RV UNIVERSITY — Go, change the world — an initiative of RV EDUCATIONAL INSTITUTIONS	USN **School of Computer Science and Engineering** B.Tech (Hons.) CP-2 Answer Scheme (Set B) Academic Year 2024-2025	
Course: PSN	**Course Code:** CS2801	**Semester:** III
Time: 2 hours	**Max Marks: 25**	**Date :** 22/09/2024

Notes/ Instructions:

a) **Answer all the questions.**
b) **Any use of laptops, phones or smartwatches and unfair means will be considered as malpractice and results in ZERO marks.**

Sl. No.	Answers and Mark distribution	Marks	L1-L6	CO
1. `	The correct answer is**: d) strong positive correlation** This is a **linear relationship** with a slope of 3. The slope indicates how much Y changes for a given change in X. ● Since the slope is positive (3), it means that as X increases, Y also increases. ● A positive slope represents a **positive correlation**.	1	L2	CO3
2.	The correct answer is**: a) -3** To find the y-coordinate of the vertex of a parabola, we first need to find the x-coordinate of the vertex using the formula: For a parabola in the form y = ax^2 + bx + c, the x-coordinate of the vertex is given by: x = -b/2a In this case, a = 2 and b = 4 , so: x = -4/2(2) = -1 Now, substitute x = -1 into the equation to find the corresponding y -coordinate: y = 2(-1)^2 + 4(-1) -1 = 2 - 4 - 1 = -3 Thus, the y -coordinate of the vertex is -3.	1	L2	CO3
3.	The correct answer is: **c)** $\sum y = a\sum x + nb$ and $\sum xy = a\sum x^2 + b\sum x$.	1	L2	CO3

4.	Line of regression of x on y: $b_{xy} = 1/5$ Line of regression of y on x: $b_{yx} = 2/3$ $\tan\theta = \left\lvert \left(\frac{1-r^2}{r}\right)\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2} \right\rvert$ $\Rightarrow \tan\theta = \left\lvert \left(\frac{1-r^2}{r}\right)\frac{1}{\frac{\sigma_x^2+\sigma_y^2}{\sigma_x\sigma_y}} \right\rvert$ $\Rightarrow \tan\theta = \left\lvert \left(\frac{1-r^2}{r}\right)\frac{1}{\frac{\sigma_x^2}{\sigma_x\sigma_y}+\frac{\sigma_y^2}{\sigma_x\sigma_y}} \right\rvert$ $\Rightarrow \tan\theta = \left\lvert \left(\frac{1-r^2}{r}\right)\frac{1}{\frac{\sigma_x}{\sigma_y}+\frac{\sigma_y}{\sigma_x}} \right\rvert$ $\therefore \tan\theta = \left\lvert \frac{1-r^2}{r\frac{\sigma_x}{\sigma_y}+r\frac{\sigma_y}{\sigma_x}} \right\rvert$ $r\frac{\sigma_x}{\sigma_y} = b_{xy}$ $r\frac{\sigma_y}{\sigma_x} = b_{yx}$ $r^2 = b_{xy}b_{yx}$ $\therefore \tan\theta = \left\lvert \frac{1-b_{xy}b_{yx}}{b_{xy}+b_{yx}} \right\rvert$ Angle between the two lines of regression, $\theta = \tan^{-1}\left\lvert \frac{1-(0.2)\left(\frac{2}{3}\right)}{0.2+\frac{2}{3}} \right\rvert$ $\Rightarrow \theta = \tan^{-1}\left\lvert \frac{\frac{3-0.4}{3}}{\frac{0.6+2}{3}} \right\rvert$ $\Rightarrow \theta = \tan^{-1}\left\lvert \frac{3-0.4}{0.6+2} \right\rvert$ $\Rightarrow \theta = \tan^{-1}\left\lvert \frac{2.6}{2.6} \right\rvert$ $\Rightarrow \theta = \tan^{-1}\lvert 1 \rvert$ $\therefore \theta = \frac{\pi}{4}$	**0.5** **0.5** **0.5** **0.5**	**L2**	**CO3**
5.	**a)**		**L3**	**CO2**

L1- Remember, L2- Understand, L3 – Apply, L4- Analyse, L5 – Evaluate, L6 – Create

	Marginal Density of X, $f_X(x)$: $$f_X(x) = \int_0^\infty f(x,y)\,dy = \int_0^\infty 2e^{-x}e^{-2y}\,dy.$$ The integral of e^{-2y} over $0 < y < \infty$ is: $$\int_0^\infty e^{-2y}\,dy = \frac{1}{2}.$$ Therefore, $$f_X(x) = 2e^{-x} \cdot \frac{1}{2} = e^{-x}, \quad 0 < x < \infty.$$ So, $$f_X(x) = e^{-x}, \quad 0 < x < \infty.$$ **Marginal Density of Y, $f_Y(y)$:** Similarly, integrate $f(x,y)$ over x: $$f_Y(y) = \int_0^\infty f(x,y)\,dx = \int_0^\infty 2e^{-x}e^{-2y}\,dx.$$ The integral of e^{-x} over $0 < x < \infty$ is: $$\int_0^\infty e^{-x}\,dx = 1.$$ Therefore, $$f_Y(y) = 2e^{-2y} \cdot 1 = 2e^{-2y}, \quad 0 < y < \infty.$$ So, $$f_Y(y) = 2e^{-2y}, \quad 0 < y < \infty.$$ **b)** $$f(x,y) = f_X(x) \cdot f_Y(y).$$ From the above, we have: $$f_X(x) = e^{-x} \quad \text{and} \quad f_Y(y) = 2e^{-2y}.$$ So, $$f_X(x) \cdot f_Y(y) = e^{-x} \cdot 2e^{-2y} = 2e^{-x}e^{-2y}.$$ This is exactly the joint density function $f(x,y)$. Therefore, X and Y are independent.	**0.5** **1** **0.5** **1** **1**		

L1- Remember, L2- Understand, L3 – Apply, L4- Analyse, L5 – Evaluate, L6 – Create

c)

The conditional density of X given $Y = 1$ is given by:

$$f(x|y=1) = \frac{f(x,1)}{f_Y(1)}.$$

First, compute $f(x,1)$:

$$f(x,1) = 2e^{-x}e^{-2(1)} = 2e^{-x}e^{-2} = 2e^{-x-2}.$$

0.5

Now, compute $f_Y(1)$:

$$f_Y(1) = 2e^{-2(1)} = 2e^{-2}.$$

Therefore, the conditional density is:

$$f(x|y=1) = \frac{2e^{-x-2}}{2e^{-2}} = e^{-x}.$$

0.5

So, the conditional density of X given $Y = 1$ is:

$$f(x|1) = e^{-x}, \quad 0 < x < \infty.$$

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6. | L4 | CO3

$$\sum x = 30, \sum y = 183, \sum x^2 = 220, \sum x^3 = 1800, \sum x^4 = 15664,$$

$$\sum xy = 1526, \sum x^2 y = 13452$$

1.5

$$183 = 5a + 30b + 220c,$$

$$1526 = 30a + 220b + 1800c,$$

1.5

$$13452 = 220a + 1800b + 15664c$$

$$a = 27/7 = 5.4, b = -241/70 = -3.4428, c = 33/28$$

2

$$y = 5.4 - 3.4428x + 1.1786x^2$$

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7. | L4 | CO3

Pearson's Correlation Coefficient

Step 1: Calculate the mean of hours studied and exam scores.

- Mean of hours studied (x̄) = (2 + 3 + 4 + 5 + 6) / 5 = 4
- Mean of exam scores (ȳ) = (65 + 72 + 80 + 85 + 90) / 5 = 78.4

1

Step 2: Calculate the deviations from the mean for each data point.

Hours Studied (x)	**Exam Score (y)**	**Deviation from x̄ (x - x̄)**	**Deviation from ȳ (y - ȳ)**
2	65	-2	-13.4
3	72	-1	-6.4
4	80	0	1.6

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5	85	1	6.6
6	90	2	11.6

Step 3: Calculate the product of the deviations for each data point.

Hours Studied (x)	Exam Score (y)	Deviation from x̄ (x - x̄)	Deviation from ȳ (y - ȳ)	Product of Deviations
2	65	-2	-13.4	26.8
3	72	-1	-6.4	6.4
4	80	0	1.6	0
5	85	1	6.6	6.6
6	90	2	11.6	23.2

Step 4: Calculate the sum of the products of deviations and the sum of squared deviations for both x and y.

- $\Sigma(x - \bar{x})(y - \bar{y}) = 26.8 + 6.4 + 0 + 6.6 + 23.2 = 63$
- $\Sigma(x - \bar{x})^2 = (-2)^2 + (-1)^2 + 0^2 + 1^2 + 2^2 = 10$
- $\Sigma(y - \bar{y})^2 = (-13.4)^2 + (-6.4)^2 + 1.6^2 + 6.6^2 + 11.6^2 = 401.2$

Step 5: Calculate Pearson's correlation coefficient (r).

$r = \Sigma(x - \bar{x})(y - \bar{y}) / \sqrt{[\Sigma(x - \bar{x})^2 * \Sigma(y - \bar{y})^2]}$ $r = 63 / \sqrt{(10 * 401.2)} \approx 0.99$

Interpretation: The correlation coefficient of 0.99 indicates a very strong positive linear relationship between hours studied and exam scores.

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a) $X \sim N(1000, 10000)$

$P(X > 1200) = P((X - \mu)/\sigma > (1200 - \mu)/\sigma)$

$= P((X - 1000)/100 > (1200 - 1000)/100)$

$= P(z > 2)$

$= 1 - P(z < 2)$

$= 1 - 0.97725 = 0.02275 = 2.28\%$

b) mean = 6 = a/λ , variance = 4 = a/λ^2

4 = a/λ^2 and 6 = a/λ , $then$

$a = 9$, and $\lambda = 1.5$ then $\beta = 1/\lambda = 2/3$

OR

</td><td>1
1
1
0.5
1.5</td><td>L4</td><td>CO2</td></tr>
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a) $X \sim B(n, p)$
$\sim B(5, \frac{1}{6})$

$P(\text{getting 2 sixes}) = {}^nC_x\, p^x (1-p)^{n-x}$

(1 Marks) —— $= {}^5C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$

$= 10 \times \frac{5^3}{6^5}$

$= 10 \times \frac{125}{6^5} = \frac{10 \times 125}{7776}$

(2 Marks) —— $= \frac{1250}{7776} \simeq 0.1607$

b) $p(\text{drawing a black ball in one draw} = \frac{N}{M+N} = p$

$p(\text{drawing a white ball}) = 1 - p = \frac{M}{M+N}$ —— 1 Marks

The probability of needing exactly n draws to get first black ball $= (1-p)^{n-1} \cdot p$

$P(\text{first black on } n^{th} \text{ draw})$

$= \left(\frac{M}{M+N}\right)^{n-1} \cdot \frac{N}{M+N}$ —— 1 Marks