Exercise 1. In any category,

1. $g \circ f$ is monic if both f and g are monic.
2. If $g \circ f$ is monic then so is f.

Proof.

1. Suppose that both f and g are monic, and let h, k, a, b, c, d be arrows and objetcs as in the diagram below:

$$\begin{tikzcd} a \arrow[r, shift left, "h"] \arrow[r, shift right, "k"'] & b \arrow[r, "f"] & c \arrow[r, "g"] & d \end{tikzcd} \tag{1}$$

   Suppose that $g \circ f \circ h = g \circ f \circ k$. Then we have:

$$\begin{aligned} g \circ (f \circ h) &= g \circ (f \circ k) && \text{associativity of } \circ \\ f \circ h &= f \circ k && \text{since } g \text{ is monic} \\ h &= k && \text{since } f \text{ is monic} \end{aligned}$$

   From this we deduce that $g \circ f$ is monic.

2. Suppose that there are objects and arrows as shown in diagram 1, that $g \circ f$ is monic, and that $f \circ h = f \circ k$. Then we have:

$$\begin{aligned} f \circ h &= f \circ k && \\ g \circ (f \circ h) &= g \circ (f \circ k) && \text{since } \operatorname{cod}(f) = \operatorname{dom}(g) \\ (g \circ f) \circ h &= (g \circ f) \circ k && \text{associativity of } \circ \\ h &= k && \text{since } g \circ f \text{ is monic} \end{aligned}$$

   From this we deduce that f is itself monic.

$\square$