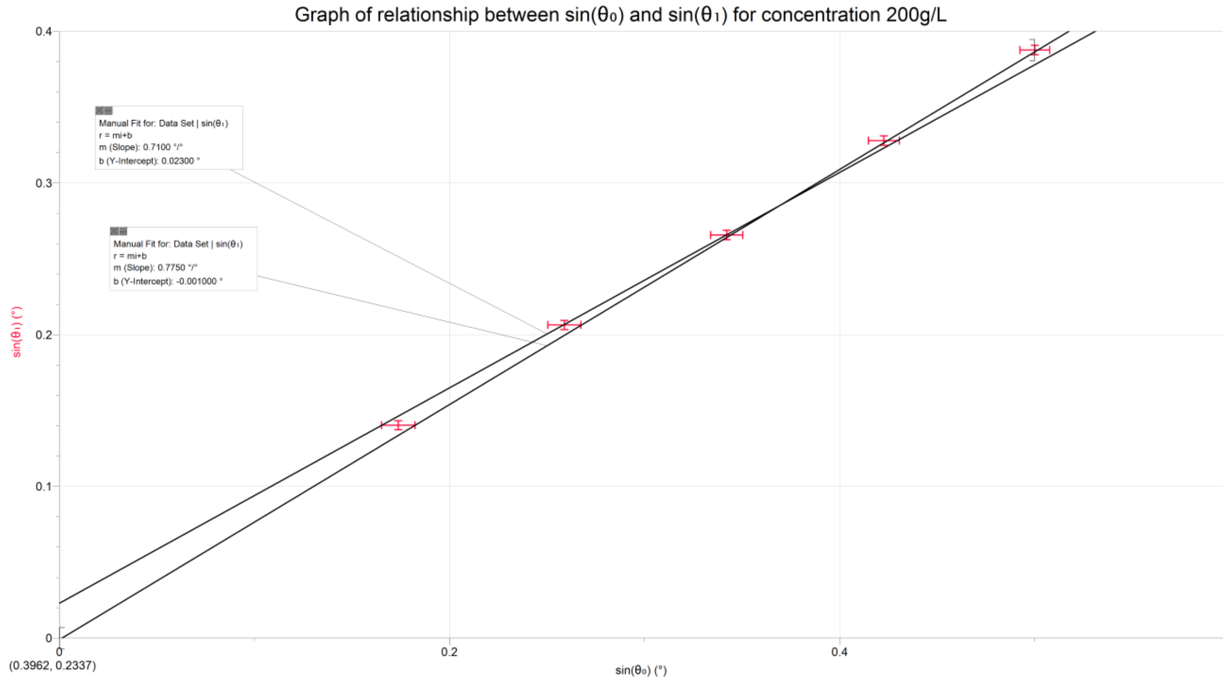


Graph of relationship between $\sin(\theta_0)$ and $\sin(\theta_1)$ for concentration 100g/L

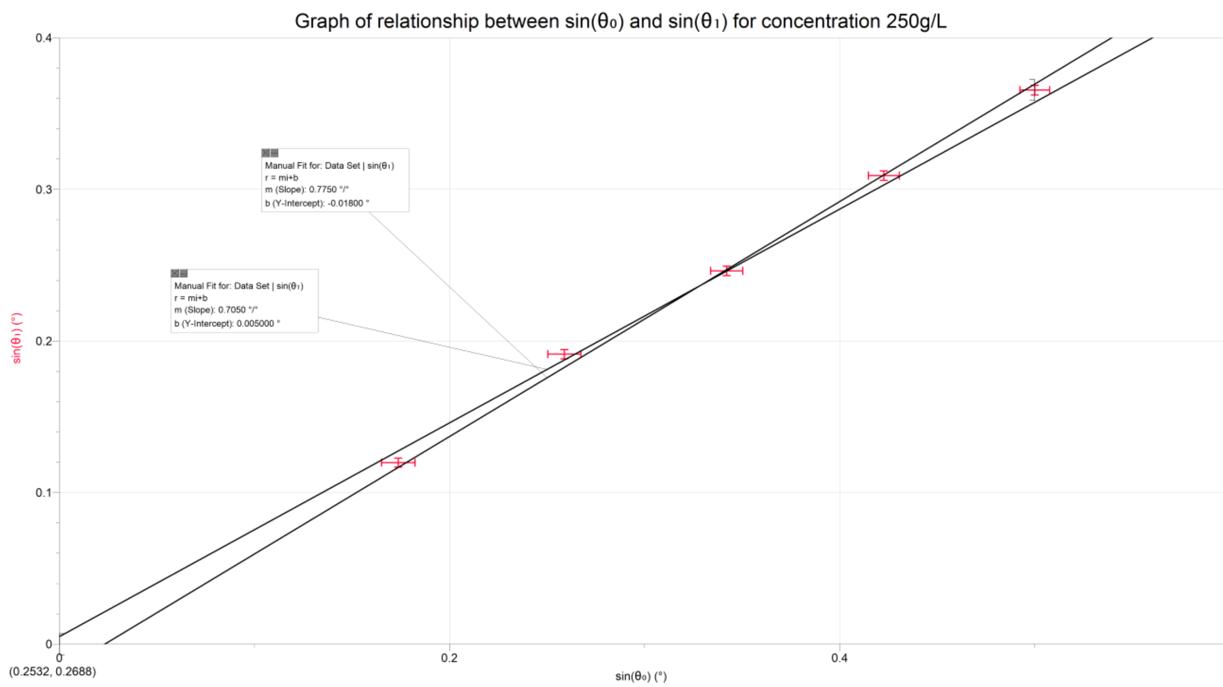


Graph of relationship between $\sin(\theta_0)$ and $\sin(\theta_1)$ for concentration 150g/L

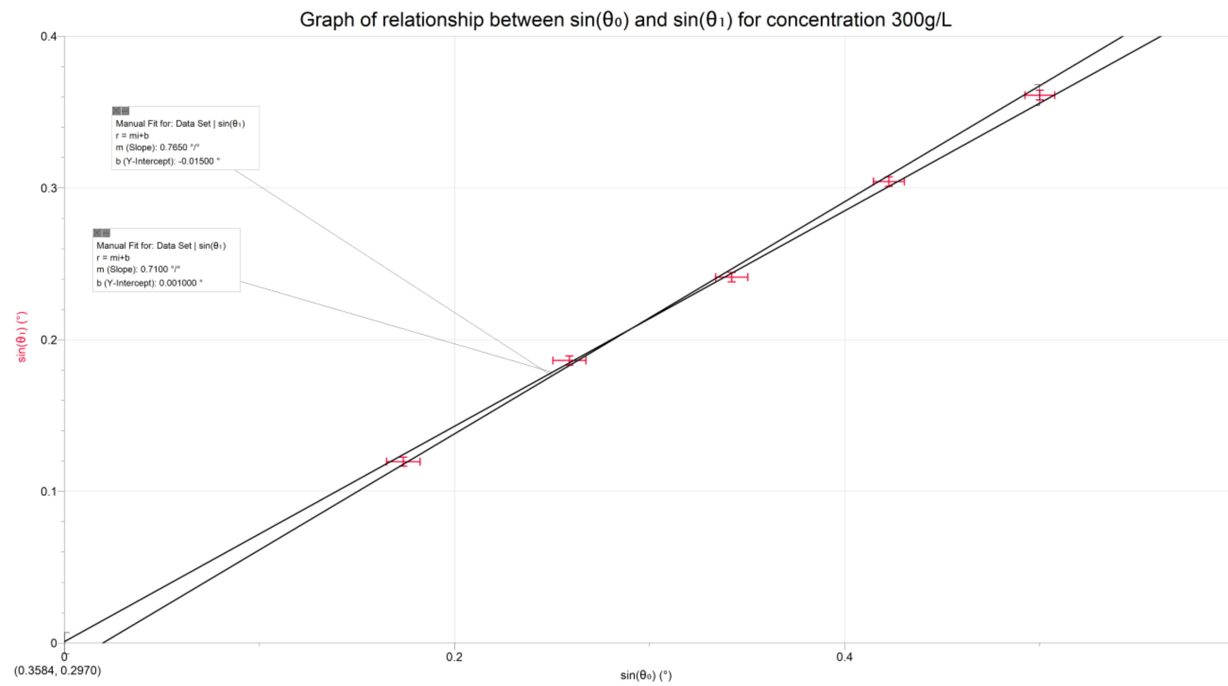




Graph of relationship between sin(θ₀) and sin(θ₁) for concentration 200g/L
Manual Fit for: Data Set | sin(θ₁)
r = mi+b
m (Slope): 0.7100 °/°
b (Y-Intercept): 0.02300 °
Manual Fit for: Data Set | sin(θ₁)
r = mi+b
m (Slope): 0.7750 °/°
b (Y-Intercept): -0.001000 °
sin(θ₁) (°)
sin(θ₀) (°)
(0.3962, 0.2337)
0
0.1
0.2
0.3
0.4



Graph of relationship between sin(θ₀) and sin(θ₁) for concentration 250g/L
Manual Fit for: Data Set | sin(θ₁)
r = mi+b
m (Slope): 0.7750 °/°
b (Y-Intercept): -0.01800 °
Manual Fit for: Data Set | sin(θ₁)
r = mi+b
m (Slope): 0.7050 °/°
b (Y-Intercept): 0.005000 °
sin(θ₁) (°)
sin(θ₀) (°)
(0.2532, 0.2688)
0
0.1
0.2
0.3
0.4



Appendix 4 (Final processed data)

concentration (g/L)	1/m	Uncertainty (concentration)	Absolute Uncertainty (1/n)	n	Percentage uncertainty (n)	Absolute uncertainty n
0	0.753	0	0.033	1.33	4.32	5.74
50	0.750	3	0.030	1.33	4.00	5.33
100	0.748	5	0.033	1.34	4.35	5.82
150	0.746	8	0.040	1.34	5.36	7.18
200	0.743	10	0.033	1.35	4.38	5.90
250	0.740	13	0.035	1.35	4.73	6.39
300	0.738	15	0.028	1.36	3.73	5.06

Appendix 5 (Final graph)