Constraint-based Modeling of Cellular Networks

Exercise 9 – Duality, OptKnock

15. 12. 2022

Duality

Primal

$$\min cx$$

subject to

$$A_{ineq}x \geq b_{ineq}$$

$$A_{eq}x = b_{eq}$$

$$x_j \geq 0$$

Dual

$$\max \left[b_{ineq}^T, b_{eq}^T\right]y$$

subject to

$$[A_{ineq}^T, A_{eq}^T]y \leq c$$

$$y_j \geq 0; 1 \leq j \leq m_1$$

There are specific relations we can use to transform primal and dual LP into each other.

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Minimization Problem	Maximization Problem
If the Constraint is	The Associated Variable is
>=	>=0
<=	<=0
=	Unrestricted
If the Variable is	The corresponding Constraint is
>=0	<=
<=0	>=
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Example

$$\min 3x_1 + 2x_2 + x_3$$

subject to

$$2x_1 + x_2 \geq 2$$

$$x_2 + 3x_3 = 1$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

$$x_3 \geq 0$$

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$$\max 2y_1 + y_2$$

subject to

$$2y_1 \leq 3$$

$$y_1 + y_2 \leq 2$$

$$3y_2 \leq 1$$

$$y_1 \geq 0$$

Implementation of simplified OptKnock

In the E. coli core model do following changes:

- Set the lower bound of ‘Fumarase’ (FUM) to zero
- Set the lower bound of ‘ATP maintenance requirement’ reaction to zero

Simplified OptKnock

$$\min \sum_i y_i$$

s.t.

$$Nv = 0$$

$$v_{bio} \geq 0.9 \cdot z^*$$

$$v_{FUM} \geq 1.5 \cdot w_{FUM}$$

$$v \geq 0$$

$\forall_i, 1 \leq i \leq n$ (n being the number of model reactions)

$$(1 - y_i) \cdot \varepsilon \leq v_i$$
$$v_i \leq (1 - y_i) \cdot v_i^{max} + y_i \cdot \varepsilon$$
$$y \in \{0,1\}$$

Simplified OptKnock

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From FBA: $z^* = \max w_{bio}$

$$Nw = 0$$

$$w_{\min} \leq w \leq w_{max}$$

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Do we have to split reversible reactions?

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Given constant: solve with $\varepsilon = 1e - 6$

OptKnock

$$(1 - y_i) \cdot \varepsilon \leq v_i \leq (1 - y_i) \cdot v_i^{max} + y_i \cdot \varepsilon$$

$\varepsilon = 1\mathrm{e} - 6$

$y_i = 1$ $\quad\quad$ $0 \leq v_i \leq \varepsilon$

$y_i = 0$ $\quad\quad$ $\varepsilon \leq v_i \leq v_i^{max}$