SET-1

RV UNIVERSITY — Go, change the world — an initiative of RV EDUCATIONAL INSTITUTIONS	USN:

School of Computer Science and Engineering

B.Tech (Hons.)

CIE-3

Academic Year 2024 – 2025

Course: Calculus and Laplace Transform	**Course Code:** CS2802	**Semester:** IV
Date: 11 April 2025	**Duration: 60 Minutes**	**Max Marks: 15 Marks**

Instructions:

- All questions are compulsory.
- **Write your USN in the space provided** as soon as you get the question paper.
- Do not answer any part of the answers in the question paper.
- Do not engage in any unfair practices.
- Non-programmable calculators are allowed.

S.No	**Question**	**Marks**	**Level**	**CO**
1	**a)** Calculate the gradient of the function $f(x, y) = x^3 + 4xy^2 - x$. [**2**] **b)** Starting from an initial guess of $(0, 0.25)$, evaluate the first two steps of the gradient descent method for the function. Take the step-size (learning rate) to be 0.1 [**3**]	5	L3	CO 3
2	**a)** Let $f(x, y) = e^x \sin(y)$. Find the **second-order Taylor series expansion** of $f(x, y)$ about the point $(0, 0)$.[**3**] **b)** Use the expansion to approximate the value of $f(0.1, 0.2)$ up to second-order terms and also determine the tangent plane at the point. [**2**]	5	L3	CO 3
Continued on next page...				

3	**a)** Evaluate the Laplace transform of $f(t) = e^{2t}\cosh 2t$ where $\cosh(2t)$ is the hyperbolic cosine function. [**2**] **b)** Using the properties of linearity of Laplace Transforms calculate the Laplace transform of the function: . [**3**] $$f(t) = 2t^2 - \sin(t) + 4e^{-2t}; \quad 0 \le t < \infty$$	5	L3	CO 4

Course Outcomes	
CO1:	Understand foundational concepts of functions, limits, continuity and derivatives to solve various engineering problems.
CO2:	Apply derivatives and integrations in problems including rate of change, surface areas, volumes, probabilities, and single variable optimization.
CO3:	Solve linear approximations and unconstrained optimization problems related to engineering applications.
CO4:	Apply Laplace transform to solve problems on time varying signals.

Marks Distribution									
L1	L2	L3	L4	L5	L6	CO1	CO2	CO3	CO4
-	-	15	-	-	-	-	-	10	5

End

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	School of Computer Science and Engineering **B.Tech (Hons.)** **CIE-3** **Academic Year** 2024 – 2025	
Course: Calculus and Laplace Transform	**Course Code:** CS2802	**Semester:** IV
Date: 11 April 2025	**Duration: 45 Minutes**	**Max Marks: 15 Marks**

Answer Keys:

Q.1)

a) Calculate the gradient of the function $f(x, y) = x^3 + 4xy^2 - x$. [**2**]

Solution:

$$\begin{aligned} f(x,y) &= x^3 + 4xy^2 - x \\ \Rightarrow \nabla f &= \begin{bmatrix} 3x^2 + 4y^2 - x \\ 8xy \end{bmatrix} \end{aligned}$$

b) Starting from an initial guess of $(0, 0.25)$, evaluate the first two steps of the gradient descent method for the function. Take the step-size (learning rate) to be 0.1 [**3**]

Solution: Given the initial guess point, $P_0 = (0, 0.25) = (0, 1/4)$, the gradient descent technique is given by:

$$\begin{aligned} P_{n+1} &= P_n - \alpha \nabla f(P_n) \\ P_0 &= \begin{bmatrix} 0 \\ 1/4 \end{bmatrix} \\ \nabla f(P_0) &= \begin{bmatrix} \frac{4}{4^2} - 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{-3}{4} \\ 0 \end{bmatrix} \\ \therefore P_1 &= P_0 - 0.1 \nabla f(P_0) \\ P_1 &= \begin{bmatrix} 0 \\ \frac{1}{4} \end{bmatrix} - \frac{1}{10} \begin{bmatrix} \frac{-3}{4} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{3}{40} \\ \frac{1}{4} \end{bmatrix} \end{aligned}$$

Now,

$$\begin{aligned} P_2 &= P_1 - \frac{1}{10} \nabla f\left(\frac{3}{40}, \frac{1}{4}\right) \\ P_2 &= \begin{bmatrix} \frac{3}{40} \\ \frac{1}{4} \end{bmatrix} - \frac{1}{10} \begin{bmatrix} -0.73 \\ 0.15 \end{bmatrix} = \begin{bmatrix} 0.148 \\ 0.235 \end{bmatrix} \end{aligned}$$

Q2).:

a) Let $f(x,y) = e^x \sin(y)$. Find the **second-order Taylor series expansion** of $f(x,y)$ about the point $(0,0)$.[**3**]

SOlution:

Taylor formula for $f(x,y)$ about a point (x_0, y_0) up to 2^{nd} order is:

$$\begin{aligned} f(x,y) &= f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0) + \\ & \frac{1}{2}\left(f_{xx}(x_0,y_0)(x-x_0)^2 + 2f_{xy}(x_0,y_0)(x-x_0)(y-y_0) + f_{yy}(x_0,y_0)(y-y_0)^2\right) \end{aligned}$$

We have:

$$\begin{aligned} f(0,0) &= 0 \\ f_x &= e^x \sin(y); \quad \Rightarrow f_x(0,0) = 0 \\ f_{xx} &= e^x \sin y; \quad \Rightarrow f_{xx}(0,0) = 0 \\ f_y &= e^x \cos(y); \quad \Rightarrow f_y(0,0) = 1 \\ f_{yy} &= -e^x \sin y \quad \Rightarrow f_{yy}(0,0) = 0 \\ f_{xy} &= e^x \cos y \quad \Rightarrow f_{xy}(0,0) = 1 \end{aligned}$$

Thus, the Taylor formula (Maclaurin) is:

$$\mathbf{f(x,y) = y + xy}$$

b) Use the expansion to approximate the value of $f(0.1, 0.2)$ up to second-order terms and also determine the tangent plane at the point.[**2**]

Solution:

From the above, the quadratic approximation of the surface at $(0,0)$ is given by:

$$f(x,y) = y + xy$$

.

Using this, the approximate value of the function at $(0.1, 0.2)$ is:

$$f(0.1, 0.2) = 0.2 + 0.1 \times 0.2 = \mathbf{0.22}$$

From the original function:

$$f(0.1, 0.2) = e^{0.1} \sin 0.2 = \mathbf{0.2196}$$

Knowing that the quadratic approximation is $f(x,y) = y + xy$, the Tangent plane is given by the Taylor formula up to the linear term. Hence:

$$\mathbf{T(x,y) = y}$$

Q 3).

a) Evaluate the Laplace transform of $f(t) = e^{2t}\cosh 2t$ where $\cosh(2t)$ is the hyperbolic cosine function. [**2**]

Answer: We know:

$$\mathcal{L}\{\cosh(2t)\} = \frac{s}{s^2 - 4}$$

Using the shift property:

$$\mathcal{L}\{e^{2t}\cosh(2t)\} = \left.\frac{s}{s^2-4}\right|_{s\to s-2} = \frac{s-2}{(s-2)^2-4}, \quad s > 4$$

b) Using the properties of linearity of Laplace Transforms calculate the Laplace transform of the function: . [**3**]

$$f(t) = 2t^2 - \sin(t) + 4e^{-2t}; \quad 0 \le t < \infty$$

Answer: By Applying the Linearity property

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{2t^2\} - \mathcal{L}\{\sin(t)\} + \mathcal{L}\{4e^{-2t}\}$$

$$\mathcal{L}\{f(t)\} = 2\mathcal{L}\{t^2\} - \mathcal{L}\{\sin(t)\} + 4\mathcal{L}\{e^{-2t}\}$$

and then using the Standard Laplace Transforms

$$\begin{aligned}\mathcal{L}\{t^2\} &= \tfrac{2}{s^3}\\ \mathcal{L}\{\sin(t)\} &= \tfrac{1}{s^2+1}\\ \mathcal{L}\{e^{-2t}\} &= \tfrac{1}{s+2}\end{aligned}$$

Substituting and then simplifying this

$$\begin{aligned}\mathcal{L}\{f(t)\} &= 2\cdot\tfrac{2}{s^3} - \tfrac{1}{s^2+1} + 4\cdot\tfrac{1}{s+2}\\ &= \tfrac{4}{s^3} - \tfrac{1}{s^2+1} + \tfrac{4}{s+2}\end{aligned}$$

End

SET-2

RV UNIVERSITY Go, change the world an initiative of RV EDUCATIONAL INSTITUTIONS	**USN:** **School of Computer Science and Engineering** **B.Tech (Hons.)** **CIE-3** **Academic Year** 2024 – 2025

Course: Calculus and Laplace Transform	**Course Code:** CS2802	**Semester:** IV
Date: 11 April 2025	**Duration: 60 Minutes**	**Max Marks: 15 Marks**

Instructions:

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- Non-programmable calculators are allowed.

S.No	Question	Marks	Level	CO
1	Using the gradient descent technique, minimize the function $$f(x,y) = (x-2)^2 + (y-3)^2$$ starting from the initial guess $(x_0, y_0) = (0,0)$ and using a learning rate $\alpha = 0.5$. Show that the method converges to the minimum in two steps.	5	L3	CO 3
2	**a)** Evaluate the Taylor series of the function $g(x,y) = \ln(x+y)$ about the point $P_0 = (1,1)$ up to second order in x and y.**[3]** **b)** Using the Taylor formula calculated above, deduce the tangent plane of the function about $(1,1)$ and obtain the approximate value of $g(1.1, 1.1)$ using it. [**2**]	5	L3	CO 3

Continued on next page...

3	**a)** Evaluate the Laplace transform of $f(t) = e^{2t}\sinh 2t$ where $\sinh(2t)$ is the hyperbolic sine function. [**2**] **b)** Using the properties of linearity of Laplace Transforms calculate the Laplace transform of the function: . [**3**] $$f(t) = 4t^2 – 3\cos(t) + 5e^{-t}$$	5	L3	CO 4

Course Outcomes	
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L1	L2	L3	L4	L5	L6	CO1	CO2	CO3	CO4
-	-	15	-	-	-	-	-	10	5

End

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Course: Calculus and Laplace Transform	**Course Code:** CS2802	**Semester:** IV
Date: 11 April 2025	**Duration: 45 Minutes**	**Max Marks: 15 Marks**

Answer Keys:

Q 1). Using the gradient descent technique, minimize the function

$$f(x, y) = (x-2)^2 + (y-3)^2$$

starting from the initial guess $(x_0, y_0) = (0, 0)$ and using a learning rate $\alpha = 0.5$. Show that the method converges to the minimum in two steps.

Solution:

$$\begin{aligned} f(x,y) &= (x-2)^2 + (y-3)^2 \\ \Rightarrow \nabla f &= \begin{bmatrix} 2(x-2) \\ 2(y-3) \end{bmatrix} \end{aligned}$$

Given the initial guess point, $P_0 = (0, 0)$, the gradient descent technique is given by:

$$\begin{aligned} P_{n+1} &= P_n - \alpha \nabla f(P_n) \\ P_0 &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ \nabla f(P_0) &= \begin{bmatrix} -4 \\ -6 \end{bmatrix} \\ \therefore P_1 &= P_0 - 0.5 \nabla f(P_0) \\ P_1 &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} -4 \\ -6 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} P_2 &= P_1 - \frac{1}{2} \nabla f\,(2,3) \\ P_2 &= \begin{bmatrix} 2 \\ 3 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \end{aligned}$$

We observe that $P_2 = P_1$, which means that the iteration is no longer changing and convergence is reached and $(2,3)$ is the minima. This can also be easily seen from the first derivative test that $f(x,y)$ has minima at $(2,3)$

Q 2).

a) Evaluate the Taylor series of the function $g(x,y) = \ln(x+y)$ about the point $P_0 = (1,1)$ up to second order in x and y.[**3**]

Solution:

Taylor series of $g(x,y)$ up to second order about (x_0, y_0) is given by:

$$\begin{aligned} g(x,y) &= g(x_0,y_0) + g_x(x_0,y_0)(x-x_0) + g_y(x_0,y_0)(y-y_0) + \\ & \frac{1}{2}\left(g_{xx}(x_0,y_0)(x-x_0)^2 + 2g_{xy}(x_0,y_0)(x-x_0)(y-y_0) + g_{yy}(x_0,y_0)(y-y_0)^2\right) \end{aligned}$$

For $g(x,y) = \ln(x+y)$, and $(x_0,y_0) = (1,1)$, we have,

$$\begin{aligned} g(1,1) &= \ln 2 \approx 0.6931 \\ g_x &= \frac{1}{x+y} \quad \Rightarrow g_x(1,1) = \frac{1}{2} \\ g_y &= \frac{1}{x+y} \quad \Rightarrow g_y(1,1) = \frac{1}{2} \\ g_{xx} &= \frac{-1}{(x+y)^2} \quad \Rightarrow g_{xx}(1,1) = \frac{-1}{4} \\ g_{yy} &= \frac{-1}{(x+y)^2} \quad \Rightarrow g_{yy}(1,1) = \frac{-1}{4} \\ g_{xy} &= \frac{-1}{(x+y)^2} \quad \Rightarrow g_{xy}(1,1) = \frac{-1}{4} \end{aligned}$$

Therefore, the Taylor Formula for $g(x,y) = \ln(x+y)$ about $(1,1)$ up to quadratic term is:

$$\begin{aligned} g(x,y) &= \ln 2 + \frac{1}{2}(x-1) + \frac{1}{2}(y-1) + \frac{1}{2}\left(\frac{-1}{4}(x-1)^2 + 2\times\frac{-1}{4}(x-1)(y-1) - \frac{1}{4}(y-1)^2\right) \\ \Rightarrow g(x,y) &= x + y - \frac{1}{8}\left(x^2 + y^2 - \frac{3}{2} + \ln 2\right) \end{aligned}$$

b) Using the Taylor formula calculated above, deduce the tangent plane of the function about $(1,1)$ and obtain the approximate value of $g(1.1, 1.1)$ using it. [**2**] From the Taylor formula in (a), the Tangent plane about $(1,1)$ is given by:

$$T(x,y) = \ln 2 + \frac{1}{2}(x-1) + \frac{1}{2}(y-1)$$

The linear approximation at $(1.1, 1.1)$ is:

$$\mathbf{T(1.1, 1.1) \approx 0.7931}$$

.

The actual value of the function at $(1.1,1.1)$ is $g(1.1,1.1) = \ln(1.1+1.1) = \ln 2.2 \approx \mathbf{0.7885}$.

Q3).

a) Evaluate the Laplace transform of $f(t) = e^{2t}\sinh 2t$ where $\sinh(x)$ is the hyperbolic sine function. [**2**]

Answer: We know:

$$\mathcal{L}\{\sinh(2t)\} = \frac{2}{s^2-4}$$

Using the shift property:

$$\mathcal{L}\{e^{2t}\sinh(2t)\} = \left.\frac{2}{s^2-4}\right|_{s\to s-2} = \frac{2}{(s-2)^2-4}, \quad s > 4$$

b) Using the properties of linearity of Laplace Transforms calculate the Laplace transform of the function: . [**3**]

$$f(t) = 4t^2 – 3\cos(t) + 5e^{-t}$$

Answer: By Applying the Linearity property

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{4t^2\} - \mathcal{L}\{3\cos(t)\} + \mathcal{L}\{5e^{-t}\}$$

$$\mathcal{L}\{f(t)\} = 4\mathcal{L}\{t^2\} - 3\mathcal{L}\{\cos(t)\} + 5\mathcal{L}\{e^{-t}\}$$

and then using the Standard Laplace Transforms

$$\begin{aligned} \mathcal{L}\{t^2\} &= \tfrac{2}{s^3} \\ \mathcal{L}\{\cos(t)\} &= \tfrac{s}{s^2+1} \\ \mathcal{L}\{e^{-t}\} &= \tfrac{1}{s+1} \end{aligned}$$

Substituting and then simplifying this

$$\begin{aligned} \mathcal{L}\{f(t)\} &= 4\cdot\tfrac{2}{s^3} - 3\tfrac{s}{s^2+1} + 5\cdot\tfrac{1}{s+1} \\ &= \tfrac{8}{s^3} - \tfrac{3s}{s^2+1} + \tfrac{5}{s+1} \end{aligned}$$

End