$\boxed{2}$ $B[\sigma) = (1-\sigma^2)I + 2[\tilde{\sigma}] + 2\sigma\sigma^T$

Let $A = (1-\sigma^2)I$, $B = 2[\tilde{\sigma}]$, $C = 2\sigma\sigma^T$

$B(\sigma)^T = (1-\sigma^2)I + 2[\tilde{\sigma}]^T + 2(\sigma\sigma^T)^T$

$B(\sigma)^T = (1-\sigma^2)I - 2[\tilde{\sigma}]^T + 2\sigma\sigma^T$

$\Downarrow$

$B(\sigma).B(\sigma)^T = (A+B+C)(A-B+C)$

$= A^2 - \overbrace{AB + AC + BA}^{\rightsquigarrow 0} - B^2 + \underbrace{BC}_{\rightsquigarrow 0}$

$+ CA - \underbrace{CB}_{\rightsquigarrow 0} + C^2$

$A^2 = (1-\sigma^2)^2 I$, $AB = BA \rightsquigarrow BA - AB = 0$

$AC = 2(1-\sigma^2)\sigma\sigma^T = CA$, $B^2 = 4[\tilde{\sigma}]^2$

$BC = CB = 0$ $([\tilde{\sigma}]\sigma\sigma^T \equiv \sigma \times \sigma\sigma^T = 0)$

$C^2 = (2\sigma\sigma^T)^2 = 4\sigma^2\sigma\sigma^T$

$\Downarrow$

$B(\sigma)B(\sigma)^T = A^2 - B^2 + C^2 + 2AC$

$= (1-\sigma^2)^2 I - 4[\tilde{\sigma}]^2 + 4\sigma^2\sigma\sigma^T + 4(1-\sigma^2)\sigma\sigma^T$

$[\tilde{\sigma}]^2 = \sigma\sigma^T - \sigma^2 I \Longrightarrow$

$$B(\sigma)\,B^T(\sigma) =$$

$$(1-\sigma^2)^2 I \underbrace{-4\sigma\sigma^T}_{(1)} + 4\sigma^2 I + \underbrace{4\sigma^2\sigma\sigma^T}_{(2)}$$

$$\underbrace{+4(1-\sigma^2)\sigma\sigma^T}_{(3)}$$

$$(1)+(2)+(3)=0$$

$$\Downarrow$$

$$B(\sigma)\,B^T(\sigma) = \left((1-\sigma^2)^2 + 4\sigma^2\right) I$$

$$\Downarrow$$

$$B^T(\sigma)\cdot\frac{1}{\left((1-\sigma^2)^2+4\sigma^2\right)} = B^{-1}(\sigma)$$