**Exercise 3.** Show that $\mathscr{B}(\mathbb{R})$ is generated by the collection of all compact subsets of $\mathbb{R}$.

*Proof.* Let $\mathscr{K}$ be the set of all compact subsets of $\mathbb{R}$ and $\mathscr{F}$ the set of all closed sets of $\mathbb{R}$. Let $F \in \mathscr{F}$, and for all $n \in \mathbb{N}$, let $F_n = F \cap [-n, n]$. As the intersection of closed sets, each $F_n$ is closed; and since it is also bounded, we conclude that $F_n$ is compact. Moreover, $F = \cup_{n \in \mathbb{N}} F_n$ is the countable union of elements of $\mathscr{K}$, so that $F \in \sigma(\mathscr{K})$.

Conversely, every compact subset of $\mathbb{R}$ is closed, so $\mathscr{K} \subset \mathscr{F}$, and $\sigma(\mathscr{K}) \subset \sigma(\mathscr{F})$. Since $\mathscr{B}(\mathbb{R}) = \sigma(\mathscr{F})$, we conclude that $\mathscr{B}(\mathbb{R}) = \sigma(\mathscr{K})$.

$\square$