Exercise 2. Let C be a relation on a set A. If $A_0 \subset A$, define the *restriction* of C to A_0 to be the relation $C \cap (A_0 \times A_0)$. Show that the restriction of an equivalence relation is an equivalence relation.

Proof.

Let $A_0 \subset A$ and $x, y, z \in A_0$. The reflexivity, symmetry and transitivity of the restriction of C to A_0 come from considering x, y, z as elements of A, applying the corresponding property of C, and remarking that all the elements involved were in A_0. This implies that the properties already hold in A_0.

$\square$