Constraint-based Modeling of Cellular Networks

Exercise 3 – Linear optimization

3. 11. 2022

Translate text into LP model

- Two products – pants and jackets
- Material: cotton and polyester

To manufacture one pant or one jacket (i.e. 1 unit), following quantities are required:

- Each unit of pants requires 1m^2 of cotton and 2m^2 of polyester
- Each unit of jackets requires 1.5m^2 of cotton and 1m^2 of polyester

The manufacturer has a total 750m^2 of cotton and 1000m^2 of polyester. The price is fixed at

- 50€ per pant
- 40€ per jacket

What is the number of pants and jackets that the manufacturer must produce to obtain a maximum sale?

Translate text into LP model

- Two products – pants and jackets
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To manufacture one pant or one jacket (i.e. 1 unit), following quantities are required:

- Each unit of pants requires 1m^2 of cotton and 2m^2 of polyester
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The manufacturer has a total 750m^2 of cotton and 1000m^2 of polyester. The price is fixed at

- 50€ per pant
- 40€ per jacket

What is the number of pants and jackets that the manufacturer must produce to obtain a maximum sale?

objective

s.t.

constraints

Q1: What are our variables?
Q2: What constrains the variables?

- Two products – pants and jackets
- Material: cotton and polyester

To manufacture one pant or one jacket (i.e. 1 unit), following quantities are required:

- Each unit of pants requires $1m^2$ of cotton and $2m^2$ of polyester
- Each unit of jackets requires $1.5m^2$ of cotton and $1m^2$ of polyester

The manufacturer has a total $750m^2$ of cotton and $1000m^2$ of polyester. The price is fixed at

- 50€ per pant
- 40€ per jacket

What is the number of pants and jackets that the manufacturer must produce to obtain a maximum sale?

objective

s.t.

constraints

Q1: What are our variables?
Q2: What constrains the variables?

- **Two products – pants (P) and jackets (J)**
- Material: cotton and polyester

To manufacture one pant or one jacket (i.e. 1 unit), following quantities are required:

- Each unit of pants requires 1m^2 of cotton and 2m^2 of polyester
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s.t.

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What is the number of pants and jackets that the manufacturer must produce to obtain a maximum sale?

objective

s.t.

constraints

Availability of material constrains product production

- Two products – pants (P) and jackets (J)
- **Material: cotton and polyester**

To manufacture one pant or one jacket (i.e. 1 unit), following quantities are required:

- Each unit of pants requires 1m^2 of cotton and 2m^2 of polyester
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The manufacturer has a total 750m^2 of cotton and 1000m^2 of polyester. The price is fixed at

- 50€ per pant
- 40€ per jacket

What is the number of pants and jackets that the manufacturer must produce to obtain a maximum sale?

objective

s.t.

? ≤ 750 **(cotton)**
? ≤ 1000 **(polyester)**

constraints

Availability of material constrains product production

- Two products – pants (P) and jackets (J)
- Material: cotton and polyester

To manufacture one pant or one jacket (i.e. 1 unit), following quantities are required:

- Each unit of pants requires 1m^2 of cotton and 2m^2 of polyester
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The manufacturer has a total 750m^2 of cotton and 1000m^2 of polyester. The price is fixed at

- 50€ per pant
- 40€ per jacket

What is the number of pants and jackets that the manufacturer must produce to obtain a maximum sale?

objective

s.t.

$1*P + 1.5*J \leq 750$ (cotton)

$? \leq 1000$ (polyester)

constraints

Availability of material constrains product production

- Two products – pants (P) and jackets (J)
- Material: cotton and polyester

To manufacture one pant or one jacket (i.e. 1 unit), following quantities are required:

- Each unit of pants requires $1m^2$ of cotton and $2m^2$ of polyester
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The manufacturer has a total $750m^2$ of cotton and $1000m^2$ of polyester. The price is fixed at

- 50€ per pant
- 40€ per jacket

What is the number of pants and jackets that the manufacturer must produce to obtain a maximum sale?

objective

s.t.

$1 * P + 1.5 * J \leq 750$ (cotton)

$\mathbf{2 * P + 1 * J \leq 1000}$ **(polyester)**

constraints

Lower and upper bounds

- Two products – pants (P) and jackets (J)
- Material: cotton and polyester

To manufacture one pant or one jacket (i.e. 1 unit), following quantities are required:

- Each unit of pants requires $1m^2$ of cotton and $2m^2$ of polyester
- Each unit of jackets requires $1.5m^2$ of cotton and $1m^2$ of polyester

The manufacturer has a total $750m^2$ of cotton and $1000m^2$ of polyester. The price is fixed at

- 50€ per pant
- 40€ per jacket

What is the number of pants and jackets that the manufacturer must produce to obtain a maximum sale?

max $z = 50\ P + 40\ J$ *objective*

s.t.

$1*P + 1.5*J \leq 750$ (cotton)

$2*P + 1*J \leq 1000$ (polyester)

$\mathbf{P \geq 0}$

$\mathbf{J \geq 0}$

constraints

The objective

- Two products – pants and jackets
- Material: cotton and polyester

To manufacture one pant or one jacket (i.e. 1 unit), following quantities are required:

- Each unit of pants requires 1m^2 of cotton and 2m^2 of polyester
- Each unit of jackets requires 1.5m^2 of cotton and 1m^2 of polyester

The manufacturer has a total 750m^2 of cotton and 1000m^2 of polyester. The price is fixed at

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- 40€ per jacket

What is the number of pants and jackets that the manufacturer must produce to obtain a maximum sale?

objective

s.t.

$$1*P + 1.5*J \leq 750 \qquad \text{(cotton)}$$
$$2*P + 1*J \leq 1000 \qquad \text{(polyester)}$$
$$P \geq 0$$
$$J \geq 0$$

constraints

The objective

- Two products – pants (P) and jackets (J)
- Material: cotton and polyester

To manufacture one pant or one jacket (i.e. 1 unit), following quantities are required:

- Each unit of pants requires 1m^2 of cotton and 2m^2 of polyester
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The manufacturer has a total 750m^2 of cotton and 1000m^2 of polyester. The price is fixed at

- 50€ per pant
- 40€ per jacket

What is the number of pants and jackets that the manufacturer must produce to obtain a maximum sale?

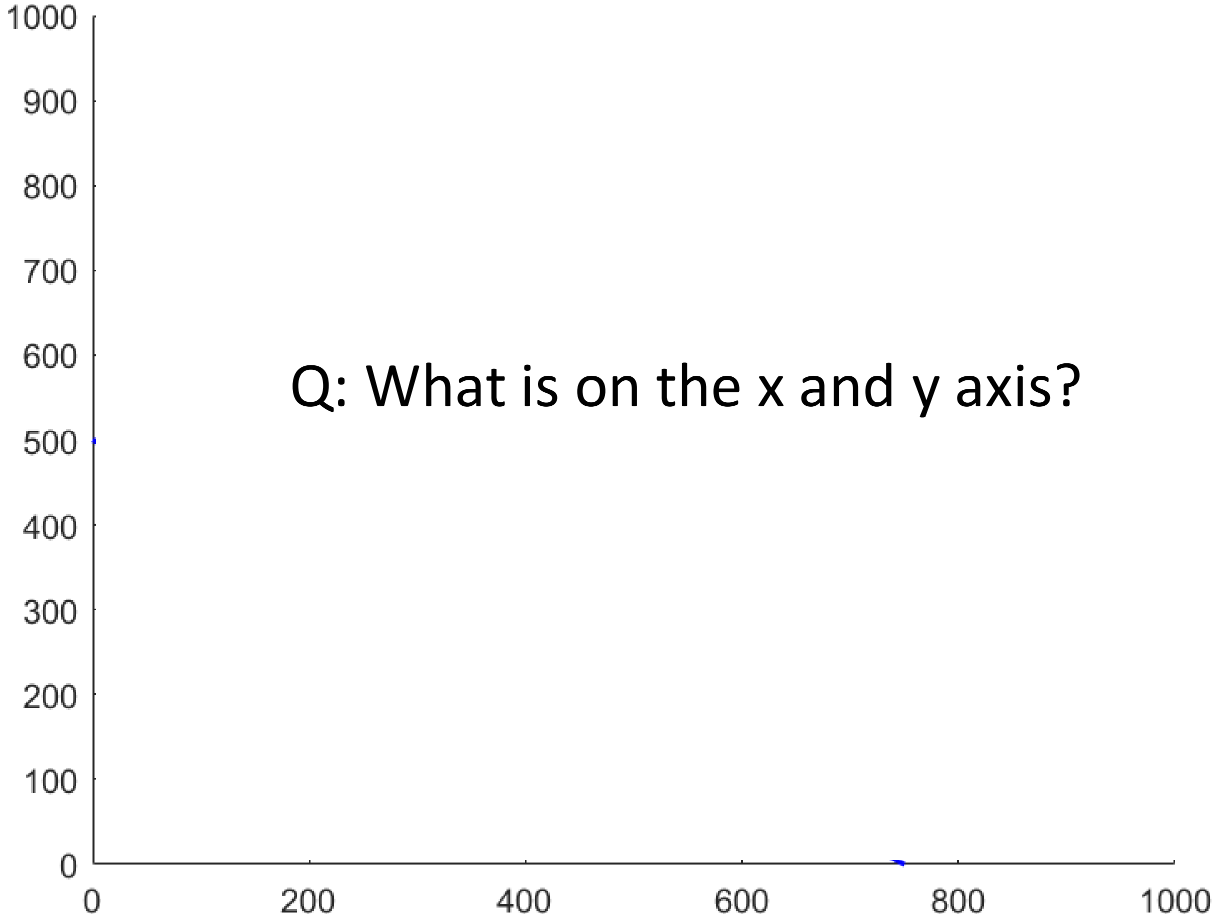
max z = 50 P + 40 J *objective*

s.t.

$$1 * P + 1.5 * J \leq 750 \quad \text{(cotton)}$$
$$2 * P + 1 * J \leq 1000 \quad \text{(polyester)}$$
$$P \geq 0$$
$$J \geq 0$$

constraints

Graphical solution



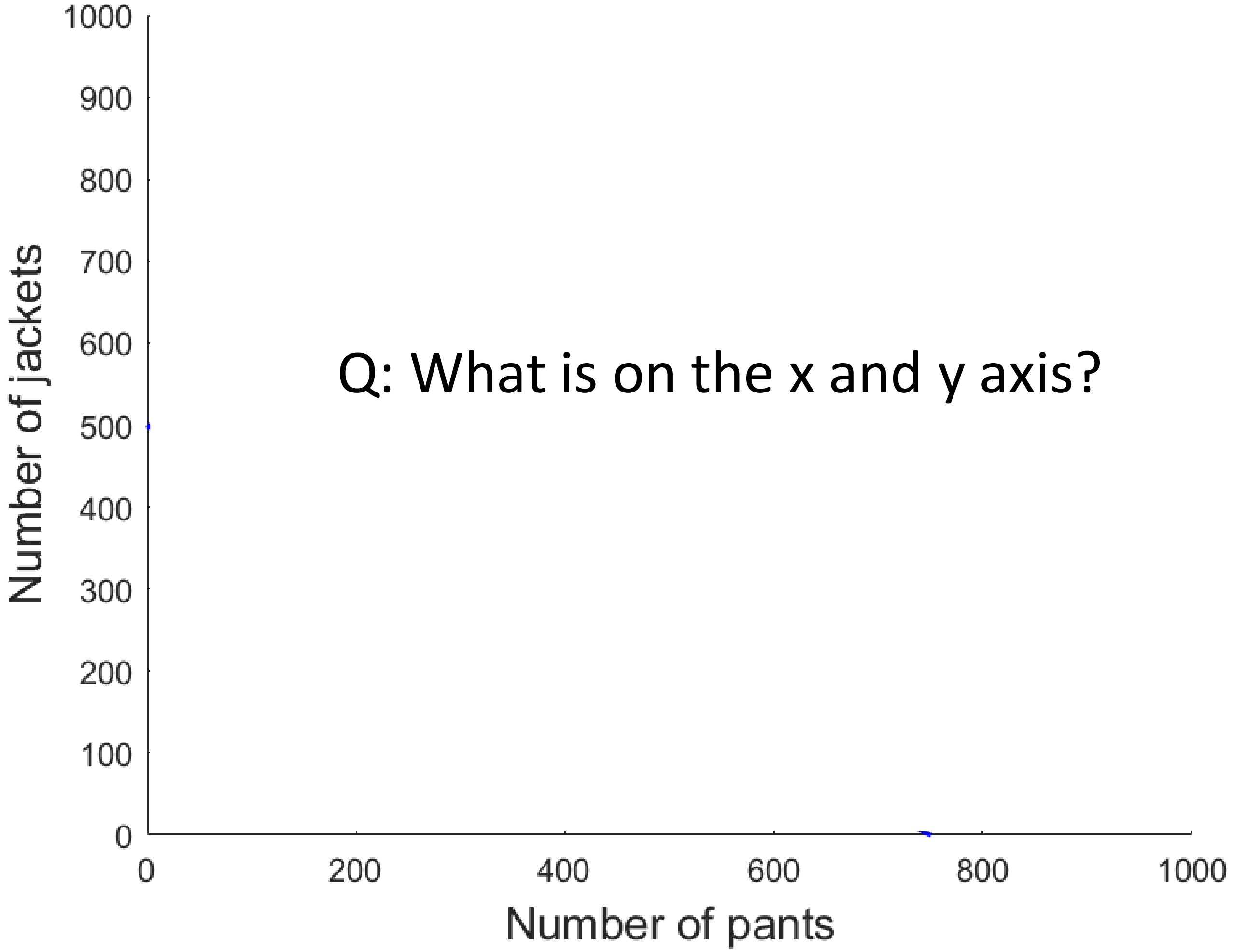
Q: What is on the x and y axis?

max z = *50 P + 40 J*

s.t.

$$1*P + 1.5*J \leq 750 \quad \text{(cotton)}$$
$$2*P + 1*J \leq 1000 \quad \text{(polyester)}$$
$$P \geq 0$$
$$J \geq 0$$

Graphical solution



Q: What is on the x and y axis?

max $z = 50\ P + 40\ J$

s.t.

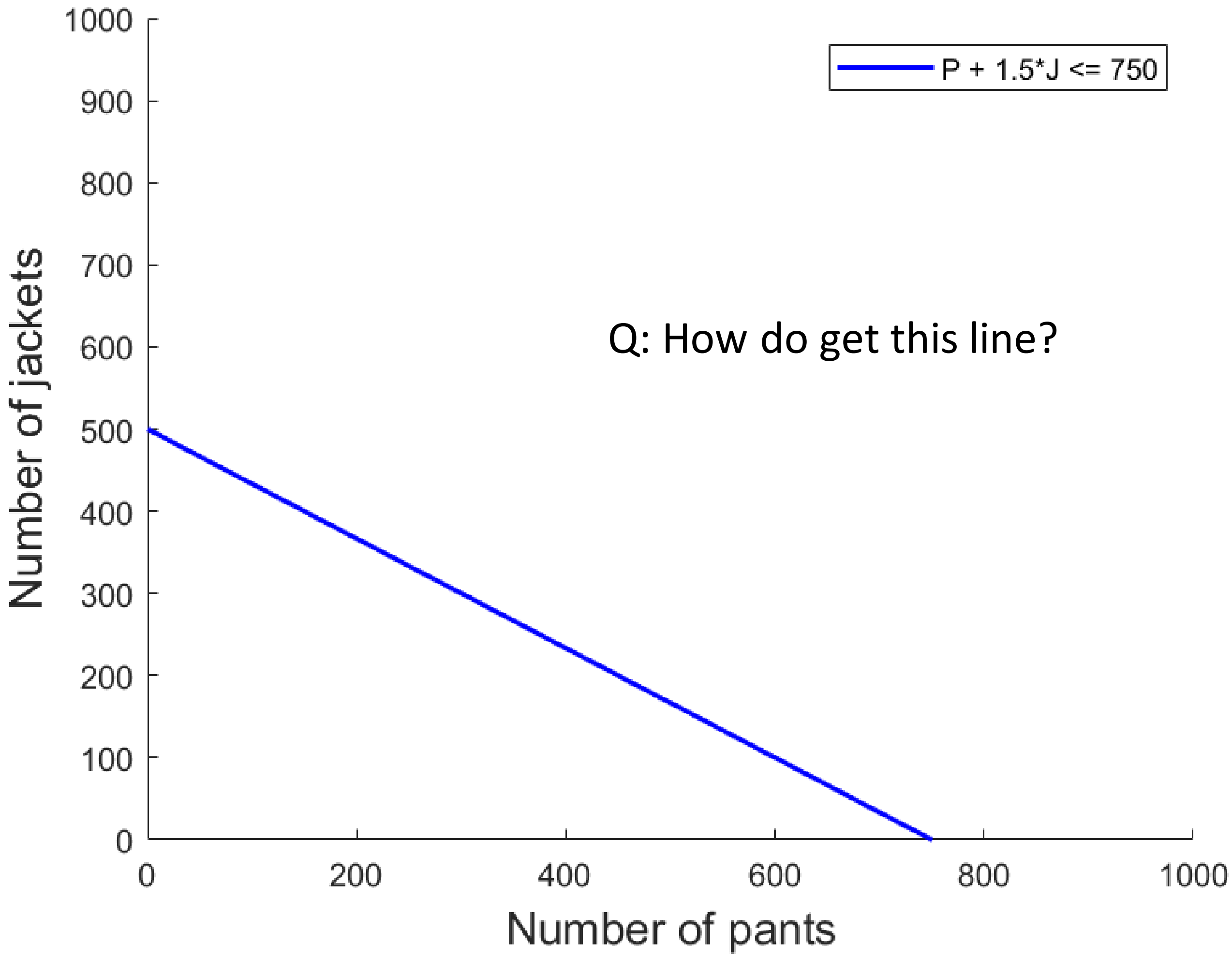
$1{*}P + 1.5{*}J \leq 750$ (cotton)

$2{*}P + 1{*}J \leq 1000$ (polyester)

$P \geq 0$

$J \geq 0$

Graphical solution – constraint 1



Q: How do get this line?

max $z = 50\ P + 40\ J$

s.t.

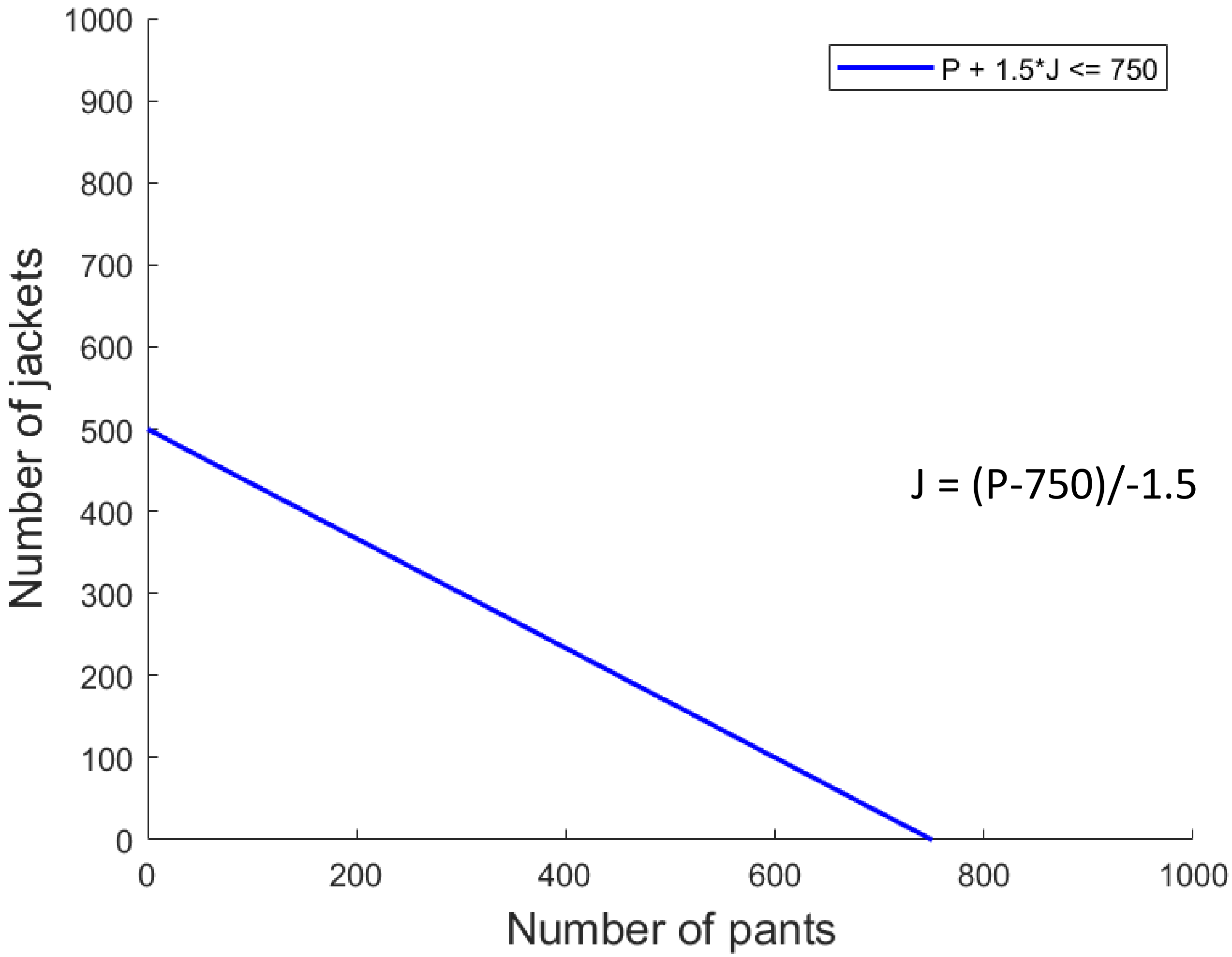
1*P + 1.5*J ≤ 750 (cotton)

2*P + 1*J ≤ 1000 (polyester)

P ≥ 0

J ≥ 0

Graphical solution – constraint 1



J = (P-750)/-1.5

max $z = 50\,P + 40\,J$

s.t.

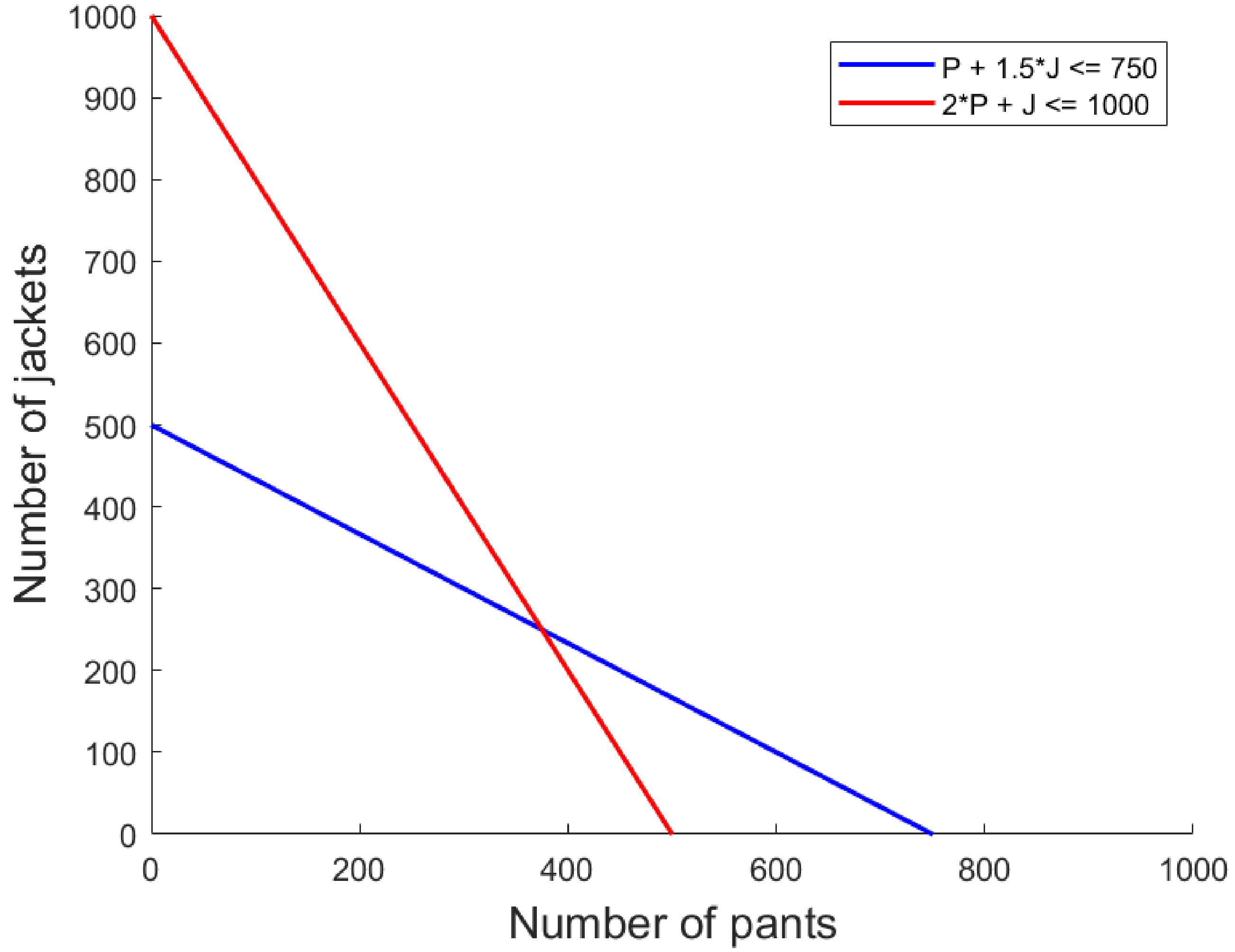
$1*P + 1.5*J \leq 750$ (cotton)

$2*P + 1*J \leq 1000$ (polyester)

$P \geq 0$

$J \geq 0$

Graphical solution – constraint 2



max $z = 50\,P + 40\,J$

s.t.

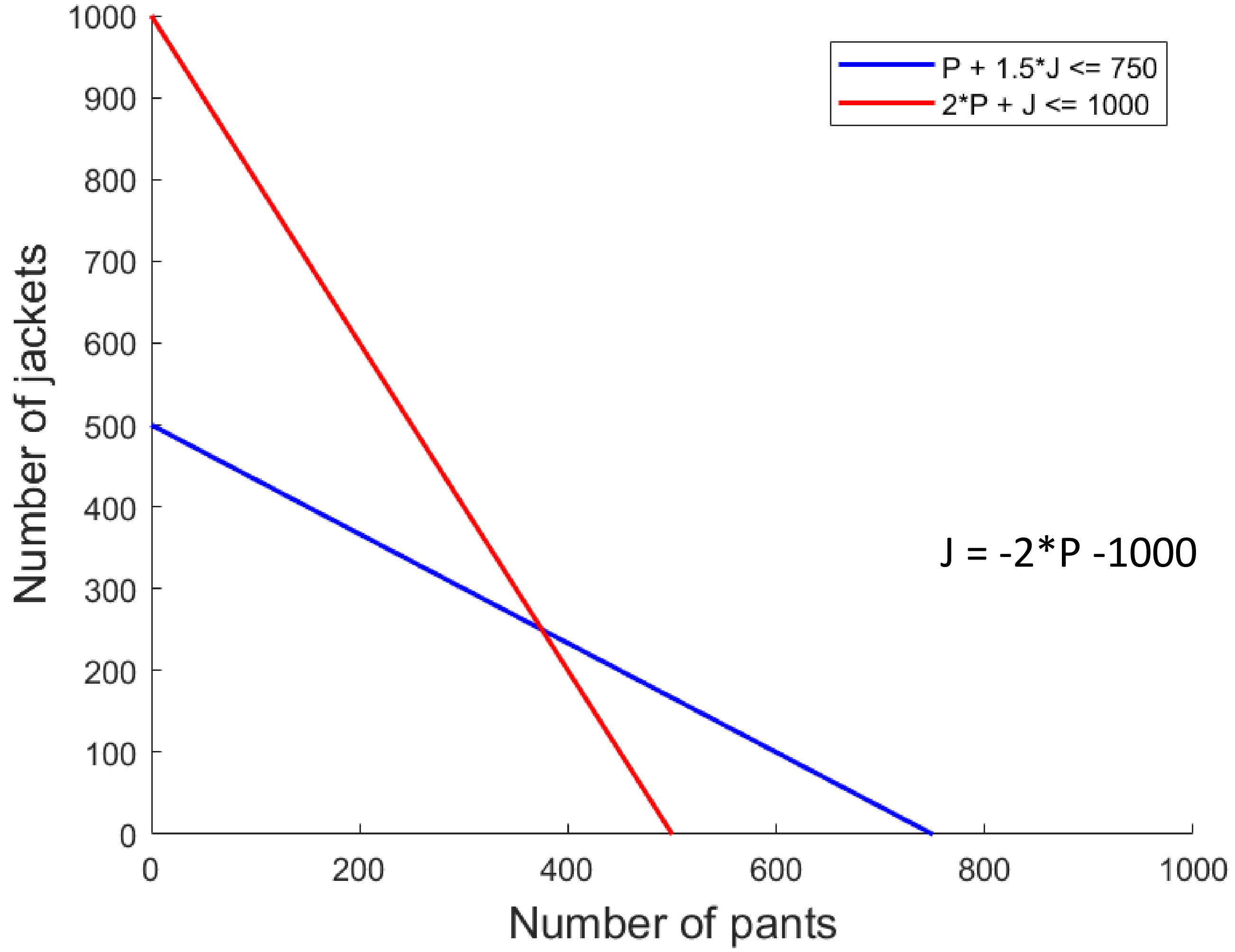
$1*P + 1.5*J \leq 750$ (cotton)

$2*P + 1*J \leq 1000$ (polyester)

$P \geq 0$

$J \geq 0$

Graphical solution – constraint 2



J = -2*P -1000

max z = *50 P + 40 J*

s.t.

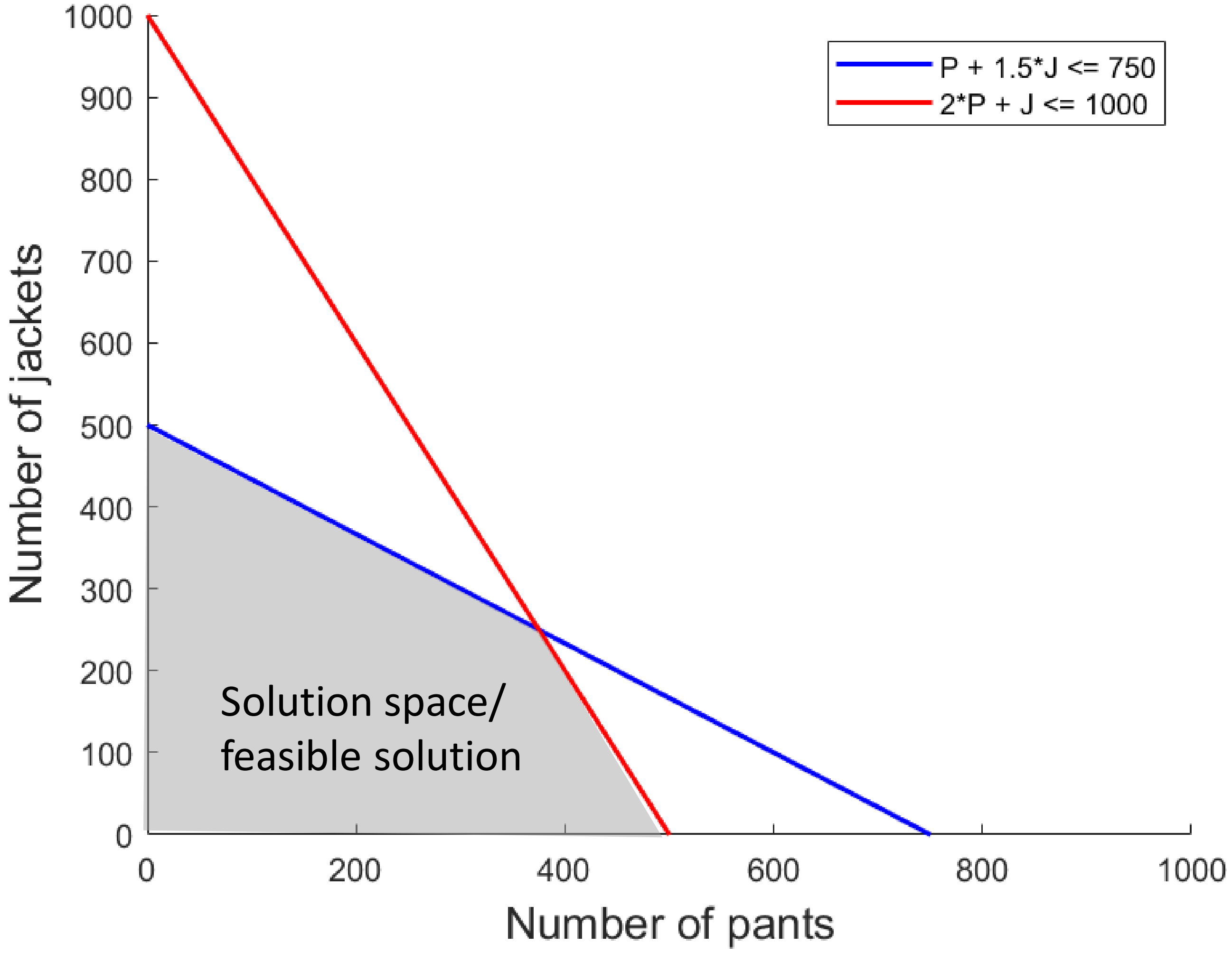
1*P + 1.5*J ≤ 750 (cotton)

2*P + 1*J ≤ 1000 (polyester)

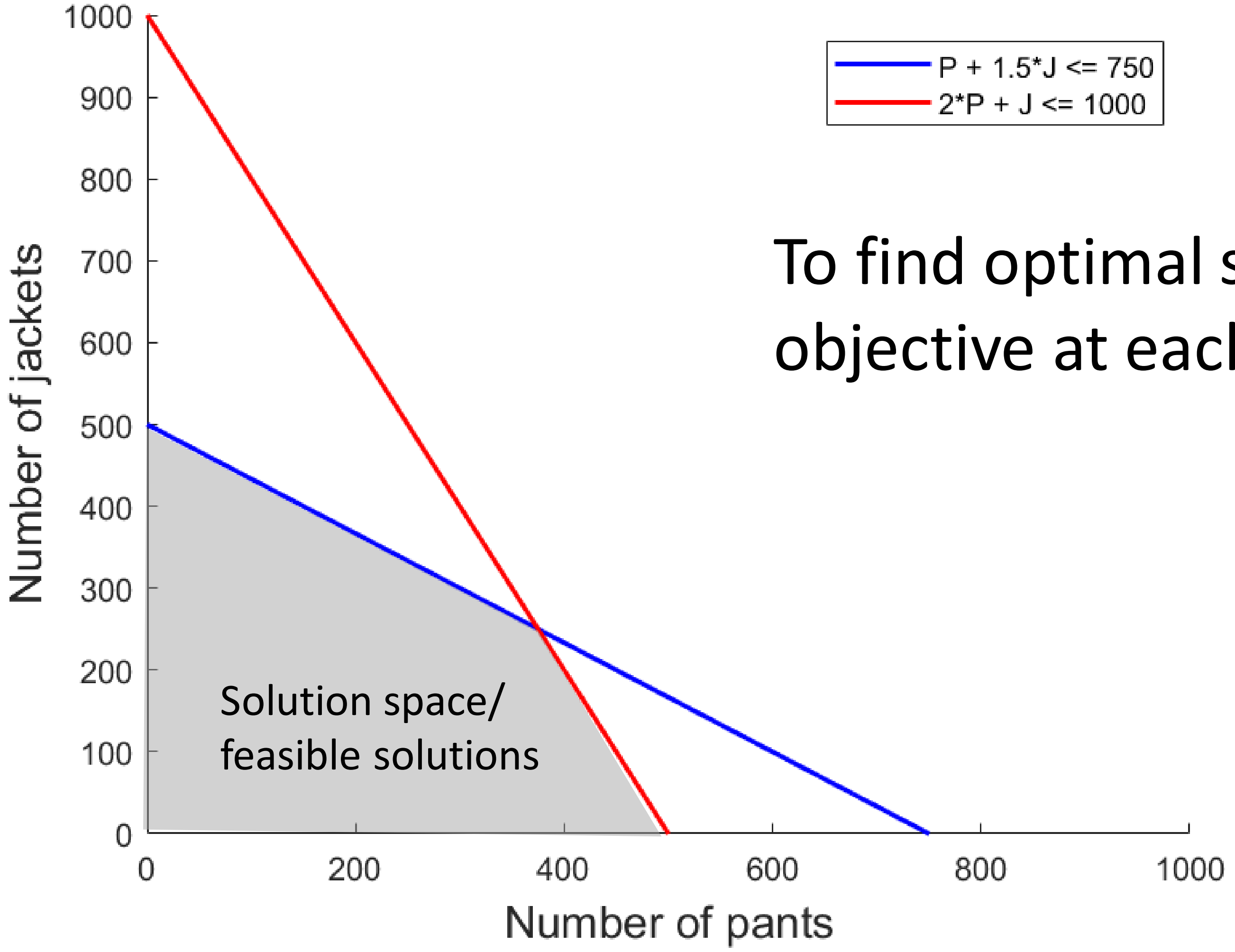
P ≥ 0

J ≥ 0

Graphical solution – solution space

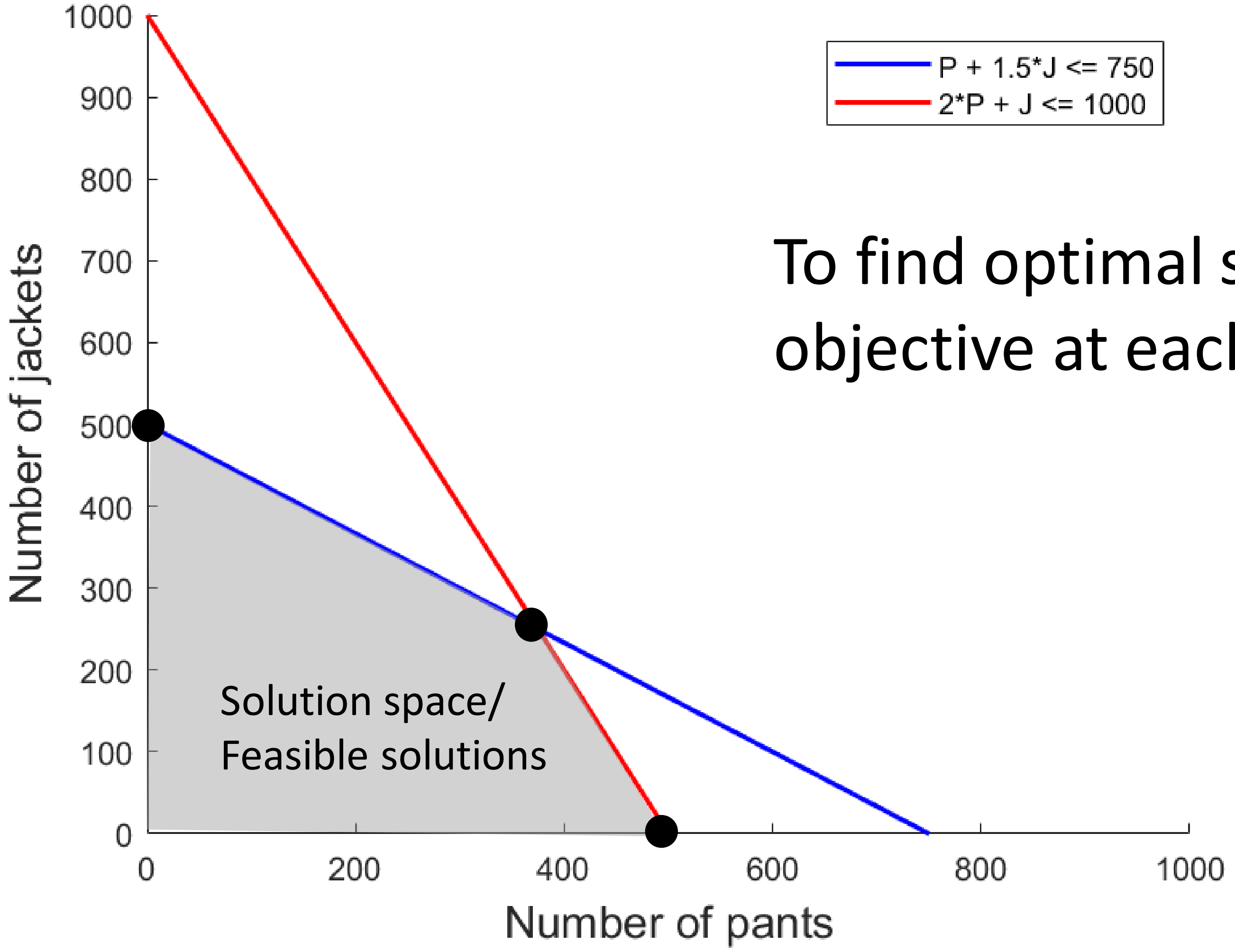


Graphical solution – optimal solution



To find optimal sale evaluate objective at each **extreme point**

Graphical solution – optimal solution



To find optimal sale evaluate objective at each **extreme point**

Graphical solution – optimal solution

z(P,J) = 50*P + 40*J (objective function)

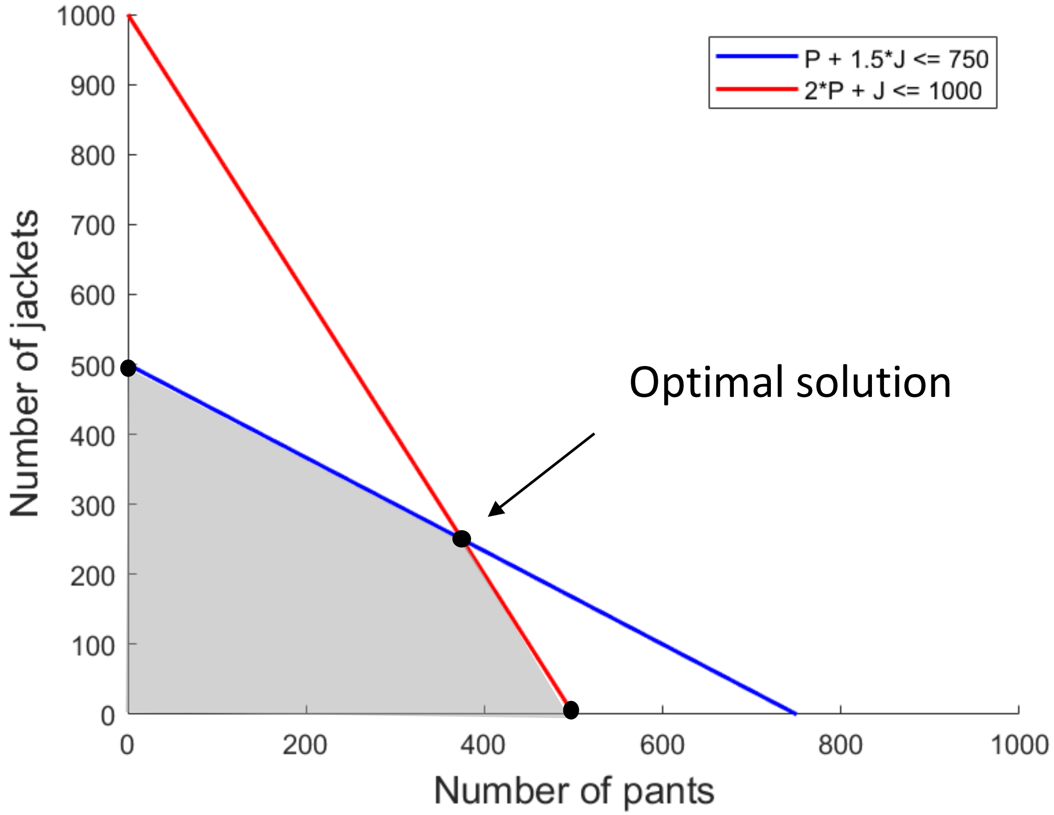
z(0,500) = 50*0 + 40*500 = 20 000€

z(500,0) = 50*500 + 40*0 = 25 000€

z(375,250) = 50*375 + 40*250 = **28 750€**

The optimal sale is achieved if 375 pants and 250 jackets are produced.

Q: Is the optimal solution unique?



Graphical solution – optimal solution

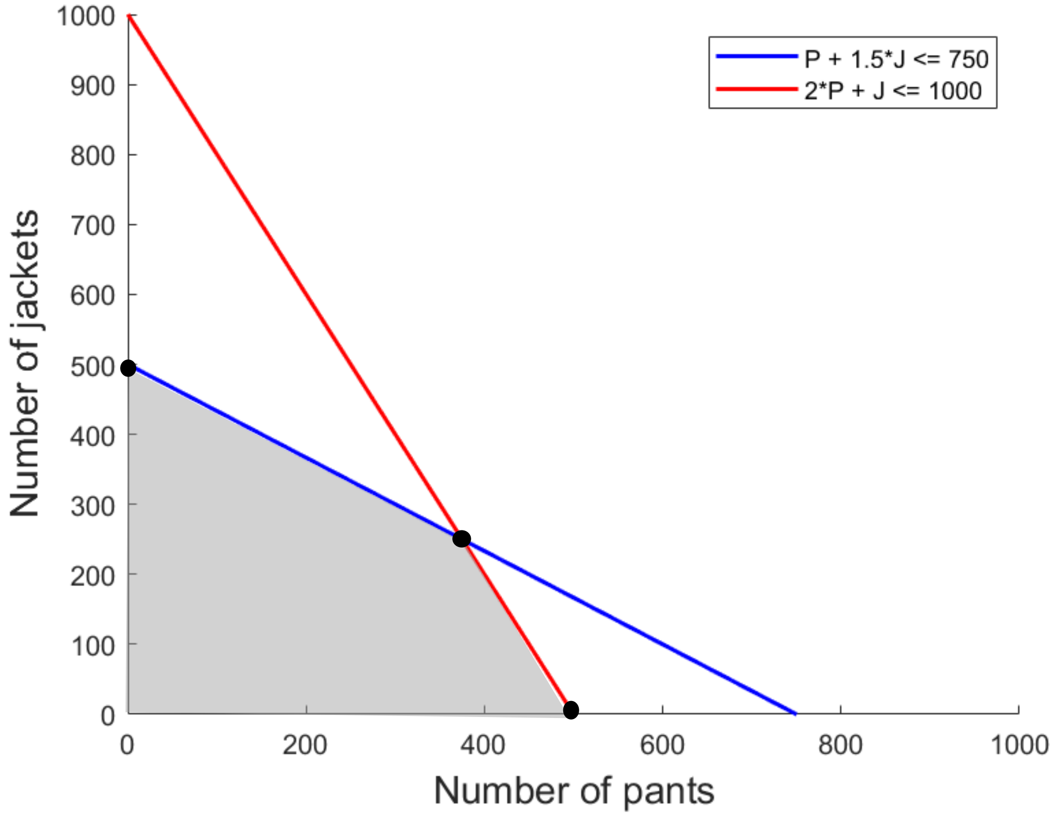
-> price change from 50€ and 40€ for pants and jackets to 20€ and 30€, respectively

z(P,J) = **20***P + **30***J

z(0,500) = 20*0 + 30*500 =

z(500,0) = 20*500 + 30*0 =

z(375,250) = 20*375 + 30*250 =



Graphical solution – constraint 2

-> price change from 50€ and 40€ for pants and jackets to 20€ and 30€, respectively

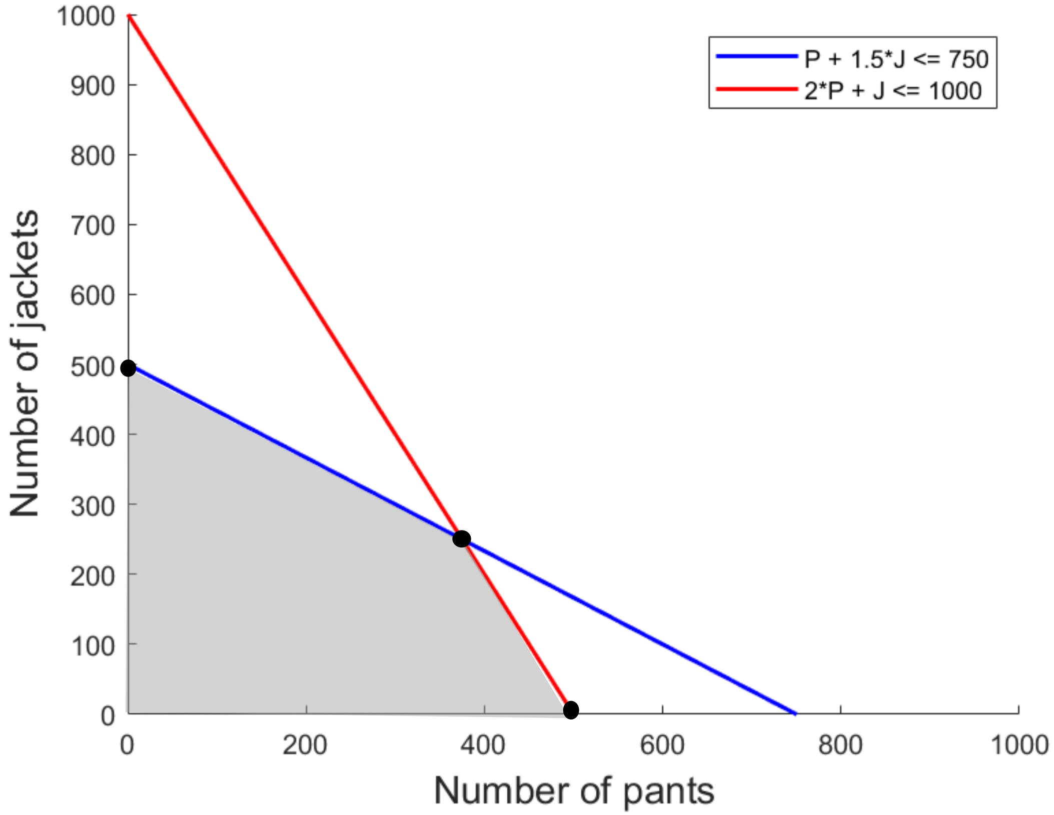
z(P,J) = **20***P + **30***J

z(0,500) = 20*0 + 30*500 = **15 000€**

z(500,0) = 20*500 + 30*0 = 10 000€

z(375,250) = 20*375 + 30*250 = **15 000€**

Q: What is the optimal solution?



Graphical solution – constraint 2

-> price change from 50€ and 40€ for pants and jackets to 20€ and 30€, respectively

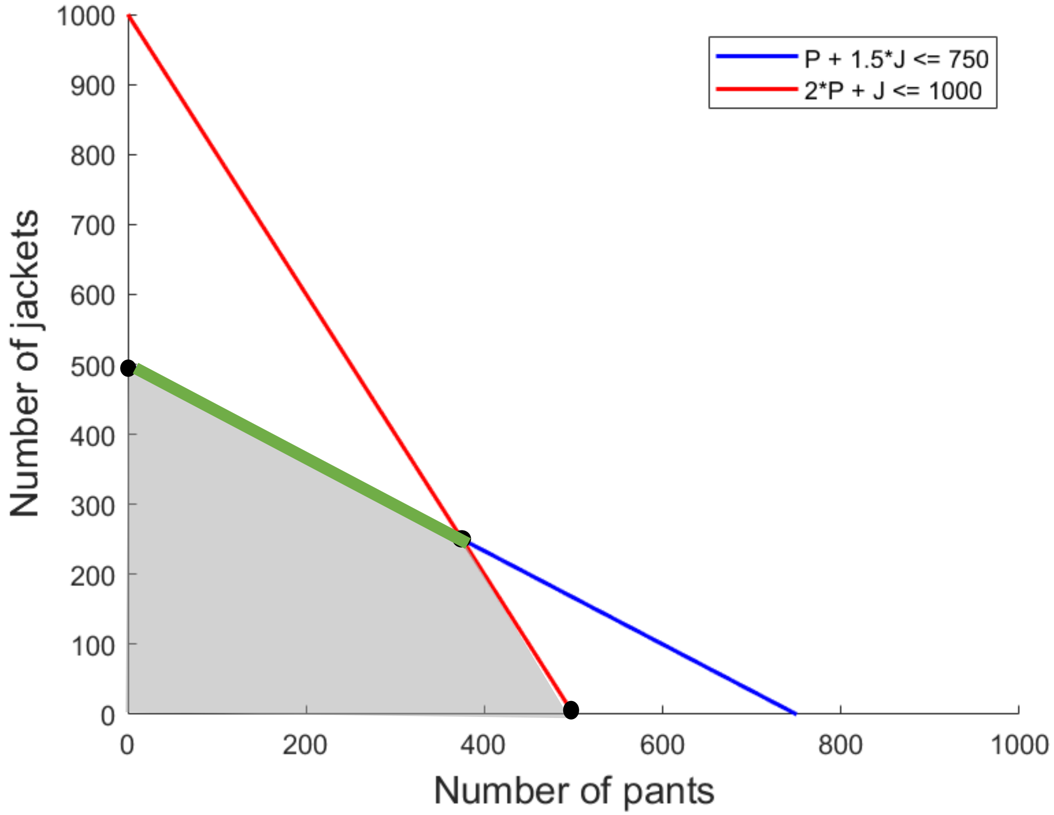
z(P,J) = **20***P + **30***J

z(0,500) = 20*0 + 30*500 = 15 000€

z(500,0) = 20*500 + 30*0 = 10 000€

z(375,250) = 20*375 + 30*250 = 15 000€

All pairs of integer values on the segment in green would give the same optimal sale!



Solving LP using Simplex

Reformulation into system of equalities!

max z = 50*P + 40*J (OBJECTIVE - MAXIMIZE PROFIT)
s.t.
1*P + 1.5*J <= 750 (AVAILABILITY OF COTTON CONSTRAINT)
2*P + 1*J <= 1000 (AVAILABILITY OF POLYESTER CONSTRAINT)
P >= 0
J >= 0

To use simplex we first convert all inequalities to equalities.

Solving LP using Simplex

Reformulation into system of equalities!

max z = 50*P + 40*J (OBJECTIVE - MAXIMIZE PROFIT)

s.t.

1*P + 1.5*J <= 750 (AVAILABILITY OF COTTON CONSTRAINT)

2*P + 1*J <= 1000 (AVAILABILITY OF POLYESTER CONSTRAINT)

P >= 0

J >= 0

To use simplex we first convert all inequalities to equalities.

Row 1: z - 50*P - 40*J - 0*s - 0*t = 0

Row 2: 1*P + 1.5*J + s = 750 (less than --> + slack, greater than --> - slack)

Row 3: 2*P + 1*J + t = 1000

P,J,s,t >= 0

Initial tableau

Basic variable: variable isolated in a constraint that can be set to the right-hand side of that constraint

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	-50	-40	0	0	0	
2	s	0	1	1.5	1	0	750	
3	t	0	2	1	0	1	1000	

Basic columns

Optimality conditions:

1. The objective row of the tableau is 0 in the basic columns, except for z
2. There is no negative entry in the objective row

Identify entering variable

Basic variable: variable isolated in a constraint that can be set to the right-hand side of that constraint
Nonbasic variable which is associated the most negative (for maximization) coefficient in the objective

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	-50	-40	0	0	0	
2	s	0	1	1.5	1	0	750	
3	t	0	2	1	0	1	1000	

Identify leaving variable

- Basic variable: variable isolated in a constraint that can be set to the right-hand side of that constraint
- Entering variable: nonbasic variable which is associated the most negative (for maximization) coefficient in the objective
- Leaving variable: variable which will be changed from a non-zero to zero value in the next solution
  -> choose the one that has smallest upper bound on entering variable

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	-50	-40	0	0	0	
2	s	0	1	1.5	1	0	750	750/1=750
3	t	0	2	1	0	1	1000	1000/2=500

Identify leaving variable

- Basic variable: variable isolated in a constraint that can be set to the right-hand side of that constraint
- Entering variable: nonbasic variable which is associated the most negative (for maximization) coefficient in the objective
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1	z	1	-50	-40	0	0	0	
2	s	0	1	1.5	1	0	750	750/1=750
3	t	0	2	1	0	1	1000	1000/2=500

Identify leaving variable

- Basic variable: variable isolated in a constraint that can be set to the right-hand side of that constraint
- Entering variable: nonbasic variable which is associated the most negative (for maximization) coefficient in the objective
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Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	-50	-40	0	0	0	
2	s	0	1	1.5	1	0	750	750/1=750
3	t	0	2	1	0	1	1000	1000/2=500

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	-50	-40	0	0	0	
2	s	0	1	1.5	1	0	750	750
3	t	0	2	1	0	1	1000	500

Second tableau

- Enter the basic variable for the second tableau.
- Update the coefficients of the tableau

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1			0					
2			0					
3	P	0	1	1/2	0	1/2	500	

Becomes basic variable

Second tableau

- Enter the basic variable for the second tableau.
- Update the coefficients of the tableau

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	-50	-40	0	0	0	
2	s	0	1	1.5	1	0	750	750
3	t	0	2	1	0	1	1000	500

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0		0			
2	s	0	0		1			
3	P	0	1	1/2	0	1/2	500	

Becomes basic variable

Second tableau

- Enter the basic variable for the second tableau.
- Update the coefficients of the tableau (T2(i,j) = T1(i,j) – T1(i,entering variable) * T2(leaving variable,j))

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0		0			
2	s	0	0		1			
3	P	0	1	1/2	0	1/2	500	

Initial tableau

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	-50	-40	0	0	0	
2	s	0	1	1.5	1	0	750	750
3	t	0	2	1	0	1	1000	500

Second tableau

- Enter the basic variable for the second tableau.
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Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-15 = -40 –(-50*1/2)	0			
2	s	0	0		1			
3	P	0	1	1/2	0	1/2	500	

Initial tableau

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	-50	-40	0	0	0	
2	s	0	1	1.5	1	0	750	750
3	t	0	2	1	0	1	1000	500

Second tableau

- Enter the basic variable for the second tableau.
- Update the coefficients of the tableau (T2(i,j) = T1(i,j) – T1(i,entering variable) * T2(leaving variable,j))

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-15 = -40 –(-50*1/2)	0			
2	s	0	0	1 = 1.5 –(1*1/2)	1			
3	P	0	1	1/2	0	1/2	500	

Initial tableau

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	-50	-40	0	0	0	
2	s	0	1	1.5	1	0	750	750
3	t	0	2	1	0	1	1000	500

Second tableau

- Enter the basic variable for the second tableau.
- Update the coefficients of the tableau (T2(i,j) = T1(i,j) – T1(i,entering variable) * T2(leaving variable,j))

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-15 = -40 –(-50*1/2)	0	25 = 0 –(-50*1/2)		
2	s	0	0	1 = 1.5 –(1*1/2)	1			
3	P	0	1	1/2	0	1/2	500	

Initial tableau

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	-50	-40	0	0	0	
2	s	0	1	1.5	1	0	750	750
3	t	0	2	1	0	1	1000	500

Second tableau

- Enter the basic variable for the second tableau.
- Update the coefficients of the tableau (T2(i,j) = T1(i,j) – T1(i,entering variable) * T2(leaving variable,j))

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-15 = -40 –(-50*1/2)	0	25 = 0 –(-50*1/2)		
2	s	0	0	1 = 1.5 –(1*1/2)	1	-1/2 = 0 –(1*1/2)		
3	P	0	1	1/2	0	1/2	500	

Initial tableau

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	-50	-40	0	0	0	
2	s	0	1	1.5	1	0	750	750
3	t	0	2	1	0	1	1000	500

Second tableau

- Enter the basic variable for the second tableau.
- Update the coefficients of the tableau (T2(i,j) = T1(i,j) – T1(i,entering variable) * T2(leaving variable,j))

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-15 = -40 –(-50*1/2)	0	25 = 0 –(-50*1/2)	25000 = 0 –(-50*500)	
2	s	0	0	1 = 1.5 –(1*1/2)	1	-1/2 = 0 –(1*1/2)		
3	P	0	1	1/2	0	1/2	500	

Initial tableau

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	-50	-40	0	0	0	
2	s	0	1	1.5	1	0	750	750
3	t	0	2	1	0	1	1000	500

Second tableau

- Enter the basic variable for the second tableau.
- Update the coefficients of the tableau (T2(i,j) = T1(i,j) – T1(i,entering variable) * T2(leaving variable,j))

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-15 = -40 –(-50*1/2)	0	25 = 0 –(-50*1/2)	25000 = 0 –(-50*500)	
2	s	0	0	1 = 1.5 –(1*1/2)	1	-1/2 = 0 –(1*1/2)	250 = 750 –(1*500)	
3	P	0	1	1/2	0	1/2	500	

Initial tableau

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	-50	-40	0	0	0	
2	s	0	1	1.5	1	0	750	750
3	t	0	2	1	0	1	1000	500

Second tableau

Optimality conditions:

1. The objective row of the tableau is 0 in the basic columns
2. There is no negative entry in the objective row

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-15	0	25	25000	
2	s	0	0	1	1	-1/2	250	
3	P	0	1	1/2	0	1/2	500	

Identify entering and leaving variable

- Basic variable: variable isolated in a constraint that can be set to the right-hand side of that constraint
- Entering variable: nonbasic variable which is associated the most negative (for maximization) coefficient in the objective
- Leaving variable: variable which will be changed from a non-zero to zero value in the next solution
  -> choose the one that has smallest upper bound on entering variable

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-15	0	25	25000	
2	s	0	0	1	1	-1/2	250	250/1 = 250
3	P	0	1	1/2	0	1/2	500	500/0.5 = 1000

Third tableau

- Enter the basic variable for the new tableau.

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1				0				
2	J	0		1	1	-1/2	250	
3				0				

previous tableau

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-15	0	25	25000	
2	s	0	0	1	1	-1/2	250	250
3	P	0	1	1/2	0	1/2	500	1000

Third tableau

- Enter the basic variable for the new tableau.

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	0				
2	J	0	0	1	1	-1/2	250	
3	P	0	1	0				

previous tableau

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-15	0	25	25000	
2	s	0	0	1	1	-1/2	250	250
3	P	0	1	1/2	0	1/2	500	1000

Third tableau

- Update the coefficients of the tableau (V(i,j) = O(i,j) – O(i,entering variable) * V(leaving variable,j))

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	0	15 = 0 –(-15*1)			
2	J	0	0	1	1	-1/2	250	
3	P	0	1	0				

previous tableau

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-15	0	25	25000	
2	s	0	0	1	1	-1/2	250	250
3	P	0	1	1/2	0	1/2	500	1000

Third tableau

- Update the coefficients of the tableau (V(i,j) = O(i,j) – O(i,entering variable) * V(leaving variable,j))

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	0	15 = 0 –(-15*1)			
2	J	0	0	1	1	-1/2	250	
3	P	0	1	0	-1/2 = 0 –(1/2*1)			

previous tableau

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-15	0	25	25000	
2	s	0	0	1	1	-1/2	250	250
3	P	0	1	1/2	0	1/2	500	1000

Third tableau

- Update the coefficients of the tableau (V(i,j) = O(i,j) – O(i,entering variable) * V(leaving variable,j))

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	0	15 = 0 –(-15*1)	17.5 = 25 -(-15*-1/2)		
2	J	0	0	1	1	-1/2	250	
3	P	0	1	0	-1/2 = 0 –(1/2*1)			

previous tableau

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-15	0	25	25000	
2	s	0	0	1	1	-1/2	250	250
3	P	0	1	1/2	0	1/2	500	1000

Third tableau

- Update the coefficients of the tableau (V(i,j) = O(i,j) – O(i,entering variable) * V(leaving variable,j))

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	0	15 = 0 –(-15*1)	17.5 = 25 -(-15*-1/2)		
2	J	0	0	1	1	-1/2	250	
3	P	0	1	0	-1/2 = 0 –(1/2*1)	1/4 = 0 –(1/2*-1/2)		

previous tableau

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-15	0	25	25000	
2	s	0	0	1	1	-1/2	250	250
3	P	0	1	1/2	0	1/2	500	1000

Third tableau

- Update the coefficients of the tableau (V(i,j) = O(i,j) – O(i,entering variable) * V(leaving variable,j))

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	0	15 = 0 –(-15*1)	17.5 = 25 -(-15*-1/2)	28750 = 25000-(-15*250)	
2	J	0	0	1	1	-1/2	250	
3	P	0	1	0	-1/2 = 0 –(1/2*1)	1/4 = 0 –(1/2*-1/2)		

previous tableau

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-15	0	25	25000	
2	s	0	0	1	1	-1/2	250	250
3	P	0	1	1/2	0	1/2	500	1000

Third tableau

- Update the coefficients of the tableau (V(i,j) = O(i,j) – O(i,entering variable) * V(leaving variable,j))

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	0	15 = 0 –(-15*1)	17.5 = 25 -(-15*-1/2)	28750 = 25000-(-15*250)	
2	J	0	0	1	1	-1/2	250	
3	P	0	1	0	-1/2 = 0 –(1/2*1)	1/4 = 0 –(1/2*-1/2)	375 = 500-(1/2*250)	

previous tableau

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-15	0	25	25000	
2	s	0	0	1	1	-1/2	250	250
3	P	0	1	1/2	0	1/2	500	1000

Optimality conditions

- ✓ The objective row of the tableau is 0 in the basic columns
- ✓ There is no negative entry in the objective row

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	0	15	17.5	28750	
2	J	0	0	1	1	-1/2	250	
3	P	0	1	0	-1/2	1/4	375	

Solution: P = ?, J = ?, z = ?

Optimality conditions

- ✓ The objective row of the tableau is 0 in the basic columns
- ✓ There is no negative entry in the objective row

Row number	Basic variable	z	P	J	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	0	15	17.5	28750	
2	J	0	0	1	1	-1/2	250	
3	P	0	1	0	-1/2	1/4	375	

Solution: P = 375, J = 250, z = 28750

Writing the LP using matrix format

$$\max z = c^T x$$

$$s.t.$$

$$Ax = b$$

$$lb \leq x \leq ub$$

max *z = 50 P + 40 J*

s.t.

1*P + 1.5*J ≤ 750 (cotton)

2*P + 1*J ≤ 1000 (polyester)

P ≥ 0

J ≥ 0

Writing the LP using matrix format

$$\max z = c^T x \qquad \max z = [50 \quad 40]\begin{bmatrix} P \\ J \end{bmatrix}$$

$$s.t.$$

$$Ax = b$$

$$lb \leq x \leq ub$$

max *z = 50 P + 40 J*

s.t.

1*P + 1.5*J ≤ 750 (cotton)

2*P + 1*J ≤ 1000 (polyester)

P ≥ 0

J ≥ 0

Writing the LP using matrix format

$$\max z = c^T x$$

$$s.t.$$

$$Ax = b$$

$$lb \le x \le ub$$

$$\max z = \begin{bmatrix} 50 & 40 \end{bmatrix} \begin{bmatrix} P \\ J \end{bmatrix}$$

$$s.t.$$

$$\begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} P \\ J \end{bmatrix} = \begin{bmatrix} 750 \\ 1000 \end{bmatrix}$$

max *z = 50 P + 40 J*

s.t.

1*P + 1.5*J ≤ 750 (cotton)

2*P + 1*J ≤ 1000 (polyester)

P ≥ 0

J ≥ 0

Writing the LP using matrix format

$$\max z = c^T x$$

$$s.t.$$

$$Ax = b$$

$$lb \leq x \leq ub$$

$$\max z = \begin{bmatrix} 50 & 40 \end{bmatrix} \begin{bmatrix} P \\ J \end{bmatrix}$$

$$s.t.$$

$$\begin{bmatrix} 1 & 1.5 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} P \\ J \end{bmatrix} \leq \begin{bmatrix} 750 \\ 1000 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \leq \begin{bmatrix} P \\ J \end{bmatrix} \leq \begin{bmatrix} Inf \\ Inf \end{bmatrix}$$

max $z = 50\ P + 40\ J$

s.t.

1*P + 1.5*J ≤ 750 (cotton)

2*P + 1*J ≤ 1000 (polyester)

P ≥ 0

J ≥ 0

Writing the LP using matrix format

$$\max z = c^T x$$

$$s.t.$$

$$Ax = b$$

$$x \geq 0$$

$$\max z = \begin{bmatrix} 50 & 40 \end{bmatrix} \begin{bmatrix} P \\ J \end{bmatrix}$$

$$s.t.$$

$$\begin{bmatrix} 1 & 1.5 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} P \\ J \end{bmatrix} \leq \begin{bmatrix} 750 \\ 1000 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \leq \begin{bmatrix} P \\ J \end{bmatrix} \leq \begin{bmatrix} Inf \\ Inf \end{bmatrix}$$

max *z = 50 P + 40 J*

s.t.

1*P + 1.5*J ≤ 750 (cotton)

2*P + 1*J ≤ 1000 (polyester)

P ≥ 0

J ≥ 0

The *linprog* solver

- part of the MATLAB Optimization Toolbox
- function definition: [x,fval,exitflag] = linprog(f,A,b,Aeq,beq,lb,ub)

Input arguments

argument	definition	Size
f	defines the objective (see c from our example)	n x 1
A	matrix encoding inequality constraints (≤)	m x n
b	Right-hand side vector for inequality constraints	m x 1
Aeq	matrix encoding equality constraints (=)	m‘ x n
beq	Right-hand side vector for equality constraints	b‘ x 1
lb	lower limits (bounds) for all variables	n x 1
ub	upper limits (bounds) for all variables	n x 1

Output

argument	definition	Size
x	Solution vector	n x 1
fval	Objective value at solution, fval = f’*x	1 x 1
exitflag	Integer indicating reason of termination	1 x 1

$$\min_x f^T \cdot x$$
$$s.t.$$
$$A \cdot x \leq b$$
$$Aeq \cdot x = beq$$
$$lb \leq x \leq ub$$

Encoding inequality constraints in *linprog*

- all inequalities must be reformulated to the form: $A\,x \le b$

examples:

- $5x + 3y \ge 60$

Encoding inequality constraints in *linprog*

- all inequalities must be reformulated to the form: $A\,x \leq b$

examples:

- 5x + 3y ≥ 60

==> -5x – 3y ≤ -60

Encoding inequality constraints in *linprog*

- all inequalities must be reformulated to the form: $A\,x \leq b$

examples:

- $5x + 3y \geq 60$

$\implies -5x - 3y \leq -60$

- $x \leq 100y$

Encoding inequality constraints in *linprog*

- all inequalities must be reformulated to the form: $A\,x \leq b$

examples:

- $5x + 3y \geq 60$

$\Longrightarrow -5x - 3y \leq -60$

- $x \leq 100y$

$\Longrightarrow x - 100y \leq 0$

Encoding inequality constraints in *linprog*

- all inequalities must be reformulated to the form: $A\,x \leq b$

examples:

- $5x + 3y \geq 60$

$\Rightarrow$ $-5x - 3y \leq -60$

- $x \leq 100y$

$\Rightarrow$ $x - 100y \leq 0$

- $x \geq -100\,(y-1)$

Encoding inequality constraints in *linprog*

- all inequalities must be reformulated to the form: $A\ x \leq b$

examples:

- $5x + 3y \geq 60$

==> $-5x - 3y \leq -60$

- $x \leq 100y$

==> $x - 100y \leq 0$

- $x \geq -100\ (y-1)$

==> $-100y - x \leq -100$

linprog arguments for our problem

function definition: x = linprog(f,A,b,Aeq,beq,lb,ub)

- f =
- A =
- b =
- Aeq =
- beq =
- lb =
- ub =

$$\min_x f^T \cdot x$$
$$s.t.$$
$$A \cdot x \leq b$$
$$Aeq \cdot x = beq$$
$$lb \leq x \leq ub$$

linprog arguments for our problem

function definition: x = linprog(f,A,b,Aeq,beq,lb,ub)

- f = [-50; -40];

→ $f \cdot (-1)$ needed since linprog *minimizes* by default, but we want to *maximize*

- A = [1 1.5;
  2 1];
- b = [750; 1000];
- Aeq = [];
- beq = [];
- lb = [0; 0];
- ub = [1000; 1000] (arbitrary large number which does not further constrain the solution)

$$\min_x f^T \cdot x$$
$$s.t.$$
$$A \cdot x \leq b$$
$$Aeq \cdot x = beq$$
$$lb \leq x \leq ub$$