Exercise 1. In any category,

1. $g \circ f$ is monic if both f and g are monic.
2. If $g \circ f$ is monic then so is f.

Proof. 1. Suppose that both f and g are monic, and let h, k, a, b, c, d be arrows and objetcs as in the diagram below:

$$a \underset{k}{\overset{h}{\rightrightarrows}} b \xrightarrow{f} c \xrightarrow{g} d \tag{1}$$

Suppose that $g \circ f \circ h = g \circ f \circ k$. Then we have:

$$\begin{aligned} g \circ (f \circ h) = g \circ (f \circ k) & \qquad \text{associativity of } \circ \\ f \circ h = f \circ k & \qquad \text{since } g \text{ is monic} \\ h = k & \qquad \text{since } f \text{ is monic} \end{aligned}$$

From this we deduce that $g \circ f$ is monic.

2. Suppose that there are objects and arrows as shown in diagram 1, that $g \circ f$ is monic, and that $f \circ h = f \circ k$. Then we have:

$$\begin{aligned} f \circ h = f \circ k & \\ g \circ (f \circ h) = g \circ (f \circ k) & \qquad \text{since } \operatorname{cod}(f) = \operatorname{dom}(g) \\ (g \circ f) \circ h = (g \circ f) \circ k & \qquad \text{associativity of } \circ \\ h = k & \qquad \text{since } g \circ f \text{ is monic} \end{aligned}$$

From this we deduce that f is itself monic.

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