

Exercise 1. Define two points (x_0, y_0) and (x_1, y_1) of the plane to be equivalent if $y_0 - x_0^2 = y_1 - x_1^2$. Check that this is an equivalence relation and describe the equivalence classes.

Proof.

Let us note this relation \mathcal{R} . The reflexivity, symmetry, and transitivity of equality imply that \mathcal{R} also has these properties. Thus \mathcal{R} is an equivalence relation.

For all $C \in \mathbb{R}$, $y - x^2 = C$ is a parabola with focus $(0, C + 1/4)$ and directrix $y = C - 1/4$. So the equivalence classes are all parabolas with C in the above equations varying over \mathbb{R} .

□