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PII: S0273-1177(17)30901-8
DOI: https://doi.org/10.1016/j.asr.2017.12.025
Reference: JASR 13558

To appear in: *Advances in Space Research*

Received Date: 26 September 2017
Revised Date: 22 November 2017
Accepted Date: 17 December 2017

Two-Phase Framework for Near-Optimal Multi-Target Lambert Rendezvous

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Abstract

This paper proposes a two-phase framework to obtain a near-optimal solution of multi-target Lambert rendezvous problem. The objective of the problem is to determine the minimum-cost rendezvous sequence and trajectories to visit a given set of targets within a maximum mission duration. The first phase solves a series of single-target rendezvous problems for all departure-arrival object pairs to generate the elementary solutions, which provides candidate rendezvous trajectories. The second phase formulates a variant of traveling salesman problem (TSP) using the elementary solutions prepared in the first phase and determines the final rendezvous sequence and trajectories of the multi-target rendezvous problem. The validity of the proposed optimization framework is demonstrated through an asteroid exploration case study.

Keywords

Rendezvous, Multi-Target, Optimization, Two-Phase, Traveling Salesman Problem, Asteroid Exploration

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Nomenclature

i / j	indices representing departure/arrival objects for a rendezvous
p	index representing an arc
A	set of arcs in a graph
c	cost associated with an arc in a graph
G	graph representing a multi-target problem
J	objective function
m_i	number of revolutions of i^{th} rendezvous, $\boldsymbol{\mu} = [m_1, \ldots, m_N]$
N	number of targets
n	mean motion of an orbit
$\mathbf{P}_{\mathrm{M}}$	optimal multi-target Lambert rendezvous problem
$\mathbf{P}_{\mathrm{ME}}$	optimal multi-target Lambert rendezvous problem with elementary solutions
$\mathbf{P}_{\mathrm{S}}$	optimal single-target Lambert rendezvous problem
Q	set of targets, $Q = \{1, \ldots, N\}$
q_i	index of i^{th} rendezvous target, $q_i \in Q$
$\mathbf{q}$	sequence of multi-target rendezvous, $\mathbf{q} = [q_1, \ldots, q_N]$
$\mathbf{r}$ / $\mathbf{v}$	position/velocity vectors
$t_{d,i}$	time at departure of i^{th} rendezvous, $\boldsymbol{\tau}_d = [t_{d,1}, \ldots, t_{d,N}]$
$t_{\max}$	maximum mission duration
$\Delta t_{ser,min}$ / $\Delta t_{ser,max}$	minimum/maximum service time on a target
$\Delta t_{tr,i}$	transfer time of i^{th} rendezvous, $\Delta\boldsymbol{\tau}_{tr} = [\Delta t_{tr,1}, \ldots, \Delta t_{tr,N}]$
V	set of nodes in a graph
$\mathbf{x}^E_{(i,j)}$	elementary solutions for an object pair (i, j)
y	decision variable for $\mathbf{P}_{\mathrm{ME}}$

I. Introduction

A multi-target rendezvous problem determines the visiting sequence and associated trajectories of a spacecraft to rendezvous with multiple objects. The problem is applicable to a number of space mission categories such as active debris removal (ADR), on-orbit servicing, and interplanetary exploration, and thus has been attracting considerable attention recently. Unlike a single-target rendezvous problem, the optimal multi-target rendezvous problem determines two different variable types (continuous and discrete) simultaneously. The problem is categorized as a mixed integer nonlinear programming (MINLP), which is known as one of the most difficult problem classes.

This paper proposes a two-phase framework to solve the multi-target Lambert rendezvous problem. The first phase of the framework generates the "elementary solutions," which are the candidate components of the final solution, by solving a series of single-target rendezvous problems for all departure-arrival object pairs. The second phase combines the elementary solutions prepared in the first phase to obtain the final solution of the original multi-target rendezvous problem – best rendezvous sequence and trajectories. In the framework, the elementary solutions (representing candidate rendezvous trajectories) are used to transform the multi-target rendezvous problem into a variant of traveling salesman problem (TSP) that has multiple arcs between each pair of nodes and time window constraints associated with each arc.

Key contribution of this work is threefold. First, we introduced a framework that decomposes the original multi-target rendezvous problem categorized as the MINLP into a series of nonlinear programming (NLP) and an integer linear programming (ILP). The proposed framework effectively reduces the complexity of trajectory optimization process required to obtain the final solution, and provides better solutions than known approaches for the problem (e.g. variants of genetic algorithm) with reasonable increase in computational resource consumption. Secondly, the resulting ILP is a routing problem class that we named as "TSP with multiple arcs and arc time windows." This problem is distinguishable from existing routing problems that consider only multiple arcs between nodes (Garaix et al., 2010; Ticha et al., 2017) or only arc time windows (Cetikaya et al., 2013) in that it deals with both of them. Finally, realistic case studies that can demonstrate the effectiveness of the proposed framework have been conducted. The multiple asteroid rendezvous mission is selected as the subject of the case study. The solutions for the case problems are obtained using the proposed framework and compared with known results found with other algorithm.

The rest of this paper is organized as follows. Section II provides reviews on past studies about the optimal rendezvous problems. Section III presents the mathematical formulation of optimal multi-target Lambert rendezvous mission. The detailed explanation on the steps of the framework to solve the problem is provided in Section IV. Case studies that can demonstrate the effectiveness of the proposed framework is presented in Section V. Finally, Section VI discusses the conclusions and potential future work of this study.

II. Literature Review

An optimal single-target rendezvous, which underlies the multi-target problem, has been addressed in a number of published studies. One of widely used approaches for this problem is the *Lambert rendezvous*, which uses the solution of the Lambert's boundary value problem to obtain the rendezvous trajectory (Battin, 1999). Past studies demonstrated that allowing multiple-revolution solutions of the Lambert's problem could reduce the fuel consumption required for the rendezvous. When we allow N_{max} for a given transfer time there exist ($2N_{max}$+1) trajectories, which enlarges the search space for the design variables characterizing the rendezvous (Prussing, 2000). Shen and Tsiotras (2003) proposed an algorithm that can determine an optimal solution of the fixed-time, two-impulse rendezvous between coplanar circular orbits among ($2N_{max}$+1) solutions quickly and efficiently by solving the multiple-revolution Lambert's problem. Zhang et al. (2011) proposed a procedure to solve the optimal two-impulse rendezvous problem for non-coplanar elliptic orbits. They considered coasting as an additional mission element and used a genetic algorithm (GA) to determine the initial and final coasting periods that minimize the propellant consumption. Chen et al. (2013) proposed a time-open constrained Lambert rendezvous problem by introducing parking time and transfer time. An interval branch-and-bound algorithm was adopted in combination with a gradient-based algorithm to find the global solution of the problem while effectively dealing with its strong nonlinearity and nonconvexity.

Optimal multi-target rendezvous problems are composed of two main tasks: determining rendezvous sequence and rendezvous trajectory optimization. Most of previous studies on the multi-target rendezvous focused on exploring the visiting sequences while optimization of the rendezvous trajectories was not seriously addressed – assumption-driven simplistic strategies were adopted or trajectory related discussions were missing. Determination of a visiting sequence for the multi-target rendezvous is similar to, but much more difficult to handle than the TSP primarily because the nodes (targets) are moving and the cost spent on an arc (rendezvous maneuver) is not fixed. Alfriend et al. (2005) formulated a geosynchronous satellites servicing mission as a relaxed TSP considering the fuel consumption only for orbital plane changes – cost associated with in-plane maneuver is ignored. Izzo et al. (2015) also estimated the cost as a relative orbital inclination and introduced two types of TSP variants for an active debris removal (ADR) mission – considering both of static and dynamic cases. Note that the optimization of the rendezvous trajectories was not the primary focus of either of the studies. Barbee et al. (2012) proposed a series method that can find a good – while not necessarily optimal – visiting order for the ADR mission with relatively low computational load. Some researchers, including Cerf (2013; 2015), Bérend and Olive (2016), introduced a drift orbit, which is an intermediate orbit that can accelerate the right ascension of the ascending node (RAAN) drift of a spacecraft. A branch and bound (B&B) algorithm was then employed to solve the optimal sequencing problem for an ADR mission using the cost of a specific three-step transfer strategy based on the Hohmann transfer.

On the other hand, there are several studies that address the Lambert rendezvous based trajectory optimization and the optimal rendezvous sequence determination simultaneously. Zhang et al. (2014; 2015) formulated the problems arising in mission designs for multi-spacecraft refueling and asteroid exploration as the multi-target rendezvous, and optimized the visiting sequence and the rendezvous trajectories simultaneously by introducing a procedure based on the hybrid encoding genetic algorithm (HEGA). Ross and D'Souza (2005) proposed a hybrid optimal control (HOC) framework composed of the inner-loop and the outer-loop to address both continuous and categorical variables, which has been widely used to formalize complex mission planning problems mathematically. In their approach, the outer-loop determines an optimal sequence of targets and the inner-loop solver optimizes the trajectories for the corresponding sequence. A number of studies that applied the HOC framework to design of space missions – such as multiple asteroid missions (Conway et al., 2007; Wall and Conway, 2009), interplanetary missions (Englander et al., 2012; Chilan and Conway, 2013), and debris removal missions (Yu et al., 2015) – can be found in the literature.

Although the MINLP-based heuristic algorithms and the HOC framework were successfully applied to the multi-target rendezvous problem, exploring optimal rendezvous trajectories between multiple targets is still challenging. It is known that the optimal multi-target rendezvous problem involves extremely large search space whose dimension increases considerably with the number of targets to visit (Izzo et al., 2007). To obtain an effective (or, close to optimal) solution of the rendezvous problem within reasonable consumption of computational resource, heuristic algorithms such as the GA and particle swarm optimization (PSO) have been widely used recently. For instance, all the referenced studies employed heuristic algorithms to optimize all design variables for Lambert rendezvous trajectories simultaneously (Zhang et al., 2014; Zhang et al., 2015; Conway et al., 2007; Wall et al., 2009; Englander et al., 2012; Chilan et al., 2013; Yu et al., 2015). These heuristic algorithms are advantageous in that they can be used for problems with very large search space, where applications of (exact) optimization-based approaches may be difficult due to the computational burden. However, the heuristic algorithms are vulnerable to premature convergence and do not guarantee the convergence to the global optimum in general. Chen et al. (2013) pointed out that the heuristic-only approach can fail to find a global solution, even for a single-target rendezvous problem. It is predictable that the effectiveness of heuristic-only approach for multi-target rendezvous problems may be diminished as the number of targets increases. Therefore, the development of a solution procedure that can smartly handle the multi-target rendezvous problem – including reduction in search space and cost-effective optimum search – is an area that requires attention.

III. Problem Definition

This section provides the mathematical formulation of the optimal multiple target Lambert rendezvous problem. We first introduce an optimal single-target rendezvous problem (**P**$_S$), which is a key component to address the multi-target problem. Then its extension to multi-target rendezvous (**P**$_M$) is discussed.

A. Single-Target Lambert Rendezvous

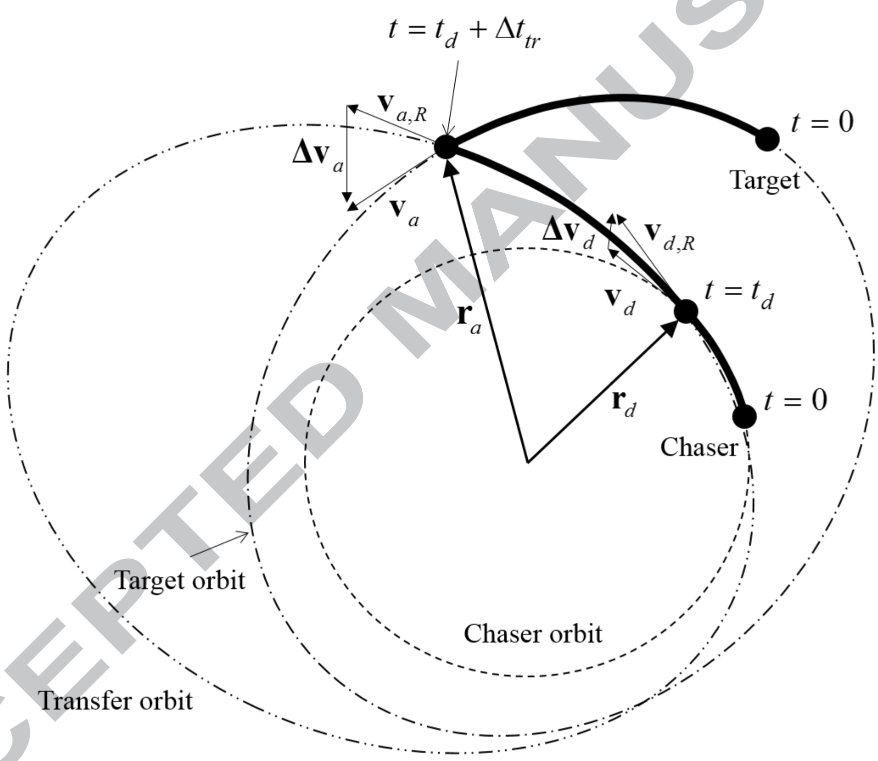

Fig. 1. Single-target Lambert rendezvous problem

A single-target time-open Lambert rendezvous problem is defined in Chen et al (2013). Fig. 1 illustrates the entire process of the single-target Lambert rendezvous. Initially (t = 0) the chaser and the target are moving in different orbits. After coasting period ($t = t_d$), the first impulsive velocity increment ($\Delta\mathbf{v}_d$) is imposed on the chaser to start an orbital maneuver, which ensures the chaser to meet with the target after transfer time Δt_{tr} ($t = t_d + \Delta t_{tr}$)[3]. The second velocity increment ($\Delta\mathbf{v}_a$) is applied

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[3] Throughout the paper, subscripts d and a represent the departure and arrival of the transfer maneuver.

on the chaser spacecraft when the positions of the two objects coincide to make its velocity identical to that of the target (rendezvous condition). Assume that the position/velocity of a chaser spacecraft in its initial orbit at time *t* are given as $\mathbf{r}^c(t)$ / $\mathbf{v}^c(t)$ and those of the target satellite are given as $\mathbf{r}^t(t)$ / $\mathbf{v}^t(t)$.

The total velocity increment necessary to complete the rendezvous can be obtained by solving a multiple-revolution Lambert's problem. The Lambert's problem determines an orbit that has a specific transfer time (Δt_{tr}) and departure/arrival position vectors ($\mathbf{r}_d$ and $\mathbf{r}_a$). The transfer angle – departing from $\mathbf{r}_d$ and arriving at $\mathbf{r}_a$ – can be smaller than 360 deg. (zero-revolution) or greater than 360 deg. (multiple-revolution) depending on the geometry and transfer time of a problem instance. There are a number of published methodologies to solve the multiple-revolution Lambert's problem (Gooding 1990), which provide the required velocity vectors for a spacecraft at the departure position ($\mathbf{v}_{d,R}^m = \mathbf{L}_d^m(\mathbf{r}_d, \mathbf{r}_a, \Delta t_{tr})$) and the arrival position ($\mathbf{v}_{a,R}^m = \mathbf{L}_a^m(\mathbf{r}_d, \mathbf{r}_a, \Delta t_{tr})$) for a given number of revolutions (*m*). The optimal single-target rendezvous problem ($\mathbf{P}_\mathrm{S}$) is defined as follows.

[$\mathbf{P}_\mathrm{S}$: Optimal Single-Target Lambert Rendezvous]

$$\min_{t_d, \Delta t_{tr}, m} J_S = (\| \Delta \mathbf{v}_d \| + \| \Delta \mathbf{v}_a \|) \tag{1}$$

subject to,

$$\mathbf{r}_d = \mathbf{r}^c(t_d) \tag{2}$$

$$\mathbf{v}_d = \mathbf{v}^c(t_d) \tag{3}$$

$$\mathbf{r}_a = \mathbf{r}^t(t_d + \Delta t_{tr}) \tag{4}$$

$$\mathbf{v}_a = \mathbf{v}^t(t_d + \Delta t_{tr}) \tag{5}$$

$$\Delta \mathbf{v}_d = \mathbf{v}_{d,R}^m - \mathbf{v}_d = \mathbf{L}_d^m(\mathbf{r}_d, \mathbf{r}_a, \Delta t_{tr}) - \mathbf{v}_d \tag{6}$$

$$\Delta \mathbf{v}_a = \mathbf{v}_a - \mathbf{v}_{a,R}^m = \mathbf{v}_a - \mathbf{L}_a^m(\mathbf{r}_d, \mathbf{r}_a, \Delta t_{tr}) \tag{7}$$

$$t_d + \Delta t_{tr} \le t_{\max} \tag{8}$$

The objective of the problem presented in Eq. (1) is to minimize sum of velocity increments (for departure and arrival) to conduct the rendezvous with the target. Design variables are the departure time for the rendezvous (t_d), the transfer time (Δt_{tr}), and the number of revolutions of the rendezvous trajectory (m). Eqs. (2)-(3) express the position and velocity of the chaser spacecraft at the beginning of the orbital transfer (at $t = t_d$) and Eqs. (4)-(5) represent the states of the target at the end of the transfer ($t = t_d + \Delta t_{tr}$). The expression for the departure and arrival velocity increments are presented in Eqs. (6)-(7). Finally, Eq. (8) represents a constraint for the maximum mission duration ($t_{\max}$).

B. Multi-Target Lambert Rendezvous

The optimal multi-target Lambert rendezvous problem is formulated by extending the single-target problem ($\mathbf{P}_\text{S}$) expressed in Eqs. (1)-(8). The problem determines the optimal rendezvous sequence and associated trajectories for the series of single-target Lambert rendezvous with given targets. Suppose that the position/velocity of a chaser spacecraft in its initial orbit at time t are given as $\mathbf{r}^c(t)$ / $\mathbf{v}^c(t)$ and those of the target (i = 1, … , N; N is the number of targets) are expressed as $\mathbf{r}^{t,i}(t) / \mathbf{v}^{t,i}(t)$. The spacecraft stays at each target for servicing where the lower and upper bounds of the service time are specified as $\Delta t_{ser,\min}$ and $\Delta t_{ser,\max}$. The optimal multi-target Lambert rendezvous problem ($\mathbf{P}_\text{M}$) is defined as follows.

[$\mathbf{P}_\text{M}$: Optimal Multi-Target Lambert Rendezvous]

$$\min_{\mathbf{x},\,\mathbf{q}} J_M = \sum_{i=1}^{N} (\|\Delta\mathbf{v}_{d,i}\| + \|\Delta\mathbf{v}_{a,i}\|) \tag{9}$$

subject to,

$$\mathbf{x} = [\boldsymbol{\tau}_d, \Delta\boldsymbol{\tau}_{tr}, \boldsymbol{\mu}] \tag{10}$$

$$\boldsymbol{\tau}_d = [t_{d,1}, \cdots, t_{d,N}] \tag{11}$$

$$\Delta\boldsymbol{\tau}_{tr} = [\Delta t_{tr,1}, \cdots, \Delta t_{tr,N}] \tag{12}$$

$$\boldsymbol{\mu} = [m_1, \cdots, m_N] \tag{13}$$

$$\mathbf{q} = [q_1, \cdots, q_N] \tag{14}$$

$$\mathbf{r}_{d,i} = \mathbf{r}^{t,q_{(i-1)}}(t_{d,i}) \quad (i \geq 2), \quad \mathbf{r}_{d,1} = \mathbf{r}^{c}(t_{d,1}) \tag{15}$$

$$\mathbf{v}_{d,i} = \mathbf{v}^{t,q_{(i-1)}}(t_{d,i}) \quad (i \geq 2), \quad \mathbf{v}_{d,1} = \mathbf{v}^{c}(t_{d,1}) \tag{16}$$

$$\mathbf{r}_{a,i} = \mathbf{r}^{t,q_i}(t_{d,i} + \Delta t_{tr,i}) \tag{17}$$

$$\mathbf{v}_{a,i} = \mathbf{v}^{t,q_i}(t_{d,i} + \Delta t_{tr,i}) \tag{18}$$

$$\Delta\mathbf{v}_{d,i} = \mathbf{L}_d^{m_i}(\mathbf{r}_{d,i}, \mathbf{r}_{a,i}, \Delta t_{tr,i}) - \mathbf{v}_{d,i} \tag{19}$$

$$\Delta\mathbf{v}_{a,i} = \mathbf{v}_{a,i} - \mathbf{L}_a^{m_i}(\mathbf{r}_{d,i}, \mathbf{r}_{a,i}, \Delta t_{tr,i}) \tag{20}$$

$$\Delta t_{ser,\min} \leq t_{d,i} - \left(t_{d,i-1} + \Delta t_{tr,i-1}\right) \leq \Delta t_{ser,\max} \quad (i \geq 2), \quad t_{d,1} \geq 0 \tag{21}$$

$$t_{d,N} + \Delta t_{tr,N} + \Delta t_{ser,\min} \leq t_{\max} \tag{22}$$

The objective of this problem is to minimize the sum of ΔV values used to complete all (N) rendezvous maneuvers, which is presented in Eq. (9). The design variables of the problem are presented in Eqs. (10)-(14). Index i represents the order of rendezvous task and variables $t_{d,i}$, $\Delta t_{tr,i}$, m_i, and q_i denote departure time, transfer time, number of revolution, and target index associated with the i^{th} rendezvous, respectively. The departure states (position and velocity) of each rendezvous are presented in Eqs. (15)-(16), the arrival states are presented in Eqs. (17)-(18), and the velocity increments at the departure and arrival of the rendezvous are expressed in Eqs. (19)-(20). Note that these equations are the "multi-target version" counterparts of Eqs. (2)-(7). Eq. (21) expresses a constraint for the service time and the maximum mission duration constraint is represented in Eq. (22). Fig. 2 illustrates the multi-target Lambert rendezvous process for the case of N = 4 and $\mathbf{q}$ = (2, 1, 3, 4).

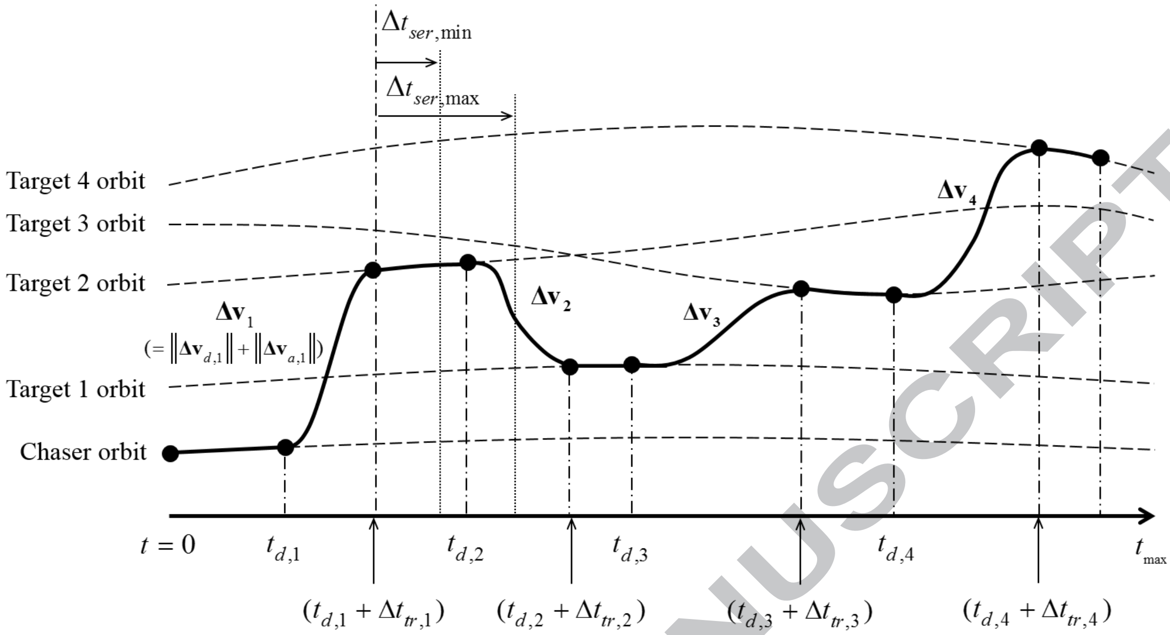

Fig. 2. Example of multi-target Lambert rendezvous process (N = 4, $\mathbf{q} = (2,1,3,4)$)

IV. Framework to Solve a Multi-Target Lambert Rendezvous Problem

As was discussed in the previous section, the optimal multi-target Lambert rendezvous problem ($\mathbf{P}_\mathrm{M}$) can be classified as MINLP, which is very difficult to handle directly. This section proposes a framework for solving the problem composed of two distinct phases. The structure of the proposed framework is outlined in Fig. 3.

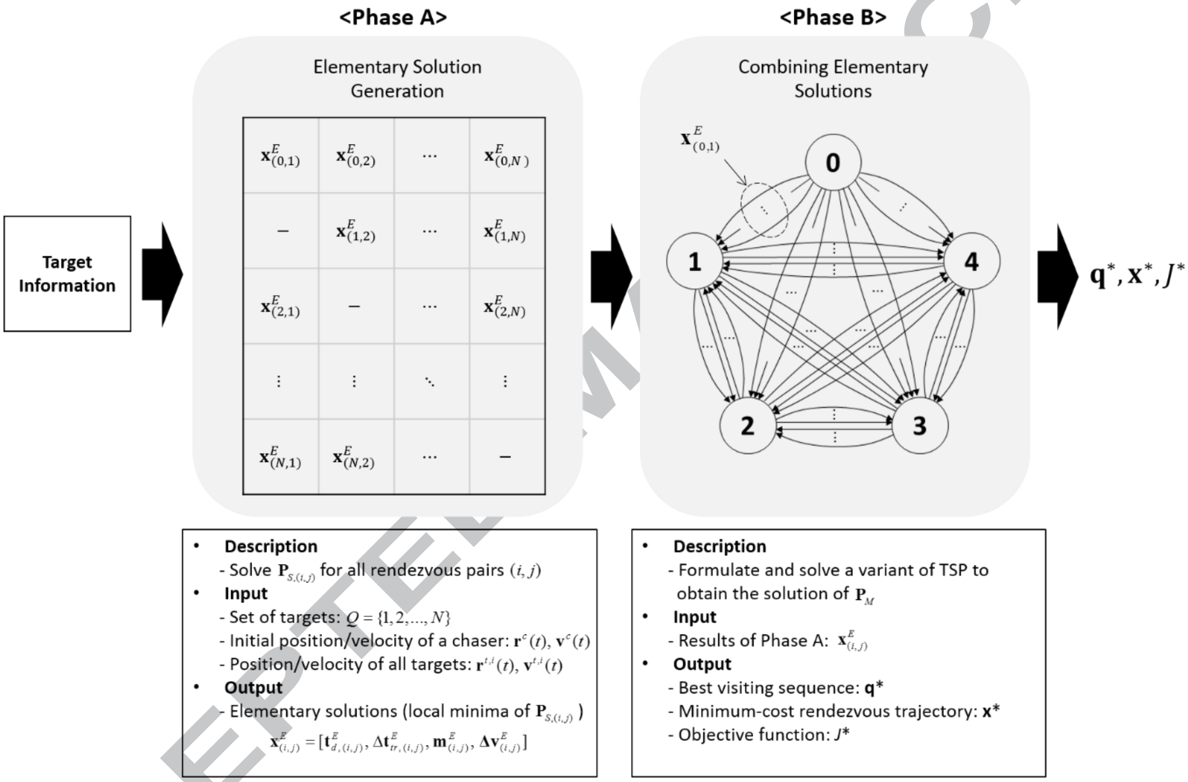

Fig. 3. Framework to solve the multi-target Lambert rendezvous problem ($\mathbf{P}_\mathrm{M}$)

In the first phase of the framework, a series of single target rendezvous problems ($\mathbf{P}_\mathrm{S}$) for every departure/arrival object pair are solved to obtain all of their local minima, which are used as the elementary solutions for the original multi-target rendezvous problem. Each elementary solution represents a candidate rendezvous trajectory between the object pair that has its own cost, departure time, and transfer time. The second phase seeks for the best rendezvous sequence ($\mathbf{q}^*$) and trajectories ($\mathbf{x}^*$) of the original problem using the elementary solutions obtained in the first phase. A new variant of TSP that considers multiple arcs (corresponding to the elementary solutions) and arc time window constraints is introduced to find the best combination of the elementary solutions. To sum up, all computations for the rendezvous trajectory optimization, which is the NLP part of the problem, are carried out in the first phase, while the second phase conducts the optimal sequencing, the ILP part of the problem. The following subsections provide the details on the steps of the proposed framework.

A. Phase A: Elementary Solution Generation

Phase A obtains all local minima of every single-target rendezvous problem ($\mathbf{P}_S$) instantiated by specified departure/arrival objects, which provide the elementary solutions of the overall problem (Bang and Ahn, 2016)[4]. We first explore the patterns in the locations of local minima of $\mathbf{P}_S$. A simple two-body dynamics with central gravity model was used to obtain the solutions of the single-target rendezvous problem.

Fig. 4 presents a sample contour plot of cost function for a single-target Lambert rendezvous (the sum of departure and arrival ΔV's) in $t_d - \Delta t_{tr}$ plane. The orbits of the chaser and the target are elliptic and non-coplanar. While the contours are diverse in their shape depending on orbital elements of the chaser and target (e.g. semi-major axis, eccentricity, and inclination angle), one can observe some patterns common to the locations of local minima. Frist, the cost function is extremely high along equally spaced straight lines (or, "walls") of two different types (type A and type B). Secondly, there are multiple minima located in regions separated by the walls. The locations of walls (illustrated as lines A, A', B and B' in Fig. 4) depend on the transfer geometry associated with the initial and final position vectors ($\mathbf{r}_d$ and $\mathbf{r}_a$) of the rendezvous maneuver.

[4] A preliminary version of the procedure to solve the single-target rendezvous optimization problem introduced in this subsection was presented in authors' previous conference paper (Bang and Ahn, 2016).

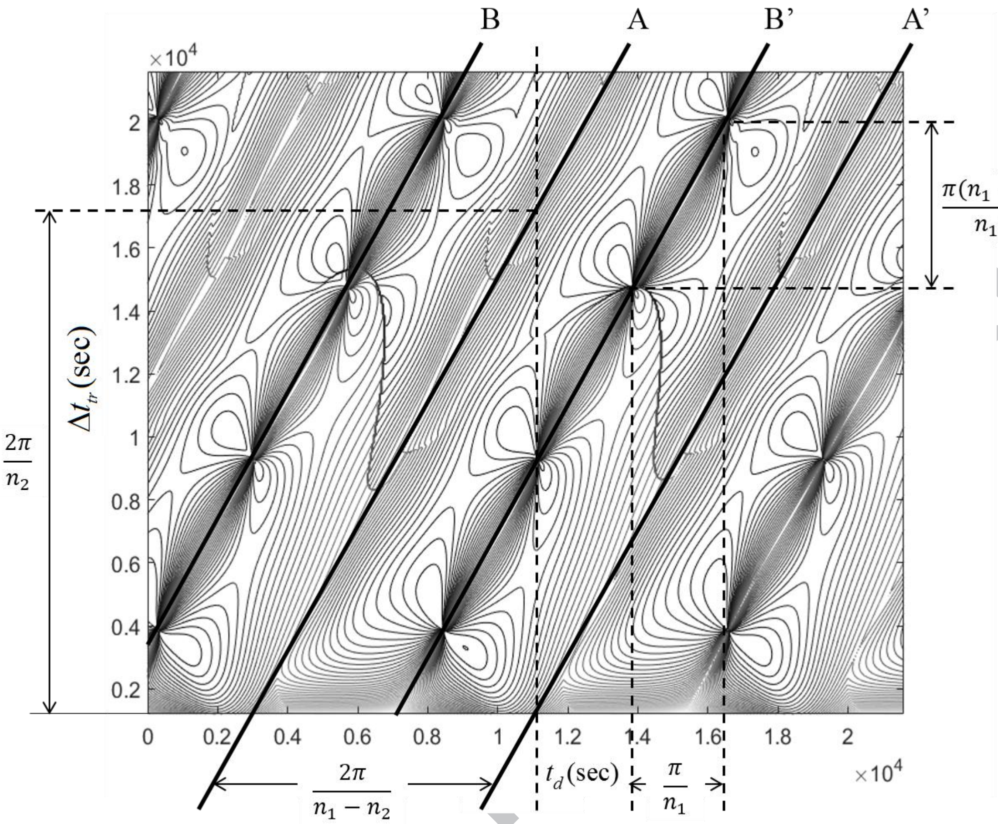

Fig. 4. Contour plot of cost function for a single-target Lambert rendezvous; Initial orbit elements of chaser / target: $[a, e, i, \Omega, \omega, \nu_0]$ = [6,678 km, 0, 0 deg., 0 deg., 0 deg., 0 deg.] / [13,878 km, 0.02, 30 deg., 20 deg., 0 deg., 90 deg.]

Fig. 5 presents the orbital transfer geometries that can cause large velocity increments. In the figure, l is the line of intersection between the orbital planes and θ_d / θ_a are the angles between l and $\mathbf{r}_d$ / $\mathbf{r}_a$, respectively. Fig. 5-(a) (zero transfer angle for a coplanar rendezvous) visualizes the geometry corresponding to type A walls (lines A and A' in Fig. 4). A rendezvous maneuver for this geometry requires very large velocity increment since the tangential velocity of the chaser should be lost at the beginning and then recovered at the end of the rendezvous. Therefore, the local minima are not located on or near the type A walls. The horizontal and vertical distances between two adjacent type A walls are given as

$$\delta t_{d,A} = \frac{2\pi}{n_1 - n_2} \tag{23}$$

$$\delta(\Delta t_{tr,A}) = \frac{2\pi}{n_2} \tag{24}$$

where n_1 and n_2 are mean motions of the chaser and the target.

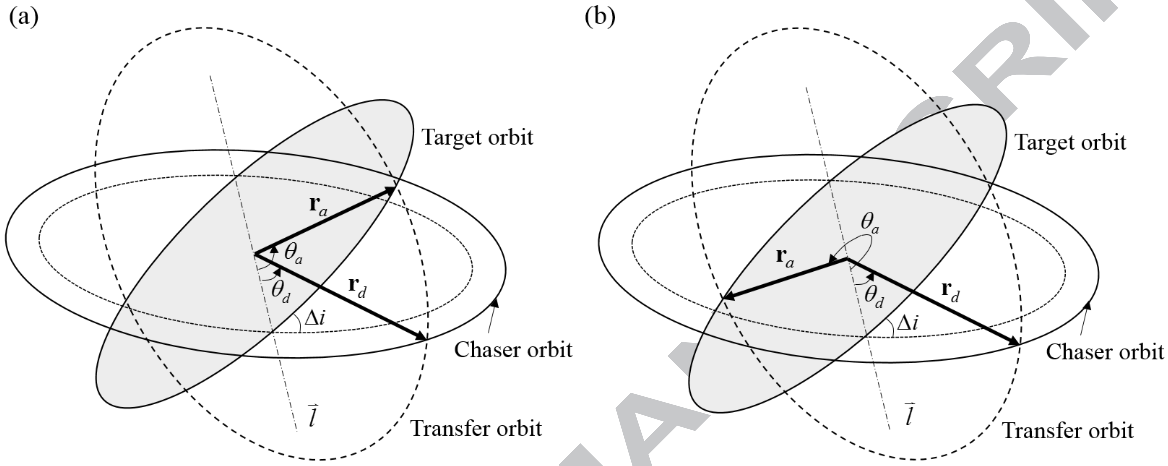

Fig. 5. Orbital transfer geometries associated with type A and type B walls

Type B walls appear only in the contour plots for non-coplanar rendezvous, which is presented in Fig. 5-(b). The value of cost function changes periodically along the lines B and B' of Fig. 4. The required ΔV is minimized when (θ_d, θ_a) equals (0°, 180°) or (180°, 0°), where $\mathbf{r}_d$ and $\mathbf{r}_a$ lie on the intersection of the two orbital planes. The local minima are placed on or near the type B walls with horizontal and vertical spaces as

$$\delta t_{d,B} = \frac{\pi}{n_1} \tag{25}$$

$$\delta(\Delta t_{tr,B}) = \frac{\pi(n_1 - n_2)}{n_1 n_2} \tag{26}$$

The procedure to find all minima of a single-target Lambert rendezvous problem is developed based on the aforementioned characteristics of the contour plot. A solution space partitioning technique is used to narrow down the exploration region and a gradient-based algorithm is implemented to obtain the minima. The solution space partitioning splits the current exploration space into multiple subspaces (exploration spaces of the next iteration step) until the subspace has at most one local minimum. Note that, when we explore the local minima of a subspace, constraints on the ranges of t_d and Δt_{tr} corresponding to the definition of the subspace are added to problem $\mathbf{P}_S$ as follows:

$$t_{d,\min} \le t_d \le t_{d,\max} \tag{27}$$

$$\Delta t_{tr,\min} \le \Delta t_{tr} \le \Delta t_{tr,\max} \tag{28}$$

A gradient-based algorithm with multiple initial points, which leverages the characteristics of the contour, is introduced to solve the single-target Lambert rendezvous problem with additional constraints ($\mathbf{P}_S$ with Eqs. (27)-(28)). When the procedure is conducted with initial guess at four vertices of an arbitrary subspace, the results can be categorized into the following three cases, which are presented in Fig. 6.

Case (a): If there is no local minimum of the single-target rendezvous problem ($\mathbf{P}_S$) inside the subspace, all the solution(s) of the constrained problem ($\mathbf{P}_S$ with additional subspace constraint) are located on the subspace boundary.

Case (b): If there is only one local minimum of $\mathbf{P}_S$ and there is no wall inside the subspace, all the solutions of the constrained problem will converge on the local minimum.

Case (c): If there are two or more local minima of $\mathbf{P}_S$ or there exists a wall inside the subspace, some solutions converge to the minima and others are located on the boundary.[5]

Cases (a) and (b) lead to the conclusions that there is no or just one local minimum in the subspace and additional split is not necessary. For Case (c), on the other hand, not all the local minima have been identified and further exploration of split solution space is required. The procedure to find all local minima of a single-target rendezvous problem is summarized in Table 1.

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[5] We set the initial grids to define the subspaces sufficiently fine and can assume that the number of type A wall inside of the subspace is no larger than one.

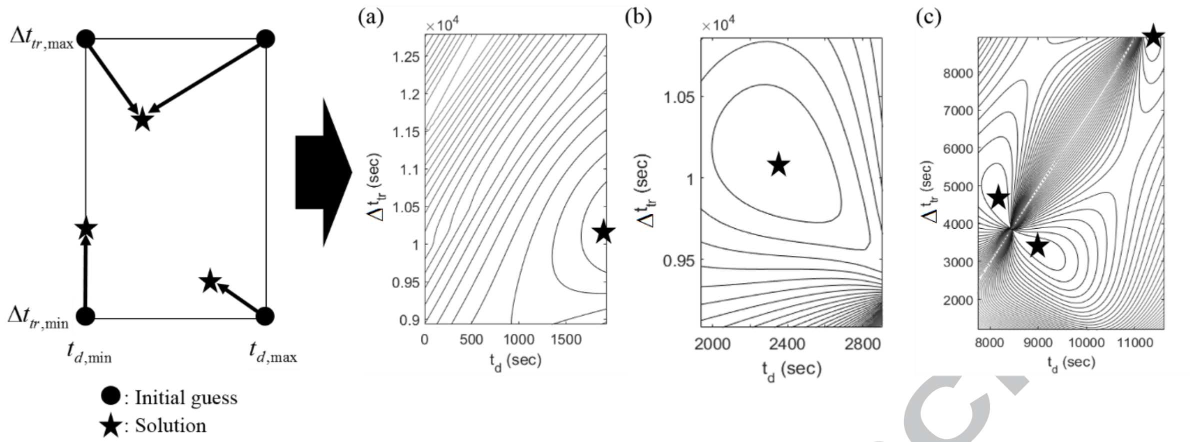

Fig. 6. Gradient-based algorithm with initial guesses at four vertices of a grid in the contour plot presented in Fig. 4

Table 1: Procedure to find all local minima of $\mathbf{P}_S$

Step	Task Description
Step 1	Partition the whole solution space into subspaces using initial grids. Note that the size of each subspace should be smaller than ($\delta t_{d,A} \times \delta(\Delta t_{tr,A})$).
Step 2	For each grid, implement the gradient-based algorithm (e.g. sequential quadratic programming (SQP)) with initial guesses of four vertex points. The lower and upper bounds for design variables are set to be identical to the boundary of the grid.
Step 3	If all the solutions are located on the boundary (Case (a)), stop exploring the subspace (eliminate the subspace) since there is no local minimum within it. Otherwise, go to Step 4.
Step 4	If all the solutions converge on a single point inside the boundary (Case (b)), save the point as a local minimum (elementary solution) and stop exploring the subspace (eliminate the subspace). Otherwise, go to the Step 5.
Step 5	Split the exploring subspace into smaller pieces because the identification of all the local minima is not guaranteed (Case (c)). Repeat steps 2 to 4 for new subspaces until every subspace is eliminated

B. Phase B: Solving Multi-Target Problem with Elementary Solutions

Phase B of the proposed framework solves the multi-target rendezvous problem ($\mathbf{P}_M$) by combining elementary solutions prepared in Phase A. The first step is to construct a graph composed of 1) nodes representing the objects, 2) arcs representing the rendezvous trajectories between two objects, and 3) costs associated with the arcs. Let $G = (V, A)$ be a directed graph where $V (= \{0\} \cup Q = \{0, 1, \ldots, N\})$

is the set of nodes (0: initial chaser orbit, $Q = \{1, \ldots, N\}$: targets) and A (= $\bigcup_{(i,j)\in V\times V,\, i\neq j} A_{(i,j)}$) is a set of arcs. Note that there is an infinite number of possible rendezvous trajectories with different costs, departure times, and transfer times between each object pair; hence, it is not appropriate to confine the path between each pair of nodes to a single arc. Instead, the elementary solutions $\mathbf{x}^E_{(i,j)}$, which represent a set of candidate rendezvous trajectories from object i to object j, are used to define a set of multiple arcs from node i to node j as

$$A_{(i,j)} = \{(i,\, j)^p \mid 1 \leq p \leq |\mathbf{x}^E_{(i,j)}|\} \tag{29}$$

Fig. 7 illustrates an example of a graph with four targets (N=4) constructed using the elementary solutions. In the graph, the proposed multi-target rendezvous problem ($\mathbf{P}_\mathrm{M}$) is equivalent to the process to determine the path in the graph that starts at node 0 and visits all other nodes with one of multiple arcs while minimizing the sum of costs (ΔV values).

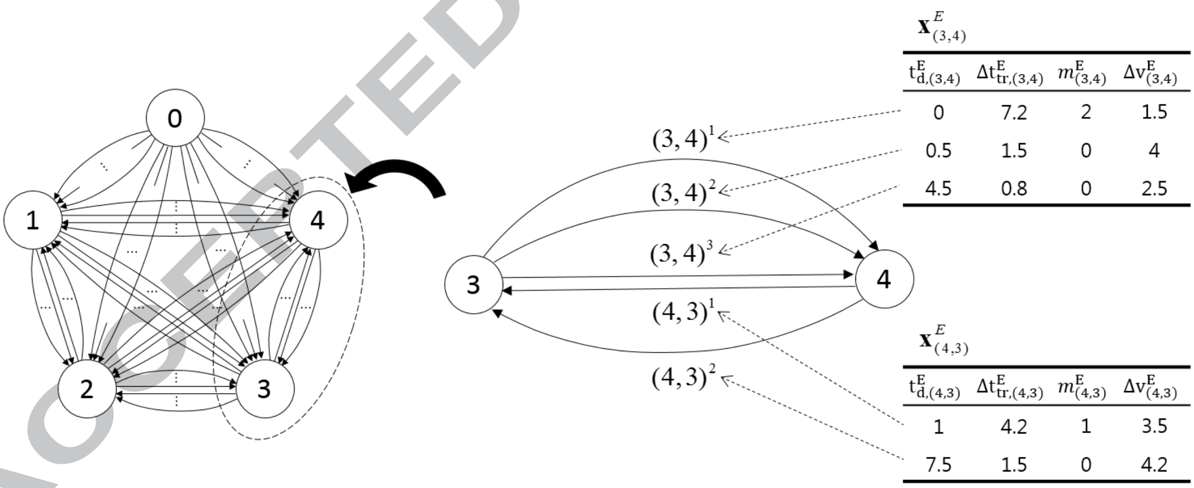

Fig. 7. Graph for multi-target rendezvous problem ($\mathbf{P}_\mathrm{M}$) – an illustrative example (N = 4)

Determination of an optimal path in the proposed graph can be interpreted as a variant of TSP formulated by considering two additional features: 1) there are multiple arcs between two nodes (i.e. departure and arrival objects), and 2) an arc is only available during a specific time interval referred to as *arc time window*. Note that each elementary solution (arc in the graph) has its own departure time (t_d). The departure time should satisfy the constraint associated with the minimum/maximum service times at the target. Otherwise the spacecraft cannot use the arc because it misses the rendezvous opportunity.

This situation is formulated as a new variant of TSP considering multiple arcs and arc time windows. The decision variable $y_{(i,j)^p}$ (binary) takes value of 1 if the spacecraft travels from node i to node j using arc $(i, j)^p$ – the p^{th} arc associated with the departure/arrival objects, and 0 otherwise. In addition, $c_{(i,j)^p}$, $t_{d(i,j)^p}$, and $\Delta t_{tr(i,j)^p}$ denote the cost, the departure time, and the transfer time associated with $(i, j)^p$, respectively. The mathematical formulation for Phase B of the proposed framework is presented as follows:

[$\mathbf{P}_{\mathrm{ME}}$: Optimal Multi-Target Rendezvous with Elementary Solutions]

$$\min_{\mathbf{y}} J_{ME} = \sum_{i \in V} \sum_{j \in V} \sum_{p=1}^{|A_{(i,j)}|} c_{(i,j)^p} y_{(i,j)^p} \tag{30}$$

subject to,

$$\sum_{i \in V} \sum_{p=1}^{|A_{(i,j)}|} y_{(i,j)^p} = 1, \forall j \in V \tag{31}$$

$$\sum_{j \in V} \sum_{p=1}^{|A_{(i,j)}|} y_{(i,j)^p} = 1, \forall i \in V \tag{32}$$

$$\sum_{j \in V} \sum_{p=1}^{|A_{(j,i)}|} (t_{d(j,i)^p} + \Delta t_{tr(j,i)^p}) y_{(j,i)^p} + \Delta t_{ser,\min} \le \sum_{j \in V} \sum_{p=1}^{|A_{(i,j)}|} t_{d(i,j)^p} y_{(i,j)^p}, \forall i \in Q \tag{33}$$

$$\sum_{j \in V} \sum_{p=1}^{|A_{(i,j)}|} t_{d(i,j)^p} y_{(i,j)^p} \le \sum_{j \in V} \sum_{p=1}^{|A_{(j,i)}|} (t_{d(j,i)^p} + \Delta t_{tr(j,i)^p}) y_{(j,i)^p} + \Delta t_{ser,\max}, \forall i \in Q \tag{34}$$

$$y_{(i,j)^p} \in \{0,1\}, \forall i, j \in V, \ \forall p \in \{1, ..., |A_{(i,j)}|\} \tag{35}$$

where $\mathbf{y}(=[\cdots, y_{(i,j)^p}, \cdots]^T)$ in Eq. (30) is the vector collecting the decision variables.

The objective function presented in Eq. (30) is to minimize sum of costs associated with the path. Eqs. (31)-(32) represent the constraints that each node is visited exactly once and there is exactly one departure from each node. Eqs. (33)-(34) represent the constraints on the admissible range of departure time associated with its minimum and maximum servicing times. Finally, the integrity constraint is presented in Eq. (35).

The proposed problem ($\mathbf{P}_{ME}$) is different from conventional TSP in that 1) there are multiple arcs (i.e., elementary solutions obtained in Phase A) assigned between departure and arrival nodes, and 2) the time windows constraints imposed on the arcs (i.e., the departure should take place within the admissible range) should be considered. The problem class is referred to as the "*Traveling Salesman Problem with Multiple Arcs and Arc Time Windows*," whose complexity is much higher than that of conventional TSP due to the increase in the design variables and constraints associated with multiple candidate arcs between nodes and the arc time windows, respectively. Note that the problem class has not yet been addressed in operations research field as far as the authors know.

The framework effectively finds a near-optimal solution of the multi-target Lambert rendezvous problem. It should be noted that there exists a trade-off relationship between the true optimality of the solution and the computational efficiency, which is reflected in generation of elementary solutions in Phase A. The proposed framework assumes that the final solution is composed of local minima of single-target rendezvous problems associated with all departure-arrival pairs, and it is not necessarily guaranteed that the solution is the true global optimum of the multi-target Lambert rendezvous problem. For example, moving away from local minimum and using a different trajectory for a rendezvous may allow new opportunity for the rendezvous with the next target. Addition of more candidate trajectories in Phase A can increase the possibility that the final solution obtained in Phase B is closer to the true optimum, which makes the computational load related to generation of elementary solutions (Phase A) and obtaining the solution of $\mathbf{P}_{ME}$ (Phase B).

V. Case Study

This section presents the case studies demonstrating the effectiveness of the proposed framework to obtain a near-optimal solution of multi-target Lambert rendezvous problem. Subsection V-A validates the algorithm to find a set of local minima for a single-target Lambert rendezvous (Phase A). Subsection V-B solves multi-target problems by adopting the whole framework introduced in this paper (Phases A and B). All the numerical results are compared with the solutions obtained by the genetic algorithm. Both Cases A and B were implemented and run on a machine with an Intel Core i7 CPU processer and 8GB RAM under the 64-bit Windows 7 operating system.

A. Case A: Validation for Single-Target Lambert Rendezvous

The proposed algorithm based on *subspace partitioning* and *gradient search* to find minima of a single target Lambert rendezvous was applied to a test problem presented in Chen et al. (2013). Table 2 summarizes the initial states of chaser/target and the constraint for the problem. The size of grids defining the initial subspace is set as: $(0.9\times\delta t_{d,A})\times(0.9\times\delta(\Delta t_{tr,A}))$. A subspace subject to additional split (Case (c) of Section IV-A) is divided into four by halving its width/height. Phase A was implemented in MATLAB and the fmincon function of MATLAB optimization toolbox was adopted as a gradient-based optimizer for local minimum search.

Table 2: Problem parameters used for Case A

Object	**Initial position**			**Initial velocity**		
	r_x, km	r_y, km	r_z, km	v_x, km/s	v_y, km/s	v_z, km/s
Chaser	7,241	4,181	0	-1.728	2.993	5.985
Target	31,760	27.391	6,027	-1.430	1.114	2.475
Constraint						**Value**
$t_{\max}$, *hr*						24

Fig. 8 illustrates the test case result. Total 37 local minima (depicted as circles) – including those located on the boundary representing the terminal time constraint – were found through the procedure introduced in Section IV-A. The figure shows that the procedure finds all local minima without any misses. In addition, the global optimum (depicted as a black square) obtained by the procedure is identical to the reported value. Chen et al. (2013) pointed out that the global optimum obtained by the GA for this problem was inconsistent (one of points depicted as crosses). This observation indicates that 1) a single run of GA does not guarantee the global optimum, and 2) a number of optimization runs are required to find the global optimum with acceptably high probability. On the contrary, the proposed approach can find the set of all local minima, which include the global optimum of $\mathbf{P}_S$. The computation time taken to identify all local minima was 47.4 seconds.

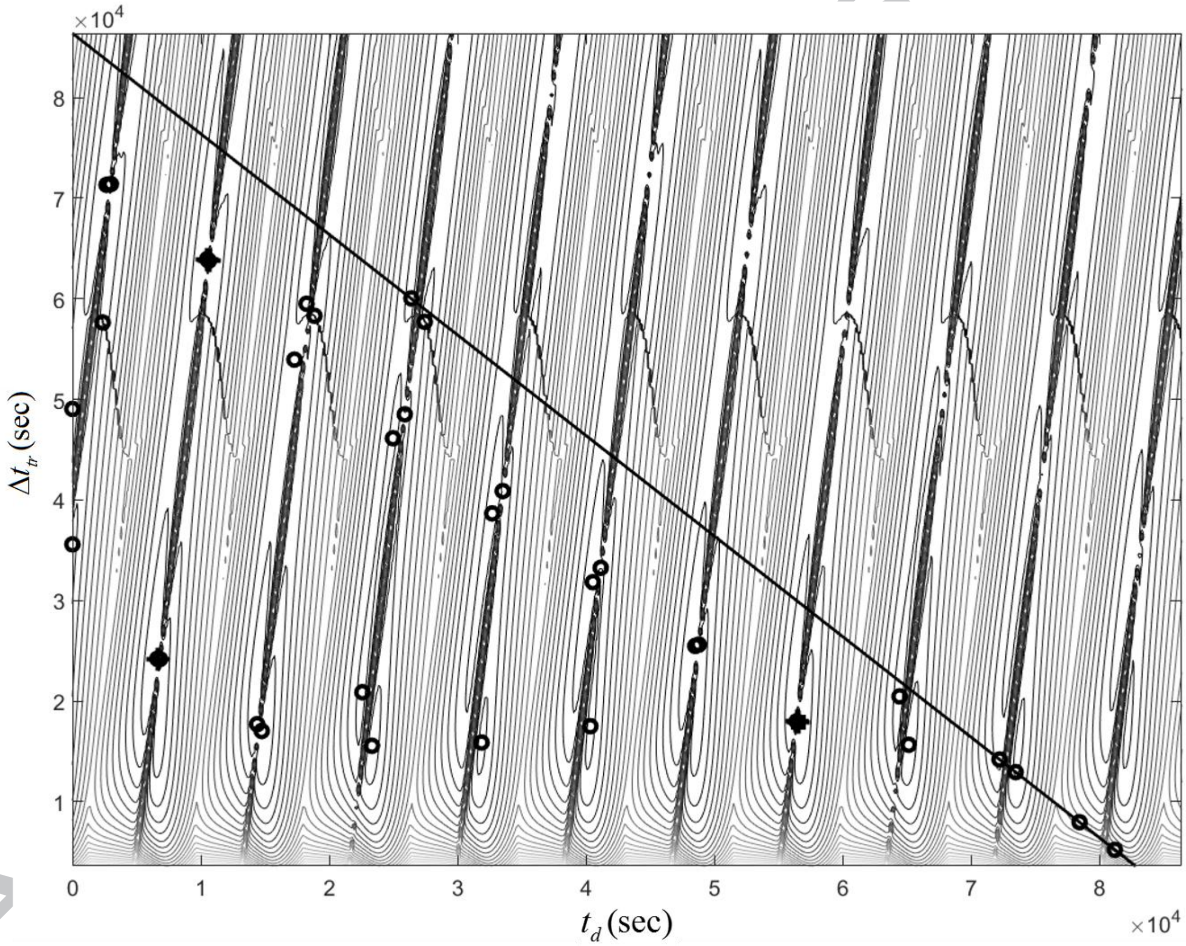

Fig. 8. Results of Case A – local minima and global optimum obtained by the proposed procedure and the GA

B. Case B: Validation for Multi-Target Lambert Rendezvous

The overall framework proposed in the paper is validated using the multi-asteroid rendezvous case presented in Zhang et al. (2015). The objective of the case is to find the best (min-ΔV) rendezvous order and trajectories to visit all asteroids within a specified mission time. Three problem instances with different number of asteroids – 4 (Case B-1), 8 (Case B-2), and 16 (Case B-3) – were considered and the framework introduced in section IV (Phases A and B) is used to solve them. The initial states of chaser/target asteroids and the constraints for the problem were summarized in Table 3 (Zhang et al. 2015). Problem $\mathbf{P}_{\mathbf{ME}}$ formulated in Phase B was implemented in MATLAB with GUROBI 7.5 solver.

Table 3: Problem parameters for Case B

Object	Initial position			Initial velocity		
	r_x, au	r_y, au	r_z, au	v_x, km/s	v_y, km/s	v_z, km/s
Chaser	-0.17206	0.96812	-0.00001	-29.815	-5.325	0.000
Target 1	-1.02848	-0.26939	0.02359	9.136	-27.020	0.029
Target 2	-0.34056	-1.00392	0.00388	27.819	-7.605	-0.060
Target 3	0.28759	1.00171	0.00877	-26.892	10.019	-0.659
Target 4	-0.90624	-0.54981	-0.01290	16.472	-21.856	-0.056
Target 5	1.08591	-0.18953	0.01340	5.717	27.219	0.525
Target 6	0.87892	-0.73790	0.00679	17.232	19.865	0.202
Target 7	0.94499	-0.21205	0.00322	7.668	30.242	0.102
Target 8	1.07411	0.17919	-0.02671	-2.489	27.494	0.004
Target 9	0.28834	1.18882	-0.05398	-23.434	4.746	1.096
Target 10	-0.13676	-0.79207	0.06676	35.975	-6.787	0.536
Target 11	0.88031	0.72432	-0.02561	-16.063	21.472	1.294
Target 12	-0.12663	1.24640	-0.01569	-25.125	-2.997	0.684
Target 13	-0.76631	-0.52576	-0.01917	19.961	-23.146	-1.172
Target 14	-0.30486	0.99227	-0.00949	-26.647	-10.308	0.302
Target 15	-0.19071	0.86048	0.04906	-33.529	-4.142	-3.042
Target 16	-0.10868	0.89039	0.00751	-32.396	-6.808	-1.266

Problem Instance	No. of Asteroids	Given Target Set
Case B-1	4	{1, 2, 3, 4}
Case B-2	8	{1, 2, …, 8}
Case B-3	16	{1, 2, …, 16}

Constraints	Value
[$\Delta t_{ser,min}$, $\Delta t_{ser,max}$], days	[7, 365.25]
$\Delta t_{tr,max}$, days	730.5
$t_{\max}$, yr (Case B-1 / Case B-2 / Case B-3)	22 / 34 / 58

Table 4 comparatively exhibits the results of the three test cases obtained using the proposed framework and the mixed-code genetic algorithm (MCGA) with search enhancement presented in Zhang et al. (2015). The optimal visiting orders for Case B-1 (rendezvous with 4 asteroids) obtained by two different methods were identical and their optimal objective function values show no significant difference. For Case B-2, however, the solutions found by different methodologies were very different – both in terms of rendezvous sequence and the best objective function ([7, 4, 1, 8, 2, 3, 5, 6] and 16.402 *km/s* for the *proposed framework* and [8, 5, 1, 4, 7, 3, 2, 6] / 19.153 *km/s* for the *MCGA with search enhancement*). The improvement in the objective function achieved by the proposed framework was 14.3 %. In Case B-3 (rendezvous with 16 asteroids), the optimal solution found by the proposed framework outperformed the reported result obtained by the MCGA with search enhancement – more significantly than previous cases. The visiting sequences obtained by two methodologies were totally different (proposed framework: [11, 1, 5, 13, 3, 7, 2, 4, 15, 16, 6, 8, 14, 12, 9, 10], MCGA: [11, 9, 1, 4, 12, 10, 3, 7, 2, 5, 6, 16, 14, 8, 15, 13]), and the reduction in objective function was 34.7 % (37.870 *km/s* versus 57.977 *km/s*). This comparison result supports the aforementioned limitation of existing heuristics based approach in its effectiveness for rendezvous with many targets. The number of Lambert routine calls for the two methods, which provide the proxy for the resource consumption, are compared as well. The results indicate that the computational resource spent by the proposed method is about 3-4 times larger than that for the MCGA.

The details on the obtained optimal solutions – target indices, departure times, transfer times, and the corresponding velocity increments for each rendezvous task – and the computation time spent in each phase are summarized in Table 5. Note that Phase A consumed the majority of computational resource (about 92-100 % of the total computation time) compared to Phase B, which suggests that solving the Lambert problem can be regarded as the bottleneck process. Therefore, accelerating the Lambert routine is the key task to improve the performance of the proposed framework, which is an interesting follow-on study subject (Ahn and Lee, 2013; Ahn et al., 2015; Lee et al., 2016).

Table 4: Objective function and computational load for Case B – Proposed method vs. Mixed-code

GA with search enhancement

Problem Instance	Proposed method		Mixed-code GA with search enhancement (Zhang et al. 2015)	
	Optimization Results	**No. of Lambert routine calls**	**Optimization Results**	**No. of Lambert routine calls**
Case B-1	$\mathbf{q}$ = (1, 2, 3, 4) J = 6.36 *km/s*	3,526,933	$\mathbf{q}$ = (1, 2, 3, 4) J = 6.40 *km/s*	960,000
Case B-2	$\mathbf{q}$ = (7, 4, 1, 8, 2, 3, 5, 6) J = 16.40 *km/s*	25,392,677	$\mathbf{q}$ = (8, 5, 1, 4, 7, 3, 2, 6) J = 19.15 *km/s*	7,680,000
Case B-3	$\mathbf{q}$ = (11, 1, 5, 13, 3, 7, 2, 4, 15 16, 6, 8, 14, 12, 9, 10) J = 37.87 *km/s*	182,852,203	$\mathbf{q}$ = (11, 9, 1, 4, 12, 10, 3, 7, 2, 5, 6, 16, 14, 8, 15, 13) J = 57.98 *km/s*	61,440,000

Table 5: Details on optimal solutions and computation times for Case B

Index	Case B-1				Case B-2				Case B-3			
	q	t_d (*days*)	Δt_{tr} (*days*)	Δv (*km/s*)	q	t_d (*days*)	Δt_{tr} (*days*)	Δv (*km/s*)	q	t_d (*days*)	Δt_{tr} (*days*)	Δv (*km/s*)
1	1	2,033.48	169.08	1.53	7	3,619.23	602.95	1.93	11	3,790.96	713.21	2.48
2	2	2,396.70	198.47	1.19	4	4,460.62	730.50	1.79	1	4,811.93	614.07	1.89
3	3	2,607.21	381.85	1.65	1	5,555.64	608.80	2.61	5	5,710.18	715.86	2.18
4	4	3,139.38	722.51	1.99	8	6,503.98	730.50	2.09	13	6,581.80	730.50	3.13
5	-	-	-	-	2	7,570.06	724.90	2.46	3	7,621.63	550.05	2.71
6	-	-	-	-	3	8,560.40	730.50	1.12	7	8,532.61	609.93	0.94
7	-	-	-	-	5	9,371.05	675.93	2.07	2	9,400.39	681.35	0.73
8	-	-	-	-	6	10,094.87	730.50	2.32	4	10,247.25	664.25	1.65
9	-	-	-	-	-	-	-	-	15	11,036.79	487.49	4.51
10	-	-	-	-	-	-	-	-	16	11,770.18	533.46	4.47
11	-	-	-	-	-	-	-	-	6	12,411.23	687.10	1.16
12	-	-	-	-	-	-	-	-	8	13,436.50	328.58	2.91
13	-	-	-	-	-	-	-	-	14	13,861.69	581.19	0.81
14	-	-	-	-	-	-	-	-	12	14,756.16	655.96	2.23
15	-	-	-	-	-	-	-	-	9	15,566.99	491.28	2.59
16	-	-	-	-	-	-	-	-	10	16,340.19	637.60	3.48
Total				6.36				16.40				37.87
	Computation time (s)			Ratio (%)	Computation time (s)			Ratio (%)	Computation time (s)			Ratio (%)
Phase A	3,171			100.00	23,010			99.92	160,385			92.23
Phase B	0.06			0.00	18.71			0.08	13,516			7.77
Total	3,171			100	23,029			100	173,901			100

Conclusions

A novel two-phase framework to find a near-optimal solution of the multi-target Lambert rendezvous problem is proposed in this paper. Elementary solutions of single target problems associated with all departure-arrival pairs are prepared in the first phase. The second phase searches for the best rendezvous sequence and trajectories by solving a variant of TSP formulated using the elementary solutions. Case studies to solve single-target and multi-target rendezvous problems were conducted to demonstrate the effectiveness of the proposed framework.

Additional studies on enhancing the performance of the proposed framework and systematic comparison with other methodologies can be considered as potential future research. Note that Phase B of the proposed framework works with various elementary solution sets – the arcs used in $\mathbf{P}_\mathbf{M}$. In this regard, the overall performance of the framework can be improved by including more trajectory options (e.g. multiple-impulse rendezvous strategy) in Phase A, which is also a promising future work. Improvement of elementary solutions by considering gravitational perturbation (e.g. J_2 term) would be helpful to claim better practicality of the proposed framework. Consideration of the target selection out of a larger candidate objects with the proposed multi-target rendezvous problem and development of a new framework that can simultaneously handle the combined problem can be another interesting subject for further study.

Acknowledgements

This work was prepared at the Korea Advanced Institute of Science and Technology, Department of Aerospace Engineering, under a research grant from the National Research Foundation of Korea (NRF-2013M1A3A3A02042461). Authors thank the National Research Foundation of Korea for the support of this work.

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