$$\max_{v,G,a} c^T v$$
s.t.
$$Nv = 0$$

$$\forall i, 1 \leq i \leq m_1, v_{i,min}(1 - a_i) \leq v_i \leq v_{i,max} a_i \text{ (internal)}$$

$$G_{i,min} a_i + (1 - a_i) \leq G_i \leq -a_i + G_{i,max}(1 - a_i)$$

$$P_{int}^T G = 0$$

$$a_i \in \{0,1\}, G_i \in R$$

$$\forall j, m_1 + 1 \leq j \leq m, v_{j,min} \leq v_j \leq v_{j,max} \text{ (exchange)}$$

$$\max_{v,G,a} c^{T}v$$

$$v,G,a \longrightarrow \text{Variable vector:} \begin{cases} c_{1} \\ c_{2} \\ c_{3} \\ c_{4} \\ c_{5} \\ c_{4} \\ c_{5} \\ c_{5} \\ c_{6} \\ c_{1} \\ c_{2} \\ c_{3} \\ c_{4} \\ c_{5} \\ c_{6} \\ c_{6} \\ c_{6} \\ c_{6} \\ c_{7} \\ c_{8} \\ c_{1} \\ c_{1} \\ c_{1} \\ c_{1} \\ c_{2} \\ c_{3} \\ c_{4} \\ c_{1} \\ c_{5} \\ c_{6} \\ c_{6} \\ c_{8} \\ c_{$$

$$\forall i, 1 \leq i \leq m_1, v_{i,min} (1 - a_i) \leq v_i \leq v_{i,max} a_i \text{ (internal)}$$

$$G_{i,min} a_i + (1 - a_i) \leq G_i \leq -a_i + G_{i,max} (1 - a_i)$$

$$P_{int}{}^T G = 0$$

$$a_i \in \{0,1\}, G_i \in R$$

$$\forall j, m_1 + 1 \leq j \leq m, v_{j,min} \leq v_j \leq v_{j,max} \text{ (exchange)}$$

$$\max_{v,G,a} c^T v$$
s.t. Equality Constraints
$$Nv = 0$$

$$\forall i, 1 \leq i \leq m_1, v_{i,min}(1 - a_i) \leq v_i \leq v_{i,max} a_i \text{ (internal)}$$

$$G_{i,min} a_i + (1 - a_i) \leq G_i \leq -a_i + G_{i,max}(1 - a_i)$$

$$P_{int}^T G = 0$$

$$a_i \in \{0,1\}, G_i \in R$$

$$\forall j, m_1 + 1 \leq j \leq m, v_{j,min} \leq v_j \leq v_{j,max} \text{ (exchange)}$$

$$\max_{v,G,a} c^T v$$
s.t. Inequality Constraints
$$Nv = 0$$

$$\forall i, 1 \leq i \leq m_1, v_{i,min}(1 - a_i) \leq v_i \leq v_{i,max} a_i \text{ (internal)}$$

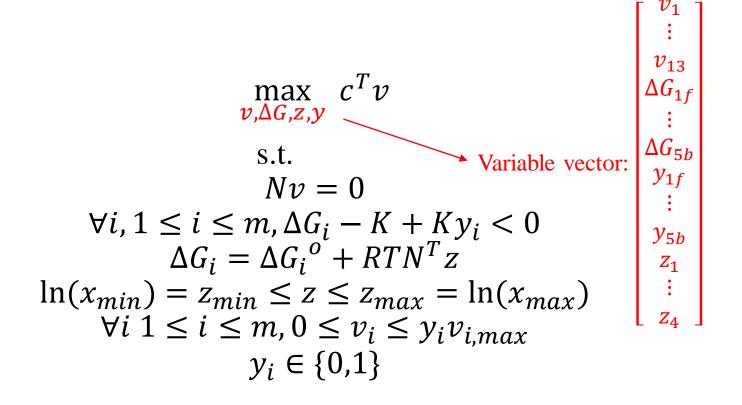
$$G_{i,min} a_i + (1 - a_i) \leq G_i \leq -a_i + G_{i,max}(1 - a_i)$$

$$P_{int}^T G = 0$$

$$a_i \in \{0,1\}, G_i \in R$$

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Thermodynamic metabolic flux analysis (TMFA)



Thermodynamic metabolic flux analysis (TMFA)

$$\max_{v,\Delta G,z,y} c^T v \qquad \textbf{Equality Constraints}$$
s.t.
$$Nv = 0$$

$$\forall i, 1 \leq i \leq m, \Delta G_i - K + K y_i < 0$$

$$\Delta G_i = \Delta G_i^o + RTN^T z$$

$$\ln(x_{min}) = z_{min} \leq z \leq z_{max} = \ln(x_{max})$$

$$\forall i \ 1 \leq i \leq m, 0 \leq v_i \leq y_i v_{i,max}$$

$$y_i \in \{0,1\}$$

Thermodynamic metabolic flux analysis (TMFA)

$$\max_{v,\Delta G,z,y} c^T v \qquad \textbf{Inequality Constraints}$$
s.t.
$$Nv = 0$$

$$\forall i, 1 \leq i \leq m, \Delta G_i - K + K y_i < 0$$

$$\Delta G_i = \Delta G_i^o + RTN^T z$$

$$\ln(x_{min}) = z_{min} \leq z \leq z_{max} = \ln(x_{max})$$

$$\forall i \ 1 \leq i \leq m, 0 \leq v_i \leq y_i v_{i,max}$$

$$y_i \in \{0,1\}$$