Exercise for Constraint-based Modeling of Cellular Networks
12 January 2023

Homework should be sent to Anika (ankueken@uni-potsdam.de)

Hand in your commented code / answers for all exercise tasks as homework.

Modified pFBA

It is expected that when cells are growing exponentially, there will be selection for the fastest growers, and then among the fastest growers, there will be a fitness advantage to cells using the least amount of enzyme.

The goal of this exercise is to minimize the weighted sum of enzyme-associated fluxes, subject to optimal biomass v_{bio}^{*}. The weight w_i for an enzyme-associated reaction i is defined by the number of genes present in the GPR rule of that reaction. The example below shows how this weight can be retrieved. Clearly, for a reaction with no associated GPR rule, the weight is zero.

Reaction	GPR Rule	Weight
1	b4025	1
2	b0351 or b1241	2
3	b0421 and b2231 and b1211	3
4	(b1210 and b1212) or (b1211 and b1212)	3

Use the *E. coli* core metabolic model to determine the minimum weighted sum of enzyme-associated fluxes at the optimum biomass in *E. coli* by solving the optimization program below.

$$\min_{v} \sum_{i} w_i \cdot |v_i|$$

s.t.

$$Nv = 0$$

$$v_{bio} = v_{bio}^{*}$$

$$v_i^{min} \le v_i \le v_i^{max},$$

$$\forall\ i \in Reactions$$

Generalization of either/or

We have the following n constraints on r variables of $x_1, x_2, \dots, x_r$, and we want any k out of n constraints to hold. How can we handle this problem?

$$a_{11} \cdot x_1 + a_{12} \cdot x_2 + \cdots + a_{1r} \cdot x_r \le b_1$$

$$a_{21} \cdot x_1 + a_{22} \cdot x_2 + \cdots + a_{2r} \cdot x_r \le b_2$$

$$\vdots$$

$$a_{n1} \cdot x_1 + a_{n2} \cdot x_2 + \cdots + a_{nr} \cdot x_r \le b_n$$