Poster title:

Parallel Quantum Annealing: A Novel Approach to Solving Multiple NP-Hard Problems Concurrently

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Poster abstract:

This study introduces an innovative application of Parallel Quantum Annealing (PQA) to concurrently solve multiple NP-hard problems on a quantum annealer, leveraging unused qubits on D-Wave's Pegasus architecture to enhance efficiency and accuracy. This application extends the traditional PQA method, which typically solves a single problem multiple times, and applies it to different problems simultaneously. This study primarily focuses on three NP-hard problems relevant to integrated circuit design: the Graph Coloring Problem (GCP), the Minimum Vertex Cover Problem (MVCP), and the Graph Partitioning Problem (GPP). After formulating these problems as Quadratic Unconstrained Binary Optimization (QUBO), they were embedded into the D-Wave quantum annealing machine, and the annealing process was run on the combined QUBO. The results were subsequently decoded into individual problems. Our results, quantified via the Time-To-Target (TTT) metric, showcased enhanced efficiency and accuracy in reaching target solutions, indicating a substantial improvement over traditional Quantum Annealing (QA) and Simulated Annealing (SA) methods. The research illustrates the potential of quantum computing for solving complex real-world problems more efficiently and emphasizes the importance of leveraging all available qubits in quantum annealers. The findings can significantly contribute to the development and optimization of quantum computing methods and have broad application implications, particularly in areas where NP-hard problems are prevalent.

Poster relevance:

This research is of high relevance to the fields of quantum computing and quantum engineering. It presents a novel application of PQA to solve multiple NP-hard problems simultaneously, which could have significant implications for the optimization of quantum computing methods and applications in various fields where NP-hard problems are prevalent.

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***Abstract*—This study introduces an innovative application of Parallel Quantum Annealing (PQA) to concurrently solve multiple NP-hard problems on a quantum annealer, leveraging unused qubits on D-Wave's Pegasus architecture to enhance efficiency and accuracy. This application extends the traditional PQA method, which typically solves a single problem multiple times, and applies it to different problems simultaneously. This study primarily focuses on three NP-hard problems relevant to integrated circuit design: the Graph Coloring Problem (GCP), the Minimum Vertex Cover Problem (MVCP), and the Graph Partitioning Problem (GPP). After formulating these problems as Quadratic Unconstrained Binary Optimization (QUBO), they were embedded into the D-Wave quantum annealing machine, and the annealing process was run on the combined QUBO. The results were subsequently decoded into individual problems. Our results, quantified via the Time-To-Target (TTT) metric, showcased enhanced efficiency and accuracy in reaching target solutions, indicating a substantial improvement over traditional Quantum Annealing (QA) and Simulated Annealing (SA) methods. The research illustrates the potential of quantum computing for solving complex real-world problems more efficiently and emphasizes the importance of leveraging all available qubits in quantum annealers. The findings can significantly contribute to the development and optimization of quantum computing methods and have broad application implications, particularly in areas where NP-hard problems are prevalent.**

Keywords*—*Quantum annealing, PQA, Combinatorial problems for real-world application

I. Introduction

Quantum annealing, a computational method leveraging quantum mechanics, offers an effective approach for solving NP-hard optimization problems. Nonetheless, optimal utilization of available qubits, particularly when handling multiple problems concurrently, remains a challenge [1, 4, 5]. Traditionally, PQA has been used to solve a single problem multiple times [1]. This study pioneers a novel application of PQA to concurrently solve multiple distinct NP-hard problems. We focus on three significant NP-hard problems relevant to integrated circuit design: the GCP, the MVCP, and the GPP [2]. Our innovative approach to PQA potentially advances the field of quantum computing by enabling simultaneous problem-solving, thus enhancing efficiency and solution accuracy.

II. Methodology

A. Problem Selection and Description

The first step was to transform these problems into a form that could be solved on a quantum annealer. This involved formulating each problem as a QUBO problem. The process of transforming these problems into QUBO form involves representing the problem variables as binary variables and the problem constraints and objective function as a quadratic function.

GCP: The QUBO formulation for the GCP is designed to ensure that each node in the graph is assigned a unique color and that no two adjacent nodes share the same color [6-8].

MVCP: The QUBO formulation for the MVCP problem is designed to minimize the sum of the binary variables associated with the nodes in the vertex cover [8].

GPP: The QUBO formulation for the GPP problem is designed to partition the graph into two sets of nodes of equal size, with the number of edges between the sets minimized [8].

B. Parallel Quantum Annealing (PQA) Approach

Our methodology utilizes PQA for simultaneous resolution of multiple NP-hard problems by efficiently exploiting the D-Wave's Pegasus architecture. Initially, the problems are transformed into QUBO problems and subsequently embedded into the D-Wave Advantage 6.2 quantum annealing machine using the "minorminer" [10] method. To ensure distinct problem allocation, each problem is assigned a unique set of qubits on the machine until full utilization.

Following the embedding, a combined QUBO is formed that consists of individual QUBOs of each problem in their embedding order. The uniqueness of variables in the combined QUBO is maintained by initializing the variables of the succeeding problem from n+1, where n is the number of embedded variables in the preceding QUBO problem. The distinct variables guarantee problem independence, thereby enabling representation on non-overlapping subgraphs of the Pegasus architecture.

The quantum annealing process is then executed on the combined QUBO, enhancing qubit utilization and solution accuracy. After annealing, the results are decoded into their respective problems for further evaluation.

III. Results

A. Overview

Our research demonstrates a notable increase in Time-To-Target (TTT) [1, 3] performance when compared to traditional QA and SA. Our innovative application of PQA consistently outperformed QA and was either comparable to or superior to SA in most cases. Notably, the PQA method was used to calculate the solutions of the three problems concurrently, while the QA and SA solutions were calculated one by one.

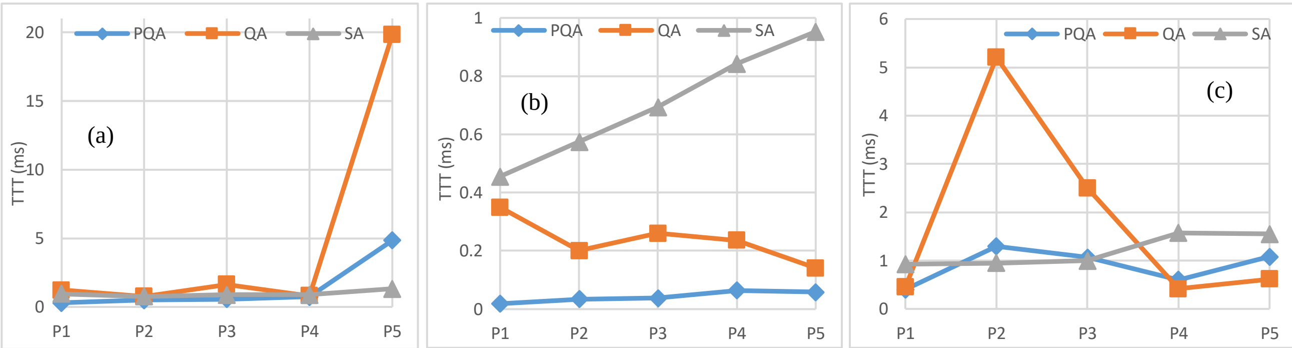

Fig. 1. Evolution of Time-To-Target (TTT) varying the problem sets we mentioned in TABLE I. (a) Relation between TTT and GCP problem sets. (b) Relation between TTT and MVC problem sets. (c) Relation between TTT and GP problem sets.

B. Problem Settings and Classical Algorithms

To evaluate the performance of our PQA method, we established five distinct problem sets, each with varying numbers of nodes and a graph density of 0.4. Classical algorithms acted as benchmarks for each problem in the sets. For the GCP, we employed the Welsh Powell algorithm [6], whereas the MVCP employed the Greedy algorithm [8], and the GPP utilized the Kernighan Lin Bisection algorithm [9]. These algorithms provided optimal solutions to compare with the quantum solutions.

TABLE I. THE PROBLEM SETS. (NODES IN THE PROBLEMS.)

Problems	**GCP**	**MVCP**	**GPP**
P1	10	20	20
P2	10	25	25
P3	12	30	30
P4	12	35	35
P5	14	40	40

C. Time-To-Target (TTT) Measure

The Time-To-Target (TTT) measure is a critical performance metric for quantum annealers, assessing the time needed to reach a solution that's 99% optimal solution. This measure is important given the probabilistic nature of quantum annealing, which doesn't guarantee the best solution at every instance. From Eq (1), one can see the $TTTpqa$,

$$TTTpqa = \frac{1}{A}\left(\frac{T_{\mathrm{QPU}} + T_{\mathrm{CPU}}}{K}\right)\frac{\log(0.01)}{\log(1 - p_K)} \quad . \qquad (1)$$

Here, A is the number of samples per annealing run, T_{QPU} is the time of quantum processing unit spends on the problem, and T_{CPU} is the classical computer's time spent on processing QUBOs and decoding results, K represents concurrently solved problems, and p_K is the average probability of obtaining a solution 99% as good as the classical solution for all concurrent problems. $TTTpqa$ is indicative of PQA's efficiency in simultaneous problem solving, enabling performance comparison between PQA and other methods. The results, as shown in Figs. 1 (a)-(c), PQA consistently outperforms both QA and SA in terms of TTT, with a noticeable edge in MVCP across all problem sets.

IV. DISCUSSION AND FUTURE WORK

This study represents a nascent exploration of PQA's potential for simultaneously addressing distinct NP-hard problems, serving as a preliminary but not an exhaustive examination. The demonstrated enhancement in TTT performance over traditional QA and SA methods holds promise. However, broader generalization to all NP-hard problems warrants additional rigorous investigation.

Future research endeavors should focus on evaluating PQA's versatility across diverse problem domains and assessing its adaptability under a multitude of problem configurations. This approach could help unveil further aspects of PQA's capability to address complex, real-world problems, thereby optimizing quantum computing resources' usage.

In summary, while this investigation embarks on an intriguing journey into uncharted territories of PQA applications, a more profound comprehension of its inherent potential remains dependent on continued research.

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Parallel Quantum Annealing: A Novel Approach to Solving Multiple NP-Hard Problems Concurrently

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1. Introduction: Solving Combinatorial Optimization Problems

♦ Combinatorial Optimization for Real-World Problem

✓ Combinatorial optimization techniques can be applied to IC layout to address various optimization problems that arise during the design process.


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- Dead Space Minimization
- Wirelength Minimization
- Floor Planning Optimization
- Routing Optimization


S. Boixo et al., Nature Phys. 10 (2014) 218.

D-Wave Systems:

- Flux Quantum Bits
- Ultra Low Temperature
- More Than 5000 Qubits

Real-World ⇨ Mathematical Modeling ⇨ Quantum Annealer ⇨ Real-World

♦ Three graph problems that are relevant to IC design

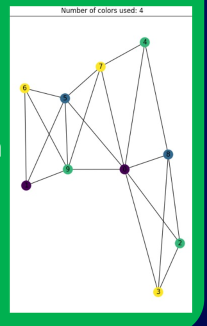
✓ Using combinatorial optimization algorithms, IC layout problems can be efficiently solved, exploring large solution spaces for near-optimal or optimal solutions.

Graph Coloring Problem (GCP):

Given an undirected graph G = (V, E), the objective of the Graph Coloring Problem is to assign colors to each vertex in V in such a way that no two adjacent vertices share the same color. The challenge is to color the graph using the smallest number of different colors possible.

Applications:

- Register allocation
- Frequency assignment in wireless communication
- Task scheduling in parallel processing

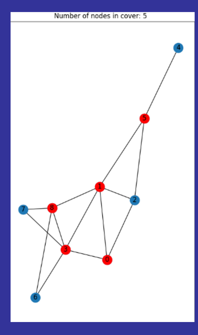


Minimum Vertex Covering Problem (MVCP):

Given an undirected graph G = (V, E), a vertex cover V' is a subset of V such that for all edges (u, v) ∈ E, at least one of the endpoints u or v is in V'. The goal is to find the smallest such subset V', which is called the minimum vertex cover.

Applications:

- Network design
- Optimizing resource allocation
- Processor allocation in parallel computing

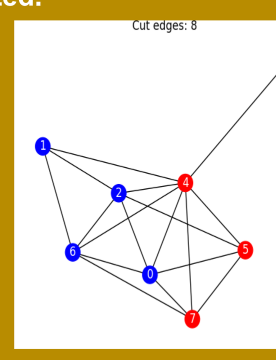


Graph Partitioning Problem (GPP):

Given an undirected graph G = (V, E) and an integer k, the Graph Partitioning Problem aims to divide the vertex set V into k equal (or as equal as possible) size partitions such that the number of edges crossing between partitions (the cut size) is minimized.

Applications:

- Load balancing in parallel computing
- Image and data segmentation



2. Solving Multiple NP-Hard Problems Simultaneously Using D-Wave Quantum Annealer

Quantum Annealing

Quantum Annealing, inspired by metallurgical annealing, is a computational technique with promising capabilities in solving optimization problems.
The D-Wave Advantage is providing a more extensive and denser programmable quantum mode comprising over 5000 qubits. This advanced hardware is purposefully designed to harness the potential of quantum annealing for solving complex computational challenges, particularly those that can be formulated as Quadratic Unconstrained Binary Optimization (QUBO) problems or Ising models.

Objective function that Quantum Annealer can solve:

Hamiltonian:

$$H = \sum_{i} h_i \sigma_i - \sum_{<i,j>} J_{ij} \sigma_i \sigma_j$$

J: Exchange Interaction Coefficient

h: External Magnetic Field

$\sigma \in \{\pm 1\}$: Spin State

QUBO (Quadratic Unconstrained Binary Optimization):

$$Q = \sum_{i} d_i x_j + \sum_{<i,j>} d_{ij} x_i x_j$$

d: User-Specified Weights

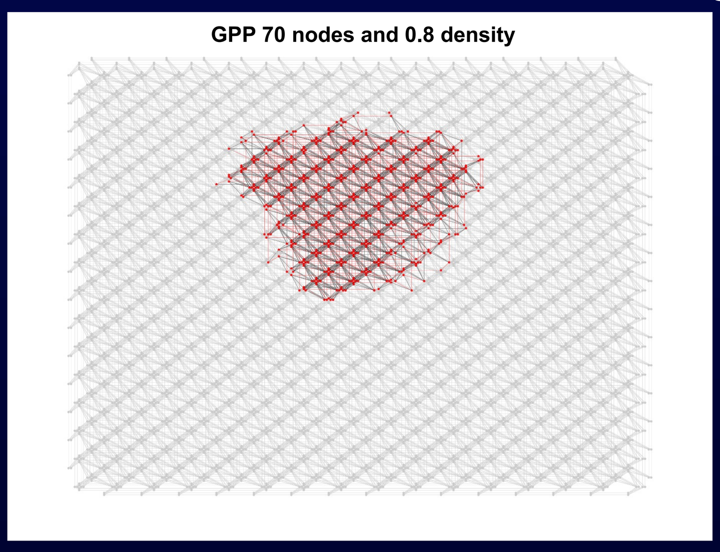
$x \in \{0, 1\}$: Binary Variable

Relation between σ and x: $x = \frac{1-\sigma}{2}$

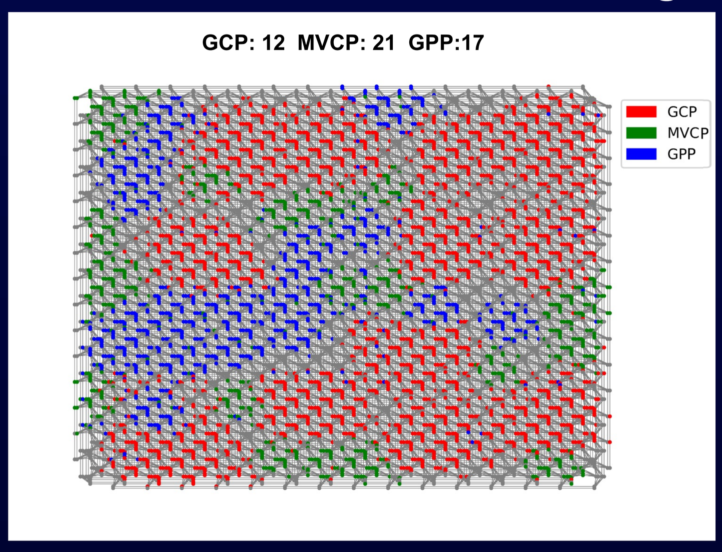
Enhancement: We have the potential to utilize all available qubits on the D-Wave's Quantum Annealer. This approach allows for concurrent problem-solving, leading to more efficient utilization of resources and an increase in overall computational efficiency.

Parallel Quantum Annealing

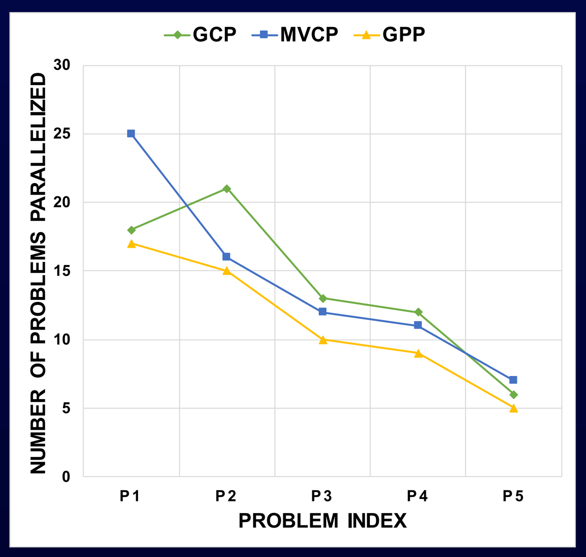
➢ Conventional Quantum Annealing



➢ Parallel Quantum Annealing



➢ Number of Problems within Same Quantum Annealer



QUBO for each problem:

$$Q_{GCP} = \sum_{i=1}^{n}\sum_{k=1}^{K} x_{ik} + P \sum_{(i,j)\in E}\sum_{k=1}^{K}(x_{ik}x_{jk} - x_{ik} - x_{jk} + 1)$$

$$Q_{MVCP} = \sum_{i=1}^{n} x_i + P \sum_{(i,j)\in E}(x_i x_j - x_i - x_j + 1)$$

$$Q_{GPP} = \sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij} x_i x_j$$

Combining

Combined QUBO

$$Q_{combined} = Q_{GCP} + Q_{MVCP} + Q_{GPP} + \cdots$$

Decoding

- Solution for GCP
- Solution for MVCP
- Solution for GPP

The processes of combining and decoding are performed on the CPU for computation.

3. Experimental Results

♦ Evaluation Metrics in Our Research: Time-To-Target (TTT)

TTT: Performance measure for evaluating QA. This metric estimates the time required to locate a solution that is at least 99% as good as the optimal classical solution, accounting for the probabilistic nature of quantum annealing, which does not guarantee finding the absolute best solution every run.
The probabilities of finding an optimal solution is recorded for each problem. These probabilities are averaged to derive the value of p_K, as follows:

$$p_K = \frac{1}{K}\sum_{i=1}^{K} p_i$$

p_i : The probability of finding a solution that's at least 99% as good as the classical solution in a single run.

K: The number of problems that are being solved concurrently.

We use this average success probability to calculate the TTT for solving all the problems concurrently (TTTens):

$$TTTens = \frac{1}{A}\left(\frac{T_{QPU} + T_{CPU}}{K}\right)\frac{\log(0.01)}{\log(1-p_K)}$$

where A is the number of anneals, T_{QPU} is the quantum processing unit's processing time, and T_{CPU} is the time the classical computer spends on combining QUBOs and decoding the results.

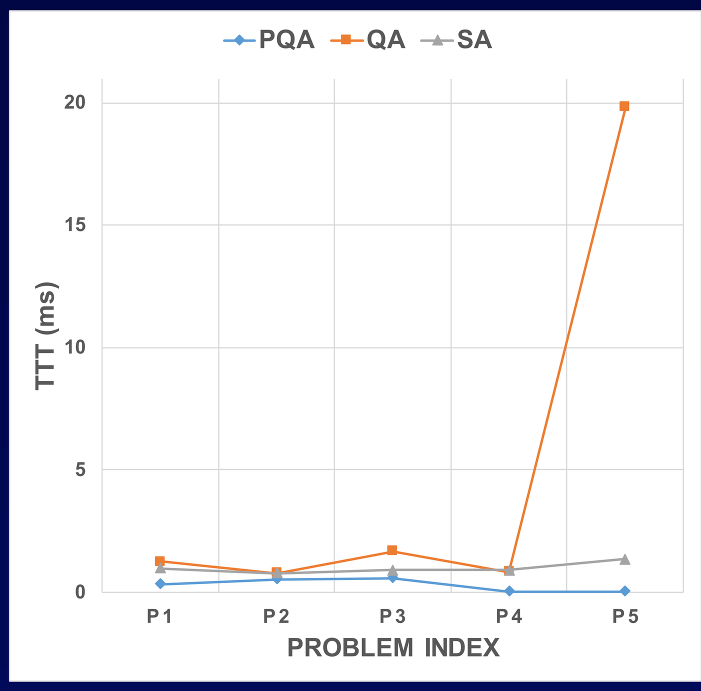
This metric demonstrates the efficiency of our approach and demonstrates the potential of PQA in tackling complex, real-world problems.

Test problem sets: Nodes in The Problems

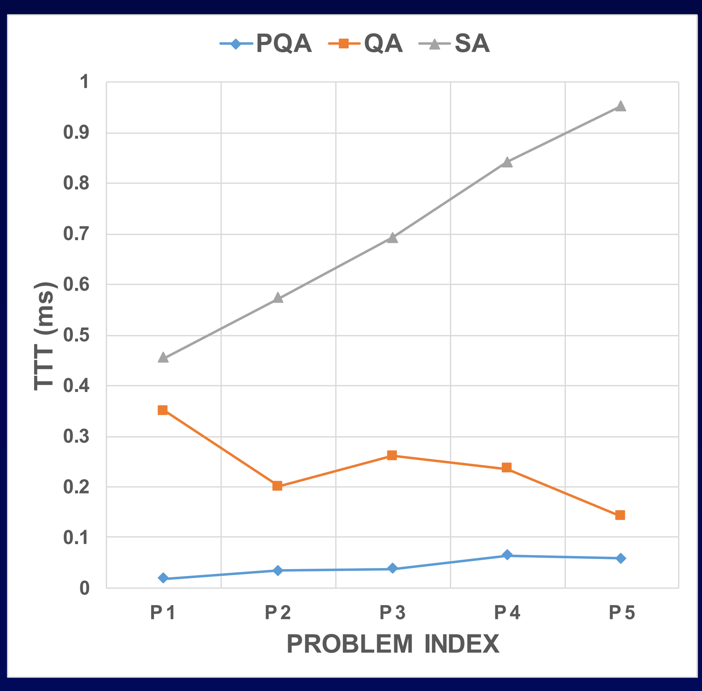
Prob.	GCP	MVCP	GPP
P1	10	20	20
P2	10	25	25
P3	12	30	30
P4	12	35	35
P5	14	40	40

♦ Analysis: Evolution of TTT Varying Test Problem Sets (P1-P5)

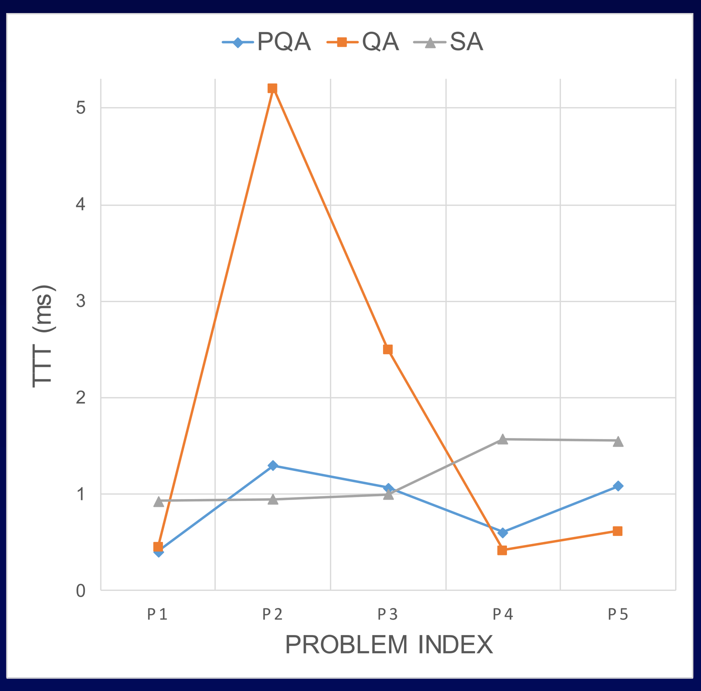
Evolution of TTT for GCP:
Number of colors used are minimized.



Evolution of TTT for MVCP:
Number of covered vertexes are minimized.



Evolution of TTT for GPP:
Number of cut edges are minimized.



✓ Our research demonstrated that Parallel Quantum Annealing (PQA) successfully solved multiple NP-hard problems concurrently, showing a significant increase in Time-To-Target (TTT) performance compared to traditional Quantum Annealing (QA) and Simulated Annealing (SA) methods.

✓ We observed a consistent outperformance of PQA over QA, and results were either comparable or superior to SA, specifically noting that the solutions for the three distinct problems were computed simultaneously in PQA while sequentially in QA and SA.

✓ Importantly, our novel approach of leveraging unused qubits on D-Wave's Pegasus architecture showed that quantum resources can be utilized more efficiently, offering a potential step-change in solving complex real-world problems with quantum computing.

4. Conclusions

➢ This preliminary study offers a promising but exploratory application of Parallel Quantum Annealing (PQA) to concurrently solve multiple NP-hard problems.

➢ Our work strongly suggests a potential use of leveraging qubits, which could lead to enhanced efficiency in problem-solving through quantum annealing.

➢ However, these results are preliminary, and the approach is novel, necessitating further rigorous exploration to verify its general applicability.