Exercise 3. $f \circ g$ is iso if f, g are, with $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

Proof. Let $a \xrightarrow{g} b$ and $b \xrightarrow{f} c$ be two iso arrows, with g^{-1} and f^{-1} their respective inverses. Then we have:

$$\begin{aligned} (f \circ g) \circ (g^{-1} \circ f^{-1}) &= \\ f \circ (g \circ g^{-1}) \circ f^{-1} &= \\ f \circ \mathbf{1}_b \circ f^{-1} &= \\ (f \circ \mathbf{1}_b) \circ f^{-1} &= \\ f \circ f^{-1} &= \\ \mathbf{1}_c & \end{aligned} \qquad \begin{aligned} (g^{-1} \circ f^{-1}) \circ (f \circ g) &= \\ g^{-1} \circ (f^{-1} \circ f) \circ g &= \\ g^{-1} \circ \mathbf{1}_b \circ g &= \\ (g^{-1} \circ \mathbf{1}_b) \circ g &= \\ g^{-1} \circ g &= \\ \mathbf{1}_a & \end{aligned}$$

so that $f \circ g$ is iso with inverse $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

□