Thermodynamically infeasible reaction loop

$$\max_{v,G,a} \; c^T v$$

s.t.

$$Nv = 0$$

$\forall i, 1 \le i \le m_1, v_{i,min}(1 - a_i) \le v_i \le v_{i,max} a_i$ (internal)

$$G_{i,min} a_i + (1 - a_i) \le G_i \le -a_i + G_{i,max}(1 - a_i)$$

$$P_{int}{}^T G = 0$$

$$a_i \in \{0,1\}, G_i \in R$$

$\forall j, m_1 + 1 \le j \le m, v_{j,min} \le v_j \le v_{j,max}$ (exchange)

Solution v is a thermodynamically feasible flux distribution

Thermodynamically infeasible reaction loop

$$\max_{v,G,a} \; c^T v \longrightarrow \text{Variable vector: } \begin{bmatrix} v_1 \\ \vdots \\ v_8 \\ G_1 \\ \vdots \\ G_5 \\ a_1 \\ \vdots \\ a_5 \end{bmatrix}$$

G, a : only for internal reacttions

s.t.

$$Nv = 0$$

$$\forall i, 1 \leq i \leq m_1, v_{i,min}(1 - a_i) \leq v_i \leq v_{i,max} a_i \text{ (internal)}$$
$$G_{i,min} a_i + (1 - a_i) \leq G_i \leq -a_i + G_{i,max}(1 - a_i)$$
$$P_{int}{}^T G = 0$$
$$a_i \in \{0,1\}, G_i \in R$$

$$\forall j, m_1 + 1 \leq j \leq m, v_{j,min} \leq v_j \leq v_{j,max} \text{ (exchange)}$$

Solution v is a thermodynamically feasible flux distribution

Thermodynamically infeasible reaction loop

$$\max_{v,G,a} \; c^T v$$

s.t.

Equality Constraints

$$Nv = 0$$

$$\forall i, 1 \leq i \leq m_1, v_{i,min}(1 - a_i) \leq v_i \leq v_{i,max} a_i \text{ (internal)}$$

$$G_{i,min} a_i + (1 - a_i) \leq G_i \leq -a_i + G_{i,max}(1 - a_i)$$

$$P_{int}{}^T G = 0$$

$$a_i \in \{0,1\}, G_i \in R$$

$$\forall j, m_1 + 1 \leq j \leq m, v_{j,min} \leq v_j \leq v_{j,max} \text{ (exchange)}$$

Solution v is a thermodynamically feasible flux distribution

Thermodynamically infeasible reaction loop

$$\max_{v,G,a} \; c^T v$$

s.t.

Inequality Constraints

$$Nv = 0$$

$$\forall i, 1 \le i \le m_1, v_{i,min}(1 - a_i) \le v_i \le v_{i,max} a_i \text{ (internal)}$$

$$G_{i,min} a_i + (1 - a_i) \le G_i \le -a_i + G_{i,max}(1 - a_i)$$

$$P_{int}{}^T G = 0$$

$$a_i \in \{0,1\}, G_i \in R$$

$$\forall j, m_1 + 1 \le j \le m, v_{j,min} \le v_j \le v_{j,max} \text{ (exchange)}$$

Solution v is a thermodynamically feasible flux distribution

Thermodynamic metabolic flux analysis (TMFA)

$$\max_{v,\Delta G,z,y} \; c^T v$$

s.t.

$$Nv = 0$$

$$\forall i, 1 \leq i \leq m, \Delta G_i - K + K y_i < 0$$

$$\Delta G_i = \Delta G_i{}^{o} + RTN^T z$$

$$\ln(x_{min}) = z_{min} \leq z \leq z_{max} = \ln(x_{max})$$

$$\forall i \; 1 \leq i \leq m, 0 \leq v_i \leq y_i v_{i,max}$$

$$y_i \in \{0,1\}$$

Variable vector: $\begin{bmatrix} v_1 \\ \vdots \\ v_{13} \\ \Delta G_{1f} \\ \vdots \\ \Delta G_{5b} \\ y_{1f} \\ \vdots \\ y_{5b} \\ z_1 \\ \vdots \\ z_4 \end{bmatrix}$

Thermodynamic metabolic flux analysis (TMFA)

$$\max_{v,\Delta G,z,y} \; c^T v$$

Equality Constraints

s.t.

$$Nv = 0$$

$$\forall i, 1 \leq i \leq m, \Delta G_i - K + K y_i < 0$$

$$\Delta G_i = \Delta G_i{}^o + RTN^T z$$

$$\ln(x_{min}) = z_{min} \leq z \leq z_{max} = \ln(x_{max})$$

$$\forall i \; 1 \leq i \leq m, 0 \leq v_i \leq y_i v_{i,max}$$

$$y_i \in \{0,1\}$$

Thermodynamic metabolic flux analysis (TMFA)

$$\max_{v,\Delta G,z,y} \; c^T v$$

Inequality Constraints

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