Exercise 3. Here is a "proof" that every relation C that is both symmetric and transitive is also reflexive: "Since C is symmetric, aCb implies bCa. Since C is transitive, aCb and bCa together imply aCa, as desired." Find the flaw in this argument.

Proof.

Let $A = \{1, 2, 3\}$ and C the relation defined by the following part of $A \times A$: $C = \big\{(1,1), (1,3), (3,3), (3,1)\big\}$. By construction, C is both symmetric and transitive, but is not reflexive since $(2,2) \notin C$. The flaw in the reasoning comes from the fact that it only considers elements of C, whereas reflexivity is $\forall x \in A, \quad xCx$.

$\square$