| | | |
|---|---|---|
| RV UNIVERSITY — Go, change the world — an initiative of RV EDUCATIONAL INSTITUTIONS | USN: | |

# School of Computer Science and Engineering

**B.Tech (Hons.)**

## CIE - 2

**Academic Year** 2024 – 2025

| **Course:** Calculus and Laplace Transform | **Course Code:** CS2802 | **Semester:** IV |
|---|---|---|
| **Date:** 14 March 2025 | **Duration: 1.5hrs** | **Max Marks: 25 Marks** |

## Instructions:

- All questions are compulsory.
- Do not answer any part of the answers in the question paper.
- For MCQs, write the letter choice and the answer.
- Non-programmable calculators are allowed.

### Part-A (5 Marks)

| S.No | Question | Marks | Level | CO |
|---|---|---|---|---|
| 1 | The area enclosed by the curve $f(x) = \sin(x)$ with the $x$-axis over $[-\pi, \pi]$ is<br>**a)** 0 **c)** 2<br>**b)** 1 **d)** $\pi$ | 1 | L2 | CO2 |
| 2 | Evaluate the integral $\int_0^1 \int_0^x dxdy$. | 2 | L2 | CO2 |
| 3 | Calculate the volume of a cube of sides $a$ located in the first octant with one corner at the origin using triple integration. | 2 | L3 | CO2 |

## Part-B (20 Marks)

| S.No | Question | Marks | Level | CO |
|---|---|---|---|---|
| 4 | **a)** Find the first three terms of the Taylor series for the function $\sin(\pi x)$ centered at $a = 1$. **[3]**<br>**b)** Use your answer to find an approximate value of $\sin\left(\pi + \frac{\pi}{10}\right)$. **[2]** | 5 | L3 | CO2 |
| 5 | For the function, $f(x) = x^3 - 2x^2 + x + 1$, over the interval $[-1, 3]$<br>**a)** Calculate the critical points and point(s) of inflection(s) (if any). **[2]**<br>**b)** Determine the concavity of the function at each critical points. **[1]**<br>**c)** Sketch a neat graph of the function using these information. **[2]** | 5 | L3 | CO2 |
| 6 | Apply double integration to calculate the area lying between the parabola, $y = 6 - x^2$ and the line $y = x$. **[5]** | 5 | L3 | CO2 |
| 7 | Given the surface, $f(x, y) = (x-2)^2 - (y-1)^2 + 2$<br>**a)** Evaluate: $f_x, f_{xx}, f_y,$ and $f_{yy}$. **[2]**<br>**b)** Calculate the critical point of $f(x, y)$. **[1]**<br>**c)** Determine the nature of the critical point(s) using the Hessian. **[2]** | 5 | L3 | CO3 |

| **Course Outcomes** | |
|---|---|
| **CO1:** | Understand foundational concepts of functions, limits, continuity and derivatives to solve various engineering problems. |
| **CO2:** | Apply derivatives and integrations in problems including rate of change, surface areas, volumes, probabilities, and single variable optimization. |
| **CO3:** | Solve linear approximations and unconstrained optimization problems related to engineering applications. |
| **CO4:** | Apply Laplace transform to solve problems on time varying signals. |

| **Marks Distribution** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| L1 | L2 | L3 | L4 | L5 | L6 | CO1 | CO2 | CO3 | CO4 |
| - | 3 | 22 | - | - | - | - | 20 | 5 | - |

**End**

| | | |
|---|---|---|
| RV UNIVERSITY<br>Go, change the world<br>an initiative of RV EDUCATIONAL INSTITUTIONS | USN: |  |

**School of Computer Science and Engineering**

**B.Tech (Hons.)**

**CIE - 2**

**Academic Year** 2024 – 2025

| **Course:** Calculus and Laplace Transform | **Course Code:** CS2802 | **Semester:** IV |
|---|---|---|
| **Date:** 13 Mar. 2025 | **Duration: 1.5hrs** | **Max Marks: 10 Marks** |

# Answer Keys:

## Part-A (5 Marks)

| S.No | Question | Marks | Level | CO |
|---|---|---|---|---|
| 1 | The area enclosed by the curve $f(x) = \sin(x)$ with the x-axis over $[-\pi, \pi]$ is<br>**a)** 0 **c)** 2<br>**b)** 1 **d)** $\pi$<br>**Answer: a) 0** | 1 | L2 | CO2 |
| 2 | $\begin{aligned}\int_0^1\int_0^x dxdy &= \int_0^1 dx\left[\int_0^x dy\right]\\ &= \int_0^1 dx\,[y]_0^x\\ &= \int_0^1 xdx = \left[\frac{x^2}{2}\right]_0^1\\ &= \mathbf{\frac{1}{2}}\end{aligned}$ | 2 | L2 | CO2 |

*Continued on next page...*

| S.No | Question | Marks | Level | CO |
|---|---|---|---|---|
| 3 | The volume of the cube in the first octant with one corner at the origin can be written as: $$\begin{aligned} V &= \int_0^a\int_0^a\int_0^a dzdydx \\ &= \int_0^a\int_0^a [z]_0^a dydx = \int_0^a\int_0^a adydx \\ &= a\int_0^a [y]_0^a dx = a\int_0^a adx = a^2\int_0^a dx \\ &= \mathbf{a^3} \end{aligned}$$ Note: The integration can be done in any order of $dx$, $dy$ and $dz$. | 2 | L3 | CO2 |

## Part-B (20 Marks)

| S.No | Question | Marks | Level | CO |
|---|---|---|---|---|
| 4 | **a)** The Taylor series of a function $f(x)$ centered at $a$ is given by: $$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$$ We are asked to find the first three terms for $f(x) = \sin(\pi x)$ around $a = 1$. <br>• First derivative: $$f'(x) = \pi\cos(\pi x)$$ • Second derivative: $$f''(x) = -\pi^2\sin(\pi x)$$ • $f(1) = \sin(\pi) = 0$ <br>• $f'(1) = \pi\cos(\pi) = \pi(-1) = -\pi$ <br>• $f''(1) = -\pi^2\sin(\pi) = 0$ $$f(x) \approx f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2$$ $$f(x) \approx 0 - \pi(x-1) + 0$$ Thus, the first three terms of the Taylor series for $\sin(\pi x)$ centered at $a = 1$ are: $$f(x) = \sin(\pi x) \approx -\pi(x-1)$$ **b)** We know that: $$\sin\left(\pi + \frac{\pi}{10}\right) = \sin\left(\pi(1 + \frac{1}{10})\right) \approx -\pi(1 + \frac{1}{10} - 1)$$ So, $$\sin\left(\pi + \frac{\pi}{10}\right) \approx -\frac{\pi}{10}$$ | 5 | L3 | CO2 |

*Continued on next page...*

| S.No | Question | Marks | Level | CO |
|---|---|---|---|---|
| 5 | For the function, $f(x) = x^3 - 2x^2 + x + 1$, over the interval $[-1, 3]$<br>**a)** Calculate the critical point(s) and point(s) of inflection(s) (if any). **[2]**<br>**b)** Determine the concavity of the function. **[1]**<br>**c)** Sketch a neat graph of the function using these informations. **[2]** | 5 | L3 | CO2 |

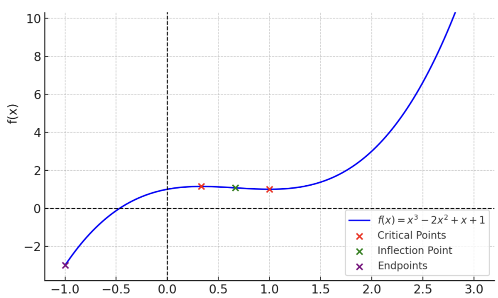
**Answer:** for critical points;

$$
\begin{aligned}
f'(x) &= \frac{d}{dx}(x^3 - 2x^2 + x + 1) = 0 \\
\implies \quad & 3x^2 - 4x + 1 = 0 \\
\implies \quad & (x-1)\left(x - \frac{1}{3}\right) = 0
\end{aligned}
$$

So, the critical points are $x = 1$ and $x = \frac{1}{3}$. Moreover, $f(x)$ is increasing on $(-1, \frac{1}{3})$ and $(1, 3)$ and decreasing on $(\frac{1}{3}, 1)$. At $x = \frac{1}{3}$ and at $x = 1$, function will have maximum and minimum values respectively, and corresponding values are $(1, 1)$ and $(\frac{1}{3}, \frac{27}{31})$.

$$
\begin{aligned}
\text{For inflection point, } f''(x) &= \frac{d}{dx}(3x^2 - 4x + 1) \\
\implies 6x - 4 &= 0 \\
\implies x &= \frac{2}{3} \\
\text{Corresponding point is: } & \left(\frac{2}{3}, \frac{29}{27}\right)
\end{aligned}
$$

$f(x)$ is concave up where $f''(x) > 0$, and $f(x)$ is concave down where $f''(x) < 0$. So, $f(x)$ is concave down on $\left(-1, \frac{2}{3}\right)$ and concave up on $\left(\frac{2}{3}, 2\right)$.




*Continued on next page...*

| S.No | Question | Marks | Level | CO |
|---|---|---|---|---|
| 6 | Apply double integration and calculate the area lying between the curves, the parabola, $y = 6 - x^2$ and the line $y = x$. [**5**]<br><br>**Answer:** We need to find the area enclosed by the parabola $y = 6 - x^2$ and the line $y = x$.<br>$\implies$ First, find their points of intersection by solving:<br>$$6 - x^2 = x$$<br>$$x^2 + x - 6 = 0$$<br>$$(x+3)(x-2) = 0$$<br>So, the curves intersect at: $x = -3 \quad \text{and} \quad x = 2$<br>$\implies$ The area can be computed using a double integral:<br>$$A = \iint_R dA$$<br>We describe the region $R$ using the boundaries of the curves:<br>• $x$ ranges from $-3$ to $2$.<br>• For a fixed $x$, $y$ ranges from the line $y = x$ to the parabola $y = 6 - x^2$.<br><br>Thus, the area is:<br>$$\begin{aligned} A &= \int_{x=-3}^{2} \int_{y=x}^{6-x^2} dydx \\ &= \int_{x=-3}^{2} [y]_{y=x}^{6-x^2} dx = \int_{x=-3}^{2} [6 - x^2 - x]dx \\ &= [6x - \frac{x^3}{3} - \frac{x^2}{2}]_{-3}^{2} = [12 - \frac{8}{3} - \frac{4}{2}] - [-18 + \frac{27}{3} - \frac{9}{2}] \\ &= [10 - \frac{8}{3}] - [-9 - \frac{9}{2}] \\ &= \frac{125}{6} \text{ sq. unit} \end{aligned}$$ | 5 | L3 | CO2 |

*Continued on next page...*

| S.No | Question | Marks | Level | CO |
|---|---|---|---|---|
| 7 | Given the surface, $f(x,y) = (x-2)^2 - (y-1)^2 + 2$ | 5 | L3 | CO3 |

**a)** Evaluate: $f_x, f_{xx}, f_y,$ and $f_{yy}$. [**2**]

**Answer:**

$$
\begin{aligned}
f(x,y) &= (x-2)^2 - (y-1)^2 + 2 \\
f_x &= 2(x-2) \\
f_x &= -2(y-1) \\
f_{xx} &= 2 \\
f_{yy} &= -2
\end{aligned}
$$

**b)** Calculate the critical point of $f(x,y)$. [**1**]

**Answer:** At critical point:

$$
\begin{aligned}
f_x &= 0 \; ; f_y = 0 \\
\Rightarrow 2(x-2) &= 0 \; ; -2(y-1) = 0 \\
\Rightarrow x = 2 \quad &; \quad y = 1
\end{aligned}
$$

Therefore the critical point is $(\mathbf{2}, \mathbf{1})$

**c)** Determine the concavity of the critical point from the Hessian. [**2**]

**Answer:** The Hessian matrix is given by:

$$
\begin{aligned}
H &= \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} \\
&= \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}
\end{aligned}
$$

The determinant of the Hessian (symmetric matrix) is

$$
H = \begin{vmatrix} 2 & 0 \\ 0 & -2 \end{vmatrix} = \mathbf{-4}
$$

Thus, since the determinant of the Hessian is $< 0$, at the critical point, the critical point is a **saddle point**.

*Continued on next page...*

| S.No | Question | Marks | Level | CO |
|---|---|---|---|---|

*Continued on next page...*

**End**