

Lab Course Machine Learning

Exercise Sheet 4

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Exercise 0: Dataset preprocessing

```
In [21]: import numpy as np
import math
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from matplotlib import cm
from matplotlib.ticker import LinearLocator
plt.rcParams['figure.figsize'] = (10, 8)
from sympy import symbols, diff
import pandas as pd
import math
import warnings
warnings.filterwarnings('ignore')
```

```
In [22]: df = pd.read_csv('tic-tac-toe.data', sep = ', ', header=None, names=["top-left-square", "top-middle-square", "top-right-square", "middle-left-square", "middle-middle-square", "middle-right-square", "bottom-left-square", "bottom-middle-square", "bottom-right-square", "Class"])
valueMap = {'negative':0, 'positive':1, 'x':2, 'o':3, 'b':4}
df = df.replace(valueMap)
```

```
Out [22]:
```

| | top-left-square | top-middle-square | top-right-square | middle-left-square | middle-middle-square | middle-right-square | bottom-left-square | bottom-middle-square | bottom-right-square | Class |
|-----|-----------------|-------------------|------------------|--------------------|----------------------|---------------------|--------------------|----------------------|---------------------|-------|
| 0 | 2 | 2 | 2 | 2 | 3 | 3 | 2 | 3 | 3 | 1 |
| 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 2 | 3 | 1 |
| 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 2 | 1 |
| 3 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 4 | 4 | 1 |
| 4 | 2 | 2 | 2 | 2 | 3 | 3 | 4 | 3 | 4 | 1 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 953 | 3 | 2 | 2 | 2 | 3 | 3 | 3 | 2 | 2 | 0 |
| 954 | 3 | 2 | 3 | 2 | 2 | 3 | 2 | 3 | 2 | 0 |
| 955 | 3 | 2 | 3 | 2 | 3 | 2 | 2 | 3 | 2 | 0 |
| 956 | 3 | 2 | 3 | 3 | 2 | 2 | 2 | 3 | 2 | 0 |
| 957 | 3 | 3 | 2 | 2 | 2 | 3 | 3 | 2 | 2 | 0 |

958 rows × 10 columns

```
In [23]: (df['Class'].value_counts()) / len(df) * 100
```

```
Out [23]:
```

| | |
|---|-----------|
| 1 | 65.344468 |
| 0 | 34.655532 |

Name: Class, dtype: float64

This show that instances of positive class(represented by 1) are more in dataset than negative class(represented by 0). This is unbalanced dataset for which we need stratification technique to make the proportionality equal in dataset.

Now for stratification , I am taking fraction from positive class while taking the whole negative class to make this proportionality equal.

```
In [24]: fraction = ((df.loc[df['Class'] == 0]).shape[0]) / ((df.loc[df['Class'] == 1]).shape[0])
td1 = df.loc[df['Class'] == 1].sample(frac=fraction)
td2 = df.loc[df['Class'] == 0]
td = pd.concat([td1, td2]).sample(frac=1)
td.head()
```

```
Out [24]:
```

| | top-left-square | top-middle-square | top-right-square | middle-left-square | middle-middle-square | middle-right-square | bottom-left-square | bottom-middle-square | bottom-right-square | Class |
|-----|-----------------|-------------------|------------------|--------------------|----------------------|---------------------|--------------------|----------------------|---------------------|-------|
| 294 | 2 | 4 | 4 | 4 | 2 | 4 | 3 | 3 | 2 | 1 |
| 689 | 2 | 3 | 2 | 4 | 3 | 4 | 4 | 3 | 2 | 0 |
| 738 | 2 | 4 | 4 | 3 | 3 | 3 | 4 | 2 | 2 | 0 |
| 525 | 4 | 2 | 4 | 4 | 2 | 4 | 3 | 2 | 3 | 1 |
| 617 | 4 | 4 | 3 | 3 | 4 | 4 | 2 | 2 | 2 | 1 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 215 | 2 | 3 | 4 | 3 | 4 | 3 | 2 | 2 | 2 | 1 |
| 105 | 2 | 2 | 4 | 3 | 2 | 3 | 3 | 2 | 4 | 1 |
| 797 | 3 | 2 | 4 | 3 | 4 | 2 | 3 | 2 | 4 | 0 |
| 632 | 2 | 2 | 3 | 2 | 3 | 3 | 3 | 4 | 2 | 0 |
| 934 | 4 | 4 | 3 | 2 | 2 | 3 | 2 | 4 | 3 | 0 |

531 rows × 10 columns

```
In [25]: td_train = td.iloc[0:math.floor(len(td)*0.8)]
td_test = td.iloc[math.floor(len(td)*0.8)+1:]
```

```
In [26]: td_train
```

```
Out [26]:
```

| | top-left-square | top-middle-square | top-right-square | middle-left-square | middle-middle-square | middle-right-square | bottom-left-square | bottom-middle-square | bottom-right-square | Class |
|-----|-----------------|-------------------|------------------|--------------------|----------------------|---------------------|--------------------|----------------------|---------------------|-------|
| 294 | 2 | 4 | 4 | 4 | 2 | 4 | 3 | 3 | 2 | 1 |
| 689 | 2 | 3 | 2 | 4 | 3 | 4 | 4 | 3 | 2 | 0 |
| 738 | 2 | 4 | 4 | 3 | 3 | 3 | 4 | 2 | 2 | 0 |
| 525 | 4 | 2 | 4 | 4 | 2 | 4 | 3 | 2 | 3 | 1 |
| 617 | 4 | 4 | 3 | 3 | 4 | 4 | 2 | 2 | 2 | 1 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 215 | 2 | 3 | 4 | 3 | 4 | 3 | 2 | 2 | 2 | 1 |
| 105 | 2 | 2 | 4 | 3 | 2 | 3 | 3 | 2 | 4 | 1 |
| 797 | 3 | 2 | 4 | 3 | 4 | 2 | 3 | 2 | 4 | 0 |
| 632 | 2 | 2 | 3 | 2 | 3 | 3 | 3 | 4 | 2 | 0 |
| 934 | 4 | 4 | 3 | 2 | 2 | 3 | 2 | 4 | 3 | 0 |

531 rows × 10 columns

```
In [27]: td_test
```

```
Out [27]:
```

| | top-left-square | top-middle-square | top-right-square | middle-left-square | middle-middle-square | middle-right-square | bottom-left-square | bottom-middle-square | bottom-right-square | Class |
|-----|-----------------|-------------------|------------------|--------------------|----------------------|---------------------|--------------------|----------------------|---------------------|-------|
| 37 | 2 | 2 | 2 | 3 | 4 | 3 | 2 | 3 | 4 | 1 |
| 570 | 4 | 3 | 3 | 2 | 4 | 3 | 2 | 2 | 2 | 1 |
| 40 | 2 | 2 | 2 | 3 | 4 | 3 | 3 | 4 | 2 | 1 |
| 4 | 2 | 2 | 2 | 2 | 3 | 3 | 4 | 3 | 4 | 1 |
| 583 | 4 | 3 | 4 | 3 | 2 | 3 | 2 | 2 | 2 | 1 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| 138 | 2 | 3 | 2 | 4 | 2 | 3 | 3 | 4 | 2 | 1 |
| 620 | 4 | 4 | 4 | 2 | 2 | 2 | 3 | 3 | 4 | 1 |
| 199 | 2 | 3 | 4 | 2 | 3 | 4 | 2 | 2 | 3 | 1 |
| 682 | 2 | 3 | 2 | 3 | 3 | 3 | 4 | 2 | 2 | 0 |
| 948 | 2 | 3 | 2 | 3 | 2 | 2 | 3 | 2 | 3 | 0 |

132 rows × 10 columns

Exercise 1: Logistic Regression with Gradient Descent

```
In [56]: def sig(X,B):
return (1 / (1 + np.exp((-1)*(np.dot(X,B)))))

def logLike(X,Y,B):
return np.sum((Y*(sig(X,B))-np.log(1+np.exp(sig(X,B)))))
# for logloss https://ml-cheatsheet.readthedocs.io/en/latest/loss_functions.html
# ref given in exercise was not running
def logloss(yHat, y):
sum=0
for i in range(len(yHat)):
if y[i] == 1:
sum = sum - np.log(yHat[i] )
else:
sum = sum -np.log(1 - yHat[i] )
return sum

def GradAscent(X,Y,μ,e,xtest,ytest):
B_old = np.zeros(shape=(len(X[0]),1))
fold_min_fnew_List=[]
numberIterations = 10000
L= logLike(X,Y,B_old)
Log_loss = []
for i in range (numberIterations):
y_hat = sig(X,B_old)
y_test_hat = sig(xtest,B_old)
Log_loss.append(logloss(y_test_hat,ytest))
B_hat =B_old +(μ* np.dot(X.T,(Y-y_hat)))
fold_min_fnew_List.append(abs(logLike(X,Y,B_old)-logLike(X,Y,B_hat)))
L_old = L
#print(L_old)
L = logLike(X,Y,B_hat)
B_old = B_hat
μ = μ*μ_minus
i +=1
return B_hat,fold_min_fnew_List,μ,Log_loss

# if(L - L_old<e):
# return B_hat,fold_min_fnew_List,Log_loss
return ('Not Converge'),[]

def stepLengthBolddriver(X,Y,μ_old,μ_plus,μ_minus,xtest,ytest):
B_old = np.zeros(shape=(len(X[0]),1))
μ = μ_old*μ_plus
y_hat = sig(X,B_old)
B_hat =B_old +(μ* np.dot(X.T,(Y-y_hat)))
fold_min_fnew_List.append(abs(logLike(X,Y,B_old)-logLike(X,Y,B_hat)))
L_old = L
L = logLike(X,Y,B_hat)
B_old = B_hat
μ = μ*μ_minus
i +=1
return B_hat,fold_min_fnew_List,μ,Log_loss

In [48]: X_train = td_train.loc[:, td_train.columns != 'Class'] # taking all columns except Class columns for X
Y_train = td_train[['Class']] # taking Class column for y

X = X_train.to_numpy() # converting to numpy matrix
Y = Y_train.to_numpy() # converting to numpy matrix
bias_column = np.ones(shape=(len(X_train),1))
X = np.append(bias_column,X,axis=1) #adding bias columns to X
μ=10**-9

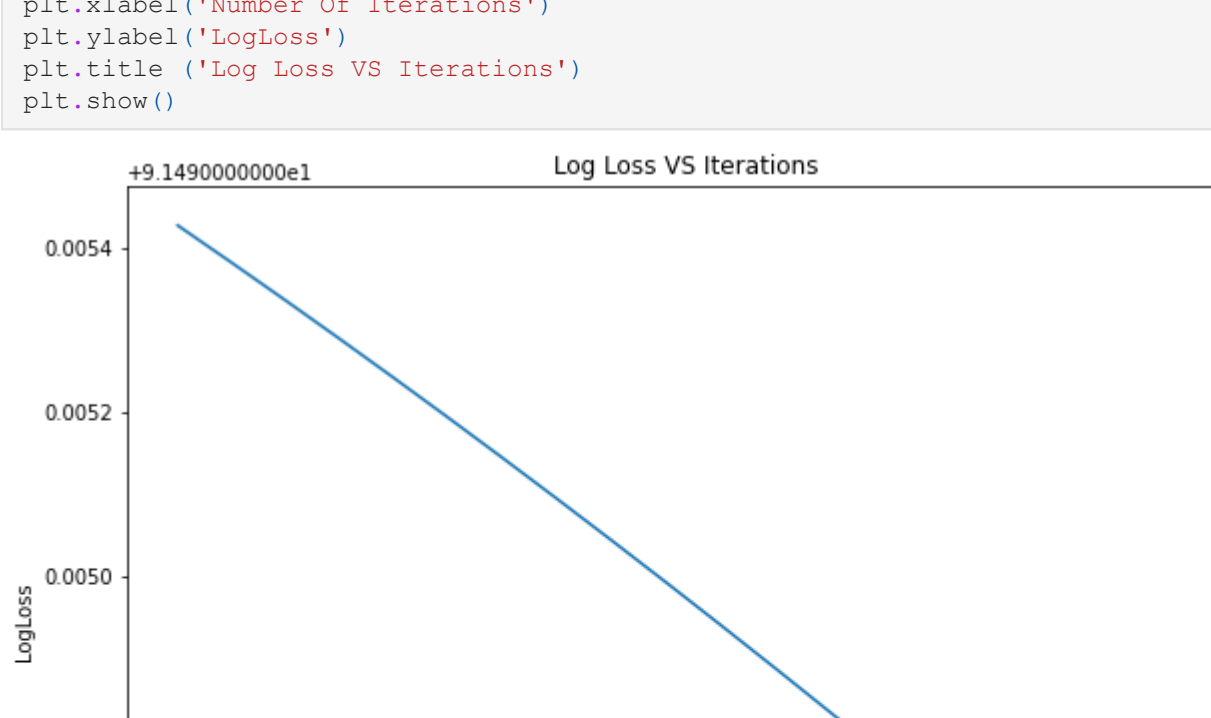
print(X.shape)
print(Y.shape)
X_test = (td_test.loc[:, td_test.columns != 'Class']).to_numpy() # taking all columns except Class columns for
bias_column = np.ones(shape=(len(X_test),1))
X_test = np.append(bias_column,X_test,axis=1)
Y_test = (td_test[['Class']]).to_numpy() # taking Class column for y

(531, 10)
(531, 1)
```

```
In [49]: B_hat,fold_min_fnew_List,Log_loss=GradAscent(X,Y,μ,10**(-2),X_test,Y_test)
fold_min_fnew_List
B_hat
```

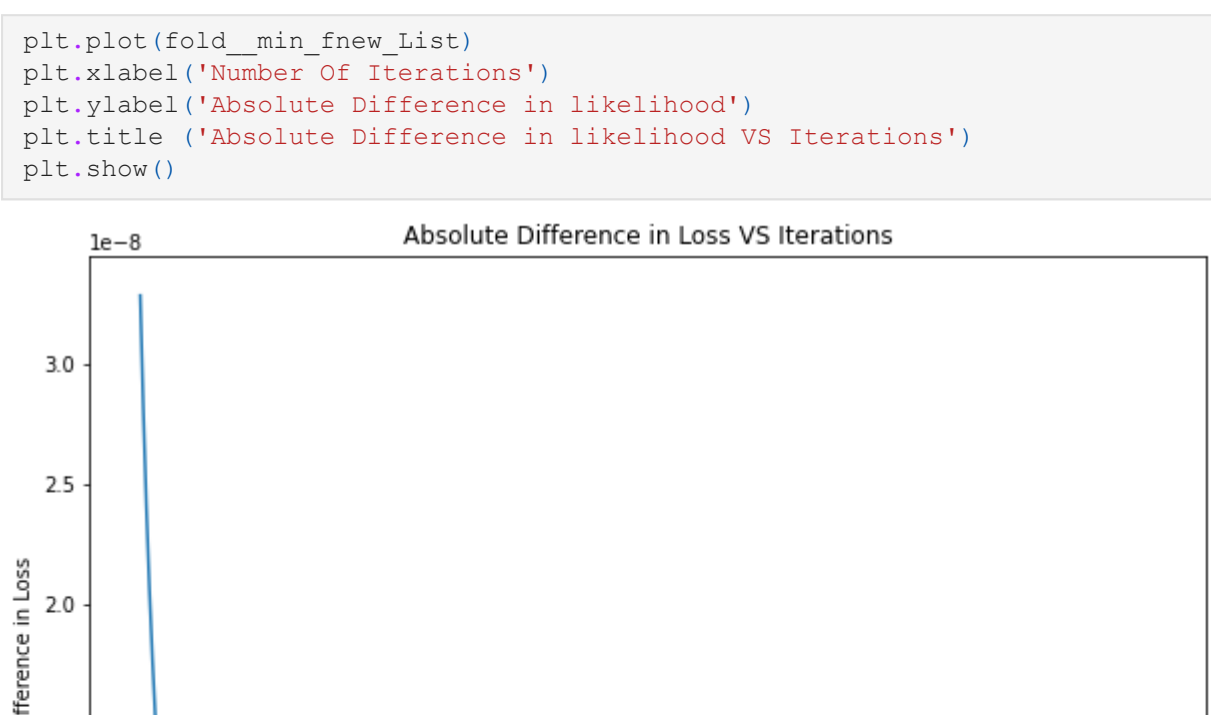
```
Out [49]: array([[ -2.40388278e-05],
[ -2.17260959e-04],
[ 1.32667784e-04],
[ -2.07224681e-04],
[ 1.32626318e-04],
[ -4.02232159e-04],
[ 1.32643677e-04],
[ -1.472861239e-04],
[ 9.76789278e-05],
[ -3.72728350e-05]])
```

```
In [53]: fold_min_fnew_List
plt.plot(Fold_min_fnew_List)
plt.xlabel('Number Of Iterations')
plt.ylabel('Absolute Difference in likelihood')
plt.title ('Absolute Difference in Loss VS Iterations')
plt.show()
```



Graph shows with number of iterations, difference of likelihood decreases.

```
In [54]: plt.plot(Log_loss)
plt.xlabel('Number Of Iterations')
plt.ylabel('LogLoss')
plt.title ('Log Loss VS Iterations')
plt.show()
```



LogLoss decreases with increase in iterations

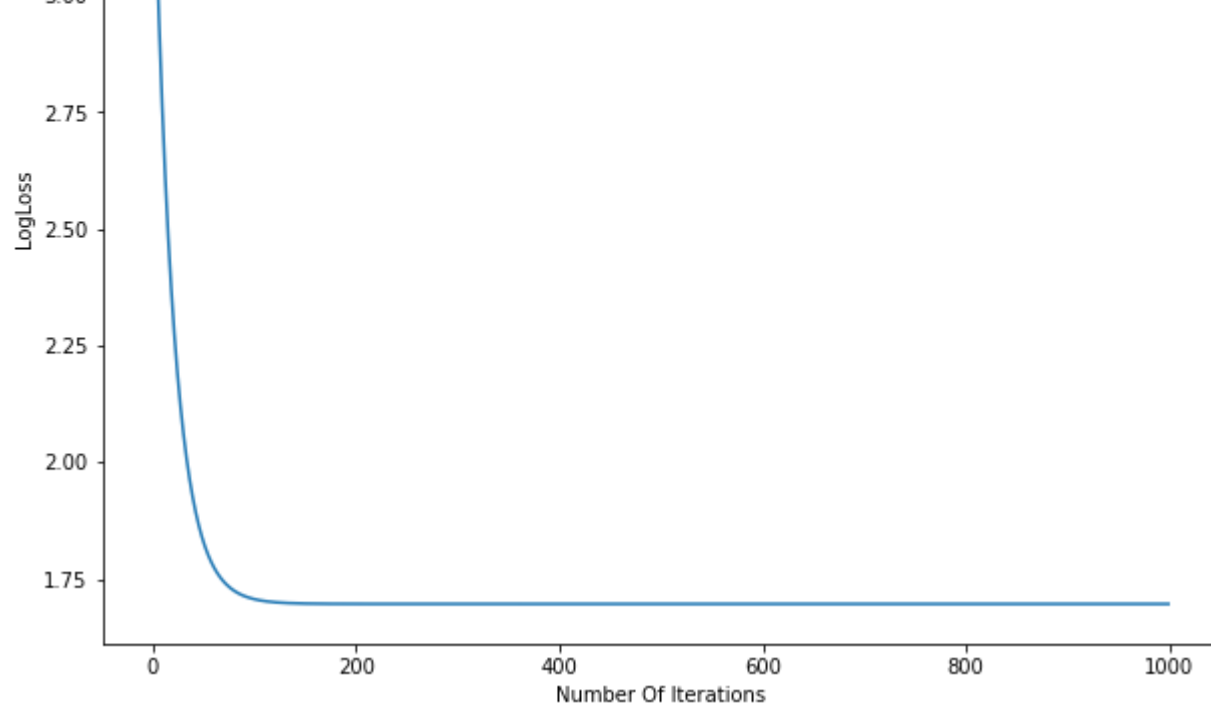
bolddriver

```
In [88]: μ=10**-11
B_hat,fold_min_fnew_List,μ,Log_loss=stepLengthBolddriver(X,Y,μ,1,0.95,X_test,Y_test)
```

```
In [89]: B_hat
```

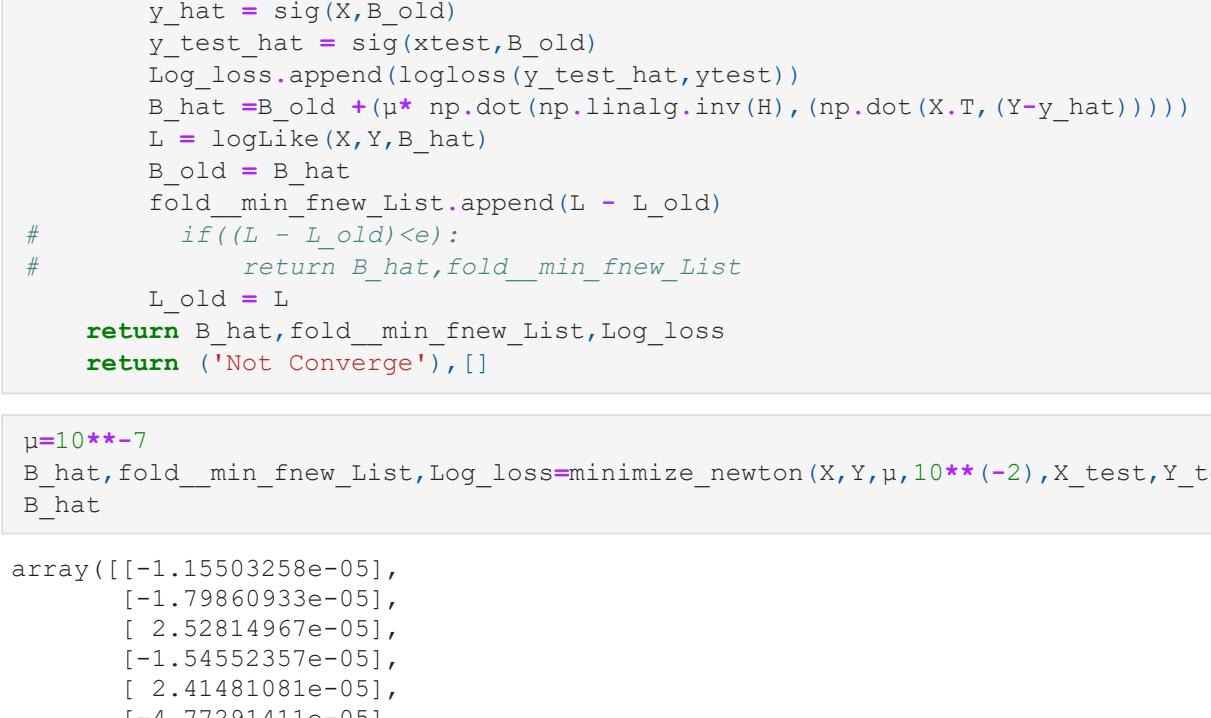
```
Out [89]: array([[ -4.99999613e-10],
[ -4.399998930e-09],
[ 2.60000107e-09],
[ -4.19999888e-09],
[ 2.60000106e-09],
[ -8.09999889e-09],
[ 2.60000106e-09],
[ -2.99999890e-09],
[ 1.90000108e-09],
[ -7.99998903e-10]])
```

```
In [90]: plt.plot(Fold_min_fnew_List)
plt.xlabel('Number Of Iterations')
plt.ylabel('Absolute Difference in likelihood')
plt.title ('Absolute Difference in likelihood VS Iterations')
plt.show()
```



With μ calculated with bolddriver, the absolute difference btw likelihood decreases quickly and after 150 iterations converges to zero.

```
In [91]: fold_min_fnew_List
plt.plot(Log_loss)
plt.xlabel('Number Of Iterations')
plt.ylabel('LogLoss')
plt.title ('Log Loss VS Iterations')
plt.show()
```



With μ calculated with bolddriver, the logloss decreases quickly and after 150 iterations converges to zero.

Exercise 2: Implement Newton Algorithm for Logistic Regression

```
In [104]: def Hessian (X,B):
ones = np.ones(shape=(len(X),1))
f = ones-sig(X,B)
W= np.diag(np.multiply(f,ones*f)).reshape(-1)
H=np.dot(X.T,np.dot(W, X), )

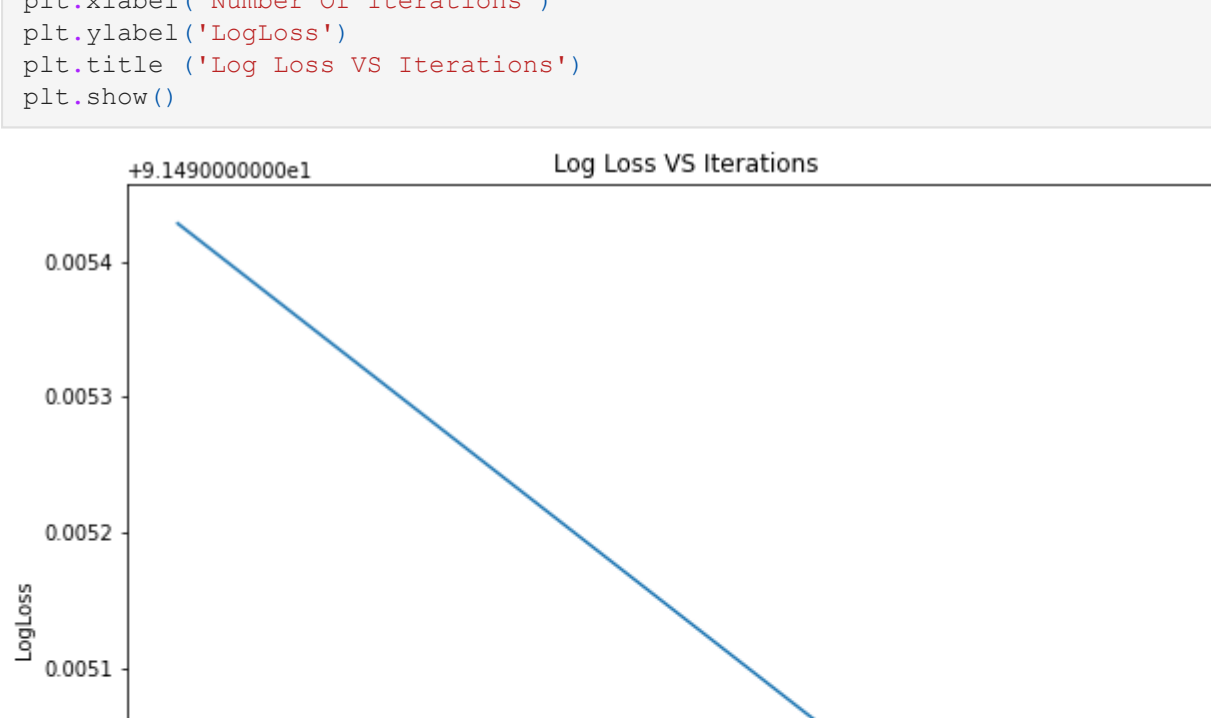
return H

def minimize_newton(X,Y,μ,e,xtest,ytest):
B_old = np.zeros(shape=(len(X[0]),1))
fold_min_fnew_List=[]
H=Hessian(X,B_old)
L_old = logLike(X,Y,B_old)
Log_loss=[]
for i in range (numberIterations):
y_hat = sig(X,B_old)
y_test_hat = sig(xtest,B_old)
Log_loss.append(logloss(y_test_hat,ytest))
B_hat =B_old +(μ* np.dot(np.linalg.inv(H), (np.dot(X.T,(Y-y_hat)))))
L = logLike(X,Y,B_hat)
B_old = B_hat
fold_min_fnew_List.append(L - L_old)
# if((L - L_old)<e):
# return B_hat,fold_min_fnew_List
L_old = L
return B_hat,fold_min_fnew_List,Log_loss
return ('Not Converge'),[]

In [105]: μ=10**-7
B_hat,fold_min_fnew_List,Log_loss=minimize_newton(X,Y,μ,10**(-2),X_test,Y_test)
B_hat
```

```
Out [105]: array([[ -1.15503258e-05],
[ -1.79860933e-05],
[ 2.52614978e-05],
[ -1.54552357e-05],
[ 2.41481081e-05],
[ -4.77291411e-05],
[ 2.03925760e-05],
[ -8.7752559e-06],
[ 1.82521246e-05],
[ 3.20211246e-06]])
```

```
In [106]: plt.plot(Fold_min_fnew_List)
plt.xlabel('Number Of Iterations')
plt.ylabel('Absolute Difference in likelihood')
plt.title ('Absolute Difference in likelihood VS Iterations')
plt.show()
```



```
In [107]: plt.plot(Log_loss)
plt.xlabel('Number Of Iterations')
plt.ylabel('LogLoss')
plt.title ('Log Loss VS Iterations')
plt.show()
```


logloss is decreases with number of iterations, it means it prediction is more accurate after each iteration.