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$$B(\sigma) = (1-\sigma^2) I_{3\times3} + 2[\tilde{\sigma}] + 2\sigma\sigma^T$$

$$B(\sigma)^T = (1-\sigma^2) I_{3\times3} - 2[\tilde{\sigma}] + 2\sigma\sigma^T \quad \left([\tilde{\sigma}]^T = -[\tilde{\sigma}] \right)$$

Let $A = (1-\sigma^2) I_{3\times3}$

$B = 2[\tilde{\sigma}]$

$C = 2\sigma\sigma^T$

$\Downarrow$

$$B(\sigma).B(\sigma)^T = (A+B+C)(A-B+C)$$
$$= A^2 + \underline{BA} + CA - \underline{AB} - B^2 - \underline{\underline{CB}} + AC + \underline{\underline{BC}} + C^2$$

$A^2 = (1-\sigma^2)^2 I_{3\times3}$

$AB = BA = 2(1-\sigma^2)[\tilde{\sigma}]$

$AC = CA = 2(1-\sigma^2)\sigma\sigma^T$

$B^2 = 4[\tilde{\sigma}]^2$

$BC = 2[\tilde{\sigma}]\sigma\sigma^T = 0 \quad ([\tilde{\sigma}]\sigma\sigma^T \equiv \sigma \times \sigma\sigma^T = 0)$

$CB = 4\sigma\sigma^T[\tilde{\sigma}] = 4([\tilde{\sigma}]^T\sigma\sigma^T)^T = 0$

$C^2 = 4.\sigma^2\sigma\sigma^T$

$\Downarrow$

$$B(\sigma)B(\sigma)^T = A^2 - B^2 + C^2 + 2AC$$
$$= (1-\sigma^2)^2 I_{3\times3} - 4[\tilde{\sigma}]^2 + 4\sigma^2\sigma\sigma^T + 4(1-\sigma^2)\sigma\sigma^T$$

given $[\tilde{\sigma}]^2 = \sigma\sigma^T - \sigma^2 I_{3\times3}$

$$B(\sigma)B(\sigma)^T = (1-\sigma^2)^2 I_{3\times3} - 4(\sigma\sigma^T - \sigma^2 I_{3\times3})$$
$$+4\sigma^2\sigma\sigma^T + 4(1-\sigma^2)\sigma\sigma^T$$
$$= (1-2\sigma^2+\sigma^4) I_{3\times3} + 4\sigma^2 I_{3\times3}$$
$$= (1+2\sigma^2+\sigma^4) I_{3\times3}$$
$$B(\sigma)B(\sigma)^T = (1+\sigma^2)^2 I_{3\times3}$$
$$B(\sigma)\left(\frac{B(\sigma)^T}{(1+\sigma^2)^2}\right) = I_{3\times3}$$
$$B(\sigma)B^{-1}(\sigma) = I_{3\times3}$$
$$\Downarrow$$
$$\boxed{B^{-1}(\sigma) = \frac{B(\sigma)^T}{(1+\sigma^2)^2}}$$