Exercise 9. Let $(X, \mathscr{A})$ be a measurable space, and let μ be a measure on $(X, \mathscr{A})$ such that $\mu(X) = 1$. Suppose that $1 \leq p_1 < p_2 < +\infty$.

(a) Show that if f belongs to $\mathscr{L}^{p_2}(X, \mathscr{A}, \mu)$, then f belongs to $\mathscr{L}^{p_1}(X, \mathscr{A}, \mu)$ and satisfies $\|f\|_{p_1} \leq \|f\|_{p_2}$. (Hint: use Hölder's inequality or Jensen's inequality.)

(b) Show that if f and $f_1, f_2, \ldots$ belong to $\mathscr{L}^{p_2}(X, \mathscr{A}, \mu)$ and $\{f_n\}$ converges to f in p_2th mean, then $\{f_n\}$ converges to f in p_1th mean.

Proof.

□