AR501
Tutorial- 1

Q1. Three identical SRS open-chain arms are grasping a common object, as shown in Figure below.
(a) Find the number of degrees of freedom of this system.
(b) Suppose there are now a total of n such arms grasping the object. How many degrees of freedom does this system have?
(c) Suppose the spherical wrist joint in each of the n arms is now replaced by a universal joint. How many degrees of freedom does this system have?

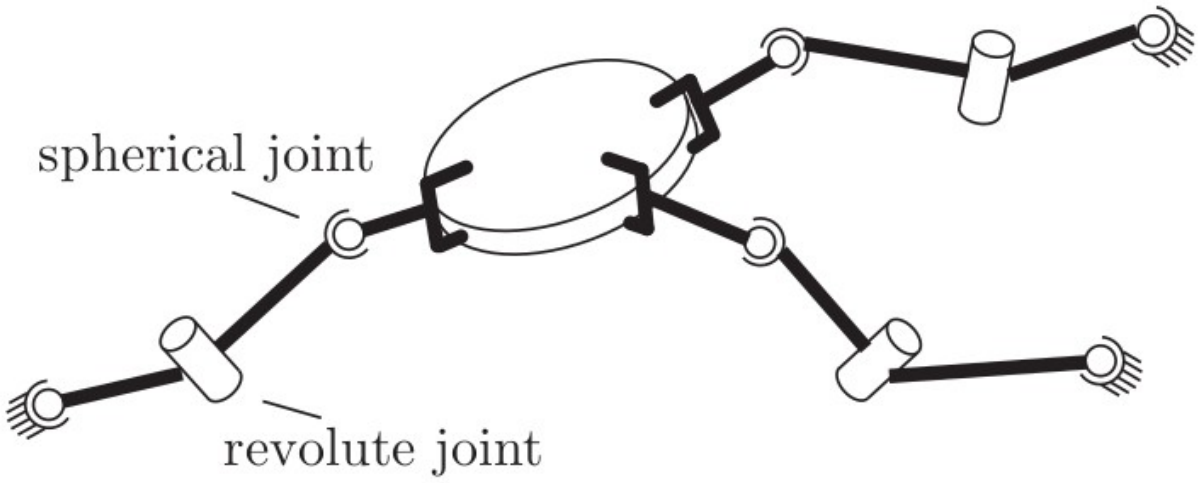


Figure. 1 Three cooperating SRS arms grasping a common object.

Q2. The dual-arm robot of is rigidly grasping a box. The box can only slide on the table; the bottom face of the box must always be in contact with the table. How many degrees of freedom does this system have?

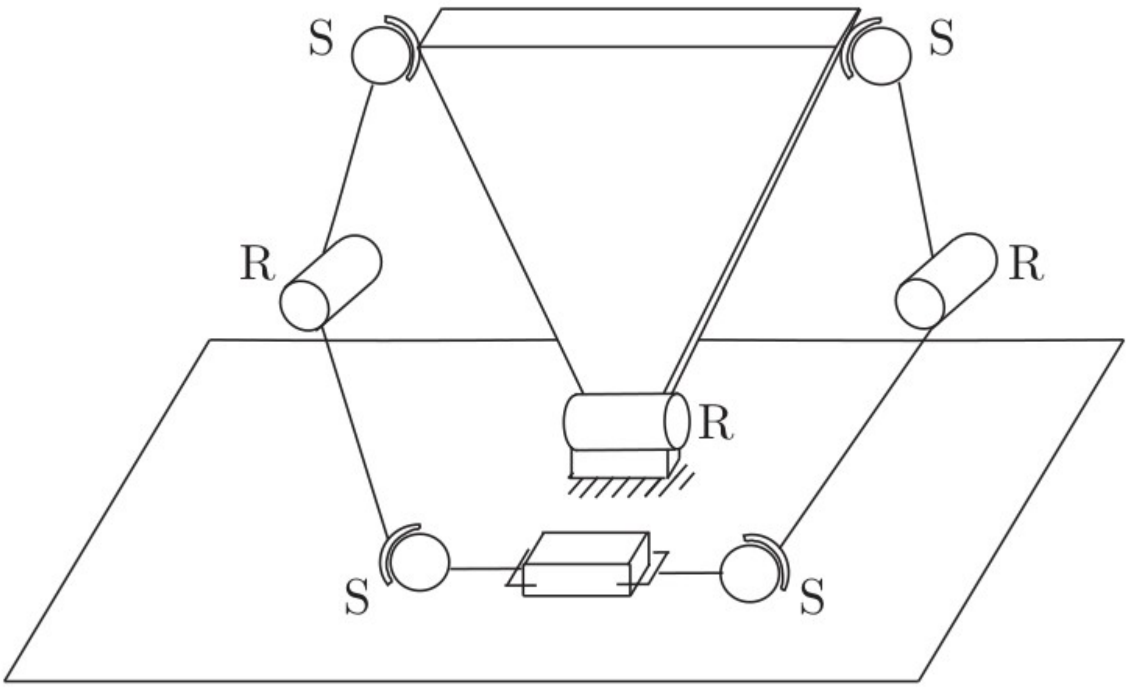


Fig. 2 Dual-Arm Robot

Q3. Determine whether the following differential constraint are holonomic or nonholonomic ?

$$(1+\cos q_1)\dot{q}_1 + (1+\cos q_2)\dot{q}_2 + (\cos q_1 + \cos q_2 + 4)\dot{q}_3 = 0.$$

table; the bottom face of the box must always be in contact with the table. How many degrees of freedom does this system have?

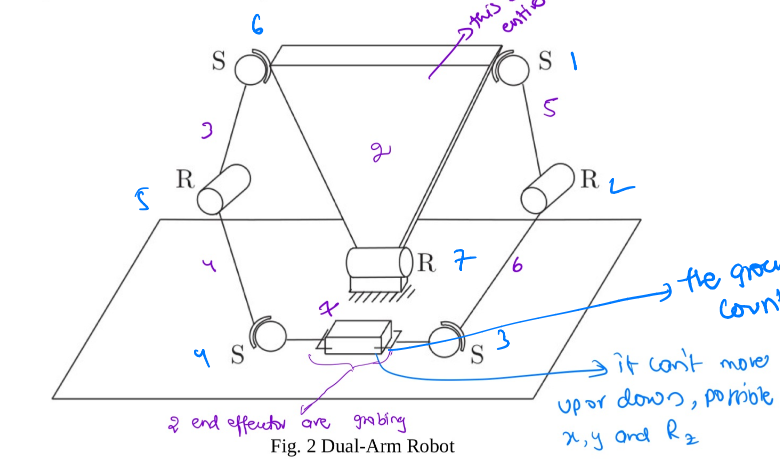


Fig. 2 Dual-Arm Robot

→ the ground is also counted as a joint

→ it can't move up or down, possible x, y and R_z

$N = 7$

$J = 8$ (joint in b/w surface and box)

→ 3 R

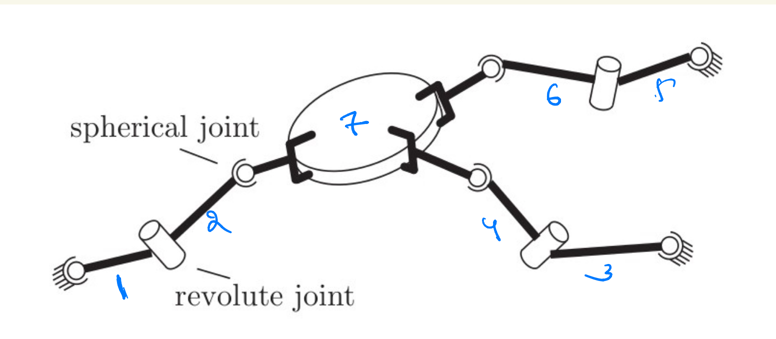
→ 4 S

→ 1 surface joint (DOF = 3)

$$6(7-1-8) + 3(1) + 4(3) + 3$$

$$2(-6) + 3 + 12 + 3$$

$$6$$



$N = 7 + 1$

$J = 6S$

$3R$

$m = 6$

$$6(8-1-9) + 6(3) + 3(1)$$

$$6(-2) + 18 + 3$$

$$-12 + 21 \Rightarrow 9$$

for generalisation → for N links = $N + 1 + \overset{\text{ground}}{1} + N = 2N + 2$

$J = 2N_s + N_R$

$\therefore$ DOF $= 6(2N + 2 - 1 - 2N_S - N_R) + (2N_S)3 + N_R$

$= 6(2N + 1 - 3N) + 6N + N$

$= 6(-n + 1) + 6N + N$

$= \cancel{-6n} + 6 + \cancel{6N} + N$

$= n + 6$

Spherical is replaced by universal.

Dof $= 6(-n+1) + 2(N_U) + (N_R)1 + (N_S)3$

$= -6n + 6 + 2N + N + 3N$

$-6N + 6 + 6N = 6$

$$(1 + \cos q_1)\dot{q}_1 + (1 + \cos q_2)\dot{q}_2 + (\cos q_1 + \cos q_2 + 4)\dot{q}_3 = 0.$$

make it into $A(\theta)\dot{\theta} = 0$ we need to find holonomic or not

$A(\theta) \in R^{k \times m}$

$(1 + \cos q_1)\dot{q}_1 + (1 + \cos q_2)\dot{q}_2 + (\cos q_1 + \cos q_2 + 4)\dot{q}_3 = 0$

$A(q)\dot{v} = 0$

$\Rightarrow$

$$\left[(1+\cos q_1)\ \ (1+\cos q_2)\ \ (\cos q_1+\cos q_2+4)b\right]\begin{bmatrix}\dot q_1\\ \dot q_2\\ \dot q_3\\ q_4\end{bmatrix} \Rightarrow$$

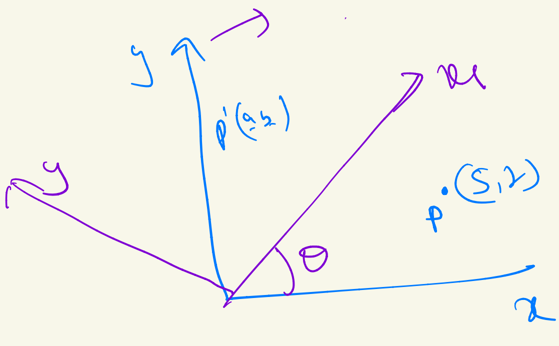
fram of refence

types

1. fixed from of refence (spatial)
2. Rotatory frame
3. Rotation + translation

} → Body frames

Rotation matrix



$$R^{x_1y_1z_1}_{XYZ} = \begin{bmatrix}\cos\theta & -\sin\theta & 0\\ +\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

↳ New to old

we want old to new

now find $p'(a,b) =$ → we need to track it

$$\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}5\\ 2\\ 0\end{bmatrix}$$

$$p' = R^T [p^3]$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 5\cos\theta & -2\sin\theta & 0 \\ 5\sin\theta & 2\cos\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_{x,y,z}^{XYZ} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

i.e $x,y,z \rightarrow XYZ$

new $\rightarrow$ fixed

$$T_{new}^{old}$$

Convention for ~~Rot~~ Translation

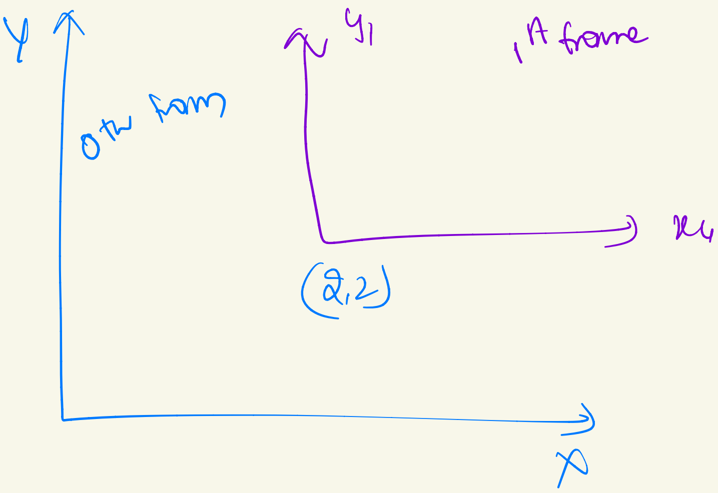
Suppose

$$R_{XYZ}^{x,y,z} = \begin{bmatrix} \cos\theta & +\sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

i.e $XYZ \rightarrow x,y,z,$

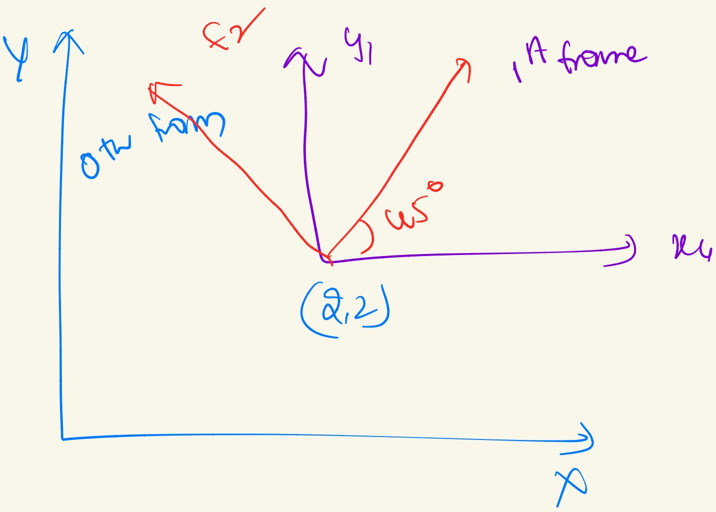
fixed $\rightarrow$ new

homogenous transformation

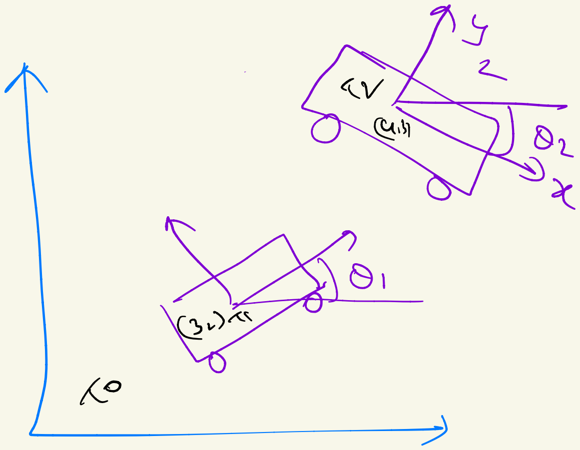


find T_0^1 $\quad 1 \rightarrow 0$

$$T_0^1 = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix} = T_0^1 = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$T_{0\,(\text{new})}^{2\ \text{old}} = \begin{bmatrix} \cos 45 & -\sin 45 & 0 & 2 \\ +\sin 45 & \cos 45 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$\theta_1 = 45^\circ$

$\theta_2 = -45^\circ$

$x_2 y_2$ (old)

$T_{x_1 y_1}$ (new)

$x_2 y_2$ (old)

$$T_{x_1 y_1 \text{(new)}} = T_1^2 = \begin{bmatrix} \cos -45 & \sin -45 & 0 & 2 \\ -\sin(-45) & \cos 45 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

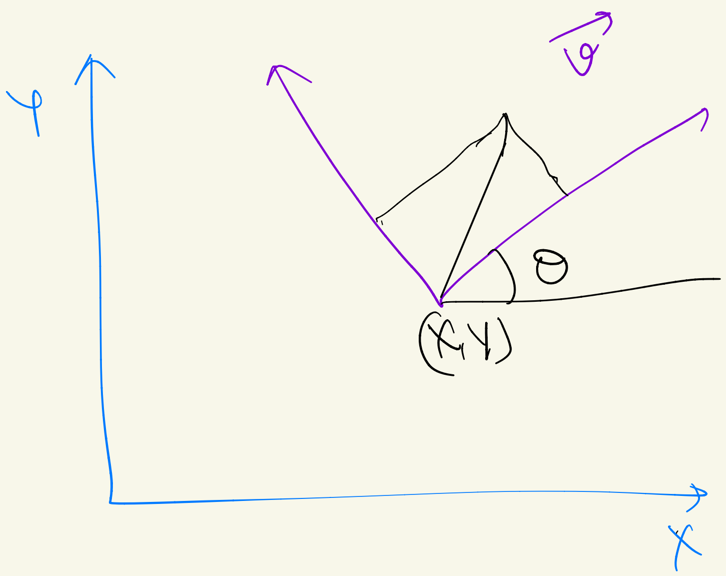
multiple frames.

$$T_0^2 = T_0^1 \, T_1^2$$

$$=$$

let $T = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix}$

$T^{-1} = \begin{bmatrix} R^T & PR^T \\ 0 & 1 \end{bmatrix}$

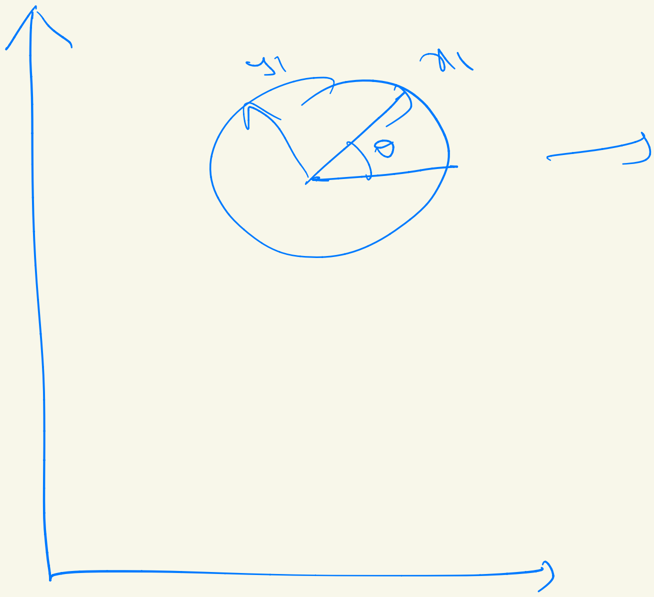


$\vec{v} \begin{bmatrix} v_1 \\ v_2 \\ 0 \end{bmatrix}$

T_{old}^{New} we need = we need $\vec{V}_{xy}$

New → old

$$\vec{V}_{xy} = \left[R_{xy\ old}^{x_1 y_1\ new} \right] \left[\vec{v} \right]$$



$\vec{v}$

$$v = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

disk is moving in 2m/s towards x

velocity of disk in its body frame.

→ it has both x, y in body frame.

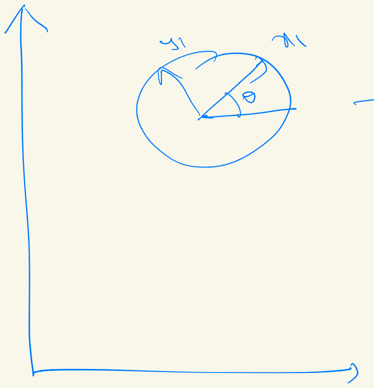
let $\vec{v}$ velocity of disk in New frame

let $\vec{v}$ velocity of disk in old frame

$\theta = 30°$

$$\vec{v} = \left[\overset{\text{old to new}}{R} \right] \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

and
$$R = \begin{bmatrix} \cos 30 & \sin 30 & 0 \\ -\sin 30 & \cos 30 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$\longrightarrow \vec{v}$

$$v = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} \cos \omega t & -\sin \omega t & 0 \\ +\sin \omega t & \cos \omega t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

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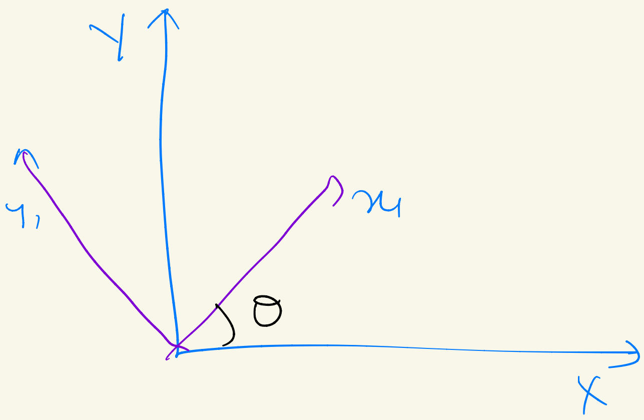
exponential co-ordinates

- Rotation
- general motion
  $\rightarrow R + T$



for exponential rotation we want

$\hat{\omega}$ = // angular momentum

θ

we need to normalize becau

$\boxed{\theta = t}$

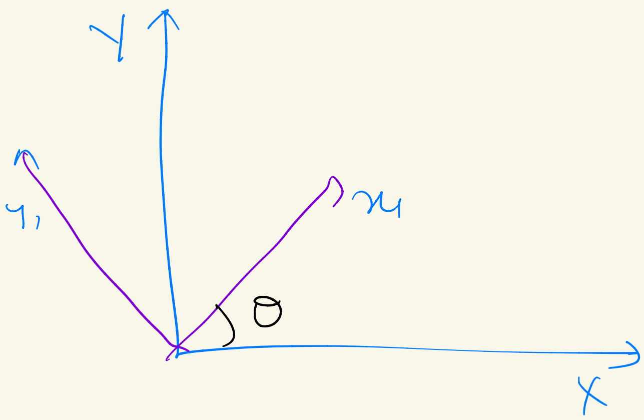
$\therefore \; \omega = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\theta = \omega t$

we want $\theta = t$

we have $\theta = \omega t$

$\therefore$ $\boxed{\omega \text{ should be} = 1}$



find exponential co-ordinate

$e^{[\omega]t} = \hat{\omega}\,\theta$

$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \theta$

to get Rotation matrix

$R = e^{[\omega]\theta}$

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

let $\theta = \pi/4$

$\omega = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

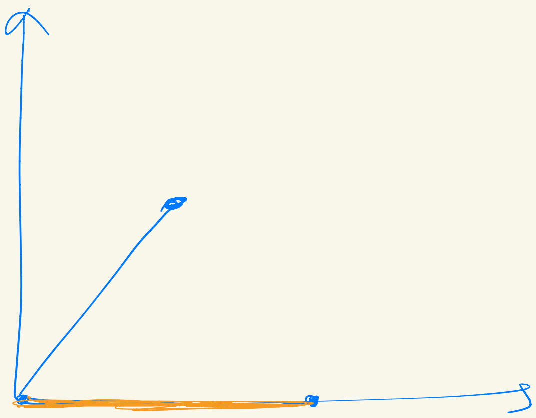
find R

G $R = I + \omega \sin\theta + (1-\cos\theta)\,\omega^2$

$= I + \omega \sin \omega t + (1-\cos \omega t)\,\omega^2$

ω is in skew symmetric form.

Skew motion of Rigid body:



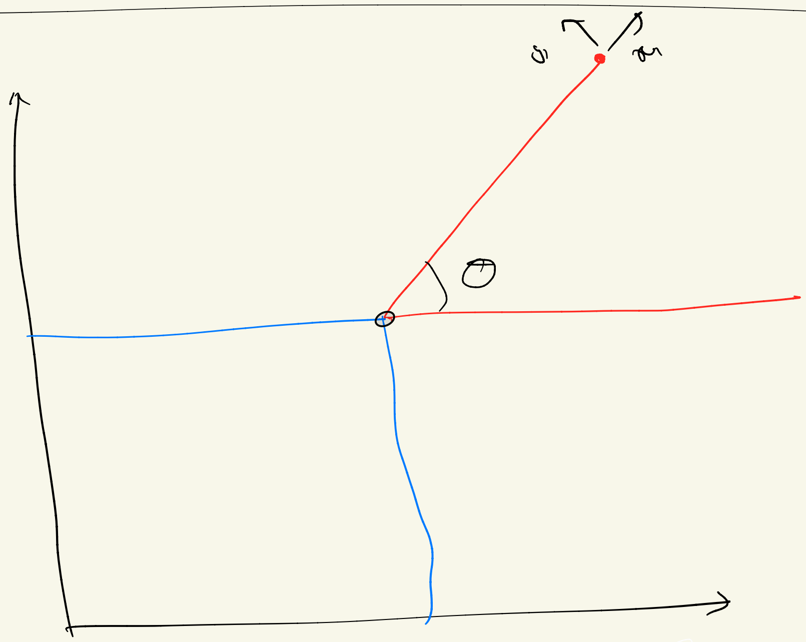
$q =$ origin $= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

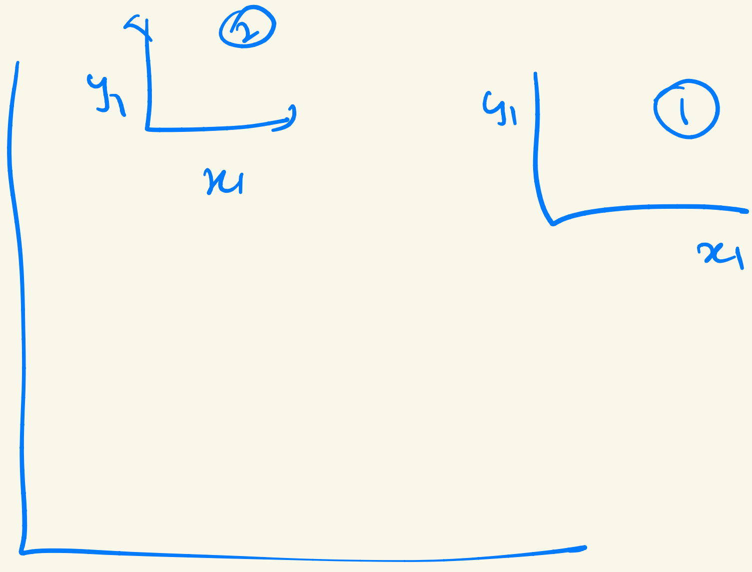
$\vec{\omega}_s = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ length of arm.

$q_b = \begin{bmatrix} -l \\ 0 \\ 0 \end{bmatrix}$

$v_b =$

$\vec{v}_b = -\vec{\omega}_b \times \vec{q}_b + \cancel{h\vec{\omega}_b}^{\,0}$





find T_0^2 $(0 \rightarrow 2)$

find T_0^2 by using screw theory $e^{[S]\theta}$

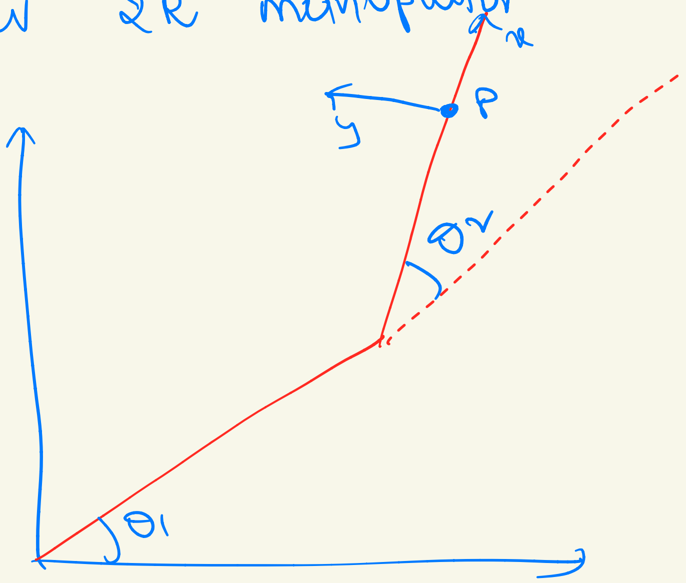
w.k.T $\quad T_0^2 = [\quad] M \rightarrow$ initial config

$$T_0^2 = e^{[S]\theta} \, T_0^1$$

forward kinematics:-

fixed frame $\longrightarrow$ required motion, velocity

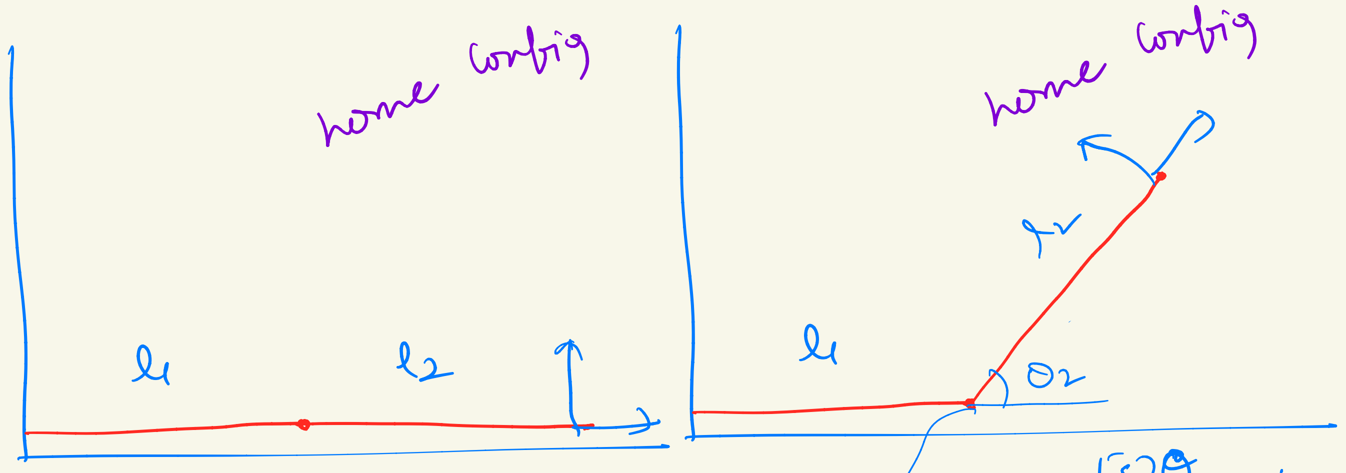
* consider 2R manupulator



$$T_2^0 = \begin{bmatrix} R_t & P_t \\ 0 & 1 \end{bmatrix}$$

$$T_0^2 = e^{S[\theta]}\,T_0^1$$

when $(\theta_1, \theta_2) = 0$ then the robot is on x-axis



$$T_2^0 = e^{[S]\theta}\,T_1^0$$

(q) pivot

$$T_1^0 = \begin{bmatrix} R & \begin{bmatrix} l_1+l_2 \\ 0 \\ 0 \end{bmatrix} \\ 0 & 1 \end{bmatrix}$$

$$q = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$$

$$\omega = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

Pitch = 0

S (screw axis) =

Spatial pitch = (V_s)

$$\begin{Bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -l_1 \\ 0 \end{Bmatrix}$$

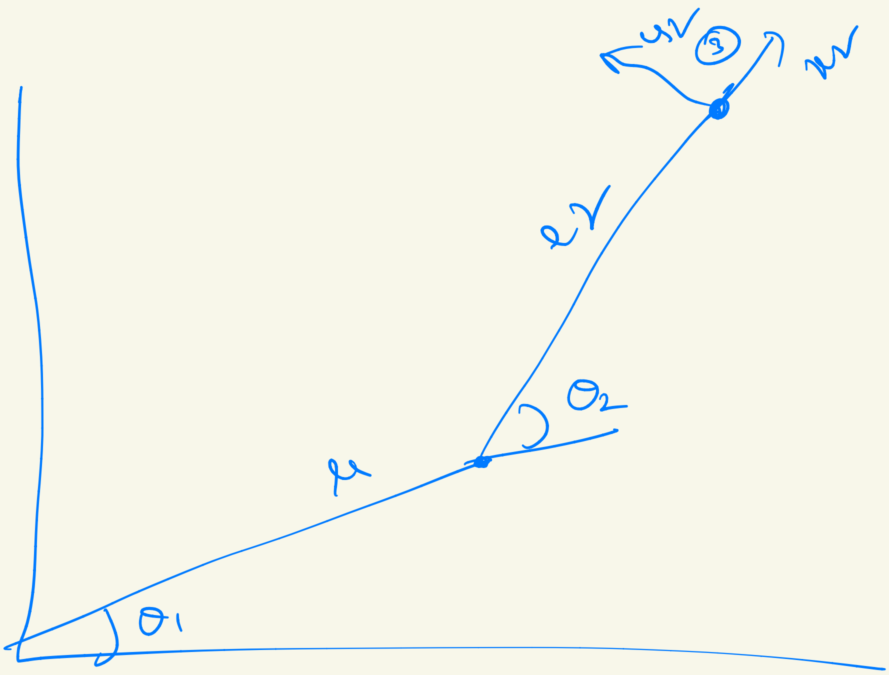
$$v = -\hat{\omega} \times q + \cancel{h\hat{\omega}}^{\,0}$$

V_s in skew symmetric form

$$S_2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \\ 0 \\ -l_1 \\ 0 \end{bmatrix} \Rightarrow [S] = \begin{bmatrix} [\hat{\omega}] & v \\ 0 & 0 \end{bmatrix}_{4\times 4}$$

$$T_0^2 = e^{S[\theta]} T_0^1$$

$$e^{S[\theta]} = \begin{bmatrix} e^{[\omega]\theta} & G(\theta)v \\ 0 & 1 \end{bmatrix}$$



we need to find

$$T_0^3 = e^{[S_1]} M$$

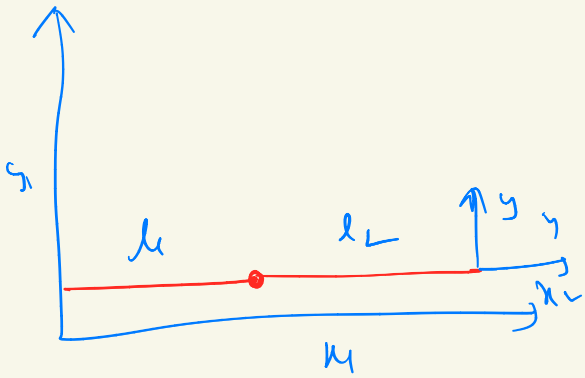
$$M = T_0^2$$

q, $\hat{\omega}$, twist.

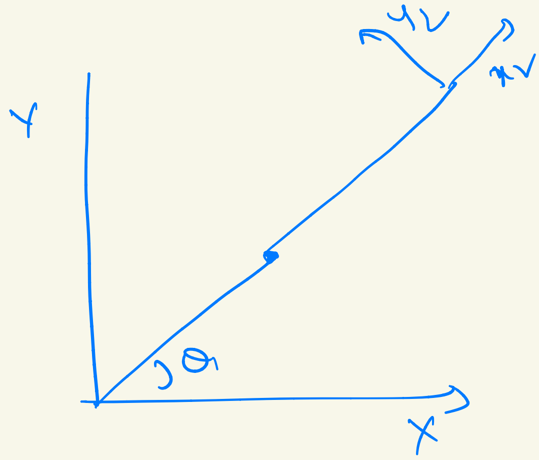
$$S_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad q = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \hat{\omega} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$T_0^3 = \begin{bmatrix} R(t) & P(t) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

$$\begin{bmatrix} \theta_1 & \theta_2 \\ \dot\theta_1 & \dot\theta_2 \\ \ddot\theta_1 & \ddot\theta_2 \end{bmatrix}$$



$\theta_1 = 0$
$\theta_2 = 0$



$\theta_1 = \theta_1$
$\theta_2 = 0$

find body twist $\mathcal{V}_b$

$$\mathcal{V}_b = \begin{Bmatrix} \omega_b \\ \vec{v}_b \end{Bmatrix}$$

$$\vec{v}_b = -\hat{\omega}_b \times q + h\hat{\omega}_b$$

$$q = \begin{bmatrix} -(l_1+l_2) \\ 0 \\ 0 \end{bmatrix}$$

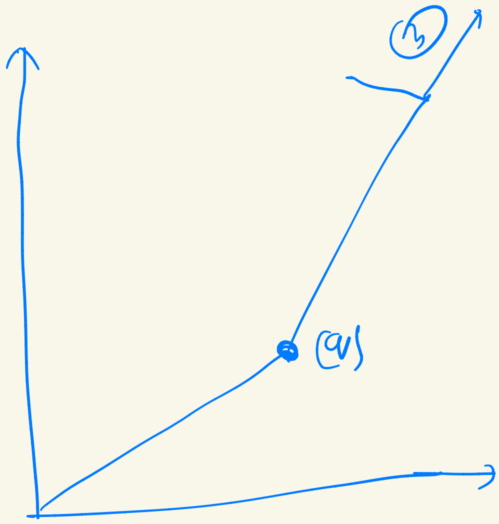
$$\vec{v}_b = \begin{Bmatrix} 0 \\ l_1+l_2 \\ 0 \end{Bmatrix}$$

$$\beta_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ l_1 + l_2 \\ 0 \end{bmatrix}$$

$$T_2^0 = \underbrace{M\, e^{[\beta_1]\theta}}_{m}$$

here we need

$$T_0^3 = M\, e^{[\beta_2]\theta_2}$$



for θ_2

$$q = \begin{Bmatrix} -l_2 \\ 0 \\ 0 \end{Bmatrix}$$

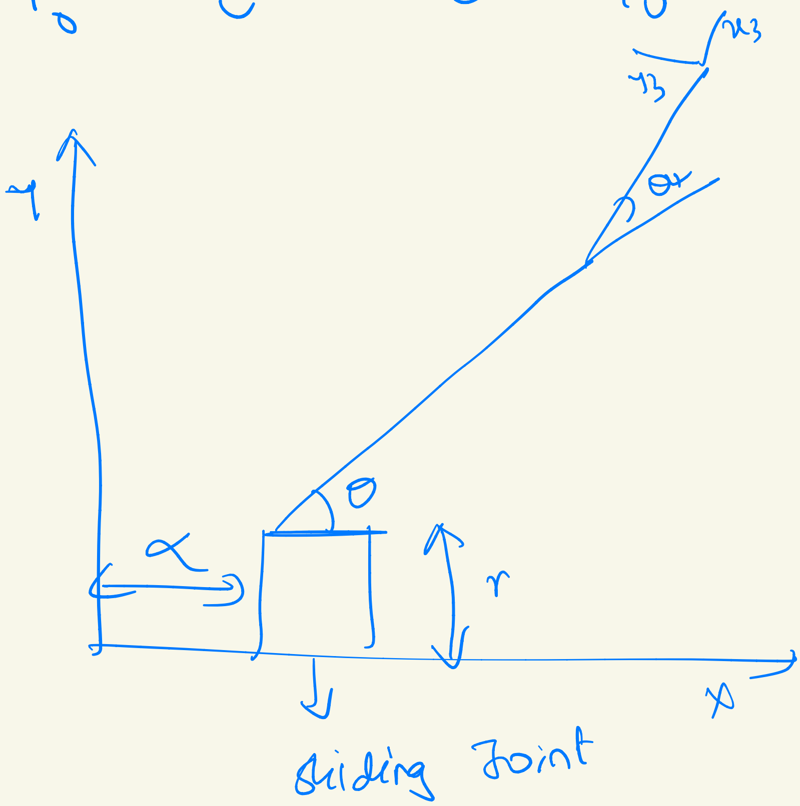
$$\hat{\omega} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

$$\beta = \begin{Bmatrix} \hat{\omega} \\ \vec{v}_b \end{Bmatrix}$$

$$\vec{v}_b = -\omega_b \times q + \cancel{h}\, \hat{\omega}_b \to 0$$

$$= \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$T_0^3 = e^{[S_1]\theta_1} e^{[S_2]\theta_2} T_0^1$



$T_0^3 = M e^{[S_1]\theta_1} e^{[S_2]\theta_2} e^{[S_3]\theta_3}$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$M \rightarrow$ home configuration.

$M =$