RV UNIVERSITY Go, change the world an initiative of RV EDUCATIONAL INSTITUTIONS	USN **School of Computer Science and Engineering** **B.Tech (Hons.)** **CP-1 Question Paper (Set A)** **Academic Year 2024-2025**	
Course: Probability, Statistics and Numerical Methods	**Course Code: CS2801**	**Semester: III**
Time: 9:15 AM - 10:45 AM (1 hour 30 minutes)	**Max Marks: 15**	**Date : 03/09/2024**

Notes/ Instructions:

a) **Answer all the questions.**
b) **This paper contains 5 questions.**
c) **Any use of laptops, phones or smartwatches and unfair means will be considered as malpractice and results in ZERO marks.**
d) **You may use simple calculators if required.**

Sl. No.	Questions	Marks	L1-L6	CO
1.	Sixty percent of new drivers have had driver education. During their first year, new drivers without driver education have a 0.08 probability of having an accident, but new drivers with driver education have only a 0.05 probability of an accident. What is the probability a new driver has had driver education, given that the driver has had no accident the first year?	**3**	**L3**	**CO1**
2.	You have a fair five-sided die with sides numbered from 1 to 5. Each roll of the die is independent of the others, and all faces have an equal chance of landing face up. Suppose the die is rolled twice. The events are as follows: D: The sum of the two rolls is 7. E: The difference between the outcomes of the two rolls is exactly 1. F: The second roll is higher than the first roll. **a)** Are events E and F independent? Justify. **b)** Are events E and F independent given that event D occurs? Justify.	**1+2**	**L3**	**CO1**

L1- Remember, L2- Understand, L3 – Apply, L4- Analyse, L5 – Evaluate, L6 – Create

3.	Let the joint probability mass function (pmf) of random variable X and Y be $$f(x,y) = \begin{cases} \frac{x+y}{21} & \text{if } x \in \{1,2,3\} \text{ and } y \in \{1,2\} \\ 0 & \text{otherwise} \end{cases}$$ Find, **a)** marginal pmf of X and Y. **b)** conditional distribution of X given Y = 2.	**2+1**	**L2**	**CO2**
4.	Let X have the probability density function $$f(x) = \begin{cases} \frac{1}{9}\left(\frac{8}{9}\right)^x & \text{for } x = 0,1,2,3,\ldots \\ 0 & \text{otherwise.} \end{cases}$$ **a)** What is the moment generating function of the random variable X? **b)** What is the expectation for X? **c)** Find the standard deviation.	**1.5+0.5 +1**	**L2**	**CO2**
5.	Three fair coins are tossed. Let X represent the number of heads obtained from the first two coin tosses, and let Y represent the number of tails obtained from the last two coin tosses. Find, **a)** Covariance (X,Y). **b)** Check how the variables X and Y are correlated and justify.	**2+1**	**L3**	**CO2**

Course Outcomes

1. Understand and demonstrate the essential concepts of probability theory and the principles of conditional probability.
2. Apply probability distribution functions to solve real world problems.

Marks Distribution									
L1	L2	L3	L4	L5	L6	CO1	CO2	CO3	CO4
-	6	9	-	-	-	✔	✔	-	-

L1- Remember, L2- Understand, L3 – Apply, L4- Analyse, L5 – Evaluate, L6 – Create

RV UNIVERSITY Go, change the world an initiative of RV EDUCATIONAL INSTITUTIONS	USN **School of Computer Science and Engineering** **B.Tech (Hons.)** **CP-1 Question Paper (Set B)** **Academic Year 2024-2025**

Course: Probability, Statistics and Numerical Methods	**Course Code: CS2801**	**Semester: III**
Time: 9:15 AM - 10:45 AM (1 hour 30 minutes)	**Max Marks: 15**	**Date : 03/09/2024**

Notes/ Instructions:

- **a) Answer all the questions.**
- **b) This paper contains 5 questions.**
- **c) Any use of laptops, phones or smartwatches and unfair means will be considered as malpractice and results in ZERO marks.**
- **d) You may use simple calculators if required.**

Sl. No.	Questions	Marks	L1-L6	CO
1.	Suppose that for the general population, 1 in 5000 people carries the human immunodeficiency virus (HIV). A test for the presence of HIV yields either a positive (+) or negative (−) response. Suppose the test gives the correct answer 99% of the time. What is P[−|H], the conditional probability that a person tests negative given that the person does have the HIV virus? What is P[H|+], the conditional probability that a randomly chosen person has the HIV virus given that the person tests positive?	**3**	**L3**	**CO1**
2.	You have a fair five-sided die. The sides of the die are numbered from 1 to 5. Each die roll is independent of all others, and all faces are equally likely to come out on top when the die is rolled. Suppose you roll the die twice. Let event A be "the total of two rolls is 10", event B be "at least one roll resulted in 5", and event C be "at least one roll resulted in 1". **a)** Are events A and C independent? Justify. **b)** Are events B and C independent given that event A occurs? Justify.	**1+2**	**L3**	**CO1**
3.	Two discrete random variables X and Y have a joint distribution of $f(x, y) = (x + y + 1)/27$, for x and y equal to 0, 1, and 2. **a)** What is the marginal of X and Y? **b)** What is $P(X \leq 1 \mid Y = 1)$?	**2+1**	**L2**	**CO2**

L1- Remember, L2- Understand, L3 – Apply, L4- Analyse, L5 – Evaluate, L6 – Create

4.	Find the moment generating function (mgf), for the following discrete probability distributions. **a)** The distribution describing a fair coin. **b)** The distribution describing a die that always comes up 3. **c)** Find the mean for the cases given in a) and b).	**1+1+1**	**L2**	**CO2**
5.	Two three faced dice are thrown. Let X be the score on the first dice and Y the score on the second dice. Let Z= $max\ \{X, Y\}$. Find, **a)** Covariance (X, Z). **b)** Check how the variables X and Z are correlated and justify it too.	**2+1**	**L3**	**CO2**

Course Outcomes

1. Understand and demonstrate the essential concepts of probability theory and the principles of conditional probability.
2. Apply probability distribution functions to solve real world problems.

Marks Distribution									
L1	L2	L3	L4	L5	L6	CO1	CO2	CO3	CO4
-	6	9	-	-	-	✔	✔	-	-

L1- Remember, L2- Understand, L3 – Apply, L4- Analyse, L5 – Evaluate, L6 – Create

	USN
RV UNIVERSITY Go, change the world an initiative of RV EDUCATIONAL INSTITUTIONS	**School of Computer Science and Engineering** **B.Tech (Hons.)** **CP-1 Question Paper** **Academic Year 2024-2025**

Course: PSN	**Course Code: CS2801**	**Semester: III**
Time: 1 hour 30 Minutes	**Max Marks: 20**	**Date : 03/09/2024**

Notes/ Instructions:

a) **Answer all the questions.**
b) **Any use of laptops, phones or smartwatches and unfair means will be considered as malpractice and results in ZERO marks.**

Sl. No.	Answers and Mark distribution	Marks	L1-L6	CO
1. `	Let A represent the new driver who has had driver education and B represent the new driver who has had an accident in his first year. Let A^c and B^c be the complement of A and B, respectively. We want to find the probability that a new driver has had driver education, given that the driver has had no accidents in the first year, that is $P(A\|B^c)$, $P(A/B^c) = \frac{P(A \cap B^c)}{P(B^c)}$ $= \frac{P(B^c/A)\,P(A)}{P(B^c/A)\,P(A) + P(B^c/A^c)\,P(A^c)}$ $= \frac{[1-P(B/A)]\,P(A)}{[1-P(B/A)]\,P(A) + [1-P(B/A^c)]\,[1-P(A)]}$ $= \frac{\left(\frac{60}{100}\right)\left(\frac{95}{100}\right)}{\left(\frac{40}{100}\right)\left(\frac{92}{100}\right) + \left(\frac{60}{100}\right)\left(\frac{95}{100}\right)}$ $= 0.6077.$ **OR**	1 1 1		**CO1**

L1- Remember, L2- Understand, L3 – Apply, L4- Analyse, L5 – Evaluate, L6 – Create

The $P[-|H]$ is the probability that a person who has HIV tests negative for the disease. This is referred to as a false-negative result. The case where a person who does not have HIV but tests positive for the disease, is called a false-positive result and has probability $P[+|H^c]$. Since the test is correct 99% of the time,

$$P[-|H] = P[+|H^c] = 0.01. \tag{1}$$

Now the probability that a person who has tested positive for HIV actually has the disease is

$$P[H|+] = \frac{P[+,H]}{P[+]} = \frac{P[+,H]}{P[+,H] + P[+,H^c]}. \tag{2}$$

We can use Bayes' formula to evaluate these joint probabilities.

$$P[H|+] = \frac{P[+|H]\,P[H]}{P[+|H]\,P[H] + P[+|H^c]\,P[H^c]} \tag{3}$$

$$= \frac{(0.99)(0.0002)}{(0.99)(0.0002) + (0.01)(0.9998)} \tag{4}$$

$$= 0.0194. \tag{5}$$

Thus, even though the test is correct 99% of the time, the probability that a random person who tests positive actually has HIV is less than 0.02. The reason this probability is so low is that the a priori probability that a person has HIV is very small.

2.

SET A

Event $D \to I + II = 7$

Event $E \to |I - II| = 1$

Event $F \to II > I$

a) To find: If $P(E \cap F) = P(E) \cdot P(F)$

Soln: $D \to (5,2), (2,5), (3,4), (4,3) \Rightarrow |D| = 4$

$E \to (1,2), (2,1), (2,3), (3,2), (3,4), (4,3), (4,5), (5,4)$ (I, II) $\Rightarrow |E| = 8$

$F : (I, II) \to \{(1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5)\} \Rightarrow 10 = |F|$

If E and F are Independent then

$P(E \cap F) = P(E) \cdot P(F)$

$\frac{4}{25} = \frac{8}{25} \cdot \frac{\cancel{10}\,2}{\cancel{25}\,5} \Rightarrow \frac{4}{25} = \frac{16}{125}$

Clearly LHS ≠ RHS & E & F are NOT Independent

1

b) To find: $P(E \cap F | D) = P(E|D) \cdot P(F|D)$

Soln:

	1	2	3	4	5 → 2nd roll
1					
2					
3					
4					
5					
↓ 1st roll					

$P(E \cap F | D) = \frac{1}{4}$

$= \frac{P((E \cap F) \cap D)}{P(D)}$

$= \frac{\frac{1}{25}}{\frac{4}{25}} = \frac{1}{4}$

llly $P(E|D) = \frac{P(E \cap D)}{P(D)}$

$= \frac{\frac{2}{25}}{\frac{4}{25}} = \frac{1}{2}$

2

and $P(F|D) = \frac{P(F \cap D)}{P(D)} = \frac{\frac{2}{25}}{\frac{4}{25}} = \frac{1}{2}$

Let us check if they are Independent.

i.e $P(E \cap F | D) = P(E|D) \cdot P(F|D)$

$\Rightarrow \frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2}$

LHS = RHS

Hence E & F are conditionally Independent Given 'D' has occured

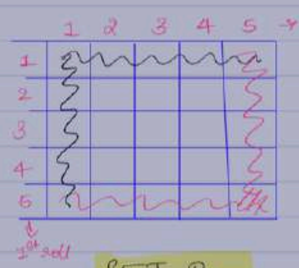
L1- Remember, L2- Understand, L3 – Apply, L4- Analyse, L5 – Evaluate, L6 – Create

OR

1) Given: Fair 5 sided die is rolled twice

Die rolls are Independent.

The sample space of the experiment can be drawn as follows



Let us consider the first roll as 'I' and second roll as 'II'

SET B

Event A → I + II = 10

Event B → At least one roll resulted in 5.

Event C → At least one roll resulted in 1.

a) To find: If $P(A \cap C) = P(A) \cdot P(C)$

Soln: $P(C) = \frac{9}{25}$; $P(A \cap C) = 0$

For Independence $P(A \cap C) = P(A) \cdot P(C)$

$0 \neq \frac{1}{25} \cdot \frac{9}{25} \Rightarrow 0 \neq 9$

∴ Events A & C are NOT Independent

(b) To find: If $P(B \cap C \mid A) = P(C \mid A) \cdot P(B \mid A)$

Soln: $P(B \cap C \mid A) = \frac{P(B \cap C \cap A)}{P(A)} = 0$

$P(C \mid A) = \frac{P(C \cap A)}{P(A)} = 0$

$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{25}}{\frac{1}{25}} = 1$

∴ $P(B \cap C \mid A) = P(C \mid A) \cdot P(B \mid A)$

$0 = 0 \cdot 1$

$0 = 0$ ∴ Events are Independent

3.

Marginal PMF of X:

$$f_X(x) = \begin{cases} \frac{5}{21} & \text{for } x = 1 \\ \frac{1}{3} & \text{for } x = 2 \\ \frac{3}{7} & \text{for } x = 3 \\ 0 & \text{otherwise.} \end{cases}$$

1

Marginal PMF of Y:

$$f_Y(y) = \begin{cases} \frac{3}{7} & \text{for } y = 1 \\ \frac{4}{7} & \text{for } y = 2 \\ 0 & \text{otherwise.} \end{cases}$$

1

	b) The conditional distribution of X given $Y=2$ is: $P(X=x \mid Y=2)=\begin{cases}\frac{1}{4} & \text{for } x=1\\ \frac{1}{3} & \text{for } x=2\\ \frac{5}{12} & \text{for } x=3\\ 0 & \text{otherwise.}\end{cases}$ **OR** **a)** **Marginal PMF of X:** $f_X(x)=\begin{cases}\frac{2}{9} & \text{for } x=0\\ \frac{1}{3} & \text{for } x=1\\ \frac{4}{9} & \text{for } x=2\\ 0 & \text{otherwise.}\end{cases}$ **Marginal PMF of Y:** $f_Y(y)=\begin{cases}\frac{2}{9} & \text{for } y=0\\ \frac{1}{3} & \text{for } y=1\\ \frac{4}{9} & \text{for } y=2\\ 0 & \text{otherwise.}\end{cases}$ **b)** $P(X\le 1 \mid Y=1)=\frac{P(X\le 1, Y=1)}{P(Y=1)}=\frac{\frac{5}{27}}{\frac{1}{3}}=\frac{\frac{5}{27}}{\frac{9}{27}}=\frac{5}{9}$	**1** **1** **1** **1**		
4.	a) $\begin{aligned} M(t) &= E\left(e^{tX}\right)\\ &= \sum_{x=0}^{\infty} e^{tx} f(x)\\ &= \sum_{x=0}^{\infty} e^{tx}\left(\frac{1}{9}\right)\left(\frac{8}{9}\right)^x\\ &= \left(\frac{1}{9}\right)\sum_{x=0}^{\infty}\left(e^t\,\frac{8}{9}\right)^x\\ &= \left(\frac{1}{9}\right)\frac{1}{1-e^t\,\frac{8}{9}} \qquad \text{if} \quad e^t\,\frac{8}{9}<1\\ &= \frac{1}{9-8e^t} \qquad \text{if} \quad t<\ln\left(\frac{9}{8}\right).\end{aligned}$ b) $\frac{d}{dt}\left(\frac{1}{9-8e^t}\right)=\frac{8e^t}{(9-8e^t)^2}.$	**1.5** **0.5**		

L1- Remember, L2- Understand, L3 – Apply, L4- Analyse, L5 – Evaluate, L6 – Create

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then at t=0, E[X]= 8,

$$\frac{d^2}{dt^2}\left(\frac{1}{9-8e^t}\right) = \frac{8e^t\left[81-64e^{2t}\right]}{(9-8e^t)^4}$$

c) the at t=0, $E(X^2) = 136$, then Var(X) = 136- 64= 72.

OR

a)
The moment-generating function $M_X(t)$ is defined as:
$$M_X(t) = \mathbb{E}[e^{tX}] = \sum_x P(X=x)e^{tx}.$$
For the fair coin:
$$M_X(t) = \frac{1}{2}e^0 + \frac{1}{2}e^t = \frac{1}{2} + \frac{1}{2}e^t.$$

b)
The moment-generating function $M_Y(t)$ is:
$$M_Y(t) = \mathbb{E}[e^{tY}] = e^{3t}$$

c) Mean for a) = 1/2
Mean for b) = 3.
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a)

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<tr><td>X</td><td>2</td><td>2</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td></tr>
<tr><td>Y</td><td>0</td><td>1</td><td>1</td><td>2</td><td>0</td><td>1</td><td>1</td><td>2</td></tr>
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Now chart
<table>
<tr><td>X/Y</td><td>0</td><td>1</td><td>2</td><td></td></tr>
<tr><td>0</td><td>0</td><td>1/8</td><td>1/8</td><td>2/8</td></tr>
<tr><td>1</td><td>1/8</td><td>2/8</td><td>1/8</td><td>4/8</td></tr>
<tr><td>2</td><td>1/8</td><td>1/8</td><td>0</td><td>2/8</td></tr>
<tr><td></td><td>2/8</td><td>4/8</td><td>2/8</td><td>1</td></tr>
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Now $E(X) = 0\cdot 2/8 + 1\cdot 4/8 + 2\cdot 2/8 = 1$
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Now $E(X) = 0\cdot\frac{2}{8} + 1\cdot\frac{4}{8} + 2\cdot\frac{2}{8} = 1$

$E(Y) = 0\cdot\frac{2}{8} + 1\cdot\frac{4}{8} + 2\cdot\frac{2}{8} = 1$

$E(XY) = 0\cdot0\cdot0 + 0\cdot1\cdot\frac{1}{8} + 0\cdot2\cdot\frac{1}{8} +$
$1\cdot0\cdot\frac{1}{8} + 1\cdot1\cdot\frac{2}{8} + 1\cdot2\cdot\frac{1}{8} +$
$2\cdot0\cdot\frac{1}{8} + 2\cdot1\cdot\frac{1}{8} + 2\cdot2\cdot0$

$= \frac{2}{8} + \frac{2}{8} + \frac{2}{8} = \frac{6}{8}$

$Cov(X,Y) = \frac{6}{8} - 1 = \frac{6-8}{8} = -\frac{1}{4}$ Ans

1

b) The random variables are negatively correlated. **1**

OR

a)

X|Z	1	2	3	
1	1/9	1/9	1/9	3/9
2	0	2/9	1/9	3/9
3	0	0	3/9	3/9
	1/9	3/9	5/9	

X/Y	1	2	
1	(1,1)	(1,2)	(
2	(2,1)	(2,2)	(
3	(3,1)	(3,2)	(

1

as $Z=1$, $X=1, Y=1$ i.e. $(1,1)$

as $Z=2$; $(2,1)\ (2,2)\ (1,2)$

as $Z=3$ $(1,3)\ (2,3)\ (3,1)\ (3,2)\ (3,3)$

1

Now $E(X) = 2$

$E(Y) = \frac{22}{9}$ $E(XY) = \frac{47}{9}$

$Cov(X,Y) = \frac{47}{9} - \frac{44}{9} = \frac{3}{9} = \frac{1}{3}$

b) The random variables are positively correlated. **1**