Constraint-based Modeling of Cellular Networks

Solution to homework of exercise 3

3. 11. 2022

Translate text into LP model

- Two products – paintings (P) and cards (C)
- Material/ Constraint: time

- Painting 2 hours
- Card 1 hours

She cannot spend more than 20 hours a week.

She should make not more than 12 paintings and cards per week.

She makes a profit of

- 50€ on paintings
- 30€ on each card

How many paintings and cards
should she make each week to maximize her profit?

max z = $50*P + 30*C$

objective

s.t.

$2*P + C \leq 20$

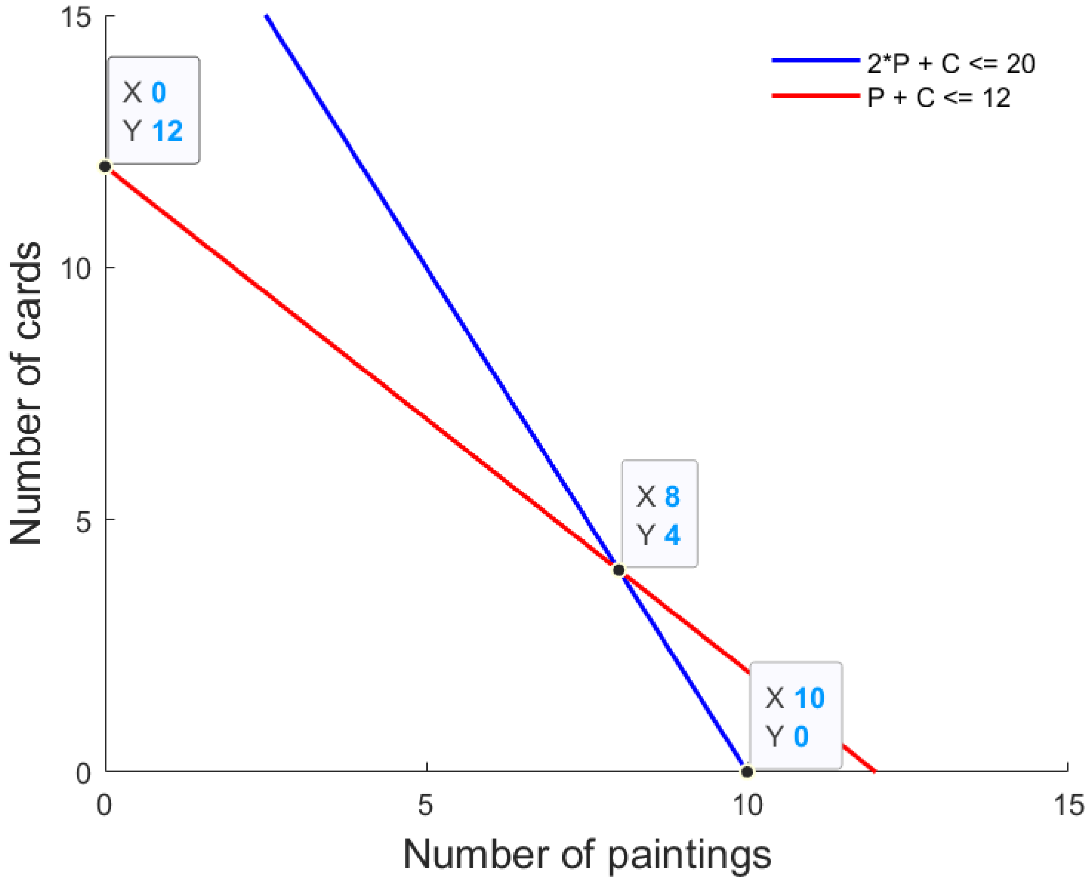
$P + C \leq 12$

$P \geq 0$

$C \geq 0$

constraints

Graphical solution



max z = *50*P + 30*C*

objective

s.t.

$2*P + C \leq 20$

$P + C \leq 12$

$P \geq 0$

$C \geq 0$

constraints

See next page for MATLAB code that creates this Figure!

Matlab Code for Figure

```
constraint_1 = @(P)(2*P-20)./-1; % time per week
constraint_2 = @(P)(-P+12); % number of paintings and cards

% define x values for plotting (paintings on x-axis)
xrange = 0:15;
% set limits for x and y axes
xlim([0,15])
ylim([0,15])
% plot two lines
hold on
line(xrange,constraint_1(xrange),'Color','b','LineWidth',1.5)
xlabel("Number of paintings",'FontSize',14)
ylabel("Number of cards",'FontSize',14)

line(xrange,constraint_2(xrange),'Color','r','LineWidth',1.5)
% plot lower and upper limits
legend('2*P + C <= 20','P + C <= 12')
legend boxoff
```

Graphical solution – optimal solution

To find optimal sale evaluate objective at each **extreme point**

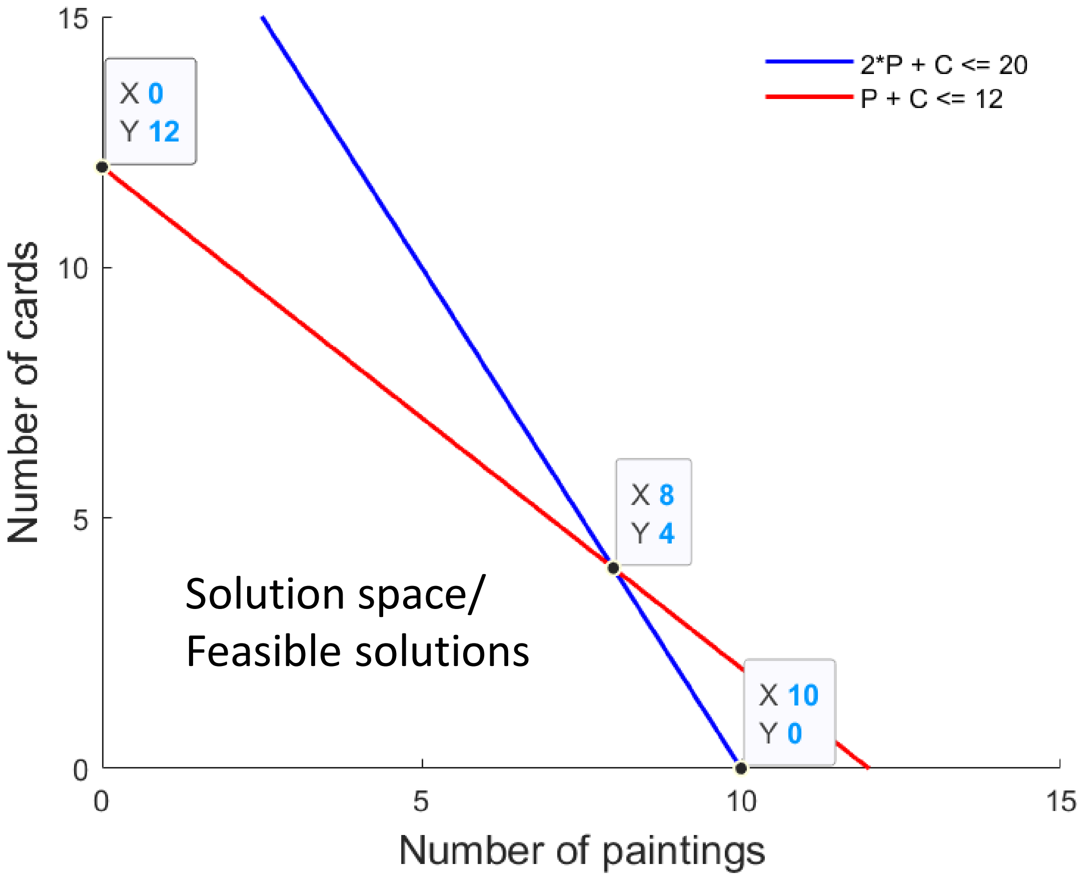
z(P,C) = 50*P + 30*C (objective function)

z(0,12) = 50*0 + 30*10 = 300€

z(10,0) = 50*10 + 30*0 = 500€

Z(8,4) = 50*8 + 30*4 = **520€**

Optimal profit when making 8 paintings and 4 cards per week.



Solving LP using Simplex

Reformulation into system of equalities!

max z = 50*P + 30*C
s.t.
2*P + C <= 20
P + C <= 12
P >= 0
C >= 0

To use simplex we first convert all inequalities to equalities.

Row 1: z - 50*P - 30*C - 0*s - 0*t = 0
Row 2: 2*P + C + s = 20
Row 3: P + C + t = 12
P,C,s,t >= 0

Initial tableau

Basic variable: variable isolated in a constraint that can be set to the right-hand side of that constraint

Row number	Basic variable	z	P	C	s	t	Right-hand side	Upper bound on entering variable
1	z	1	-50	-30	0	0	0	
2	s	0	2	1	1	0	20	
3	t	0	1	1	0	1	12	

Basic columns

Optimality conditions:

1. The objective row of the tableau is 0 in the basic columns
2. There is no negative entry in the objective row ✖

Identify entering and leaving variable

Row number	Basic variable	z	P	C	s	t	Right-hand side	Upper bound on entering variable
1	z	1	-50	-30	0	0	0	
2	s	0	2	1	1	0	20	20/2=10
3	t	0	1	1	0	1	12	12/1=12

Second tableau

- Fill basic variables

Previous table

Row number	Basic variable	z	P	C	s	t	Right-hand side	Upper bound on entering variable
1	z	1	-50	-30	0	0	0	
2	s	0	2	1	1	0	20	10
3	t	0	1	1	0	1	12	12

Row number	Basic variable	z	P	C	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0			0		
2	P	0	1	0.5	0.5	0	10	
3	t	0	0			1		

Becomes basic variable

Second tableau

- Update the coefficients of the tableau (T2(i,j) = T1(i,j) – T1(i,entering variable) * T2(leaving variable,j))

Row number	Basic variable	z	P	C	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-5	25	0	500	
2	P	0	1	0.5	0.5	0	10	
3	t	0	0	0.5	-0.5	1	2	

Initial tableau

Row number	Basic variable	z	P	C	s	t	Right-hand side	Upper bound on entering variable
1	z	1	-50	-30	0	0	0	
2	s	0	2	0.5	1	0	20	10
3	t	0	1	1	0	1	12	12

Second tableau

Optimality conditions:

1. The objective row of the tableau is 0 in the basic columns
2. There is no negative entry in the objective row

Row number	Basic variable	z	P	C	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-5	25	0	500	
2	P	0	1	0.5	0.5	0	10	
3	t	0	0	0.5	-0.5	1	2	

Identify entering and leaving variable

- Basic variable: variable isolated in a constraint that can be set to the right-hand side of that constraint
- Entering variable: nonbasic variable which is associated the most negative (for maximization) coefficient in the objective
- Leaving variable: variable which will be changed from a non-zero to zero value in the next solution
  -> choose the one that has smallest upper bound on entering variable

Row number	Basic variable	z	P	C	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-5	25	0	500	
2	P	0	1	0.5	0.5	0	10	10/0.5=20
3	t	0	0	0.5	-0.5	1	2	2/0.5=4

Third tableau

- Enter the basic variable for the new tableau.

Row number	Basic variable	z	P	C	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	0				
2	P	0	1	0				
3	C	0	0	1	-1	2	4	

previous tableau

Row number	Basic variable	z	P	C	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-5	25	0	500	
2	P	0	1	0.5	0.5	0	10	10/0.5=20
3	t	0	0	0.5	-0.5	1	2	2/0.5=4

Third tableau

- Update the coefficients of the tableau (V(i,j) = O(i,j) – O(i,entering variable) * V(leaving variable,j))

Row number	Basic variable	z	P	C	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	0	20	10	520	
2	P	0	1	0	1	-1	8	
3	C	0	0	1	-1	2	4	

previous tableau

Row number	Basic variable	z	P	C	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	-5	25	0	500	
2	P	0	1	0.5	0.5	0	10	10/0.5=20
3	t	0	0	0.5	-0.5	1	2	2/0.5=4

Optimality conditions

- ✓ The objective row of the tableau is 0 in the basic columns
- ✓ There is no negative entry in the objective row

Row number	Basic variable	z	P	C	s	t	Right-hand side	Upper bound on entering variable
1	z	1	0	0	20	10	520	
2	P	0	1	0	1	-1	8	
3	C	0	0	1	-1	2	4	

Solution: P = 8, C = 4, z = 520€

Solving LP with additional constraint C≥5 using Simplex

Reformulation into system of equalities!

max z = 50*P + 30*C
s.t.
2*P + C <= 20
P + C <= 12
C >= 5
P >= 0
C >= 0

To use simplex we first convert all inequalities to equalities.

Row 1: z - 50*P - 30*C - 0*s - 0*t = 0
Row 2: 2*P + C + s = 20
Row 3: P + C + t = 12
Row 4: C – u = 5
P,C,s,t >= 0

Inital solution – Big M-method

To obtain an initial solution we add artificial variable w to row 4

Row 1: z - 50*P - 30*C - 0*s - 0*t = 0
Row 2: 2*P + C + s = 20
Row 3: P + C + t = 12
Row 4: C – u + w = 5
P,C,s,t >= 0

Initial feasible solution is
P = C = u = 0, s = 20, t = 12, w = 5

w = 5 – C + u

Only solutions with w=0 are feasible therefore we penalize w with large constant such that it is driven to be zero. Hence, we change objective to
z - 50*P - 30*C - 0*s - 0*t + 5000*w = 0

Substitute w in objective with 5-C+u
z-50*P-30*C-0*s-0*t+5000*(5-C+u) = z-50*P-5030*C-0*s-0*t+5000*u +25000

Initial tableau for LP with additional constraint C≥5

$$z-50*P-5030*C-0*s-0*t+5000*u\ \ +25000$$
$$2*P + C + s = 20$$
$$P + C + t = 12$$
$$C - u + w = 5$$

Row number	Basic variable	z	P	C	s	t	u	w	Right-hand side	Upper bound on entering variable
1	z	1	-50	-5030	0	0	5000	0	-25000	
2	s	0	2	1	1	0	0	0	20	
3	t	0	1	1	0	1	0	0	12	
4	w	0	0	1	0	0	-1	1	5	

Solve the problem using *linprog* (version 1)

$$\min_x f^T \cdot x$$
$$s.t.$$
$$A \cdot x \leq b$$
$$Aeq \cdot x = beq$$
$$lb \leq x \leq ub$$

```
A = [2  1; % constraint 1 - time per week
     1  1]; % constraint 2 - number per week

b = [20; 12]; % right-hand side of equations

lb = [0;0];% number of cards and paintings
           % cannot have negative numbers
ub = [12; 12]; % from constraint 2,
               % but solution is the same if larger values are used

c = [50; 30]; % coefficients in objective

[X,FVAL,EXIT] = linprog(-c,A,b,[],[],lb,ub)
```

Solve the problem using *linprog* (version 2)

$$\min_x f^T \cdot x$$
$$s.t.$$
$$A \cdot x \leq b$$
$$Aeq \cdot x = beq$$
$$lb \leq x \leq ub$$

```
A = [2  1; % constraint 1 - time per week
     1  1]; % constraint 2 - number per week

b = [20; 12]; % right-hand side of equations

% additional constraint C >= 5
lb = [0; 5];% number of cards and paintings
           % cannot have negative numbers
ub = [12; 12]; % from constraint 2,
               % but solution is the same if larger values are used

c = [50; 30]; % coefficients in objective

[X,FVAL,EXIT] = linprog(-c,A,b,[],[],lb,ub)
```