Problem description for the
9th Global Trajectory Optimisation Competition

The Kessler Run

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> "Its the ship that made the Kessel run in less than 12 parsecs"
>
> — Han Solo

1 Background

It is the year 2060 and the commercial exploitation of Low Earth Orbits (LEOs) went well beyond the trillion of Euros market size. Following the unprecedented explosion of a Sun-synchronous satellite, the Kessler effect triggered further impacts and the Sun-synchronous LEO environment was severely compromised. Scientists from all main space agencies and private space companies isolated a set of 123 orbiting debris pieces that, if removed, would restore the possibility to operate in that precious orbital environment and prevent the Kessler effect to permanently compromise it. You are thus called to design a series of missions able to remove all critical debris pieces while minimizing the overall cumulative cost of such an endeavour. Each single mission cost (in EUR) will depend on how early the mission is submitted to the "authorities" (regardless of their actual launch epoch) and on the spacecraft initial mass.

The website `https://kelvins.esa.int/` has to be used to submit all missions in the format described in this document: each submission will be automatically verified and scored.

2 The problem

Design n **missions** able to cumulatively remove all the 123 orbiting debris defined in the file `debris.csv`. One **mission** is a multiple-rendezvous spacecraft trajectory where a subset of size N of the 123 orbiting debris is removed by the delivery and activation of N de-orbit packages. The following cost function has to be minimized:

$$J = \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} \left[c_i + \alpha \left(m_{0_i} - m_{dry} \right)^2 \right]$$

where C_i is the cost charged by the contracted launcher supplier for the i-th **mission** and it is composed of a base cost c_i (increasing linearly during the competition time frame) and a term $\alpha \left(m_{0_i} - m_{dry} \right)^2$ favouring a lighter spacecraft. At the beginning of the i-th mission m_{0_i} is the spacecraft mass and m_{dry} its dry mass. The α parameter is set to be $\alpha = 2.0 \cdot 10^{-6}$ [MEUR / Kg²]: each Kg of launch mass saved results in a discount over the mission cost (but also in less ΔV capability).

The i-th mission starting epoch is denoted with t_i^s and its end epoch with t_i^f. A mission starts with a launch delivering, at t_i^s, one spacecraft at a chosen debris and ends when all the N de-orbit packages on-board have been delivered and activated. An orbiting debris is considered as removed if: a) its position and velocity at some epoch t coincides with the spacecraft position and velocity vector and b) for the following $t_w \geq 5$ days the spacecraft stays in proximity of the debris while delivering and activating a de-orbit package of mass $m_{de} = 30$ [kg].

After, the spacecraft is free to ignite its propulsion system again and leave towards the next debris (note that only in-between debris transfers the spacecraft is subject to the full J_2 perturbation and its dynamics is described by the equations of motion reported in the Appendix. During the removal operations (i.e. for a time t_w) the position and velocity of the spacecraft will, instead, be considered to be those of the debris as computed by the ephemerides)

The basic cost c_i of each mission (i.e. not including the α term), increases linearly during the competition month and is computed as follows:

$$c_i = c_m + \frac{t_{submission} - t_{start}}{t_{end} - t_{start}} (c_M - c_m)$$

where $t_{submission}$ is the epoch at which the i-th mission is validated (via the Kelvins web-site) and t_{end} and t_{start} are the end and the beginning epochs of the GTOC9 competition. The exact value of c_i will be visible at all time on the website. The minimal basic cost c_m is 45 MEUR, while the maximum cost c_M is 55 MEUR.

Each orbiting debris that is not removed by any of the missions submitted by one team will be considered, at the end of the competition, removed by a dedicated launch costing 55.0018 MEUR

3 Orbital Manoeuvres

The only manoeuvres allowed to control the spacecraft trajectory are instantaneous changes of the spacecraft velocity (its magnitude being denoted by ΔV. After each such manoeuvre, the spacecraft mass is to be updated using Tsiolkovsky equation:

$$m_f = m_i \exp\left(-\frac{\Delta V}{v_e}\right),$$

where $v_e = I_{sp} g_0$. A maximum of 5 impulsive velocity changes is allowed during each transfer between two successive debris.

4 Spacecraft

Each spacecraft initial mass m_0 is the sum of its dry mass, the weights of the $N \geq 1$ de-orbit packages to be used and the propellant mass: $m_0 = m_{dry} + N m_{de} + m_p$. All spacecraft have a dry mass of $m_{dry} = 2000$ [kg] and a maximum initial propellant mass of $m_p = 5000$ [kg] (less propellant may be used, in which case the launch costs will decrease). Each de-orbit package has a weight of $m_{de} = 30$ [kg]

5 Operational Constraints

The debris removal operations during each of the multiple-rendezvous trajectories are complex and demand some control over the schedule of the debris visits:

1. The overall time between two successive debris rendezvous, within the same mission, must not exceed 30 days. So that if the arrival epoch to the debris $\mathcal{D}_a$ is $t_{\mathcal{D}_a}$ and the arrival to the next debris $\mathcal{D}_b$ is $t_{\mathcal{D}_b}$, then $t_{\mathcal{D}_b} - t_{\mathcal{D}_a} \leq 30$ [days].

2. In order to avoid operating different missions in parallel, a time of at least 30 days must be accounted for between any two missions so that if $t_i^s > t_j^s$ then $t_j^f + 30 \leq t_i^s$ [days], $\forall i \neq j$.

3. All mission events (arrivals, departures, etc..) must take place in an allowed window, so that indicating with t_{event} the epoch of any event, then $23467 \leq t_{event} \leq 26419$ [MJD2000].

4. The osculating orbital periapsis r_p cannot be smaller than $r_m = 6600000$ [m] (this is only checked immediately after arrival, departures and deep space manoeuvres events, not in-between)

	$Value$	$Units$
α	$2.0 \cdot 10^{-6}$	$MEUR/Kg^2$
c_m	45	MEUR
c_M	55	MEUR
t_w	5	$days$
m_{de}	30	Kg
m_{dry}	2000	Kg
m_p	5000	Kg
r_{p_m}	6600000	m
μ	$398600.4418 \cdot 10^9$	m^3/sec^2
J_2	$1.08262668 \cdot 10^{-3}$	–
r_{eq}	6378137	m
I_{sp}	340	sec
g_0	9.80665	m/sec^2
Day	86400	sec
Year	365.25	$days$
JD = MJD2000 + 2451544.5	–	–
MJD = MJD2000 + 51544	–	–

Table 1: Values to be used for problem constants and conversions. JD is the Julian date. MJD the Modified Julian Date and MJD2000 the Modified Julian Date 2000

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6 Submission

Each **mission** is described by a stand-alone text file. One file contains one mission. Each file must be submitted separately. Each member of a team is allowed to submit files on behalf of the whole team.

As soon as a file is submitted two steps will happen in our servers: a) **preliminary validation** and b) **scoring step**. During the preliminary validation, the submission is considered as a stand-alone mission and only some constraints are thus checked. If a problem arises at this stage an error code and a description of the problem will appear to help the user debug the problem. If the preliminary validation is passed the submitted file will be stored in our servers and appear in the submission list of the team's users. This does not mean that the submission is valid: problems may arise during the following **scoring step** as the mission will be tested against all previously submitted missions that were marked as valid (e.g. for repeating debris, mission overlaps, etc...). If some constraint violation is detected during the **scoring step** , the submission will be marked as invalid and appear in red in the team's users submission list, but no error will be explicitly signaled. A non valid submission has no consequence to the team score.

Upon successful completion of both steps, the updated team score, as well as the total number of debris removed so far will be immediately visible in the leader-board and thus available to all competing teams.

6.1 File format

Each line of the file contains an **event** which, essentially, correspond to an impulsive ΔV manoeuvre. Impulses can be given at arrival or at departure to/from a given debris, as well as in deep space. Each line thus contains the epoch, the spacecraft state, the Cartesian components of the instantaneous velocity increment $\Delta \mathbf{V}$ (assumed to be delivered immediately after the **event**) and the **event** id (a deep space manoeuvre is denoted by -1, the arrival and departure to/from a specific debris with the debris id). Valid submissions are available for inspection in the folder submission_examples contained in the data file gtoc9-data.zip. The file example_submission1.txt, as an example, contains a possibly valid mission able to remove the debris with ids 23, 3 and 51. A direct transfer is made between debris pieces with ids 23 and 3, while one deep space manoeuvre is made between ids 3 and 51. The file, thus, has the following structure:

line in file	t [mjd2000]	x [m]	y [m]	z [m]	v_x [m/s]	v_y [m/s]	v_z [m/s]	m [kg]	ΔV_x [m/s]	ΔV_y [m/s]	ΔV_z [m/s]	event id
0	float,	float,	float,	float,	float,	float,	float,	float,	0,	0,	0,	23
1	float,	float,	float,	float,	float,	float,	float,	float,	float,	float,	float,	23
2	float,	float,	float,	float,	float,	float,	float,	float,	float,	float,	float,	3
3	float,	float,	float,	float,	float,	float,	float,	float,	float,	float,	float,	3
4	float,	float,	float,	float,	float,	float,	float,	float,	float,	float,	float,	-1
5	float,	float,	float,	float,	float,	float,	float,	float,	float,	float,	float,	51
6	float,	float,	float,	float,	float,	float,	float,	float,	0,	0,	0,	51

6.2 Preliminary validation

Each file submitted will be validated and scored automatically. A number of validation checks will be made as soon as the file is received, if any fails a the following error codes will be returned:

1. The file size is checked. If exceeding 1MB the submission will be marked as invalid.

2. The file structure is checked. If not a csv file containing 12 values per line, the submission will be marked as invalid.

3. The number of lines is checked. If exceeding 856 or less than 2 the submission will be marked as invalid.

Let us indicate the file content using, as a subscript, the file line so that, for example, t_3 denotes the value of the epoch in the fourth line (the first line has subscript 0, the last f). We also indicate with $\mathbf{r}_{\mathcal{D}_k}(t)$, $\mathbf{v}_{\mathcal{D}_k}(t)$ the position and velocity of the debris with $id = k$ at the epoch t. Finally we use $\mathbf{r}_i = [x_i, y_i, z_i]$, $\mathbf{v}_i = [v_{x_i}, v_{y_i}, v_{z_i}]$ to indicate the position and velocity as read from the file at line i and $\Delta\mathbf{V}_i = [\Delta V_{x_i}, \Delta V_{y_i}, \Delta V_{z_i}]$ to indicate the velocity increment vector as read from the file at line i. Using these conventions, the following further checks will be made, if any fails the submission will be marked as invalid:

4. The event ids are checked: $-1 \leq id_i \leq 122$, $\forall i$

5. The pericenter is checked: $a_i(1 - e_i) > 6600000$, $\forall i$, where a_i and e_i are the osculating orbital elements computed from $\mathbf{r}_i$ and $\mathbf{v}_i$.

6. The initial mass is checked: $m_0 \geq 2030$. The embarked propellant mass is checked $m_p = m_0 - m_{dry} - 30N \leq 5000$ (N is here assumed to be equal to the number of different debris visited). The final mass is checked: $m_f \geq 2000$.

7. The epochs are checked (monotonically increasing): $t_j > t_i$, $\forall i$, $\forall j > i$.

8. The initial and final velocity increments are checked: $|\Delta\mathbf{V}_0| = |\Delta\mathbf{V}_f| = 0$.

9. The first and last two events id are checked to be an arrival and a departure to/from the same debris: $id_0 = id_1 \neq -1$ and $id_f = id_{f-1} \neq -1$

10. The event sequence is checked: if $id_i \neq -1$ then either $id_{i-1} = id_i$ or $id_{i+1} = id_i$ $\forall i \in [2,...,f-2]$.

11. The event sequence is checked: exactly two events (arrival and departure) must be associated to each $id \neq -1$

12. For all arrival events, the rendezvous conditions are checked as: $|\mathbf{r}_i - \mathbf{r}_{\mathcal{D}_i}(t_i)| < \epsilon_r$, $|\mathbf{v}_i - \mathbf{v}_{\mathcal{D}_i}(t_i) + \Delta\mathbf{V}_i| < \epsilon_v$

13. For all deep space manoeuvre events and arrival events (except the first), the mass update is checked as $\left| m_i - m_{i-1} \exp\left(-\frac{|\Delta \mathbf{V}_{i-1}|}{v_e} \right) \right| \leq \epsilon_m$.

14. For all arrival events, the waiting time constraint is checked as $t_{i+1} - t_i \geq 5$ days.

15. For all arrival events, except the first one, the time elapsed since the previous debris rendezvous is checked: $t_i - t_j \leq 30$, where j is the line corresponding to the previous arrival event.

16. For all departure events, the rendezvous conditions are checked as: $|\mathbf{r}_i - \mathbf{r}_{\mathcal{D}_i}(t_i)| < \epsilon_r$, $|\mathbf{v}_i - \mathbf{v}_{\mathcal{D}_i}(t_i)| < \epsilon_v$

17. For all departure events the mass update is checked as $\left| m_i - m_{i-1} \exp\left(-\frac{|\Delta \mathbf{V}_{i-1}|}{v_e} \right) + 30 \right| \leq \epsilon_m$. (delivery of the de-orbit package is included as well as the mass loss from the ΔV)

18. For all deep space manoeuvre events and arrival events (except the first) the spacecraft state $\mathbf{r}$, $\mathbf{v}$ is first computed numerically integrating the equation of motion from initial conditions $\mathbf{r}_{i-1}$ and $\mathbf{v}_{i-1}$ for a time of $T = t_i - t_{i-1}$ [days] and then checked as $|\mathbf{r} - \mathbf{r}_i| < \epsilon_r$ and $|\mathbf{v} - \mathbf{v}_i| < \epsilon_v$.

19. All epochs must be in the allowed window. $t_i \geq 23467$, $t_i \leq 26419$, $\forall i$.

6.3 Scoring

If the file passes the **validation step**, a **scoring step** is applied. During this process the submitted file can still be detected as invalid w.r.t. to the previously submitted files as some operational constraints will be now checked (e.g. debris visited only once). In case any is violated the submission will be considered invalid and not scored.

If an error occurs in this phase no message will be communicated, but the submission will simply be marked as invalid in the user submission page.

Otherwise the score J of the team will be updated on the leader-board as well as the number of debris removed so far.

6.4 Testing for preliminary validation

To help the teams figure out the correct file format to submit, any submission with **filename** containing the string TRIAL will be ignored after the preliminary validation step.

Try, as a test, to submit the file TRIAL-my_mission.txt which contains a mission that passes the preliminary validation. Then try to submit the same file, changing some numbers it contains.

6.5 Undoing submissions

Submissions can be withdrawn by the team, but only in reverse submission order. This may be useful in cases where a mission was submitted early in the competition timeline in the hope to exploit a favorable price, but later the team has found that the submitted mission is no longer of use. To withdraw the last valid submission, (and thus revert to the previous valid one) any team member may submit a dummy file that passes the preliminary validation (e.g. `example_submission1.txt`) and put in the **title** field of the submission page the text "[delete last valid]". The result of this action will be to mark as invalid the last valid submission. The dummy file will also appear as a dummy invalid submission on your user page.

By issuing this command repeatedly all submissions can be undone in reverse order.

7 Appendices

A Equations of motion for the spacecraft

Each spacecraft dynamics is described, between two manoeuvres, by the following set of Ordinary Differential Equations (ODEs):

$$\ddot{\mathbf{x}} = -\frac{\mu x}{r^3} \left\{ 1 + \frac{3}{2} J_2 \left(\frac{r_{eq}}{r} \right)^2 \left(1 - 5 \frac{z^2}{r^2} \right) \right\}$$

$$\ddot{\mathbf{y}} = -\frac{\mu y}{r^3} \left\{ 1 + \frac{3}{2} J_2 \left(\frac{r_{eq}}{r} \right)^2 \left(1 - 5 \frac{z^2}{r^2} \right) \right\}$$

$$\ddot{\mathbf{z}} = -\frac{\mu z}{r^3} \left\{ 1 + \frac{3}{2} J_2 \left(\frac{r_{eq}}{r} \right)^2 \left(3 - 5 \frac{z^2}{r^2} \right) \right\}$$

that describe a Keplerian motion perturbed by main effects of an oblate Earth, i.e. J_2. Note that between an arrival and a departure **event** the spacecraft is co-orbiting with the debris piece and hence its position and velocity is not described by the above equations, rather it coincides with the orbiting debris'.

B The orbital ephemerides

Each debris orbit is defined by the values $t_0, a, e, i, \Omega_0, \omega_0, M_0$ as read from the file `debris.csv`. At any given epoch t the position and velocity vectors of each debris piece must be computed by updating its osculating Keplerian elements using the mean motion and the precession rates and then converting, as in the Keplerian case, the updated osculating elements to position and velocity. Note that by doing so we are neglecting the velocity component deriving from $\dot{\Omega}$ and $\dot{\omega}$, for the purpose of this competition this is deemed as appropriate as it removes complexity from the equations without introducing any significant change on the search space landscape.

The procedure detailed below (assumes consistent units everywhere) shows all necessary equations.

1 - Compute the osculating Keplerian parameters

After having defined the mean motion $n = \sqrt{\frac{\mu}{a^3}}$, the semilatus rectum $p = a(1 - e^2)$ and the precession rates:

$$\dot{\Omega} = -\frac{3}{2} J_2 \left(\frac{r_{eq}}{p} \right)^2 n \cos i$$

$$\dot{\omega} = \frac{3}{4} J_2 \left(\frac{r_{eq}}{p} \right)^2 n(5 \cos^2 i - 1)$$

compute the right ascension of the ascending node Ω from:

$$\Omega - \Omega_0 = \dot{\Omega}(t - t_0),$$

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the argument of perigee ω from:

$$\omega - \omega_0 = \dot{\omega}(t - t_0),$$

and the mean anomaly from:

$$M - M_0 = n(t - t_0)$$

2 - Compute position and velocity as in the Keplerian case

The Kepler's equation is used to compute the eccentric anomaly E from the mean anomaly:

$$E - e\sin E = M$$

while the true anomaly θ can be obtained from the relation:

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2},$$

Compute the flight path angle γ from:

$$\tan\gamma = \frac{e\sin\theta}{1 + e\cos\theta},$$

the norm of the radius vector from:

$$r = \frac{a(1 - e^2)}{1 + e\cos\theta}$$

and the velocity norm from:

$$v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}},$$

The Cartesian coordinates of the position vector $\mathbf{r}$ and velocity vector $\mathbf{v}$ can then be computed from:

$$x = r[\cos(\theta + \omega)\cos\Omega - \sin(\theta + \omega)\cos i \sin\Omega]$$

$$y = r[\cos(\theta + \omega)\sin\Omega + \sin(\theta + \omega)\cos i \cos\Omega]$$

$$z = r[\sin(\theta + \omega)\sin i]$$

$$v_x = v[-\sin(\theta + \omega - \gamma)\cos\Omega - \cos(\theta + \omega - \gamma)\cos i \sin\Omega]$$

$$v_y = v[-\sin(\theta + \omega - \gamma)\sin\Omega + \cos(\theta + \omega - \gamma)\cos i \cos\Omega]$$

$$v_z = v[\cos(\theta + \omega - \gamma)\sin i]$$

Table 2: Examples of orbital ephemerides computations.

deb id	epoch [MJD2000]	r_x [m]	r_y [m]	r_z [m]	v_x [m/s]	v_y [m/s]	v_z [m/s]
49	2.037664379998013E+04	-1.821352972868430E+06	-1.480732939140827E+06	-6.813437431330045E+06	-6.803794342724509E+03	-1.924704779749390E+03	2.232710487827681E+03
86	2.199002307265206E+04	1.976247700188033E+06	3.232774258285246E+06	6.120781250304952E+06	-1.052232676283728E+03	-6.354127728958516E+03	3.690861969295572E+03
14	2.195267652894209E+04	1.718590403710658E+05	-4.144115141529548E+06	5.838923225019871E+06	-1.887136274878838E+03	5.890478027257130E+03	4.190443985359899E+03
7	2.186496327845370E+04	-5.516476511489800E+06	5.874310457847400E+05	4.337773040831194E+06	4.684523520294676E+03	8.060302918788046E+02	5.834196543724089E+03
56	2.266025970506062E+04	-1.783147791131331E+06	-2.827567104459726E+06	-6.425865272777785E+06	-5.327033561348323E+03	-3.949243767248435E+03	3.194272376650931E+03
53	2.162774385353694E+04	6.969828836489021E+06	9.967694620525107E+05	-9.934041327231396E+05	-9.970982664859371E+02	-1.311523920363018E+03	-7.309143805556092E+03
45	2.075002578586824E+04	9.683039395430545E+05	5.454986834489745E+05	-7.157078786344362E+06	9.180515758407775E+02	-7.327194860016563E+03	-3.877069266554420E+02
105	2.240571554950649E+04	1.406101196822654E+06	1.957060209228951E+06	-6.747917188775276E+06	6.482967731474705E+03	2.971686486025229E+03	2.178560743681855E+03
110	2.204879081888735E+04	-1.164290866823851E+05	-3.260594455643641E+06	6.177419383216485E+06	-2.422530172750027E+03	-6.342292678430323E+03	-3.370159035391017E+03
114	2.212222338012047E+04	-2.540950669240206E+05	-5.424597422103481E+06	4.660428439709237E+06	-1.742672624675980E+03	-4.734614554753444E+03	-5.468201641543479E+03
104	2.142582120299275E+04	-4.361815803376273E+06	2.673368640265360E+06	4.968079858133554E+06	5.094901980261741E+03	-1.427094281574711E+03	5.286993980698330E+03
121	2.245262052279784E+04	8.971517792624621E+04	-5.158652926257161E+05	6.958556246975000E+06	-2.857128617119371E+03	-6.968000022658087E+03	-1.473200781880025E+02
100	2.281075926435661E+04	2.837147924629459E+06	-2.601387174719586E+06	5.930869832873454E+06	-5.895132297797102E+03	2.573953319032953E+03	3.967716941221940E+03
51	2.292431022228073E+04	-3.788883041488286E+06	4.048518886715053E+05	6.196028081660959E+06	6.274869983853391E+03	1.138641736036124E+03	3.758741442360347E+03
99	2.064574332350334E+04	2.839746199577784E+06	-3.277914826603945E+06	5.778500047742808E+06	-5.099978696882559E+03	3.178836327918069E+03	4.300424278860319E+03
46	2.025781477921072E+04	7.866968522497224E+04	-1.555753397501009E+06	-7.004258321334701E+06	-5.301152323845247E+03	-5.126079366174669E+03	1.073616524716189E+03
3	2.024287233722016E+04	-5.472681686690684E+06	-4.139106809222321E+06	1.961081581374216E+06	1.076967540328029E+03	2.012793690480879E+03	7.102105383958025E+03
18	2.002236619138996E+04	2.324628347302265E+06	-6.765537587345250E+06	6.034269361999463E+05	-1.245421093061683E+03	2.148774222156119E+02	7.309723756639946E+03
23	2.191887346558434E+04	2.292567004733084E+06	-3.063010436774577E+06	-5.990289424053151E+06	1.996885395951417E+03	-6.168960895932770E+03	3.797838089297656E+03
15	2.117393059381893E+04	-3.106487724551213E+06	-4.357239584639464E+06	-5.660147066676341E+06	-2.286707400074488E+03	-5.559070508923006E+03	4.458081839848156E+03
29	2.158292675678325E+04	-3.983834294960220E+06	2.688655777318943E+06	6.702786507840586E+06	2.061267268341338E+03	6.662134390134589E+03	-2.513499095024149E+03
60	2.246602224516183E+04	6.416361116560075E+06	3.468093000664969E+06	3.202817458341109E+06	-3.272876510537006E+03	-1.318938540293548E+03	6.567514403347020E+03
112	2.209368427825353E+04	-2.252164484239048E+06	5.967064376827653E+06	3.191867764663101E+06	-2.046322497526261E+03	3.507135770744188E+03	-6.596555536557287E+03
16	2.210841547047867E+04	-6.081373481028727E+06	3.484614496490478E+06	-9.042117897870549E+05	-3.374397673339674E+02	1.451481920411059E+03	7.372127421412792E+03
66	2.132571721867566E+04	8.202363849169365E+06	6.545147466770171E+05	7.033405834515233E+05	6.230430316486590E+03	-4.192609840169661E+03	-3.242984386892650E+02
53	2.214651393980229E+04	-1.153873842607186E+06	-1.222781973810500E+06	6.850073570278545E+06	-7.365025471928830E+03	-7.441023230147379E+02	-1.499418804440361E+03
17	2.005933463388659E+04	2.245687915754477E+06	6.389266433844509E+06	2.276042058320934E+06	1.848232351414249E+03	1.898453104546111E+03	-6.998763730684710E+03
97	2.152334680478337E+04	-5.314949592636377E+06	4.582243359013861E+06	9.335748696642752E+05	1.392588472300622E+03	1.075815843838875E+03	7.378928368068175E+03
35	2.018749869543062E+04	5.842866334279581E+04	-3.726032382394898E+06	-1.916860122790610E+06	-2.192232138454113E+03	3.404993575208846E+03	-7.103142753157433E+03
73	2.241432288459700E+04	1.639827774880750E+06	-8.653289928730364E+05	6.926384685530734E+06	-7.194005203175222E+03	-8.336586357858549E+02	1.609333824461351E+03

Table 3: Example of a numerical integration of the spacecraft equations of motion. The state is computed every six hours.

epoch [MJD2000]	r_x [m]	r_y [m]	r_z [m]	v_x [m/s]	v_y [m/s]	v_z [m/s]
2.3567000000000000E+04	-9.0656779992979474E+04	-4.8397431127596954E+06	-5.0408120071376814E+02	-7.6805804020022015E+02	5.4710987127502662E+03	-5.1022193482389539E+03
2.3567250000000000E+04	1.0920601198177545E+06	-1.9719312043068873E+06	6.6852892646843726E+02	-4.0833841182019057E+02	-7.2083121930906815E+03	-2.0579927050199744E+03
2.3567500000000000E+04	-2.5351548984495568E+05	6.8352157692217119E+06	-1.7021213695013500E+06	1.2010894720186029E+03	1.8437526270760009E+03	7.1938182245087291E+03
2.3567750000000000E+04	-8.1623675555577948E+06	-4.9613406263258317E+06	-4.9369182195369545E+06	-8.6929599260473924E+02	5.3467906956042934E+03	-5.2168278978595226E+03
2.3568000000000000E+04	1.1274400939171189E+06	-1.8090272463331309E+06	6.7253204383930536E+06	-2.7406087103360500E+02	-7.2656466733704958E+03	-1.9047360134927003E+03
2.3568250000000000E+04	-3.7998608548162162E+05	6.7918240787807805E+06	-1.8464586607563380E+06	1.1646945577493977E+03	2.0177712316921316E+03	7.1527926969203372E+03
2.3568500000000000E+04	-7.2211356737964950E+06	-5.0789988339745114E+06	-4.8307543769701021E+06	-9.6652335690875759E+02	5.2182297923030628E+03	-5.3290426157189049E+03
2.3568750000000000E+04	1.1576255544683798E+06	-1.6447120269365069E+06	6.7622408178596273E+06	-1.3833360689783812E+02	-7.2990595088631771E+03	-1.7506091567220674E+03
2.3569000000000000E+04	-5.0498042397399090E+05	6.7430523928129701E+06	-1.9899504564349221E+06	1.1228082590823278E+03	2.1901257215015030E+03	7.1084607211957000E+03
2.3569250000000000E+04	-6.2433826007295109E+06	-5.1925782474316098E+06	-4.7223693379975623E+06	-1.0595912553244107E+03	5.0855685658895218E+03	-5.4388111247683701E+03
2.3569500000000000E+04	1.1825698911360730E+06	-1.4791815907045981E+06	6.7960334805398947E+06	-1.4225178065988426E+00	-7.3357367175292738E+03	-1.5956826075898455E+03
2.3569750000000000E+04	-6.2830798680215038E+05	6.6889592573358798E+06	-2.1325306229242897E+06	1.0754938271713874E+03	2.3606105672199778E+03	7.0608433885153981E+03
2.3570000000000000E+04	-5.2305050896485243E+06	-5.3019447009006450E+06	-4.6118129820265360E+06	-1.1483564390614936E+03	4.9489649882690092E+03	-5.5460821394031354E+03
2.3570250000000000E+04	1.2022336564404522E+06	-1.3126331936533761E+06	6.8266829504073849E+06	1.3652486106940441E+02	-7.3665651850870017E+03	-1.4400271715157978E+03
2.3570500000000000E+04	-7.4978505079004892E+05	6.6296101809435673E+06	-2.2741332309912886E+06	1.0228223590408763E+03	2.5290228122775306E+03	7.0099633411215573E+03
2.3570750000000000E+04	-4.1842984289148438E+05	-5.4066952932675566E+06	-4.4991361869061411E+06	-1.2326814429700061E+03	4.8085822335528637E+03	-5.6508054918370754E+03
2.3571000000000000E+04	1.2165846178750116E+06	-1.1452659404237082E+06	6.8541752026790688E+06	2.7527963182773267E+02	-7.3915071650392765E+03	-1.2837139565510861E+03
2.3571250000000000E+04	-8.6692122673635150E+05	6.5650775477192309E+06	-2.4146928527170066E+06	9.6487169865384976E+02	2.6951622810692349E+03	6.9558447587366227E+03
2.3571500000000000E+04	-3.1060636065291107E+05	-5.5075297393781990E+06	-4.3843908060504533E+06	-1.3124349806059329E+03	4.6645884641336452E+03	-5.7529321580860560E+03
2.3571750000000000E+04	1.2255978004072069E+06	-9.7727961979423999E+05	6.8784976732967310E+06	4.1463180945464859E+02	-7.4105361053653487E+03	-1.1268143434972103E+03
2.3572000000000000E+04	-9.8642013801807712E+05	6.4954405209128922E+06	-2.5541445390472459E+06	9.0172732793921455E+02	2.8588331850272032E+03	6.8985133445744050E+03
2.3572250000000000E+04	-1.9975052130408915E+05	-5.6035081729352772E+06	-4.2676296450699540E+06	-1.3874917115777894E+03	4.5171566097350496E+03	-5.8524142835584589E+03
2.3572500000000000E+04	1.2292555193111568E+06	-8.0887488885153818E+05	6.8996392605740670E+06	5.5437059869690177E+02	-7.4236331230733367E+03	-9.6939956007514560E+02
2.3572750000000000E+04	-1.1012242876995557E+06	6.4207849375020703E+06	-2.6924238502972671E+06	8.3348174764371770E+02	3.0193877103695939E+03	6.8379631044626860E+03
2.3573000000000000E+04	-8.6027928807004937E+04	-5.6947936628594799E+06	-4.1489064379318329E+06	-1.4577328273483336E+03	4.3664641397176110E+03	-5.9492050825231220E+03
2.3573250000000000E+04	1.2275474032923782E+06	-6.4025291618454247E+05	6.9175903316033296E+06	6.9428467182733573E+02	-7.4307870268748310E+03	-8.1154263069013518E+02
2.3573500000000000E+04	-1.2134477649446905E+06	6.3412031937878430E+06	-2.8294668636860792E+06	7.6023445031708798E+02	3.1779896263379746E+03	6.7743228610592013E+03
2.3573750000000000E+04	3.0391889429417894E+04	-5.7812811808310254E+06	-4.0282758264218E+06	-1.5230460087856643E+03	4.2126928929273774E+03	-6.0432594913463199E+03
2.3574000000000000E+04	1.2204704078796285E+06	-4.7161511446174904E+05	6.9323427258611269E+06	8.3416244697522700E+02	-7.4319943300747427E+03	-6.5331438722877203E+02
2.3574250000000000E+04	-1.3229174397221520E+06	6.2567941221932806E+06	-2.9652103167638117E+06	6.8209177927958910E+02	3.3331011941370343E+03	6.7075216775361214E+03
2.3574500000000000E+04	1.4933570856491048E+05	-5.8628719770361800E+06	-3.9057933165701596E+06	-1.5833256230806307E+03	4.0560285174125388E+03	-6.1345329357100973E+03
2.3574750000000000E+04	1.2080288190691883E+06	-3.0316287274664378E+05	6.9438897585350234E+06	9.7379236667631278E+02	-7.4272592534085643E+03	-4.9478739850110901E+02
2.3575000000000000E+04	-1.4294636957144595E+06	6.1676628594177263E+06	-3.0995941100560336E+06	5.9916678437606538E+02	3.4849900881131512E+03	6.6376259001628432E+03
2.3575250000000000E+04	2.7062683786851529E+05	-5.9394737115388345E+06	-3.7815152909266436E+06	-1.6384728705029850E+03	3.8966608642817810E+03	-6.2229826113910985E+03
2.3575500000000000E+04	1.1903242721984800E+06	-1.3509728886838377E+05	6.9522262232182948E+06	1.1129631763049404E+03	-7.4165937182734415E+03	-3.3603396069923389E+02
2.3575750000000000E+04	-1.5329209082240660E+06	6.0739207061384087E+06	-3.2325480633862871E+06	5.1157906415329342E+02	3.6334783047226779E+03	6.5646681103919018E+03
2.3576000000000000E+04	3.9406353671821644E+05	-6.0110005771390097E+06	-3.6554898456896358E+06	-1.6883959220939419E+03	3.7334783096045660E+03	-6.3085668789837246E+03
2.3576250000000000E+04	1.1671056112084414E+06	3.2381097767744042E+04	6.9573483893950231E+06	1.2514620204971299E+03	-7.4000173299258040E+03	-1.7712646344650310E+02
2.3576500000000000E+04	-1.6331276167697462E+06	5.9756849784499155E+06	-3.3640188313657460E+06	4.1945460105847394E+02	3.7783923996055505E+03	6.4868628121249773E+03
2.3576750000000000E+04	5.1952835499668121E+05	-6.0773734135455815E+06	-3.5278022829865497E+06	-1.7330100480792596E+03	3.5705917497814453E+03	-6.3912454122829995E+03