Exercise 1. For any $\mathscr{C}$-objects, show that:

i) $a \cong a$

ii) if $a \cong b$, then $b \cong a$

iii) if $a \cong b$ and $b \cong c$, then $a \cong c$

Proof.

i) The law of identity for a gives $\mathbf{1}_a \circ \mathbf{1}_a = \mathbf{1}_a$, showing that $\mathbf{1}_a$ is an isomorphism $a \to a$, which implies that $a \cong a$.

ii) If $a \cong b$, then let $a \xrightarrow{f} b$ be an isomorphism from a to b. Then there exists and arrow $b \to a$ noted f^{-1} such that $f \circ f^{-1} = \mathbf{1}_b$ and $f^{-1} \circ f = \mathbf{1}_a$. This in turn means that f^{-1} is an isomorphism, so that $b \cong a$.

iii) Let f (resp. g) be an isomorphism from a to b (resp. from b to a). Then $g \circ f$ is an isomorphism from a to c (its inverse is $f^{-1} \circ g^{-1}$), so that $a \cong c$.

□