Exercise 1. Prove that all terminal $\mathscr{C}$-objects are isomorphic.

Proof. Let a and b be terminal $\mathscr{C}$-objects. Then the identity $\mathbf{1}_a \colon a \to a$ (resp. $\mathbf{1}_b \colon b \to b$) is the unique arrow $a \to a$ (resp. $b \to b$). Since a (resp. b) is terminal, there exists a unique arrow $j \colon b \to a$ (resp. $i \colon a \to b$). Then the composite arrow $j \circ i$ (resp. $i \circ j$) is an arrow from a to a (resp. b to b), and therefore equal to $\mathbf{1}_a$ (resp. $\mathbf{1}_b$). Therefore we have:

$$i \circ j = \mathbf{1}_b \qquad\qquad j \circ i = \mathbf{1}_a$$

which shows that a and b are isomorphic.

□