Exercise 1. Show that there is a bijective correspondence of $A \times B$ with $B \times A$.

Proof. If either $A = \varnothing$ or $B = \varnothing$, then both $A \times B$ and $B \times A$ are empty. Since there is no function with image the empty set, both A and B must be nonempty for a bijection between them to exist.

Suppose then that both A and B are nonempty, and define:

$$\begin{aligned} \phi : A \times B &\to B \times A \\ (x, y) &\mapsto (y, x) \end{aligned} \qquad \begin{aligned} \psi : B \times A &\to A \times B \\ (y, x) &\mapsto (x, y) \end{aligned}$$

Then $\phi \circ \psi = i_{B \times A}$ and $\psi \circ \phi = i_{A \times B}$, so ϕ and ψ are bijections.

$\square$