Exercise for Constraint-based Modeling of Cellular Networks
15 December 2022

Homework should be sent to Anika (ankueken@uni-potsdam.de)

Hand in your commented code / answers for all exercise tasks as homework by 5.1.2023 before the exercise.

Duality

a) Write the dual problem for the following optimization problem

$$\text{minimize } C = 0.5v_1 + 0.3v_2 + 0.2v_3$$
$$\text{subject to}$$
$$v_1 + v_3 \geq 5$$
$$v_2 + v_3 \geq 10$$
$$v_3 \leq 2$$
$$v \geq 0.$$

b) Use Matlab to check that both problems (primal and dual) have the same optimum.

c) Calculate the shadow prices for the optimization problem above.

d) Given a metabolic model with all irreversible reactions, the objective is to maximize the flux towards the biomass reaction when the model is at the steady state. Compare the number of variables and constraints in the primal problem and in the dual problem.

Simplified OptKnock

Use the *E. coli* core model to determine the optimal reaction eliminations for increasing flux through fumarase reaction. Follow the steps below.

1. Set the lower bound of 'Fumarase' (FUM) and 'ATP maintenance requirement' reaction to zero.
2. Calculate the optimum biomass flux z^*, under the steady state constraints.
3. Calculate the optimum flux through fumarase reaction w_{FUM}, under steady state constraints at the optimum biomass.
4. Check if reversible reactions in the model have to be replaced by two irreversible reactions to solve the MILP below.
5. The problem is to find the minimum number of eliminations such that while having at least 90% of the maximum biomass in the *E. coli* model, flux through fumarase reaction should be increased by 50%.

   Implement the following MILP:

$$\min \sum_i y_i$$

s.t.

$$Nv = 0$$

$$v_{bio} \geq 0.9 \cdot z^*$$

$$v_{FUM} \geq 1.5 \cdot w_{FUM}$$

$$v \geq 0$$

$\forall_i, 1 \leq i \leq n$ (n being the number of model reactions)

$$(1 - y_i) \cdot \varepsilon \leq v_i$$
$$v_i \leq (1 - y_i) \cdot v_i^{max} + y_i \cdot \varepsilon$$
$$y \in \{0,1\}$$

6. Solve the MILP with $\varepsilon = 1e - 6$.