Unit-4 : Algebraic Structures

* **Algebraic System:-**

Let 'S' be a set and +, * are two binary operators then the algebraic system is denoted as $<S,+>$ (or) $<S,+,*>$ (or) $<S,*>$

* **Properties of Algebraic System:-**

Let 'S' denotes a set of positive integers and a, b, c are three integers, * and + are two Binary operators then the following are the properties of algebraic System.

1.* **Closure Property:-**

Let 'S' be a set of all positive integers.

(i) $a+b \in S$ [wrt Addition]

(ii) $a*b \in S$ [wrt Multiplication]

2. **Associative Property:-**

(i) $a+(b+c) = (a+b)+c$

(ii) $a*(b*c) = (a*b)*c$

3. **Identity Property:-**

(i) $a+e = e+a = a$

(ii) $a*e = e*a = a$

where $\{e = 0, 1\}$

'0' is an Identity element wrt Binary Addition

'1' is " " " " Binary Multiplication

4. Inverse Property:-

(i) $a * a^{-1} = a^{-1} * a = e$

(ii) $a + (-a) = (-a) + a = e$ $\{e = 0, 1\}$

→ a^{-1} is the multiplicative inverse of a
$-a$ is the Additive inverse of a.

5. Commutative Property:-

(i) $a * b = b * a$

(ii) $a + b = b + a$

6. Distributive Property:-

i) $a * (b + c) = (a * b) + (a * c)$

ii) $a + (b * c) = (a + b) * (a + c)$

* Group:-

→ Let $<S, +>$ (or) $<S, *>$ are two Algebraic Systems then the algebraic Systems $<S, +>$ or $<S, *>$ is said to be a 'Group'. If it can satisfy the following properties

1) Closure Property
2) Associative Property
3) Identity Property
4) Inverse Property

* **Semi-group:-**

Let $<S,+>$ or $<S,*>$ are two algebraic systems then the algebraic systems $<S,+>$ (or) $<S,*>$ is said to be a "Semi-group" if it can satisfies the following properties:

1. Closure Property
2. Associative Property

* **Monoid:-** Let $<S,+>$ or $<S,*>$ are two algebraic systems then the algebraic systems $<S,+>$ or $<S,*>$ is said to be Monoid, if it can satisfies the following properties

   i, Closure
   ii, Associative
   iii, Identity

* **Abelian Group:-** Let $<S,+>$ or $<S,*>$ are two algebraic systems then the algebraic systems $<S,+>$ or $<S,*>$ is said to be 'Abelian group', if it can satisfies the following properies.

   i) Closure Property
   ii) Associative "
   iii) Identity "
   iv) Inverse "
   v) Commutative property

* **Sub-group** :- Let $<S, +>$ and $<G, +'>$ are two groups then $<S, +'>$ is Said to be a Sub-group of $<G, +>$, if it Can Satisfy all The properties of $<G, +>$

* **Homo-morphism** :- Let $<S, +>$ and $<S, *>$ are two algebraic systems then a mapping from one algebraic system to another algebraic System is Known as Homo-morphism

ie. $f: <S, *> \rightarrow <S, +>$

* **Abelian Homo-morphism** :-
Let $<A, *>$ and $<A, +>$ are two Abelian group then a mapping from one abelian group to another abelian group is known as Abelian Homo-morphism.

ie. $f: <A, *> \rightarrow <A, +>$

Example.1 :- Show That $<Z, +>$ is an Abelian group.

Sol:

$<Z, +>$

$Z = \{$ Set of all integers $\}$

$a = 1, \; b = 2, \; c = 3$

$a, b, c \in Z$

(i)

(i) Closure Property :-

$a+b = 1+2 = 3 \in Z$

$a+b \in Z$

$(Z,+)$ satisfies closure

ii) Associative Property:-

$a+(b+c) = (a+b)+c$

$a+(b+c) = 1+(2+3)$

$= 1+5$

$= 6$

$(a+b)+c = (1+2)+3 = 3+3 = 6$

$(Z,+)$ satisfies Associative

iii) Identity Property:-

$a+e = e+a = a$; $e = 0$ for Binary Addition

$a+e = 1+e = 1+0 = 1 = a$

$e+a = e+1 = 0+1 = 1 = a$

$(Z,+)$ Satisfies Identity

iv) Inverse Property:-

$a+(-a) = (-a)+a = e$; $e = 0$

$a+(-a) = 1+(-1) = 0 = e$

$(-a)+a = (-1)+1 = 0 = e$.

$(Z,+)$ Satisfies Inverse.

v) Commutative Property:-

$a+b = b+a$

$a+b = 1+2 = 3$; $b+a = 2+1 = 3$

$\Rightarrow (a+b) = b+a$

$(Z,+)$ Satisfies Commutative.

$\therefore \langle Z,+ \rangle$ is an Abelian group.

Ex:- (2) Show that $\langle Z, * \rangle$ is an Abelian group

Sol:- $\langle Z, * \rangle$

$Z = \{$ set of all integers $\}$

$a = 1, \; b = 2, \; c = 3$

$a, b, c \in Z$

(i) Closure Property:- $a * b \in Z$.

$a * b = 1 * 2 = 2$

$2 \in Z \Rightarrow a * b \in Z$

$(Z, *)$ Satisfies closure Property

(ii) Associative Property:-

$a * (b * c) = (a * b) * c$

$a * (b * c) = 1 * (2 * 3) = 1 * 6 = 6$

$(a * b) * c = (1 * 2) * 3 = 2 * 3 = 6$

$(Z, *)$ Satisfies Associative.

iii) Identity Property:-

$a * e = e * a = a$; $e = 1$ for binary mul.

$a * e = 1 \times 1 = 1 = a$

$e * a = 1 \times 1 = 1 = a$

$(Z, *)$ Satisfies Identity.

iv) Inverse Property:-

$a * a^{-1} = a^{-1} * a = e$

$e = 1$ for binary multiplication

$a * a^{-1} = 1 \times 1 = 1 = e$

$a^{-1} * a = 1 \times 1 = 1 = e$

($\because a^{-1}$ is multiplicative inverse of a)

$(Z, *)$ Satisfies Inverse property.

v) Commutative Property:-

$a * b = b * a$

$a * b = 1 * 2 = 2$

$b * a = 2 * 1 = 2$

$(Z, *)$ Satisfies Commutative.

$(Z, *)$ Satisfies closure, Associative, Identity, Inverse & Commutative

$\therefore \langle Z, * \rangle$ is an Abelian group.

Ex:-③ Show that $\langle S = \{1, \omega, \omega^2\}, * \rangle$ is an Abelian group, where $\omega^3 = 1$

Sol:- $\langle S, * \rangle$ where $S = \{1, \omega, \omega^2\}$; $a = 1, b = \omega, c = \omega^2$

$a, b, c \in S$

(i) Closure Property

$a * b \in S$

$1 * \omega = \omega$

$\omega \in S$.

$\langle S, * \rangle$ Satisfies.

(ii) Associative

$a * (b * c) = (a * b) * c$

$a * (b * c) = 1 * (\omega * \omega^2)$

$= 1 * \omega^3 = 1 * 1 = 1$

$(a * b) * c = (1 * \omega) * \omega^2$

$= \omega * \omega^2 = 1$

$\langle S, * \rangle$ Satisfies.

(iii) Identity

$a * e = e * a = a$; $e = 1$

$a * e = 1 * 1 = 1$

$e * a = 1 * 1 = 1$

$\langle S, * \rangle$ Satisfies Identity

(iv) Inverse

$a * a^{-1} = a^{-1} * a = e$; $e = 1$

$a * a^{-1} = 1 * 1 = 1 = e$

$a^{-1} * a = 1 * 1 = 1 = e$

$\langle S, * \rangle$ Satisfies

(v) Commutative

$a * b = b * a$

$a * b = 1 * \omega = \omega$

$b * a = \omega * 1 = \omega$

$\langle S, * \rangle$ Satisfies

$\therefore \langle S = \{1, \omega, \omega^2\}, * \rangle$ Is an Abelian group

Ex: ④ Show that $< Z_5^+, +_5 >$ is an Abelian group

here Z_5^+ mean all positive integers upto 5.

$Z_5^+ = \{1, 2, 3, 4, 5\}$; $+_5$ ⇒ if answer is greater than 5, then ÷ with 5 and remainder is our answer

$+_5$ is a binary operator with a remainder of 5

Ex: 8 (8>5)

$5) 8 (1$
$\underline{5}$
③

③ is answer

(i) Closure Property

let $a = 1, b = 2, c = 3$.

$< Z, + >$

$\boxed{a + b \in S}$

$1 + 2 = 3 \in S$

$< Z_5^+, +_5 >$ Satisfies closure Property

(ii) Associative property :- $a + (b + c) = (a + b) + c$

$a + (b + c) = 1 + (2 + 3) = 1 + 5 = 6 \Rightarrow 1$ $\quad 5) 6 (1$

$(a + b) + c = (1 + 2) + 3 = 3 + 3 = 6 \Rightarrow 1.$ $\quad (\because 6 > 5)$

$< Z_5^+, +_5 >$ Satisfies Associative.

(iii) Identity Property :- $a + e = e + a = a$; $e = 0$

$a + e = 1 + e = 1 + 0 = 1 = a$

$e + a = 0 + 1 = 1 = a$

$< Z_5^+, +_5 >$ Satisfies Identity.

(iv) Inverse Property :- $a + (-a) = (-a) + a = e$; $e = 0$

$a + (-a) = 1 + (-1) = 0 = e$

$(-a) + a = (-1) + 1 = 0 = e$

$< Z_5^+, +_5 >$ Satisfies Inverse

(v) Commutative :- $a + b = b + a$

$a + b = 1 + 2 = 3$; $b + a = 2 + 1 = 3$

$< Z_5^+, +_5 >$ Satisfies Commutative

$\therefore < Z_5^+, +_5 >$ is an Abelian Group.

Ex:- (4) Show that $< Z_5^+, +_5 >$ is an Abelian group

here Z_5^+ mean all positive integers upto 5.

$Z_5^+ = \{1, 2, 3, 4, 5\}$; $+_5$ => if answer is greater than 5, then ÷ with 5 and remainder is our answer

$+_5$ is a binary operator with a remainder of 5

Ex:- 8 (8>5)

$5) 8 (1$
5
$\overline{3}$

3 is answer

(i) Closure Property

let $a=1, b=2, c=3$

$< Z, + >$

$\boxed{a+b \in S}$

$1+2 = 3 \in S$

$< Z_5^+, +_5 >$ Satisfies closure Property

(ii) Associative property :- $a+(b+c) = (a+b)+c$

$a+(b+c) = 1+(2+3) = 1+5 = 6 \Rightarrow 1$

$(a+b)+c = (1+2)+3 = 3+3 = 6 \Rightarrow 1.$

$5) 6 (1$
5
$\overline{1}$

$(\because 6>5)$

$< Z_5^+, +_5 >$ Satisfies Associative.

(iii) Identity Property :- $a+e = e+a = a$; $e=0$

$a+e = 1+e = 1+0 = 1 = a$

$e+a = 0+1 = 1 = a$

$< Z_5^+, +_5 >$ Satisfies Identity.

(iv) Inverse Property :- $a+(-a) = (-a)+a = e$; $e=0$

$a+(-a) = 1+(-1) = 0 = e$

$(-a)+a = (-1)+1 = 0 = e$

$< Z_5^+, +_5 >$ Satisfies Inverse

(v) Commutative :- $a+b = b+a$

$a+b = 1+2 = 3$; $b+a = 2+1 = 3$

$< Z_5^+, +_5 >$ Satisfies Co

(5) Show that $< Z_5^+, \times_5 >$ is an Abelian group

'Z' means all integers ; Z^+ means all +ve integers

$Z_5^+ = \{1, 2, 3, 4, 5\}$

$a = 1, b = 2, c = 3$

(i) <u>Closure property</u> :-

$a \times b \in Z_5^+$

$1 \times 2 = 2 \in Z_5^+$

$< Z_5^+, \times_5 >$ Satisfies closure.

(ii) <u>Associative Property</u> :-

$a \times (b \times c) = (a \times b) \times c$

$a \times (b \times c) = 1 \times (2 \times 3)$

$= 6$

$5) \, 6 \, (1$

$= 1$

$(a \times b) \times c = (1 \times 2) \times 3$

$= 6$

Ans $= 1$

$< Z_5^+, \times_5 >$ Satisfies Associative

(iii) <u>Identity Property</u> :-

$a \times e = e \times a = a ; e = 1$

$1 \times 1 = 1$

$1 \times 1 = 1$

$< Z_5^+, \times_5 >$ Satisfies Identity.

(iv) <u>Inverse Property</u>

$a \times a^{-1} = a^{-1} \times a = e$

$e = 1$ for bin. multip

$a \times a^{-1} = 1 \times 1^{-1} = 1 \times 1 = 1$

$= e$

$a^{-1} \times a = 1 \times 1 = 1 = e$

$< Z_5^+, \times_5 >$ Satisfies Inverse

(v) <u>Commutative</u> :-

$a \times b = b \times a$

$a \times b = 1 \times 2 = 2$

$b \times a = 2 \times 1 = 2$

$< Z_5^+, \times_5 >$ Satisfies Commutative

$\therefore < Z_5^+, \times_5 >$ is an Abelian group.

Ex: (6) Given 's' is set of positive integers and a, b, c ∈ S then show that $a * b = a + b + 1$ is an Abelian group, where '$*$' is binary operator.

Sol: Given, $a, b, c \in S$

$S = \{$ set of all positive integers $\}$

$a = 1, b = 2, c = 3$

(i) Closure Property :-

$a * b \in S$ $a + b + 1 \in S$

$\Rightarrow 1 * 2 = 2 \in S$ $1 + 2 + 1 = 4 \in S$

$\therefore a + b + 1 \in S$

$\therefore$ $\langle S, * \rangle$ Satisfies closure Property

(ii) Associative Property :-

$$a * (b * c) = (a * b) * c$$

$$a * (b * c) = \underset{a}{a} * (\underset{b}{b + c + 1})$$

($\because$ Given $a * b = a + b + 1$)

$$= a + (b + c + 1) + 1$$

$$= a + b + c + 2$$

$$(a * b) * c = (\underset{a}{a + b + 1}) * \underset{b}{c}$$

$$= (a + b + 1) + c + 1$$

$$= a + b + c + 2$$

$\therefore$ It Satisfies Associative Property.

(iii) Identity Property:-

$$a * e = e * a = a \quad ; \quad e = 1$$

$a * e = a * 1 = a$

$e * a = 1 * a = a$

$\langle S, * \rangle$ satisfies Identity.

(iv) Inverse Property:-

$$a * (a^{-1}) = (a^{-1}) * a = e \quad ; \quad e = 1$$

$a * a^{-1} = a * 1/a = 1 * 1 = e$

$a^{-1} * a = 1/a * a = 1 * 1 = e$

$\langle S, * \rangle$ Satisfies Inverse.

(v) Commutative Property:-

$$a * b = b * a$$

$a * b = a + b + 1$

$= 1 + 2 + 1 = 4$

$b * a = b + a + 1$

$= 2 + 1 + 1 = 4$

$\langle S, * \rangle$ isatisfies commutative.

$\therefore \langle S, * \rangle$ is an Abelian group

Ex:- 7 Show that $a * b = a + b + ab$ is an Abelian group, where $*$ is a binary operator.

Ex:- 8 Draw the multiplication table for $S = \{1, \omega, \omega^2\}$ where $\omega^3 = 1$

Sol:

$S \times S$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

Ex:- 9 Draw multiplication table for $\langle Z_5^+, +_5 \rangle$

$(Z_5^+, +_5)$	1	2	3	4	5
1	2	3	4	5	1 $(1+5)$
2	3	4	5	1	2
3	4	5	1	2	3
4	5	1	2	3	4
5	1	2	3	4	5

$2+4=6$

$5)6(1$

$\underline{5}$

$1 = 1$

$5+5=10$

$5)10(2$

$\underline{10}$

$0 = 5$

(8) Draw the multiplication table for $S = \{1, \omega, \omega^2\}$ where $\omega^3 = 1$

Sol:

$S \times S$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

(9) Draw multiplication table for $\langle Z_5^+, +_5 \rangle$

$(Z_5^+, +_5)$	1	2	3	4	5
1	2 (1+1)	3 (1+2)	4 (1+3)	5 (1+4)	1 (1+5)
2	3	4	5	1	2
3	4	5	1	2	3
4	5	1	2	3	4
5	1	2	3	4	5

here,

$2+4=6$

$5)6(1$

$(1) = 1$

$5+5=10$

$5)10(2$

10

$(0) = 5$

Ex: (10) Draw multiplication table for $\langle Z_5^+, \times_5 \rangle$

$Z_5^+, \times_5$	1	2	3	4	5
1	1	2	3	4	5
2	2	4	1	3	5
3	3	1	4	2	5
4	4	3	2	1	5
5	5	5	5	5	5

Ex:- (11) Draw the multiplication table for $\langle S, * \rangle$ where $S = \{-2, -1, 0, 1, 2\}$.

here there is no limit [illegible] if the value doesn't contain in set then there [illegible] no element.

$$
\begin{array}{c|ccccc}
 & -2 & -1 & 0 & 1 & 2 \\
\hline
-2 & - & 2 & 0 & -2 & - \\
-1 & 2 & 1 & 0 & -1 & -2 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & -2 & -1 & 0 & 1 & 2 \\
2 & - & -2 & 0 & 2 & -
\end{array}
$$

→ no value

* Lattice :-

In Hassie diagram, if each and every ordered pair Contains the minimal element or least elements (or) Maximal element or greater element then the Hassie diagram is said to be 'Lattice'.

Ex:- ① Check whether the following Hassie diagram is lattice or not.

	Minimal	Maximal
(1, 2)	1	2
(1, 3)	1	3
(1, 6)	1	6
(2, 3)	1	6
(2, 6)	2	6
(3, 6)	3	6

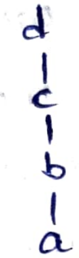
∴ The given diagram is Lattice.

Ex: ② check whether the following Hassie diagram is lattice or not.

	Minimal	Maximal
(1, 2)	1	2
(1, 3)	1	3
(1, 4)	1	4
(2, 3)	1	Not Possible
(2, 4)	2	4
(3, 4)	1	Not Possible

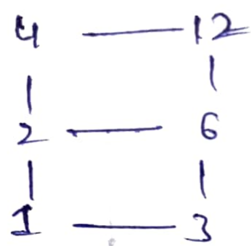
Given is "Not a Lattice"

Ex-(3) check whether the given diagram is lattice or not.



	Minimal	Maximal
(a,b)	a	b
(a,c)	a	c
(a,d)	a	d
(b,c)	b	c
(b,d)	b	d
(c,d)	c	d

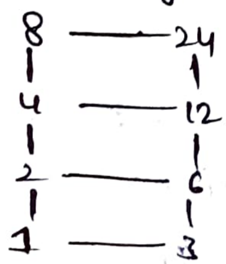
Ex-(4) check whether the given diagram is lattice or not.



	Minimal	Maximal
(1,2)		
(1,3)		
(1,4)		
(1,6)		
(1,12)		
(2,3)		
(2,4)		
(2,6)		
(2,12)		
(3,4)		

(3,6)		
(3,12)		
(4,6)		
(4,12)		
(6,12)		

Ex-6 check whether the given diagram is lattice ornot



	Minimal	Maximal
(1,2)	1	2
(1,3)	1	3
(1,4)	1	4
(1,6)	1	6
(1,8)	1	8
(1,12)	1	12
(1,24)	1	24
(2,3)	1	6
(2,4)	2	4
(2,6)	2	6
(2,8)	2	8
(2,12)	2	12
(2,24)	2	24
(3,4)	1	4
(3,6)	3	6
(3,8)	1	~~24~~ 8
(3,12)	3	12
(3,24)	3	24
([illegible])		
(4,6)	2	12
(4,8)	4	8

	Minimal	Maximal
(4,12)	4	12
(4,24)	4	24
• (6,8)	2	8
(6,12)	6	12
(6,24)	6	24
• (8,12)	4	24
(8,24)	8	24
(12,24)	12	24

∴ Given Hasse diagram is Lattice.

* Properties of Lattice :-

→ Let 'N' represents set of positive integers and a, b, c are integer values, ie. $a \in N, b \in N, c \in N$. '$\wedge$' and '$\vee$' are two binary operators. Then the following are the properties of Lattice.

1) **Closure Property** :- If $a \in N, b \in N$ then i) $a \wedge b \in N$

ii) $a \vee b \in N$.

2) **Associative Property** :-

(i) $a \vee (b \vee c) = (a \vee b) \vee c$

(ii) $a \wedge (b \wedge c) = (a \wedge b) \wedge c$

3) **Identity Property** :- With respect to "$\vee$" operator Zero is an Identity element and w.r.t "$\wedge$" operator One(1) is an Identity element

(i) $a \vee 0 = a$

(ii) $a \wedge 1 = a$

4) **Inverse Property** :-

(i) $a \vee (-a) = 0$

(ii) $a \wedge (a^{-1}) = 1$

5) **Commutative Property** :-

(i) $a \vee b = b \vee a$

(ii) $a \wedge b = b \wedge a$

6) **Distributive Property** :-

(i) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

(ii) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

Note :- In the above Lattice properties, the "$\wedge$" connective represents Maximal element and "$\vee$" Connective represents Minimal element in Hassie diagram.

Ex :- Find the following expression Values from the given Hassie diagram.

Sol:- $1 \wedge 2 = 2$ (Maximal)

$1 \wedge 6 = 6$ "

$2 \wedge 3 = 6$ "

$1 \vee 3 = 1$ (Minimal)

$2 \vee 3 = 1$ "

* **Ring** :- An Algebraic System $< R, +, * >$ is said to be a 'Ring' w.r.t Binary Addition and Binary Multiplication, if it can satisfies the following Properties.

i) $< R, + >$ is an Abelian group.

ii) $< R, * >$ is a Semi-group.

iii) The Binary addition (+) operator is distributed over Binary multiplication ($*$) operator

$$(a+b)*c = (a*c)+(b*c)$$

(or)

$$(a*b)+c = (a+c)*(b+c)$$

Ex: Show that $< Z, +, * >$ is a Ring.

Sol: Let us consider $a, b, c \in Z$, where Z is equal to Set of Integers.

$$a=1, b=2, c=3$$

(i) $< Z, + >$ is an Abelian group.

1. Closure Property : $a \in Z, b \in Z$.

$a+b \in Z$ ie $1+2=3 \in Z$

$< Z, + >$ Satisfies Closure Property

2. Associative : $a+(b+c) = (a+b)+c$.

$a+(b+c) = 1+(2+3) = 1+5 = 6$

$(a+b)+c = (1+2)+3 = 3+3 = 6$

$\therefore < Z, + >$ Satisfies Associative.

3. Identity : $a+e = e+a = a$; $e=0$ for '+'

$a+e = 1+0 = 1$; $e+a = 0+1 = 1$

$\therefore < Z, + >$ Satisfies Identity.

4. Inverse : $a+(-a) = (-a)+a = e$, $e=0$ for '+'

$a+(-a) = 1+(-1) = 0$; $(-1)+1 = 0$ $= 0$

$< Z, + >$ Satisfies Inverse property.

Commutative Property : $a+b=b+a$

$a+b = 1+2 = 3$

$b+a = 2+1 = 3 \Rightarrow 3=3$

$\langle Z,+\rangle$ Satisfies Commutative.

$\therefore \langle Z,+\rangle$ is an Abelian group.

ii) $\langle Z,*\rangle$ is a Semi group :

1. closure property : $a \in Z, b \in Z$

$a*b = 1*2 = 2 \in Z$

$\therefore \langle Z,*\rangle$ Satisfies closure.

2. Associative : $a*(b*c) = (a*b)*c$

$a*(b*c) = 1*(2*3) = 1*6 = 6$

$(a*b)*c = (1*2)*3 = 2*3 = 6 \Rightarrow 6=6$

$\therefore \langle Z,*\rangle$ Satisfies Associative.

$\therefore \langle Z,*\rangle$ is a Semi-group.

(iii)

$(a+b)*c = (a*c)+(b*c)$

~~LHS~~ $(a+b)*c = (1+2)*3 = 3*3 = 9$

$(a*c)+(b*c) = (1*3)+(2*3) = 3+6 = 9$

$\therefore$ It Satisfies The above condition.

$\therefore \langle Z,+\rangle$ is a Ring.

Ex: Show that $\langle Z_5^+, +_5, *_5\rangle$ is a Ring.

* **Field :-**

An algebraic System $<F, +, *>$ is Said to be a Field with respect to binary addition and Binary Multiplication if it Can Satisfy the following Properties.

i) $<F, +>$ is an Abelian group

ii) $<F_0, *>$ is an Abelian group where F_0 is a Set of Non-zero elements of 'F'.

iii) '+' operator is distributive over "*"

$(a+b)*c = (a*c)+(b*c)$

$(a*b)+c = (a+c)*(b+c)$

Note :- Every field is a Ring w.r.t Binary Addition (+) and Binary multiplication.

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