LaTeX in VSCode Example

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Abstract

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1 Averages

Let $x_1, \ldots, x_n$ be numeric recorded data values.

Definition 1.1. Average (mean):

$$\overline{x} = \frac{x_1 + \cdots + x_n}{n} = \frac{\Sigma_{i=1}^{n} x_i}{n} = \frac{\text{SUM}([x])}{\text{COUNT}([x])} = \text{AVG}([x])$$

Note that

$$\overline{x} = \frac{1}{n}x_1 + \cdots + \frac{1}{n}x_n$$

A weighted average is an average which in which some of the data is given higher or lower priority. More rigorously, if we have weights $w_1, \ldots, w_n$ where each w_i is between 0 and 1, and $w_1 + \cdots + w_n = 1$, Then

Definition 1.2. Weighted Average:

$$\overline{x} = w_1 x_1 + \cdots + w_n x_n = \Sigma_{i=1}^{n} w_i x_i = \text{SUM}([w] \times [x])$$

Note that the mean can be viewed as a weighted average with each weight being $\frac{1}{n}$.
What weights should we choose?

1.1 Our favourite weighted average

An important use-case of weighted averages is the “correct” way to take averages of averages (or percentages). If we **have** two **averages already** $\overline{x}$ and $\overline{y}$, and the **number of records used to compute those averages**, n and m, as in

$$\overline{x} = \frac{x_1 + \cdots + x_n}{n}$$

$$\overline{y} = \frac{y_1 + \cdots + y_m}{m}$$

Then we can compute the average of both via the formula:

Example. Weighted Average of averages

$$\overline{z} = \frac{n\overline{x} + m\overline{y}}{n+m} = \frac{\text{SUM}([\text{Number of records}] \times [\text{Averages}])}{\text{SUM}([\text{Number of records}])}$$

Where $\overline{z}$ is the average for the row-level numeric data values for x and y combined (union).
Note that this is the same as

$$\overline{z} = \frac{n}{n+m}\overline{x} + \frac{m}{n+m}\overline{y}$$

Where you can notice that $\frac{n}{n+m} \times 100$ and $\frac{m}{n+m} \times 100$ are the percentages of the total number of records!

Why this formula? We know that

$$\frac{n\overline{x} + m\overline{y}}{n+m} = \frac{\cancel{n}\frac{x_1 + \cdots + x_n}{\cancel{n}} + \cancel{m}\frac{y_1 + \cdots + y_m}{\cancel{m}}}{n+m} = \frac{x_1 + \cdots + x_n + y_1 + \cdots + y_m}{n+m}$$

Which is exactly the formula for the average of the combined data field of x and y! Therefore, this is the correct way to take averages of averages.
This is **not** equal to

$$\text{naive average} = \frac{\overline{x} + \overline{y}}{2}$$

Note that similar logic can be applied for when you have more than two averages.

2 Quadratic Formula via Sympy Example

Start:

$$ax^2 + bx + c = 0$$

Start (simplified):

$$ax^2 + bx + c = 0$$

Divide by a:

$$\frac{ax^2 + bx + c}{a} = \frac{0}{a}$$

Divide by a (simplified):

$$\frac{ax^2 + bx + c}{a} = 0$$

Move constant to RHS:

$$-\frac{c}{a} + \frac{ax^2 + bx + c}{a} = -\frac{c}{a} + \frac{0}{a}$$

Move constant to RHS (simplified):

$$\frac{x\left(ax + b\right)}{a} = -\frac{c}{a}$$

Add $(\frac{b}{2a})^2$ to both sides:

$$\left(-\frac{c}{a} + \frac{ax^2 + bx + c}{a}\right) + \frac{b^2}{4a^2} = \left(-\frac{c}{a} + \frac{0}{a}\right) + \frac{b^2}{4a^2}$$

Add $(\frac{b}{2a})^2$ to both sides (simplified):

$$\frac{ax\left(ax + b\right) + \frac{b^2}{4}}{a^2} = \frac{-ac + \frac{b^2}{4}}{a^2}$$

Perfect square form (unsimplified RHS):

$$\left(x + \frac{b}{2a}\right)^2 = \left(-\frac{c}{a} + \frac{0}{a}\right) + \frac{b^2}{4a^2}$$

Perfect square form (unsimplified RHS) (simplified):

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-ac + \frac{b^2}{4}}{a^2}$$

Perfect square form (simplified RHS):

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-ac + \frac{b^2}{4}}{a^2}$$

Perfect square form (simplified RHS) (simplified):

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-ac + \frac{b^2}{4}}{a^2}$$

Square root (plus branch):

$$x + \frac{b}{2a} = \frac{\sqrt{-4ac + b^2}}{2a}$$

Square root (plus branch) (simplified):

$$x + \frac{b}{2a} = \frac{\sqrt{-4ac + b^2}}{2a}$$

Square root (minus branch):

$$x + \frac{b}{2a} = -\frac{\sqrt{-4ac + b^2}}{2a}$$

Square root (minus branch) (simplified):

$$x + \frac{b}{2a} = -\frac{\sqrt{-4ac + b^2}}{2a}$$

Solve for x (plus):

$$x = \frac{-b + \sqrt{-4ac + b^2}}{2a}$$

Solve for x (plus) (simplified):

$$x = \frac{-b + \sqrt{-4ac + b^2}}{2a}$$

Solve for x (minus):

$$x = \frac{-b - \sqrt{-4ac + b^2}}{2a}$$

Solve for x (minus) (simplified):

$$x = \frac{-b - \sqrt{-4ac + b^2}}{2a}$$

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$