Exercise 1. Let X be a topological space; let A be a subset of$X$. Suppose that for each $x \in A$ there is an open set U containing x such that $U \subset A$. Show that A is open in X.

Proof. For each $x \in A$, there exists an open set U_x containing x and such that $U_x \subset A$. From this we deduce that $\cup_{x \in A} U_x \subset A$. Conversely, for all $x \in A$, $\{x\} \subset U_x$, so $A = \cup_{x \in A} \{x\} \subset \cup_{x \in A} U_x$, and finally

$$A = \bigcup_{x \in A} U_x$$

The set A is an union of open sets, and therefore an open set itself. $\square$