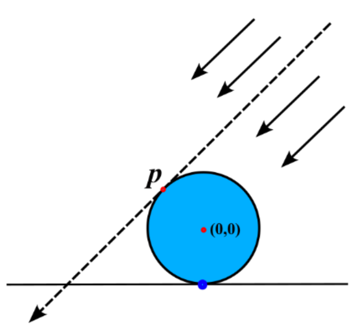
SET 1

	School of Computer Science and Engineering **B.Tech (Hons.)** **Internal Assessment -1** **Academic Year** 2024 – 2025	
Course: Calculus and Laplace Transform	**Course Code:** CS2802	**Semester:** IV
Date: 10/02/2025 (**Time:** 9:00AM)	**Duration: 50 Minutes**	**Max Marks: 10 Marks**

Instructions:

- All questions are compulsory.
- Do not write any part of the answer in the question paper.
- Non-programmable calculators are allowed
- **Please be seated in your allotted seat 10 minutes before the exam, i,e., by 8:50AM.**

S.No	Question	Marks	Level	CO
1	The daily temperature $T°$C in a city is modeled by $T(h) = 10 + 15\cos\left(\frac{h\pi}{12}\right)$ where h is the hour of the day. Compute the domain and range of function $T(h)$.	2	L3	CO 1
2	The concentration of a particular medicine in the bloodstream (in mg/l), t hours after taking the medicine is given by: $$C(t) = \frac{50t}{t^2+4}$$ Determine the absorption rate of the medicine into the tissue as a function of time. Compute the absorption rate of medicine into the tissue after 24 hours?	2	L3	CO 1
3	Sun is shining down at an angle of 45° on a ball of radius 10 cm that is on the surface of a flat horizontal plane. If the center of the ball is taken as the origin, calculate the tangent point p and determine the tangent line equation (shown in dash line) at p. 	2	L3	CO 1

Continued on next page...

4	Let $f(x)$ represent the post-tax income of a person earning ₹x annually. Define $$f(x) = \begin{cases} x & ; x \leq 1275000 \\ 0.9x & ; x > 1275000 \end{cases}$$ Check the continuity at $x = 1275000$. If a person earning ₹12,74,999 gets their full salary but someone earning ₹$12,75,001$ loses ₹$1,27,500$ to taxes, determine if this is a fair system.	2	L3	CO 1
5	A gas bubble is expanding and contracting due to the variations in the temperature such that the radius of the bubble as function of temperature T is given by: $$r(T) = 2\sin(20 - 5T)$$ Calculate the rate at which the volume of the gas bubble is changing with respect to temperature. Given volume of a sphere is: $\frac{4\pi r^3}{3}$	2	L3	CO1

Course Outcomes:

CO1: Apply foundational concepts of functions, limits, continuity and derivatives to solve various engineering problems.

Marks Distribution										
L1	L2	L3	L4	L5	L6	CO1	CO2	CO3	CO4	CO5
-	-	10	-	-	-	10	-	-	-	-

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RV UNIVERSITY Go, change the world	**RV University, Bengaluru** **School of Computer Science and Engineering** **B.Tech (Hons.)** **Internal Assessment 1** **Answer Keys** **Academic Year 2024-2025**

Course: Calculus and Laplace Transforms		**Course Code: CS2802**	**Semester : IV**
Date : 10 Feb '25	**Duration: 50 minutes**	**Max Marks: 10**	Set -1

Sl. No	**Answers and marking scheme**	**Marks**
1.	The daily temperature T°C in a city is modeled by $T(h) = 10+15\cos\left(\frac{h\pi}{12}\right)$ where h is the hour of the day. Compute the domain and range of function $T(h)$. **Solution:** - Since h represents time in a day, $h \in [0, 24]$. - The cosine function oscillates between -1 and 1, so: $$T(h) = 10 + 15(-1 \leq \cos(\frac{\pi}{12}h) \leq 1).$$ $$10 - 15 \leq T(h) \leq 10 + 15.$$ $$-5 \leq T(h) \leq 25.$$ - The range is $[-5, 25]$.	**2**
2.	The concentration of a particular medicine in the bloodstream (in mg/l), t hours after taking the medicine is given by: $$C(t) = \frac{50t}{t^2+4}$$ Determine the absorbtion rate of the medicine into the tissue as function of time. Compute the absorption rate of medicine into the tissue after 24 hours? **ANSWER** $$C(t) = \frac{50t}{t^2+4}$$	**2**

L1- Remember, L2- Understand, L3 – Apply, L4- Analyse, L5 – Evaluate, L6 – Create

	$$\frac{dC(t)}{dt} = \frac{50(t^2+4) - 50t(2t)}{(t^2+4)^2}$$ At 24 hours, $$\left.\frac{dC}{dt}\right\|_{t=24} = -0.085$$	
3.	Sun is shining down at an angle of 45◦ on a ball of radius 10 cm that is on the surface of a flat horizontal plane. If the center of the ball is taken as the origin, calculate the tangent point p and determine the tangent line equation (shown in dash line) at p. **ANSWER** $$f(x) = \sqrt{100 - x^2}$$ $$f'(x) = \frac{1}{2}(100 - x^2)^{-\frac{3}{2}}(-2x) = -x(100 - x^2)^{-\frac{3}{2}}$$ Circle with radius 10 cm and origin at centre: $x^2 + y^2 = 100$ $$\frac{dy}{dx} = -\frac{x}{y} = \tan\frac{\pi}{4} = 1$$ $$\Rightarrow x = -y$$ $$\Rightarrow y^2(1^2 + 1) = 100$$ $$y^2 = \frac{100}{2} \Rightarrow y = \sqrt{\frac{100}{2}} = 7.07 \text{ and } x = -7.07$$ Tangent line: $T(x) = f(p) + f'(p)(x - p)$ $$T(x) = f(-7.07) + 1 \times (x + 7.07)$$ $$T(x) = 14.14 + x$$	2
4.	Let $f(x)$ represent the post-tax income of a person earning ₹x annually. Define $$f(x) = \begin{cases} x & ; x \leq 1275000 \\ 0.9x & ; x > 1275000 \end{cases}$$ Check the continuity at $x = 1275000$. If a person earning ₹12,74,999 gets their full salary but someone earning ₹12, 75, 001 loses ₹1, 27, 500 to taxes, determine if this is a fair system.	2

L1- Remember, L2- Understand, L3 – Apply, L4- Analyse, L5 – Evaluate, L6 – Create

3. $f(r) = f(r-1) + f(1)$

$f(r) = f(r-1) + 5$

$f(r) = f(r-2) + 5 + 5$

$= f(r-2) + 2 \times 5$

$= f(r-3) + 3 \times 5$

$= f(1) + (r-1) \times 5$

$\therefore \sum_{n=1}^{P} f(n) = \sum_{n=1}^{P} (5n) = \frac{5P(P+1)}{2}$

4. Continuity check :

- $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$
- LHL $\lim_{x \to 12,75,000^-} f(x) = 12,75,000$
- RHL $\lim_{x \to 12,75,000^+} f(x) = 0.9(12,75,000) = 11,47,500$

$\therefore$ LHL $\neq$ RHL function is discontinuous @ $x = 12,75,000$

$\Rightarrow$ person getting 12,75,000 gets full salary, but someone earning just 1 more experiences a sudden drop in the income. Hence not fair.

5. $f'(x) = (3x^2+2x)e^{x^3} \cdot 3x^2 + e^{x^3}(6x+2) - \left[\frac{x^2\cos x - 2x\sin x}{x^4}\right] + \frac{5}{5x+1}$

5. A gas bubble is expanding and contracting due to the variations in the temperature such that the radius of the bubble as function of temperature T is given by:

$$r(T) = 2\sin(20 - 5T)$$

Compute the rate at which the volume of the gas bubble is changing with respect to temperature. Given volume of a sphere is: $V = \frac{4}{3}\pi r^3$. **2**

ANSWER

	$$\frac{dV\big(r(T)\big)}{dT} = \frac{dV}{dr}\cdot\frac{dr}{dT}$$ $$= \ 4\pi r^2 \frac{dr}{dT}$$ $$= 4\pi\, r^2\ (2\cos(20-5T\ )(-5))$$ $$= \ -40\,\pi\ 4\sin^2(20-5T)\ 2\cos(20-5T)$$ $$= \ \boldsymbol{-320\,\pi\ \sin^2(20-5T)\ 2\cos(20-5T)}$$	

SET 2

	School of Computer Science and Engineering B.Tech (Hons.) Internal Assessment -1 **Academic Year** 2024 – 2025	
Course: Calculus and Laplace Transform	**Course Code:** CS2802	**Semester:** IV
Date: 10/02/2025(**Time:** 9:00AM)	**Duration: 50 Minutes**	**Max Marks: 10 Marks**

Instructions:

- All questions are compulsory.
- Do not write any part of the answer in the question paper.
- Non-programmable calculators are allowed
- **Please be seated in your allotted seat 10 minutes before the exam, i,e., by 8:50AM.**

S.No	Question	Marks	Level	CO
1	The height H (in meters) of a passenger on a Ferris wheel is given by the function: $$H(t) = 20 + 15\sin\left(\frac{\pi t}{15}\right)$$ where t is the time in minutes after the ride starts, and the Ferris wheel completes one full rotation in 30 minutes. Determine the Domain and Range.	2	L3	CO 1
2	The temperature T in degrees Celsius is related to the temperature F in degrees Fahrenheit by the function $T(F) = \frac{5(F-32)}{9}$ and the temperature K in Kelvin is related to the temperature T in degrees Celsius by the function $K(T) = T + 273.15$. Express the temperature K in Kelvin as a function of temperature F in degrees Fahrenheit.	2	L3	CO 1
3	The position of a particle moving along a straight line is given by $s(t) = t^2 - 4t + 3$ where $s(t)$ is the displacement in meters and t is the time in seconds. Calculate the equation of the tangent line to the particle's path at $t = 1$.	2	L3	CO 1
4	The gross domestic product (GDP), $G(t)$ of a country (in trillions of dollars) at time t years is modeled by: $G(t) = 5(1 - e^{-0.03t})$. Evaluate $$\lim_{t\to\infty} G(t)$$ and analyse the continuity over $[0, \infty]$	2	L3	CO 1

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5	The position at time $t \geq 0$ of a particle moving along a coordinate line is $s = 10\cos\left(2t^2 + \frac{\pi}{4}\right)$. Find the particle's velocity and acceleration when $t = 0$ s	2	L3	CO1

Course Outcomes:

CO1: Apply foundational concepts of functions, limits, continuity and derivatives to solve various engineering problems.

Marks Distribution										
L1	L2	L3	L4	L5	L6	CO1	CO2	CO3	CO4	CO5
-	-	10	-	-	-	10	-	-	-	-

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	RV University, Bengaluru **School of Computer Science and Engineering** **B.Tech (Hons.)** **Internal Assessment 1** **Answer Keys** **Academic Year 2024-2025**		
Course: Calculus and Laplace Transforms		**Course Code: CS2802**	**Semester : IV**
Date : 10 Feb '25	**Duration: 50 minutes**	**Max Marks: 10**	Set-2

Sl. No	**Answers and marking scheme**	**Marks**
1.	The height H (in meters) of a passenger on a Ferris wheel is given by the function: $$H(t) = 20 + 15\sin\left(\frac{\pi t}{15}\right)$$ where t is the time in minutes after the ride starts, and the Ferris wheel completes one full rotation in 30 minutes. Determine the Domain and Range. **ASNWER** H(t)=20+15sin(πt/15). The function H(t) describes the height of a passenger over time while riding the Ferris wheel. Since the ride lasts for **one full rotation in 30 minutes**, the time ttt is restricted to: 0≤t≤30. Domain is [0,30]. From the function, the sine function oscillates between **-1 and 1.** The minimum value of H(t) occurs when sin(πt/15)=−1, H(minimum)=20+15(−1)=5. The maximum value of H(t) occurs when	**2**

L1- Remember, L2- Understand, L3 – Apply, L4- Analyse, L5 – Evaluate, L6 – Create

	sin(πt/15)=1, H(maximum)=20+15(1)=35. The Range is [5,35].	
2.	The temperature T in degrees Celsius is related to the temperature F in degrees Fahrenheit by the function $T(F) = \dfrac{5(F-32)}{9}$ and the temperature K in Kelvin is related to the temperature T in degrees Celsius by the function $K(T) = T + 273.15$. Express the temperature K in Kelvin as a function of temperature F in degrees Fahrenheit. **ANSWER** To find the composite function $K(F)$, we substitute $T(F)$ into $K(T)$: $$\begin{aligned} K(F) &= K(T(F)) \\ &= T(F) + 273.15 \\ &= \frac{5}{9}(F-32) + 273.15 \\ &= \frac{5}{9}F - \frac{160}{9} + 273.15 \\ &= \frac{5}{9}F + 273.15 - \frac{160}{9} \\ &= \frac{5}{9}F + 273.15 - 17.78 \\ &= \frac{5}{9}F + 255.37 \end{aligned}$$ Thus, the temperature in Kelvin as a function of Fahrenheit is: $$K(F) = \frac{5}{9}F + 255.37$$	**2**
3.	The position of a particle moving along a straight line is given by $s(t) = t^2 - 4t + 3$ where $s(t)$ is the displacement in meters and t is the time in seconds. Calculate the equation of the tangent line to the particle's path at $t = 1$. **ANSWER**	**2**

<table>
<tr><td></td><td>s´(t)=2t−4, s´(1)=−2. Now, we need to find the position of the particle at t=1, s(1)=0, So, the point on the curve at (1.0).

the point−slope form of the equation of a line is: y−y1 =m(x−x1)

the point is (1,0), the slope is −2, y=−2t+2</td><td></td></tr>
<tr><td>4.</td><td>The gross domestic product (GDP), $G(t)$ of a country (in trillions of dollars) at time t years is modeled by: $G(t) = 5(1 - e^{-0.03t})$. Evaluate
$$\lim_{t\to\infty} G(t)$$
and analyse the continuity over $[0, \infty]$

ANSWER

Solution:
1. Finding the Limit as $t \to \infty$:
As $t \to \infty$, the term $e^{-0.03t}$ approaches zero:
$$\lim_{t\to\infty} G(t) = 5\left(1 - \lim_{t\to\infty} e^{-0.03t}\right) = 5(1-0) = 5.$$
Therefore:
$$\lim_{t\to\infty} G(t) = 5.$$
2. Continuity of $G(t)$ on $[0, \infty)$:
The function $G(t) = 5(1 - e^{-0.03t})$ is composed of continuous functions: the exponential function $e^{-0.03t}$, the constant function 1, and their linear combinations. Since these functions are continuous for all real numbers, $G(t)$ is continuous on $[0, \infty)$.</td><td>2</td></tr>
<tr><td>5.</td><td>The position at time $t \geq 0$ of a particle moving along a coordinate line is $s = 10\cos\left(2t^2 + \frac{\pi}{4}\right)$. Find the particle's velocity and acceleration when $t = 0$ s

ANSWER</td><td>2</td></tr>
</table>

ans: Given: $s = 10\cos\left(2t^2 + \frac{\pi}{4}\right)$

$t = 10s$ & $t = 20s$

To find: $v = ?$; $a = ?$

Soln:

$$s = 10\cos\left(2t^2 + \frac{\pi}{4}\right)$$

$$v = \frac{ds}{dt} = \frac{d}{dt}\left(10\cos\left(2t^2 + \frac{\pi}{4}\right)\right)$$

$$= -10\sin\left(2t^2 + \frac{\pi}{4}\right) \cdot 4t$$

$$\Rightarrow v = -40t\sin\left[2t^2 + \frac{\pi}{4}\right]$$

at $t = 0s$

$v_{10} = 0\ ms^{-1}$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[-40t\sin\left[2t^2 + \frac{\pi}{4}\right]\right]$$

$$a = -40\left[\sin\left(2t^2 + \frac{\pi}{4}\right) + \cos\left[2t^2 + \frac{\pi}{4}\right]4t\right]$$

at $t = 0$

$$a = -40\sin\frac{\pi}{4}$$

$$a = -40 \cdot \frac{1}{\sqrt{2}} \Rightarrow a = -20\sqrt{2}\ ms^{-2}$$

SET 3

	School of Computer Science and Engineering **B.Tech (Hons.)** **Internal Assessment -1** **Academic Year** 2024 – 2025	
Course: Calculus and Laplace Transform	**Course Code:** CS2802	**Semester:** IV
Date: 10/02/2025	**Max Marks: 10 Marks**	

Instructions:

- All questions are compulsory.
- Do not write any part of the answer in the question paper.
- Non-programmable calculators are allowed
- **Please be seated in your allotted seat 10 minutes before the exam, i,e., by 8:50AM.**

S.No	Question	Marks	Level	CO
1	A farmer is designing a rectangular garden where the number of plants y, depends on the continuous arrangement of rows, represented by the variable x, where x can take any positive real number value. The equation that models the number of plants is $$f(x) = \frac{x^2 - x + 1}{x^2 + x + 1}$$ Determine the possible values of x (the domain) and the possible number of plants y (the range) that the farmer can have in the garden, assuming that x can be any positive real number.	2	L3	CO 1
2	Given the function $f(x) = 2x^3 - 4$, determine a function $g(x)$ such that $(f \circ g)(x) = x + 2$	2	L3	CO 1
3	A cricket ball is rolling along the pitch, and its position at any time t (in seconds) is given by $s(t) = t^{\frac{3}{2}} - 4t^{\frac{1}{2}} + 3$ where $s(t)$ represents the displacement in meters. Find the equation of the tangent line to the ball's path at $t = 4$.	2	L3	CO 1
4	A scientist is studying the behavior of a machine that operates based on periodic signals. The efficiency of the machine at any given time x (measured in hours) depends on the function: $$f(x) = \begin{cases} \frac{\sin \lceil x \rceil}{\lceil x \rceil} & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$$ where $\lceil x \rceil$ denotes the greatest integer less than or equal to x. Help the scientist to examine whether the efficiency function is continuous at integer values of $x = 0$.	2	L3	CO 1

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5	A cricket analyst is studying the trajectory of a ball hit by a batsman. The height y of the ball at any horizontal distance x from the batsman is modeled by the equation $y = \left(x + \sqrt{(1+x^2)}\right)^2$. Find $\frac{dy}{dx}$, which represents the instantaneous slope of the ball's trajectory, helping determine when the ball will reach its peak and when it will start descending.	2	L3	CO1

Course Outcomes:

CO1: Apply foundational concepts of functions, limits, continuity and derivatives to solve various engineering problems.

Marks Distribution										
L1	L2	L3	L4	L5	L6	CO1	CO2	CO3	CO4	CO5
-	-	10	-	-	-	10	-	-	-	-

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RV University, Bengaluru

School of Computer Science and Engineering

B.Tech (Hons.)

Internal Assessment 1

Answer Keys

Academic Year 2024-2025

Course: Calculus and Laplace Transforms		**Course Code: CS2802**	**Semester : IV**
Date : 10 Feb '25	**Duration: 50 minutes**	**Max Marks: 10**	Set-3

Sl. No	**Answers and marking scheme**	**Marks**
1.	A farmer is designing a rectangular garden where the number of plants y, depends on the continuous arrangement of rows, represented by the variable x, where x can take any positive real number value. The equation that models the number of plants is $$f(x) = \frac{x^2 - x + 1}{x^2 + x + 1}$$ Determine the possible values of x (the domain) and the possible number of plants y (the range) that the farmer can have in the garden, assuming that x can be any positive real number. **ANSWER:**	**2**

	Domain of the function:- put the denominator = 0 $x^2+x+1=0$ Since $D = b^2-4ac = 1-4 = -ve$ (i.e. <0) it has no real solution $\Rightarrow$ domain $= \mathbb{R}$ Range of the function:- $f'(x) = \dfrac{(x^2+x+1)(x^2-x+1)' - (x^2-x+1)(x^2+x+1)'}{(x^2+x+1)^2}$ $= \dfrac{(x^2+x+1)(2x-1) - (x^2-x+1)(2x+1)}{(x^2+x+1)^2}$ $= \dfrac{(2x^3+2x^2+2x-x^2-x-1)-(2x^3-2x^2+2x+x^2-x+1)}{(x^2+x+1)^2}$ $= \dfrac{\cancel{2x^3}+2x^2+\cancel{2x}-x^2-\cancel{x}-\cancel{1}-\cancel{2x^3}+2x^2-\cancel{2x}-x^2+\cancel{x}-\cancel{1}}{(x^2+x+1)^2}$ Now $f'(x)=0$ (to find critical points) $\Rightarrow 4x^2-2x^2-2=0$ $\Rightarrow 2x^2-2=0 \Rightarrow x=\pm 1$ Now $f(1)=\frac{1}{3}$ and $f(-1)=3$ so range $= [\frac{1}{3}, 1]$ Answer	
2.	Given the function $f(x) = 2x^3 - 4$, determine a function $g(x)$ such that $(f \circ g)(x) = x + 2$ **ANSWER**	**2**

	ans: Given: $f(x) = 2x^3 - 4$ $f(g(x)) = x + 2$ To find: $g(x) = ?$ Solution: $f(g(x)) = x + 2 \quad \dots (1)$ $f(x) = 2x^3 - 4 \quad \dots (2)$ From (1) & (2) we get $f(g(x)) = 2[g(x)]^3 - 4 = x + 2$ $2[g(x)]^3 = x + 6$ $g(x) = \left(\frac{x+6}{2}\right)^{1/3}$	
3.	A cricket ball is rolling along the pitch, and its position at any time t (in seconds) is given by $s(t) = t^{\frac{3}{2}} - 4t^{\frac{1}{2}} + 3$ where $s(t)$ represents the displacement in meters. Find the equation of the tangent line to the ball's path at $t = 4$. **ANSWER**	**2**

L1- Remember, L2- Understand, L3 – Apply, L4- Analyse, L5 – Evaluate, L6 – Create

Ques) Since $s = t^{3/2} - 4t^{1/2} + 3$

then $\frac{ds}{dt} = \frac{3}{2} t^{1/2} - \frac{4}{2} t^{-1/2} = \frac{3}{2} t^{1/2} - 2t^{-1/2}$

$\frac{ds}{dt}\Big|_{t=4} = \frac{3}{2}(4)^{1/2} - 2(4)^{-1/2}$

$= \frac{3}{2} \times 2 - \frac{2}{2} = 2$

Now equation of tangent for (x_0, y_0)

$y - y_0 = \frac{dy}{dx}\Big|_{x=x_0} (x - x_0)$

thus when $t=4$, $s=3 \Rightarrow (t,s) = (4,3)$

eqn:

$s - 3 = 2(t-4)$

$s - 3 = 2t - 8$

$s = 2t - 8 + 3$

$s = 2t - 5$ (Answer)

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4. A scientist is studying the behavior of a machine that operates based on periodic signals. The efficiency of the machine at any given time x (measured in hours) depends on the function:

$$f(x) = \begin{cases} \dfrac{\sin \lceil x \rceil}{\lceil x \rceil} & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$$

where $\lceil x \rceil$ denotes the greatest integer less than or equal to x. Help the scientist to examine whether the efficiency function is continuous at integer values of $x = 0$. **2**

ANSWER

	LHL: $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} \frac{\sin[0-h]}{[0-h]} = \frac{\sin(-1)}{-1} = \sin 1$ RHL $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} \frac{\sin[0+h]}{[0+h]} = \frac{\sin 0}{0}$ N.D. Hence $f(x)$ is not continuous at $x=0$.	
5.	A cricket analyst is studying the trajectory of a ball hit by a batsman. The height y of the ball at any horizontal distance x from the batsman is modeled by the equation $y = \left(x + \sqrt{(1+x^2)}\right)^2$. Find $\frac{dy}{dx}$, which represents the instantaneous slope of the ball's trajectory, helping determine when the ball will reach its peak and when it will start descending. **ANSWER** $\frac{dy}{dx} = 2\left(x+\sqrt{1+x^2}\right)\cdot\left(1+\frac{1}{2\sqrt{1+x^2}}\cdot 2x\right)$ Ans.	2

L1- Remember, L2- Understand, L3 – Apply, L4- Analyse, L5 – Evaluate, L6 – Create