UNIT-I
Amplitude Modulation

Introduction:-

* The main objective of a communication system is to transfer information from one place to another place by using electrical signals.

* An information signal is transmitted from transmitter to Receiver

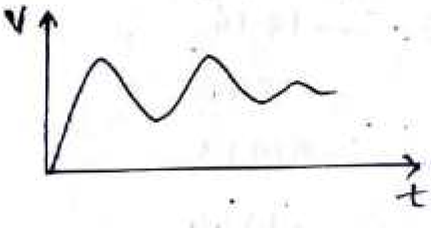
* Based on the nature of signal transmitted, communication system is of two types.

  1. Analog communication system
  2. Digital communication system

1. Analog communication system:- The message signal used in this system is analog in nature.

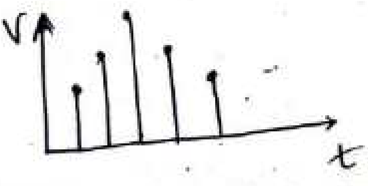
   A signal whose amplitude varies continuously with respect to time period is analog signal

   Eg: sine wave, coswave.



2. Digital communication system:- The message signal is used in this system is digital in nature.

   A signal in which the sequence of numbers represent the amplitude at any instant of time is called digital signal. It is discrete in nature.



* In this subject, we are dealing with transmission of following four types of signals.

1. Voice (subset of Audio) → 300- 3.5 KHZ → Telephone.

2. Audio → 20- 20 KHZ → Radio

3. Video → 0 - 4.5 MHZ → TV

4. Binary data → (Frequency depends on pulse width) → Internet.

* The electronic equipment which are used for communication purpose are called communication equipments.

* The most humans communicate through spoken words, body movements, facial expressions are effective communication tools

* The first electrical communication established between two distinct points i.e Telegraphy message, it is invented by John Mars in 1838

Telephone → 1876
Radio → 1895
Television → 1923
Colour TV → 1954.

* First satellite was launched in 1962.

Elements of communication system (or) Block diagram of communication system.

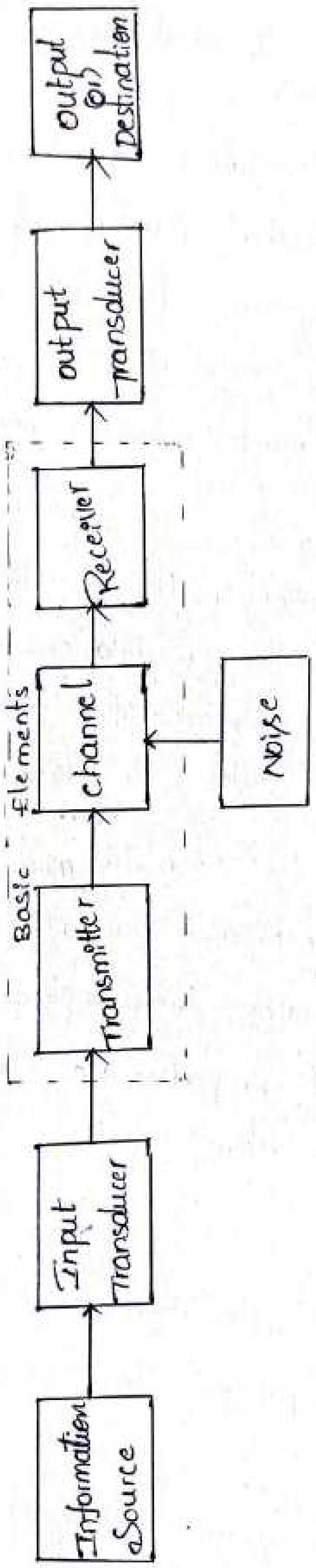


Fig: Elements of communication System.

The basic elements of communication system are transmitter, communication channel, Receiver.

Information source:- The system which generates the source of information in the form of text message, voice, audio, video etc is called information source.

Input Transducer:- A transducer is a device which converts one form of energy into another form of energy.

The message from the information source may or may not be electrical signal. If it is not a electrical signal, the input transducer which converts non electrical signal into electrical signal.

Transmitter:- The transmitter transfers the electrical signal through the channel. In transmitter, modulation process is done. It converts low frequency signal into high frequency signal. Because channel passes only the high frequency signals.

Channel:- the channel is basically medium and it is establish electrical connection between transmitter to Receiver, the connection may be Wire, cable, optical fiber, etc.

The channel properties are influences the performance of a communication system.

Receiver:-

Receiver receives the signal, At receiver demodulation process is done, it separates the carrier signal and message signal.

Output Transducer:- It converts electrical signal into non-electrical signal. For eg:- Loud speaker → converts electrical signal into sound signal

Destination:- It is the final stage. It receives the original information

Modulation:-

It is the process of changing the characteristic parameters of the carrier signal is varied inaccordance with the instantaneous value of the message signal

(or)

It is the process of super imposition of the message signal into the carrier signal.

Types of modulation:

The modulation types are basically 2 types.

(1) continuous wave modulation.

(2) pulse modulation.

* continuous wave modulation is also known as Analog modulation
* the pulse modulation is also known as digital (or) discrete (or) discontinuous modulation.

Need for Modulation:-

* Any signal is to transmitt directly without modulation having several limitations
* To avoid this limitations, we require modulation technique at transmitter side and demodulation technique at receiver side.

the limitations are

1. Reduction of Antenna Height
2. Avoid mixing of signals
3. Increases the range of communication
4. Allows the multiplexing
5. Allows the adjustment in Bandwidth.

1. Reduction of Antenna Height: to transmit the radio signals, the antenna height must be a $\frac{\lambda}{4}$

$$\boxed{H = \frac{\lambda}{4}}$$

where 'λ' is the wavelength $\lambda = \frac{c}{f}$

$$\therefore H = \frac{c/f}{4}$$

$$\boxed{H = \frac{c}{4f}}$$

where 'c' is the velocity of light in space
$C = 3 \times 10^{8} m/sec.$

For eg:-

If $f = 50 KHz$

$$\therefore H = \frac{3 \times 10^{8}}{4(50 \times 10^{3})}$$

$$\boxed{H = 1.5 \text{ km}}$$

If $f = 100 KHz$

$$\therefore H = \frac{3 \times 10^{8}}{4(100 \times 10^{3})}$$

$$\boxed{H = 0.75 \text{ Km}}$$

If $f = 2 MHz$

$$\therefore H = \frac{3 \times 10^{8}}{4(2 \times 10^{6})}$$

$$\boxed{H = 37.5 \text{ m}}$$

From the above examples,

* If the height of the antenna is very high, then the freq is low, therefore we can't get the proper communication

* In communication, always require better propagation.

* If the frequency is increases, then the height of the antenna is decreases.

2. Avoid mixing of signals:-

All sound signals are concentrated within the range from 20HZ to 20KHZ. There is a problem of mixing of signals with same frequency range during transmission. then it is difficult to separate at the receiver end

This problem can be overcome by modulating carrier signal by different carrier frequencies. once the signals have been transmitted, a tuned circuit at the receiver end, we can tune the selected signals by using tuner.

3. Increases the range of communication.

The base band signal is a low frequency signal, hence radiation is very poor and signal gets highly attenuated. this signal is not possible to transmit longer distances. By using modulation, the signals can travel long distance transmission. It can also improves the quality of signals without any noise in the signal.

4. Allows the multiplexing :-

* The multiplexing is a process in which two (or) more signals transmitted over a same channel simultaneously.

* For considering number of inputs, we get the overlapping of signa

* To avoid this overlapping incase of designing multiplexer, we are using modulation

5. Allows the adjustment in Bandwidth

* Each and every signal having bandwidth, if it is a better Bandwidth, then it produces the better output signal.

* Adjustment in Bandwidth is possible by using modulation

Electronic communication System Frequency Spectrum

* The electronic communication system frequency spectrum categorised into several types based on the needs of the consumer requirement, communication system carrier freq range.

* In general radio frequency (RF) range is 3KHZ - 300GHZ

* The frequency spectrum divided into 10 categories.

Frequency	Designation	Abbrevation
30 - 300HZ	Extreme Low Frequency	E.L.F
300 - 3000HZ	Voice (or) Low frequency	V.F
3K - 30KHZ	Very Low Frequency	V.L.F.
30K - 300K	Low frequency	L.F
0.3M - 3MHZ	Medium frequency	M.F
3MHZ - 30MHZ	High frequency	H.F
30MHZ - 300MHZ	Very high frequency	V.H.F.
0.3GHZ - 3GHZ	ultra high frequency	U.H.F
3GHZ - 30 GHZ	Super High frequency	S.H.F
30GHZ - 300 GHZ	Extra high frequency	E.H.F
88 - 108 MHZ	F.M	

Classification of Modulation

* The modulation techniques are classified into two types based upon the carrier signal. there are.

(1) continuous wave modulation

(2) pulse wave modulation.

* the continuous wave modulation is classifed into 2 types.

(a) Amplitude modulation (AM)

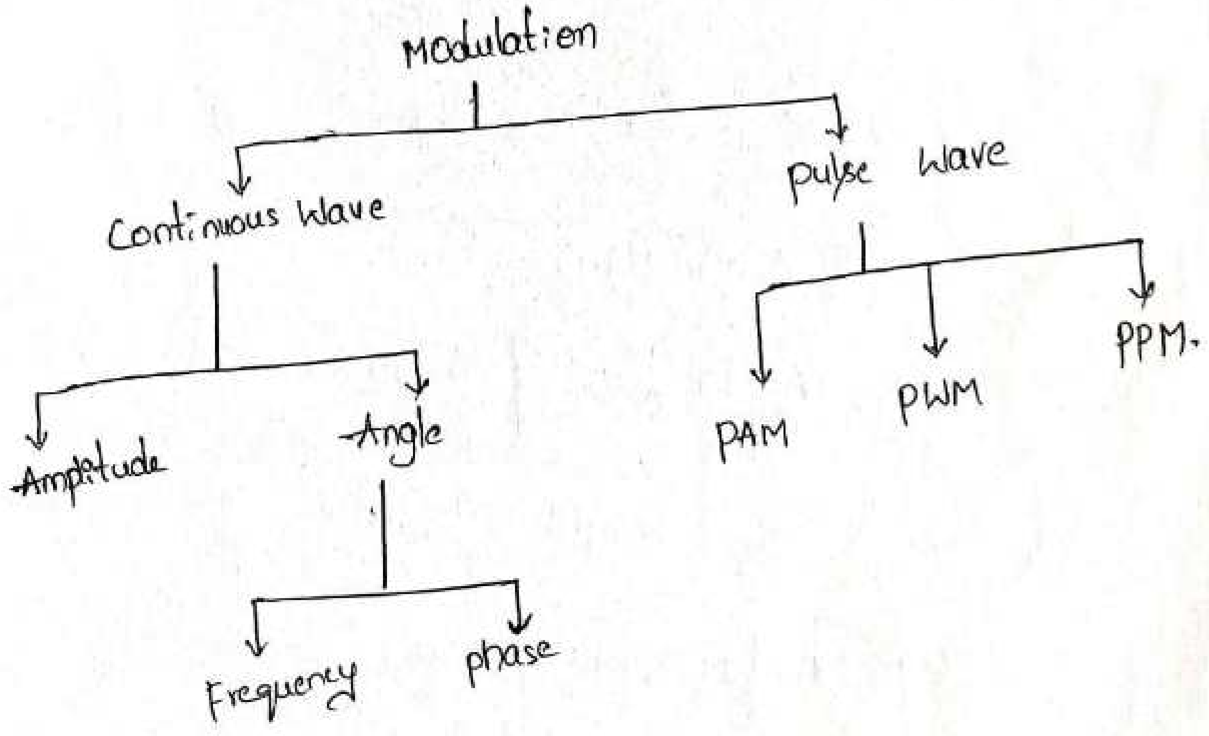
(b) Angle modulation → Frequency modulation (FM), phase modulation. (PM)

* If the carrier is pulse type, then that modulation is pulse wave modulation, and it is furthur classified into 3 types.

(i) pulse Amplitude Modulation (PAM)

(ii) pulse width Modulation (PWM)

(iii) pulse position Modulation. (PPM)



Amplitude Modulation:-

the amplitude of the carrier signal is varied inaccordance with the instantaneous value of the message signal (or) modulating signal (or) base band signal.

Time domain Analysis of Amplitude Modulation

* In Time domain, the message signal and carrier signal mathematical equations are.

message signal $\rightarrow m(t) = A_m \cos\omega_m t \longrightarrow (1)$

Carrier signal $\rightarrow c(t) = A_c \cos\omega_c t \longrightarrow (2)$

where

A_m is message signal Amplitude.

ω_m is message signal frequency

A_c is carrier signal Amplitude

ω_c is carrier signal frequency

* In modulation, the carrier signal Amplitude varies according to the message signal values, hence to obtain mathematical equation in time domain is given by.

$$s(t) = (A_c + m(t)) \cos\omega_c t \longrightarrow (3)$$

$$s(t) = A_c\left[1 + \frac{1}{A_c}.m(t)\right] \cos\omega_c t.$$

If $\frac{1}{A_c} = K_a$ = Amplitude sensitivity factor, then the eq is

$$\therefore s(t) = A_c\left[1 + K_a m(t)\right] \cos\omega_c t \longrightarrow (4)$$

$$= A_c\left[1 + K_a A_m \cos\omega_m t\right] \cos\omega_c t$$

$$= A_c\left[1 + \frac{A_m}{A_c} \cos\omega_m t\right] \cos\omega_c t \longrightarrow (5)$$

where $\boxed{\frac{A_m}{A_c} = \mu = \text{Modulation Index}}$

$$\therefore s(t) = A_c\left[1+\mu\cos\omega_m t\right]\cos\omega_c t \longrightarrow (6)$$

$$= A_c\cos\omega_c t + A_c\mu\cos\omega_m t\cdot\cos\omega_c t$$

$$= A_c\cos\omega_c t + \frac{A_c\mu}{2}\cdot 2\left[\cos\omega_c t\cdot\cos\omega_m t\right]$$

$$\boxed{2\cos A\cdot\cos B = \cos(A+B)+\cos(A-B)}$$

$$s(t) = A_c\cos\omega_c t + \frac{A_c\mu}{2}\left[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t\right]$$

$$= A_c\cos 2\pi f_c t + \frac{A_c\mu}{2}\left[\cos 2\pi(f_c+f_m)t + \cos 2\pi(f_c-f_m)t\right]$$

$$\boxed{s(t) = \underbrace{A_c\cos 2\pi f_c t}_{1} + \underbrace{\frac{A_c\mu}{2}\cos 2\pi(f_c+f_m)t}_{2} + \underbrace{\frac{A_c\mu}{2}\cos 2\pi(f_c-f_m)t}_{3}} \rightarrow (7)$$

From the above equation, the modulated wave contain the following three terms.

1. carrier signal with frequency 'f_c' having amplitude A_c
2. The upper side Band (USB) signal with frequency "f_c+f_m" having amplitude $\frac{A_c\mu}{2}$
3. The lower side Band (LSB) signal with frequency "f_c-f_m" having amplitude $\frac{A_c\mu}{2}$

Time domain waveform

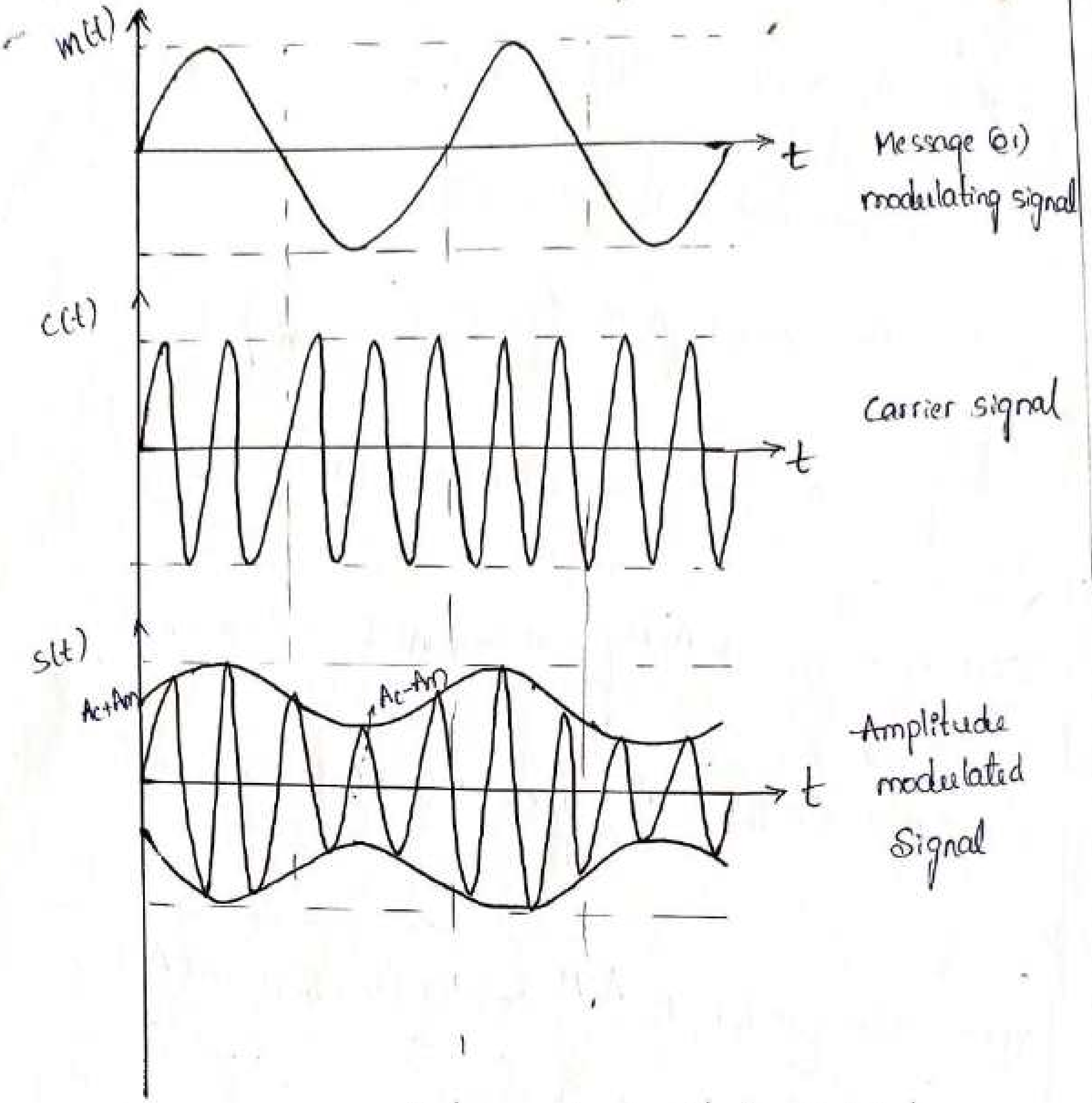


Fig: Graphical representation of Amplitude modulation.

* Based upon <u>modulation index</u>, the modulation is classified into 3 cases.

  a) $\mu < 1 \Rightarrow$ under modulation

  b) $\mu = 1 \Rightarrow$ critical modulation (100% modulation)

  c) $\mu > 1 \Rightarrow$ over modulation.

* From Amplitude modulated wave, we can define maximum and minimum Amplitudes.

Maximum Amplitude $V_{max} = A_c + A_m \longrightarrow (1)$

Minimum Amplitude $V_{min} = A_c - A_m \longrightarrow (2)$

From eq (1)

$$V_{max} = A_c + A_m = A_c\left(1 + \frac{A_m}{A_c}\right)$$

where $\mu = \frac{A_m}{A_c}$

$$\therefore V_{max} = A_c(1+\mu) \longrightarrow (3)$$

From eq (2)

$$V_{min} = A_c - A_m = A_c\left(1 - \frac{A_m}{A_c}\right)$$

$$\therefore V_{min} = A_c(1-\mu) \longrightarrow (4)$$

From eq (3) & (4)

$$\frac{V_{max}}{V_{min}} = \frac{A_c(1+\mu)}{A_c(1-\mu)}$$

$$V_{max} - \mu V_{max} = V_{min} + \mu V_{min}.$$

$$V_{max} - V_{min} = \mu V_{max} + \mu V_{min}$$

$$V_{max} - V_{min} = \mu(V_{max} + V_{min})$$

$$\boxed{\therefore \mu = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}}$$

Time domain waveform for ~~center~~ critical modulation ($\mu = 1$)

$$\mu = \frac{A_m}{A_c}$$

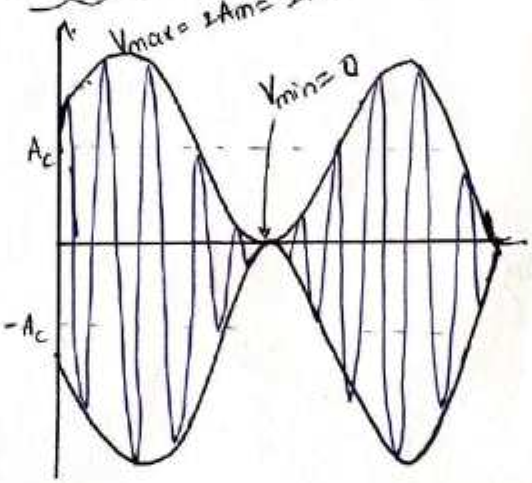
If $\mu = 1$ then $A_m = A_c$

$$\therefore V_{max} = A_c + A_m$$

$$\boxed{\therefore V_{max} = 2A_m = 2A_c}$$

$$V_{min} = A_c - A_m$$

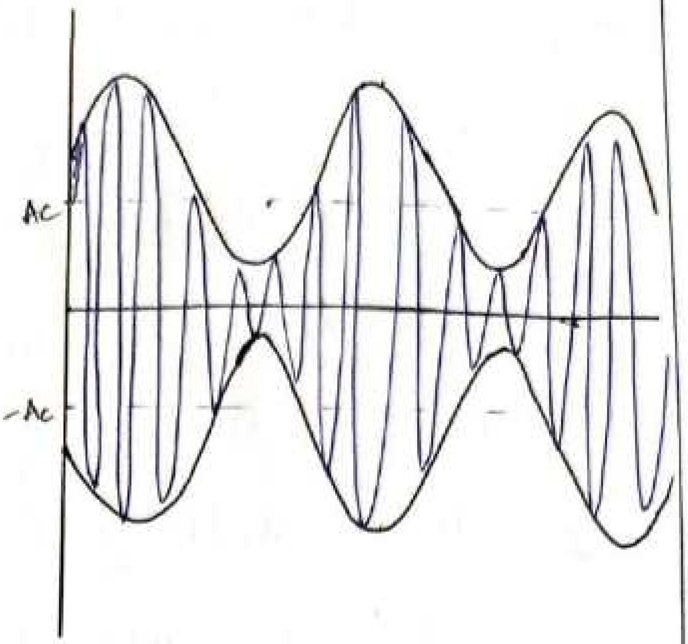
$$\boxed{V_{min} = 0}$$



Time domain Waveform for under modulation ($\mu<1$)

$$\mu = \frac{A_m}{A_c}$$

if $\mu<1$ i.e $\mu = \frac{1}{2} = \frac{A_m}{A_c}$



$\therefore \boxed{2A_m = A_c}$

$V_{max} = A_c + A_m$

$= 2A_m + A_m$

$V_{max} = 3A_m$ (or) $\frac{3A_c}{2}$

$V_{min} = A_c - A_m$

$= 2A_m - A_m$

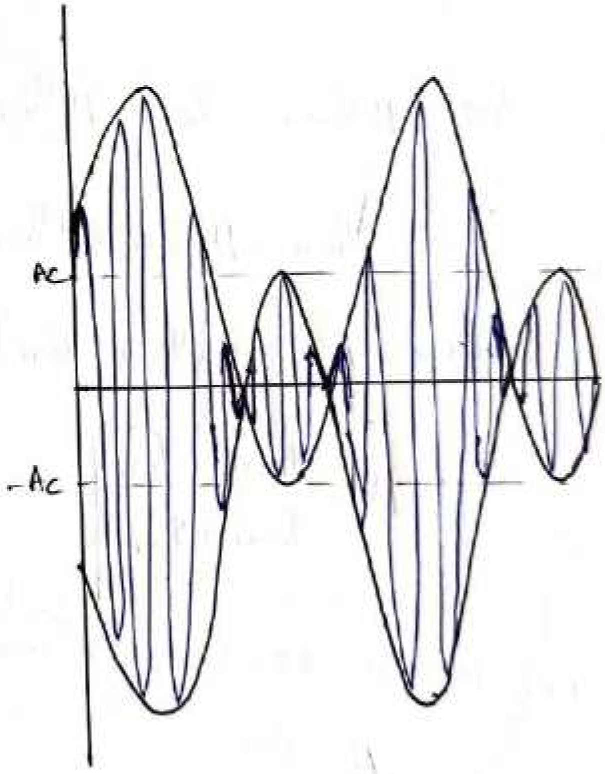
$V_{min} = A_m$ (or) $\frac{A_c}{2}$

Time domain waveform for over modulation ($\mu>1$)

$$\mu = \frac{A_m}{A_c}$$

if $\mu>1$ i.e $\mu = 2 = \frac{A_m}{A_c}$

$\boxed{\therefore 2A_c = A_m}$



$V_{max} = A_c + A_m$

$= \frac{A_m}{2} + A_m$

$= \frac{3A_m}{2}$ (or) $3A_c$

$V_{min} = A_c - A_m$

$= \frac{A_m}{2} - A_m$

$V_{min} = -\frac{A_m}{2}$ (or) $-A_c$

$\boxed{V_{min} = -ve}$

Frequency domain Analysis for Amplitude Modulation

* To find its spectrum (or) frequency domain representation, initially we have to take time domain equation for AM wave, is given by

$$S(t) = A_c\left[1+K_a m(t)\right]\cos\omega_c t$$ → This eq is present in time domain analysis of AM (eq no. 4)

$$\boxed{\therefore\ s(t) = A_c\cos\omega_c t + A_c K_a m(t)\cos\omega_c t}$$

* We know that, the Fourier Transform (F.T) of a cosine signal consists of two impulses at $-f_c$ & f_c, then the Fourier transform (F.T) of $\cos 2\pi f_c t$ will be written as

$$\cos 2\pi f_c t \xleftrightarrow{F.T} \frac{\delta(f-f_c)+\delta(f+f_c)}{2}$$

$$m(t)\cos 2\pi f_c t \xleftrightarrow{F.T} \frac{M(f-f_c)+M(f+f_c)}{2}$$

∴ Apply F.T on $S(t)$, then we get.

$$S(f) = A_c\left[\frac{\delta(f-f_c)+\delta(f+f_c)}{2}\right] + A_c K_a\left[\frac{M(f-f_c)+M(f+f_c)}{2}\right]$$

Interms of modulation index (μ), the modulated eq is given by

$$S(t) = A_c\left[1+\mu\cos\omega_m t\right]\cos\omega_c t$$ → This eq is present in time domain analysis of AM (eq no. 6)

$$s(t) = A_c\cos\omega_c t + A_c\mu\cos\omega_m t\cdot\cos\omega_c t$$

$$s(t) = A_c \cos\omega_c t + \frac{A_c \mu}{2} \cdot 2 (\cos\omega_c t \cdot \cos\omega_m t)$$

$$\boxed{2\cos A \cdot \cos B = \cos(A+B) + \cos(A-B)}$$

$$s(t) = A_c \cos\omega_c t + \frac{A_c \mu}{2} \left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t\right]$$

$$s(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu}{2} \left[\cos 2\pi (f_c + f_m)t + \cos 2\pi (f_c - f_m)t\right]$$

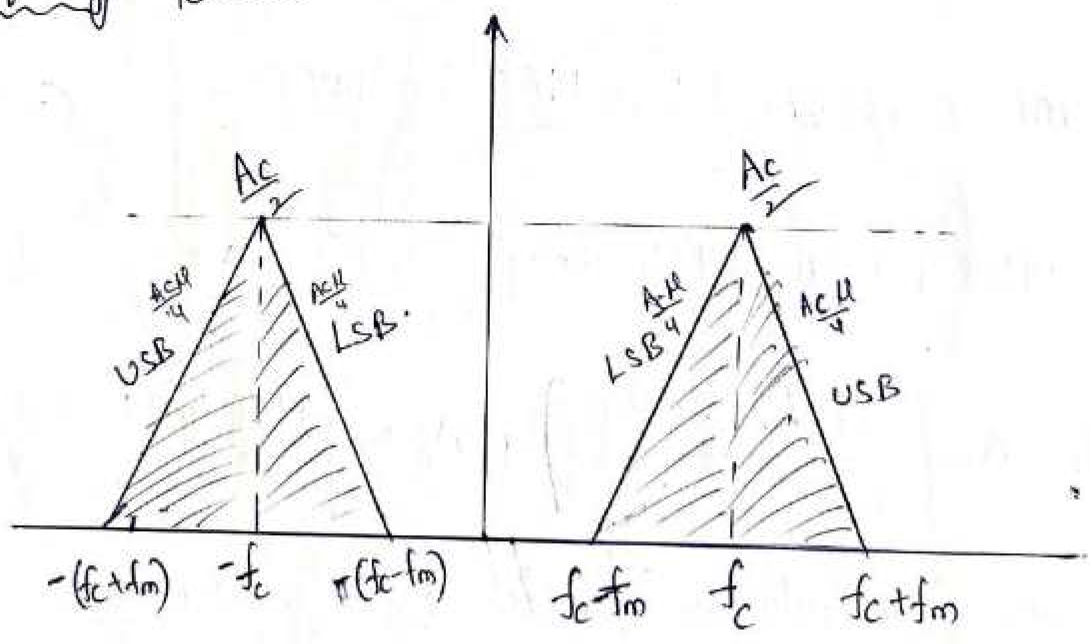
$\therefore$ Apply Fourier Transform on $s(t)$, then we get

$$S(f) = A_c \left[\frac{\delta(f - f_c) + \delta(f + f_c)}{2}\right] + \frac{A_c \mu}{2} \left[\frac{\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))}{2}\right]$$

$$+ \frac{A_c \mu}{2} \left[\frac{\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))}{2}\right]$$

$$S(f) = \frac{A_c}{2} \left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{A_c \mu}{4} \left[\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))\right] + \frac{A_c \mu}{4} \left[\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))\right]$$

Frequency spectrum



Power Calculation in Amplitude Modulation

Mathematical equation for AM is given by

$$s(t) = [A_c + m(t)] \cos\omega_c t$$

$$= A_c \left[1 + \frac{1}{A_c} \cdot A_m \cos\omega_m t\right] \cos\omega_c t$$

$$= A_c \left[1 + \frac{A_m}{A_c} \cos\omega_m t\right] \cos\omega_c t$$

$$= A_c \left[1 + \mu \cos\omega_m t\right] \cos\omega_c t.$$

$$= A_c \cos\omega_c t + A_c \mu \cos\omega_c t \cdot \cos\omega_m t.$$

$$= A_c \cos\omega_c t + \frac{A_c \mu}{2}\left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t\right]$$

$$s(t) = \underbrace{A_c \cos\omega_c t}_{\text{Carrier power}} + \underbrace{\frac{A_c \mu}{2}\cos(\omega_c + \omega_m)t}_{\text{U.S.B power}} + \underbrace{\frac{A_c \mu}{2}\cos(\omega_c - \omega_m)t}_{\text{L.S.B power}}$$

The sinusoidal signal power is given by.

$$P = \frac{V^2}{R} \qquad \text{where } V = \frac{V_{rms}}{\sqrt{2}}$$

$$P = \frac{\left(\frac{V_{rms}}{\sqrt{2}}\right)^2}{R}$$

$$P = \frac{V_{rms}^2}{2R}$$

The Average (or) total power of the modulated signal is given by

$$P_t = P_c + P_{U.S.B} + P_{L.S.B}.$$

Carrier power $P_c = \frac{A_c^2}{2R}$

Upper side band power $P_{U.S.B} = \frac{\left(\frac{A_c \mu}{2\sqrt{2}}\right)^2}{R} = \frac{A_c^2 \mu^2}{8R}.$

Lower side band power $P_{L.S.B.} = \frac{\left(\frac{A_c \mu}{2\sqrt{2}}\right)^2}{R} = \frac{A_c^2 \mu^2}{8R}.$

$$\therefore P_t = \frac{A_c^2}{2R} + \frac{A_c^2 \mu^2}{8R} + \frac{A_c^2 \mu^2}{8R}$$

$$= \frac{A_c^2}{2R}\left[1 + \frac{\mu^2}{4} + \frac{\mu^2}{4}\right]$$

$$= \frac{A_c^2}{2R}\left[1 + \frac{2\mu^2}{4}\right] \qquad P_c = \frac{A_c^2}{2R}$$

$$= \frac{A_c^2}{2R}\left[1 + \frac{\mu^2}{2}\right]$$

$$= \frac{A_c^2}{2R}\left[\frac{2+\mu^2}{2}\right]$$

$$\boxed{P_t = P_c\left[1 + \frac{\mu^2}{2}\right]}$$

If the modulation index $\mu = 0$ (No modulation)

$$\therefore \boxed{P_t = P_c}$$

If the modulation index $\mu = 1$ (full modulation)

$$\therefore P_t = P_c\left[1 + 0.5\right]$$

$$\boxed{P_t = 1.5\, P_c}$$

Current Relations in Amplitude Modulation

- The total power of the modulated signal is given by

$$P_t = P_c\left[1 + \frac{\mu^2}{2}\right]$$

$$\frac{P_t}{P_c} = 1 + \frac{\mu^2}{2}$$

$$\frac{I_t^2 (R)}{I_c^2 (R)} = 1 + \frac{\mu^2}{2}$$

$$\left(\frac{I_t}{I_c}\right)^2 = 1 + \frac{\mu^2}{2} \Rightarrow \frac{I_t}{I_c} = \sqrt{1 + \frac{\mu^2}{2}} \Rightarrow \boxed{I_t = I_c \sqrt{1 + \frac{\mu^2}{2}}}$$

Interms of voltage

$$\frac{P_t}{P_c} = 1 + \frac{\mu^2}{2}$$

$$\frac{V_t^2/R}{V_c^2/R} = 1 + \frac{\mu^2}{2}$$

$$\left(\frac{V_t}{V_c}\right)^2 = 1 + \frac{\mu^2}{2} \Rightarrow \frac{V_t}{V_c} = \sqrt{1 + \frac{\mu^2}{2}} \Rightarrow \boxed{\therefore V_t = V_c\sqrt{1 + \frac{\mu^2}{2}}}$$

Modulation Efficiency:-

The modulation efficiency specifies how much amount of power is utilizing to perform modulation. It is denoted as η.

$$\boxed{\eta = \frac{\text{Side Band power}}{\text{Total power}}}$$

Side Band power $P_{U.SB} + P_{L.S.B} = \frac{A_c^2 \mu^2}{8R} + \frac{A_c \mu^2}{8R}$

$$= \frac{2 A_c^2 \mu^2}{8R}$$

$$\boxed{\text{Side Band power} = \frac{A_c^2 \mu^2}{4R}}$$

Total power $P_t = P_c \left[1 + \frac{\mu^2}{2}\right]$ $\qquad P_c = \frac{A_c^2}{2R}$

$$\boxed{\text{Total power} = \frac{A_c^2}{2R}\left[1 + \frac{\mu^2}{2}\right]}$$

$$\therefore \eta = \frac{\frac{A_c^2 \mu^2}{4R}}{\frac{A_c^2}{2R}\left[1 + \frac{\mu^2}{2}\right]} \Rightarrow \frac{\frac{\mu^2}{2}}{\left[1 + \frac{\mu^2}{2}\right]} \Rightarrow \frac{\frac{\mu^2}{2}}{\left[\frac{2 + \mu^2}{2}\right]}$$

$$= \frac{\mu^2}{2}\left[\frac{2}{2 + \mu^2}\right]$$

$$\boxed{\therefore \eta = \frac{\mu^2}{2 + \mu^2}}$$

Problems

1. A broadcast AM transmitter radiates 50kW of power. What will be the radiated power if 85% of modulation and what is the side band power.

Sol

$P_c = 50KW$

$P_t = ?$

$P_{SB} = P_{U.S.B} + P_{L.S.B} = ?$

$\mu = 85\% = 0.85$

$P_t = P_c\left[1 + \frac{\mu^2}{2}\right]$

$= 50\times10^3\left[1 + \frac{(0.85)^2}{2}\right]$

$\boxed{P_t = 68KW}$

$P_{SB} = \frac{A_c^2\mu^2}{8R} + \frac{A_c^2\mu^2}{8R}$

$= \frac{A_c^2\mu^2}{4R}$

$P_c = \frac{A_c^2}{2R}$

$= \frac{A_c^2}{2R}\left[\frac{\mu^2}{2}\right]$

$= P_c\left[\frac{\mu^2}{2}\right]$

$= 50\times10^3\left[\frac{(0.85)^2}{2}\right]$

$\boxed{P_{SB} = 18KW}$

2. The maximum voltage of cathode ray oscilloscope displaying modulated signal on the screen is 4.6 divisions and the minimum voltage of CRO screen display is 0.6 divisions. Calculate the modulation Index.

Sol

Maximum Voltage $= 4.6$

Minimum Voltage $= 0.6$

$\therefore \mu = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$

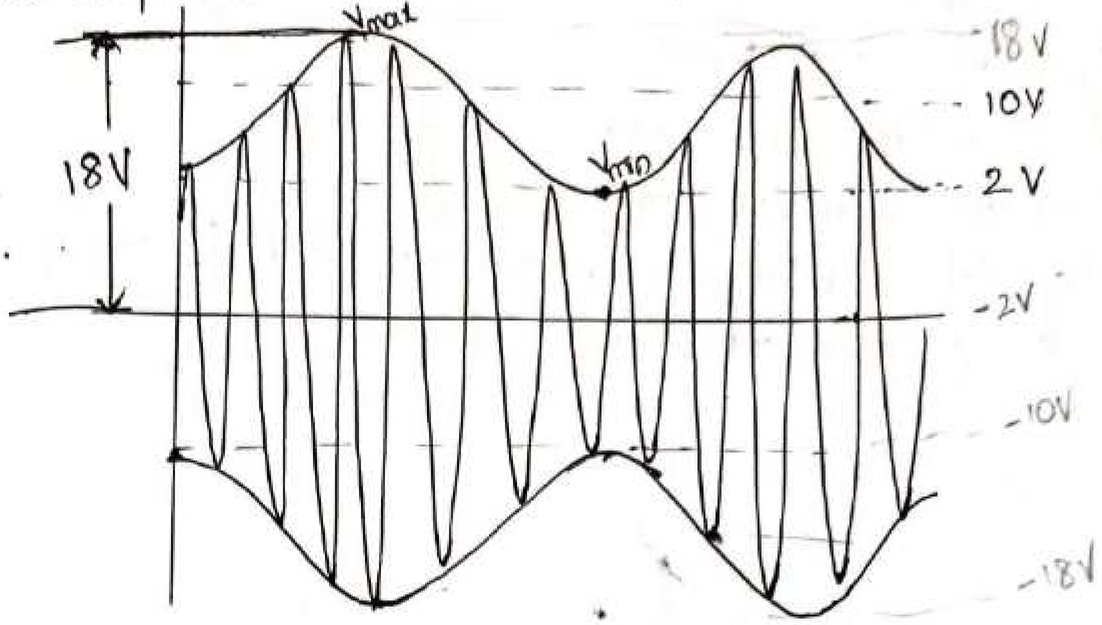
$\mu = \frac{4.6 - 0.6}{4.6 + 0.6}$

$\mu = \frac{4}{5.2}$

$\mu = 0.769$

$\boxed{\mu = 76.9\%}$

(3) The amplitude modulated wave is



Calculate peak amplitude of the unmodulated carrier,
peak changing amplitude of the envelope.
Modulation Index
percentage of modulation Index.

Sol

peak amplitude of unmodulated carrier = 10 V.

~~peak changing amplitude of the envelope~~ V_{max} = 18 V. , V_{min} = 2 V

$\therefore$ Modulation Index = $\dfrac{V_{max} - V_{min}}{V_{max} + V_{min}}$

$$= \frac{18-2}{18+2} = \frac{16}{20} = \frac{8}{10} = \boxed{0.8}.$$

peak changing amplitude of the envelope = (A_m) = ?

(or) message signal.

$V_{max} = A_c + A_m$

$V_{min} = A_c - A_m$

$V_{max} + V_{min} = A_c + A_m + A_c - A_m$

$= 2A_c \Rightarrow A_c = \dfrac{V_{max} + V_{min}}{2}$

$V_{max} - V_{min} = A_c + A_m - A_c + A_m$

$= 2A_m \Rightarrow A_m = \dfrac{V_{max} - V_{min}}{2}$

$\therefore A_m = \dfrac{18-2}{2} = \dfrac{16}{2} = 8V$

Percentage of modulation Index = $0.8 \times 100 = \boxed{80\%}$

Q. An audio signal $15 \sin 2\pi(1500t)$ and carrier signal $16 \sin 2\pi(100{,}000t)$

(i) sketch the audio signal

(ii) sketch the carrier signal.

(iii) construct modulated waveform.

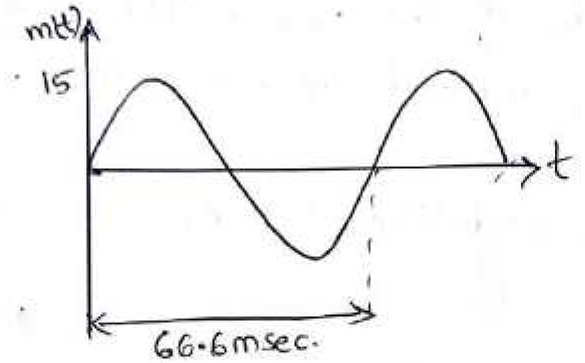
(iv) calculate modulation index and also find frequency of message signal and carrier signal.

Sol

$m(t) = 15 \sin 2\pi (1500t)$

$c(t) = 16 \sin 2\pi (100{,}000t)$

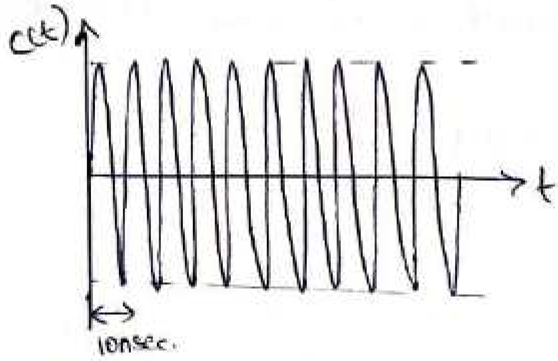
(i) sketch the audio signal



$f_m = 1500$

$T = \frac{1}{f_m} = \frac{1}{1500} = 66.6\text{ms}$

(ii) sketch the carrier signal.



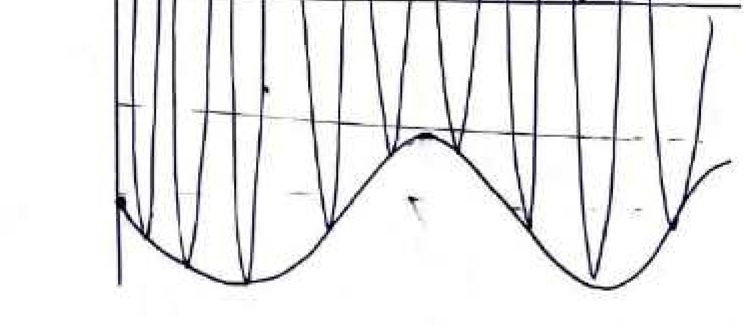
$f_c = 100{,}000$

$T = \frac{1}{f_c}$

$= \frac{1}{100000}$

$T = 10\text{nsec}.$

(iii) construct modulated waveform.



(iv) Modulation index

$$\mu = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

$$= \frac{75-45}{75+45} = 0.25$$

$$\boxed{\mu\% = 25\%}$$

(or) $\mu = \frac{A_m}{A_c} = \frac{15}{16} = 0.93$

$\boxed{\mu\% = 93\%}$

Frequency of message signal = 1500

Frequency of carrier signal = 100,000

12

(5). A 400 watts carrier is modulated to a depth of 75%. calculate the total power in the modulated wave.

Sol

$P_c = 400$ watts

$\mu = 0.75$

$\therefore P_t = P_c\left[1+\frac{\mu^2}{2}\right]$

$= 400\left[1+\frac{(0.75)^2}{2}\right] \Rightarrow 400\left[1+0.28125\right] \Rightarrow 400\left[1.281\right]$

$\boxed{P_t = 512.5 \text{ watts}}$

(6). A broadcasting radio transmitter radiates 5 kW power. when the modulation percentage is 60%. How much the carrier signal power.

Sol

$P_t = 5\,kW$

$P_t = 5000$ watts

$\mu\% = 60\%$

$\mu = 0.6$

$\therefore P_t = P_c\left[1+\frac{\mu^2}{2}\right]$

$$P_c = \frac{P_t}{1+\frac{\mu^2}{2}}$$

$$= \frac{5000}{1+\frac{(0.6)^2}{2}}$$

$$= \frac{5000}{1.18}$$

$$= 4237.2$$

$\therefore P_c = 4.237\,kW.$

(7) A 300 watts carrier is simultaneously modulated by a two audio waves with modulation percentages 50% and 60% respectively. what is the total radiated side band power.

sol

$$P_t = P_c + P_{SB}$$

$$P_{SB} = P_t - P_c$$

$P_c = 300$ watts

$\mu_1 = 0.5$

$\mu_2 = 0.6$

$$\therefore P_t = P_c\left[1 + \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2}\right]$$

$$= 300\left[1 + \frac{(0.5)^2}{2} + \frac{(0.6)^2}{2}\right]$$

$$= 300\left[1 + 0.125 + 0.18\right]$$

$$= 300\left[1.305\right]$$

$$P_t = 391.5 \text{ watts}$$

$$\therefore P_{SB} = P_t - P_c \Rightarrow 391.5 - 300 \Rightarrow \underline{\underline{91.5 \text{ watts}}}$$

(8) ~~A~~ the Antenna current of AM broad casting transmitter modulated to a depth of 40% by an audio sinusoidal wave is 11-Amps. It is increased to 12-Amperes as a result of sinusoidal modulation by another audio sine wave, what is the modulation index due to second sine wave.

sol

$I_{t_1} = 11\,A$ $\quad$ $I_{t_2} = 12\,A$

$\mu_1 = 0.4$ $\quad$ $\mu_2 = ?$

In Multi Tone modulation

$$I_t = I_c\sqrt{1 + \frac{\mu_t^2}{2}}$$

$$\frac{I_t}{I_c} = \sqrt{1 + \frac{\mu_t^2}{2}}$$

We know that

$$\mu_t^2 = \mu_1^2 + \mu_2^2$$

$$\mu_2^2 = \mu_t^2 - \mu_1^2$$

$$\mu_2 = \sqrt{\mu_t^2 - \mu_1^2}$$

$$I_C = \frac{I_{t_1}}{\sqrt{1+\frac{\mu_t^2}{2}}}$$

$$I_C = \frac{11}{\sqrt{1+\frac{0.16}{2}}} \Rightarrow \frac{11}{\sqrt{1.08}} \Rightarrow \frac{11}{1.039}$$

$$\therefore I_C = 10.5A$$

$$\frac{I_{t_2}}{I_C} = \sqrt{1+\frac{\mu_t^2}{2}}$$

$$\frac{12}{10.5} = \sqrt{1+\frac{\mu_t^2}{2}}$$

$$(1.14)^2 = 1+\frac{\mu_t^2}{2}$$

$$0.299 = \frac{\mu_t^2}{2}$$

$$\mu_t^2 = 0.598$$

$$\mu_t = \sqrt{0.598} = 0.773.$$

$$\therefore \mu_2 = \sqrt{\mu_t^2 - \mu_1^2}$$

$$= \sqrt{(0.773)^2-(0.4)^2} \Rightarrow \sqrt{0.437} \Rightarrow 0.661$$

$\therefore$ Modulation percentage is 66%

Modulation efficiency interms of critical, under, over modulation

$$\eta = \frac{\text{side band power}}{\text{Total transmitted power}}$$

$P_t = P_c + P_{SB}$

We know that

$$\% \eta = \frac{\mu^2}{2+\mu^2} \times 100$$

If $\mu = 0.5$ (under modulation ($\mu < 1$)), then $\eta = 0.11$

$\therefore P_{SB} = 11\%$

$P_c = 89\%$

If $\mu = 1$ (critical modulation ($\mu = 1$)), then $\eta = 0.33$

$\therefore P_{SB} = 33.33\%$

$P_c = 66.66\%$

If $\mu = 2$ (over modulation ($\mu > 1$)) ; then $\eta = 0.66\%$

$\therefore P_{SB} = 66.66\%$

$P_c = 33.33\%$

If $\mu = 0$, then $\eta = 0$

$\therefore P_{SB} = 0\%$

$P_c = 100\%$

* out of this total power (P_t), the useful message (or) base band power is the power carried by the side bands i.e P_{SB}.
* P_c is transmitted along with side band power only. for convenient and detection. Hence, the P_{SB} is the only useful power present in the AM wave.
* The maximum transmission efficiency of AM is only 33.33% i.e ($\frac{1}{3}$rd) of the total power carried by the side bands and rest of the power i.e $\frac{2}{3}$rd of the carrier power is wasted.

Single Tone Modulation

* In this method, the message (or) base band signal is only single frequency component, hence the resultant modulated output is called Single Tone Modulation.
* Here, the message signal contains a single frequency, and amplitude of message signal is varying and amplitude of carrier signal is constant.

The mathematical representation of single frequency message signal is

$m(t) = A_m \cos\omega_m t$

$c(t) = A_c \cos\omega_c t$

$\mu = \frac{A_m}{A_c}$

$\therefore S(t) = [A_c + m(t)] \cos\omega_c t$

$= A_c \left[1 + \frac{1}{A_c} m(t)\right] \cos\omega_c t$

$= A_c \left[1 + \frac{1}{A_c} \cdot A_m \cos\omega_m t\right] \cos\omega_c t$

$= A_c [1 + \mu \cos\omega_m t] \cos\omega_c t$

$= A_c \cos\omega_c t + A_c \mu \cos\omega_c t \cdot \cos\omega_m t$

$= A_c \cos\omega_c t + \frac{A_c \mu}{2} \cdot 2 [\cos\omega_c t \cdot \cos\omega_m t]$

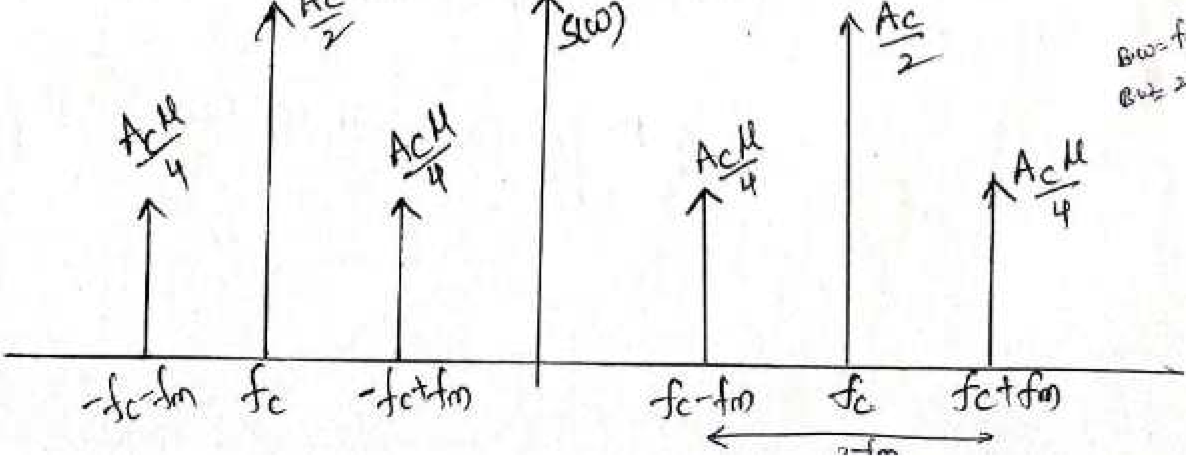
$= A_c \cos\omega_c t + \frac{A_c \mu}{2} [\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t]$

$= A_c \cos 2\pi f_c t + \frac{A_c \mu}{2} (\cos 2\pi(f_c + f_m)t + \cos 2\pi(f_c - f_m)t]$

Apply Fourier Transform on $s(t)$, then we get

$$S(t) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{A_c \mu}{2}\left[\frac{\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))}{2}\right] + \frac{A_c \mu}{2}\left[\frac{\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))}{2}\right]$$

$$= \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{A_c \mu}{4}\left[\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))\right] + \frac{A_c \mu}{4}\left[\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))\right]$$



$BW = f_c + f_m - f_c + f_m$

$BW = 2f_m$

Multi Tone Amplitude Modulation

* The message signal (or) modulating signal (or) base band signal contains multiple frequencies (or) more than one frequency, then the resultant modulated output signal is called Multi Tone Amplitude Modulation.

* the mathematical representation of message signal and carrier signal is

$$m(t) = A_{m_1} \cos \omega_{m_1} t + A_{m_2} \cos \omega_{m_2} t$$

$$c(t) = A_c \cos \omega_c t.$$

* Here the message signal contains a multiple frequency and amplitude of message signal is varing and amplitude of carrier signal is constant and frequency is varying.

$$s(t) = [A_c + m(t)] \cos \omega_c t$$

$$= A_c \left[1 + \frac{1}{A_c} m(t)\right] \cos \omega_c t.$$

$$= A_c \left[1 + \frac{A_{m_1}}{A_c} \cos \omega_{m_1} t + \frac{A_{m_2}}{A_c} \cos \omega_{m_2} t\right] \cos \omega_c t.$$

$$= A_c \left[1 + \mu_1 \cos \omega_{m_1} t + \mu_2 \cos \omega_{m_2} t\right] \cos \omega_c t.$$

$$= A_c \cos \omega_c t + A_c \mu_1 \cos \omega_{m_1} t \cdot \cos \omega_c t + A_c \mu_2 \cos \omega_{m_2} t \cdot \cos \omega_c t$$

$$s(t) = A_c \cos \omega_c t + \frac{A_c \mu_1}{2} \left[\cos(\omega_c + \omega_{m_1})t + \cos(\omega_c - \omega_{m_1})t\right]$$

$$+ \frac{A_c \mu_2}{2} \left[\cos(\omega_c + \omega_{m_2})t + \cos(\omega_c - \omega_{m_2})t\right]$$

Apply Fourier Transform on s(t), then we get.

$$S(\omega) = \frac{A_c}{2}\left[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)\right] + \frac{A_c\mu_1}{2}\left[\frac{\delta(\omega-(\omega_c+\omega_{m_1})+\delta(\omega+(\omega_c+\omega_{m_1})}{2}\right]$$

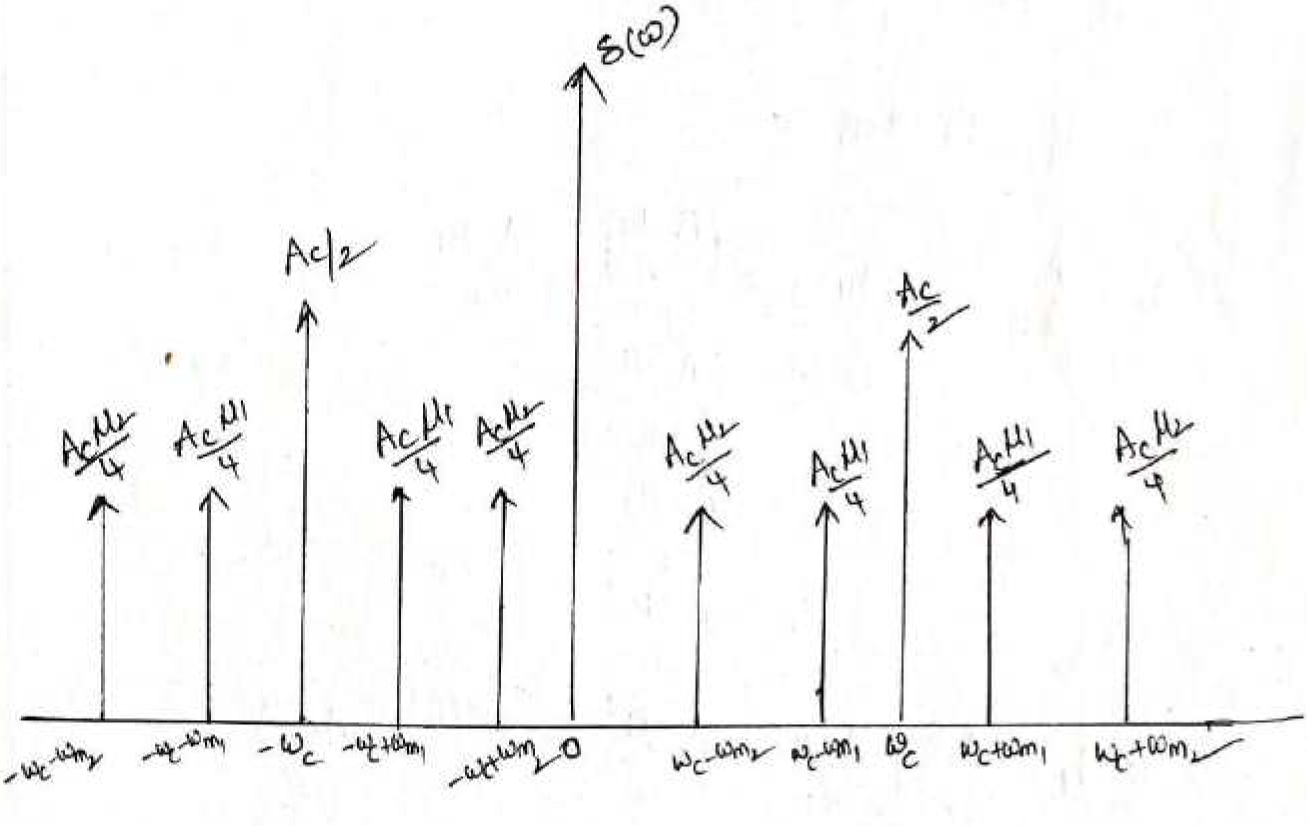
$$+\frac{A_c\mu_1}{2}\left[\frac{\delta(\omega-(\omega_c-\omega_{m_1})+\delta(\omega+(\omega_c-\omega_{m_1})}{2}\right]$$

$$+\frac{A_c\mu_2}{2}\left[\frac{\delta(\omega-(\omega_c+\omega_{m_2})+\delta(\omega+(\omega_c+\omega_{m_2})}{2}\right]+\frac{A_c\mu_2}{2}\left[\frac{\delta(\omega-(\omega_c-\omega_{m_2})+\delta(\omega+(\omega_c-\omega_{m_2})}{2}\right]$$

$$S(\omega) = \frac{A_c}{2}\left[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)\right] + \frac{A_c\mu_1}{4}\left[\delta(\omega-(\omega_c+\omega_{m_1})+\delta(\omega+(\omega_c+\omega_{m_1})\right]$$

$$+\frac{A_c\mu_1}{4}\left[\delta(\omega-(\omega_c-\omega_{m_1})+\delta(\omega+(\omega_c-\omega_{m_1})\right]$$

$$+\frac{A_c\mu_2}{4}\left[\delta(\omega-(\omega_c+\omega_{m_2})+\delta(\omega+(\omega_c+\omega_{m_2})\right]+\frac{A_c\mu_2}{4}\left[\delta(\omega-(\omega_c-\omega_{m_2})+\delta(\omega+(\omega_c-\omega_{m_2})\right]$$



* In multi tone amplitude modulation technique, the message signal having more frequency components, then the side bands of modulated output signal is increases.

* If $\omega_{m_1} < \omega_{m_2}$, then the two tone modulation required Bandwidth is

$$\therefore B.W = 2\omega_{m_2}$$

Power calculation in Two tone amplitude Modulation

The mathematical equation for two tone amplitude modulation is given by.

$$s(t) = A_c \cos\omega_c t + \frac{A_c \mu_1}{2}\left[\cos(\omega_c + \omega_{m_1})t + \cos(\omega_c - \omega_{m_1})t\right]$$
$$+ \frac{A_c \mu_2}{2}\left[\cos(\omega_c + \omega_{m_2})t + \cos(\omega_c - \omega_{m_2})t\right]$$

The total power of Amplitude modulation is

$$P_t = P_c + P_{SB}$$

$$P_c = \frac{A_c^2}{2R}, \quad P_{U.S.B} = \frac{\left(\frac{A_c \mu_1}{2\sqrt{2}}\right)^2}{R} = \frac{A_c^2 \mu_1^2}{8R} \text{ for one freq}$$

$$P_{L.S.B} = \frac{A_c^2 \mu_1^2}{8R} \text{ for one freq}$$

$$\left.\begin{aligned} P_{U.S.B} &= \frac{A_c^2 \mu_2^2}{8R} \\ P_{L.S.B} &= \frac{A_c^2 \mu_2^2}{8R} \end{aligned}\right\} \text{for second freq.}$$

$$\therefore P_t = \frac{A_c^2}{2R} + \frac{A_c^2 \mu_1^2}{8R} + \frac{A_c^2 \mu_1^2}{8R} + \frac{A_c^2 \mu_2^2}{8R} + \frac{A_c^2 \mu_2^2}{8R}$$

$$= \frac{A_c^2}{2R}\left[1 + \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2}\right]$$

$$P_t = P_c\left[1 + \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2}\right]$$

If $\mu_t^2 = \mu_1^2 + \mu_2^2 + \ldots$ then

$$\boxed{\therefore P_t = P_c \left[1 + \frac{\mu_t^2}{2}\right]}$$

Problems

1. The output power of an AM transmitter is 1KW, when sinusoidally modulated to a depth of 100%. Calculate the power of each side band. when the modulated depth is reduced to 50%

sol

$P_t = 1KW$ when $\mu = 1$

$P_{U.S.B} = P_{L.S.B} = ?$ when $\mu = 0.5$

$$P_{SB} = \frac{A_c^2\mu^2}{8R} + \frac{A_c^2\mu^2}{8R}$$

$$= \frac{A_c^2\mu^2}{4R}$$

$$P_{SB} = \frac{A_c^2}{2R}\left[\frac{\mu^2}{2}\right]$$

$$\boxed{P_{SB} = P_c\left[\frac{\mu^2}{2}\right]}$$

$$P_t = P_c\left(1 + \frac{\mu^2}{2}\right]$$

$$1KW = P_c\left[1 + \frac{1}{2}\right]$$

$$10^3 = P_c\left[\frac{3}{2}\right]$$

$$\boxed{P_c = 0.67KW}$$

$\therefore P_{SB} = P_c\left[\frac{\mu^2}{2}\right]$ when $\mu = 0.5$

$$= 0.67 \times 10^3\left[\frac{(0.5)^2}{2}\right]$$

$=$

An audio frequency signal 10 sin 2π 500t is used to amplitude modulate of a carrier if 50 sin 2π 10^5 t. Assume μ = 0.2, then calculate

(i) side Band frequencies
(ii) Amplitude of each side band
(iii) B.W.
(iv) Total power delivered to 600Ω.

Sol

Given $m(t) = 10 \sin 2\pi 500t$

$A_m = 10$

$f_m = 500$

$c(t) = 50 \sin 2\pi 10^5 t$

$A_c = 50$

$f_c = 10^5$

$\mu = 0.2$.

(i) side Band frequencies

$f_{U.S.B} = f_c + f_m \Rightarrow 10^5 + 500 = \underline{\underline{100.5 KHz}}$

$f_{L.S.B} = f_c - f_m \Rightarrow 10^5 - 500 = \underline{\underline{99.5 KHz}}$

(ii) Amplitude of each side Band

$\Rightarrow \frac{A_c \mu}{2}$

$= \frac{50 \times 0.2}{2} \Rightarrow \underline{\underline{5V}}$

(iii) $B.W = f_c + f_m - f_c + f_m$

$= 2f_m \Rightarrow 2(500) = \underline{\underline{1KHz}}$

(iv) total power

$P_t = P_c \left[1 + \frac{\mu^2}{2}\right]$

$= \frac{A_c^2}{2R}\left[1 + \frac{\mu^2}{2}\right]$

$= \frac{(50)^2}{2(600)}\left[1 + \frac{(0.2)^2}{2}\right] \Rightarrow \frac{25}{12}[1.2] \Rightarrow \underline{\underline{250 watts}}$

(3) The maximum and minimum amplitudes of AM wave are 12V, 3V for a carrier signal voltage of V_c volts at 6MHz determine.

(i) Modulation Index

(ii) carrier signal power

(iii) Total Average power (Assume $R = 100\Omega$)

Sol

$V_{max} = 12\,V$

$V_{min} = 3\,V$

(i) Modulation Index $\mu = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{12-3}{12+3} = \frac{9}{15} = 0.6.$

(ii) carrier signal power $P_c = \frac{A_c^2}{2R}$

$$V_{max} = A_c + A_m$$

$$= A_c\left[1 + \frac{A_m}{A_c}\right]$$

$$V_{max} = A_c[1+\mu]$$

$$12 = A_c[1+0.6]$$

$$A_c = \frac{12}{1.6}$$

$$\boxed{A_c = 7.5\,V}$$

$$\therefore \boxed{P_c = \frac{(7.5)^2}{2(100)} = 0.281\text{ watts}}$$

(iii) $P_t = P_c\left[1 + \frac{\mu^2}{2}\right]$

$$= 0.281\left[1 + \frac{(0.6)^2}{2}\right]$$

$$\boxed{P_t = 0.331\text{ watts}}$$

(4) An AM Transmitter has unmodulated carrier power of 10KW, it can be modulated by sinusoidal modulating voltage to a maximum depth of 40% without over loading if the modulation index is reduced to 30%. what is the extent upto the unmodulated carrier power can be increased to avoid overloading.

Sol

$P_c = 10KW$

$\mu = 0.4$

$P_t = P_c\left[1+\frac{\mu^2}{2}\right]$

$= 10\times10^3\left[1+\frac{(0.4)^2}{2}\right] \Rightarrow 10^4\left[1+\frac{0.16}{2}\right] \Rightarrow 10.8\ KW.$

$P_c = ?$

$\mu = 0.3$

$\therefore P_t = P_c\left[1+\frac{\mu^2}{2}\right]$

$10.8\times10^3 = P_c\left[1+\frac{(0.3)^2}{2}\right]$

$\boxed{\therefore P_c = 10.33KW}$

(5) An Audio signal $10\sin 18.85\times10^3 t$. is modulate with the carrier $50\sin 1570.8\times10^3 t$. what are the frequencies in AM spectrum and also calculate the modulation Index.

Sol

$m(t) = 10\sin 18.85\times10^3 t.$

$c(t) = 50\sin 1570.8\times10^3 t.$

Here $A_m = 10$

$A_c = 50$

modulation index $(\mu) = \frac{A_m}{A_c} = \frac{10}{50} \Rightarrow \frac{1}{5} \Rightarrow 0.2.$

freq's in AM spectrum

$2\pi f_m = 18.85\times10^3$

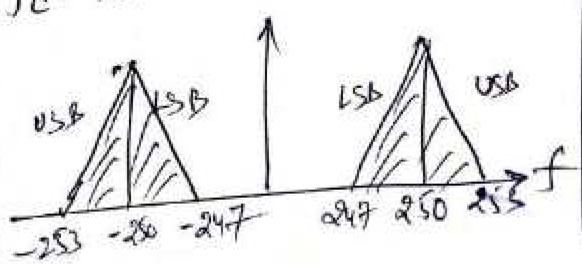
$f_m = 3KHz$

$2\pi f_c = 1570.8\times10^3$

$f_c = 250KHz$

$f_c + f_m = 250+3 = 253KHz$

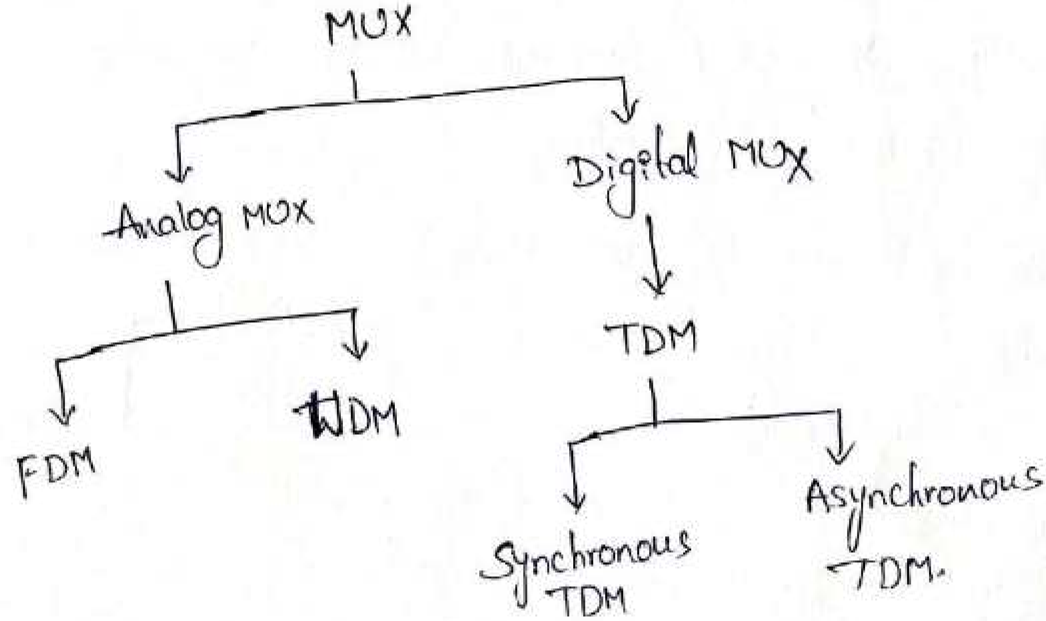
$f_c - f_m = 250-3 = 247KHz$



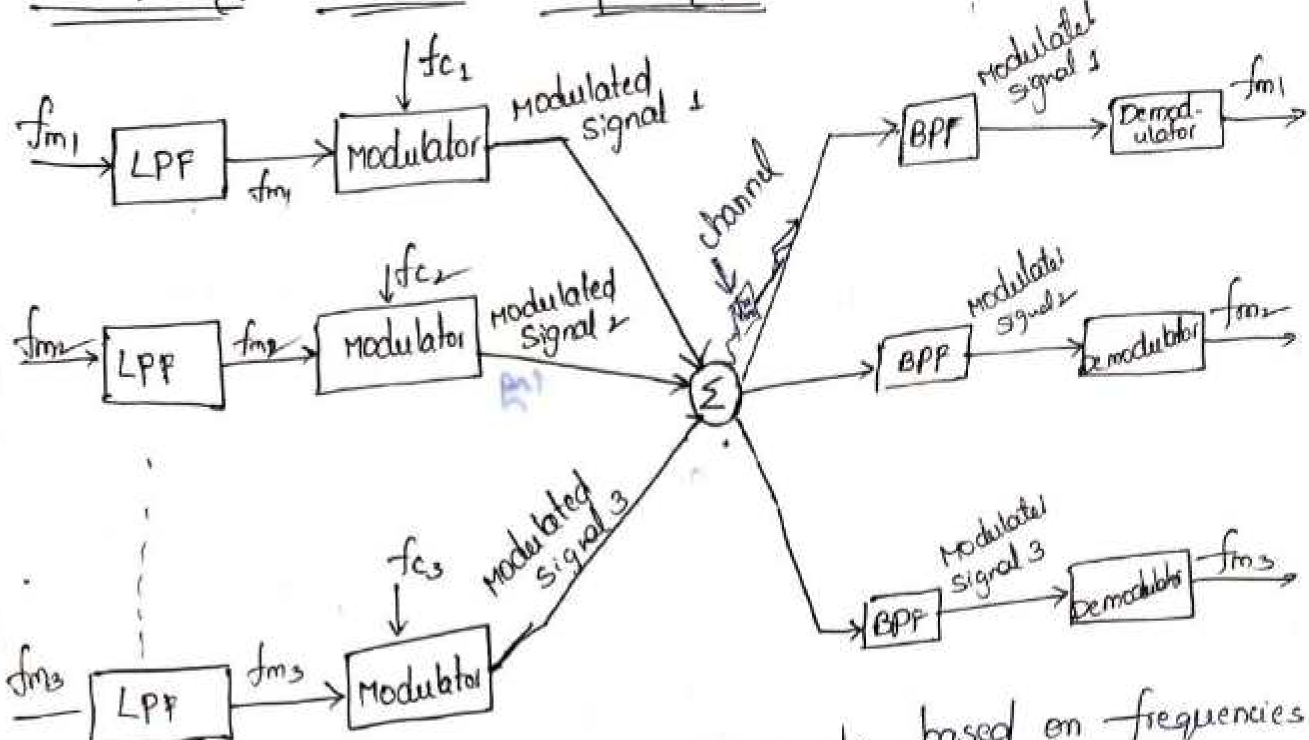
Multiplexing:-

* The multiplexing is a process of combining more signals into a one signal over a same (shared) channel.
* The multiplexing divides a communication channels into several logic channels, allotting each one for different message signals to be transferred.
* If the combination signals are analog, hence the multiplexing is analog multiplexing.
* If the combination signals are digital, hence the multiplexing is called Digital Multiplexing
* The reverse processing of multiplexing i.e. extract the number of channels from one signal is called de-multiplexing

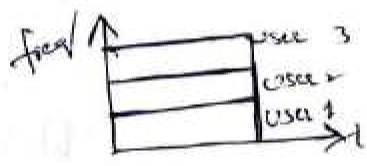
Types of Multiplexing



Frequency Division Multiplexing (FDM)



* The technique of separating the signals based on frequencies referred as FDM.
* At transmitter end, the input message signals are passed through LPF which removes the high frequency components.
* The filtered message signals are modulated with their carrier signal by the help of modulation
* The output modulators are combine and transmitted through a single channel.
* At the receiver end, the BPF separates occupancy basics.
* Finally the original message signals are recovered back by individual demodulators.
* The most commonly FDM techniques is used for analog systems only.



Eg:- Television

* In FDM, there is a chance of crosstalk between one user to another user because all signals are transmitted simultaneously.

Wavelength Division Multiplexing:- (WDM).

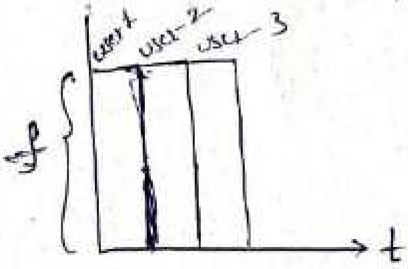
* the WDM technique is used to transmitt defferent wavelengths through a light spectrum.

* If the wavelength of the signal increases, then the frequency of the signal decreases i.e $\lambda = \frac{c}{f}$.

Time Division Multiplexing (TDM):-

* It is a digital multiplexing technique, and the information is transferred in the form of packets (or) frames.

* In time division multiplexing, all the users utilizes full bandwidth, but each user allot a fixed Time slot.



* In TDM, each user occupy complete channel bandwidth and fixed time slots and there is no cross talk on TDM.

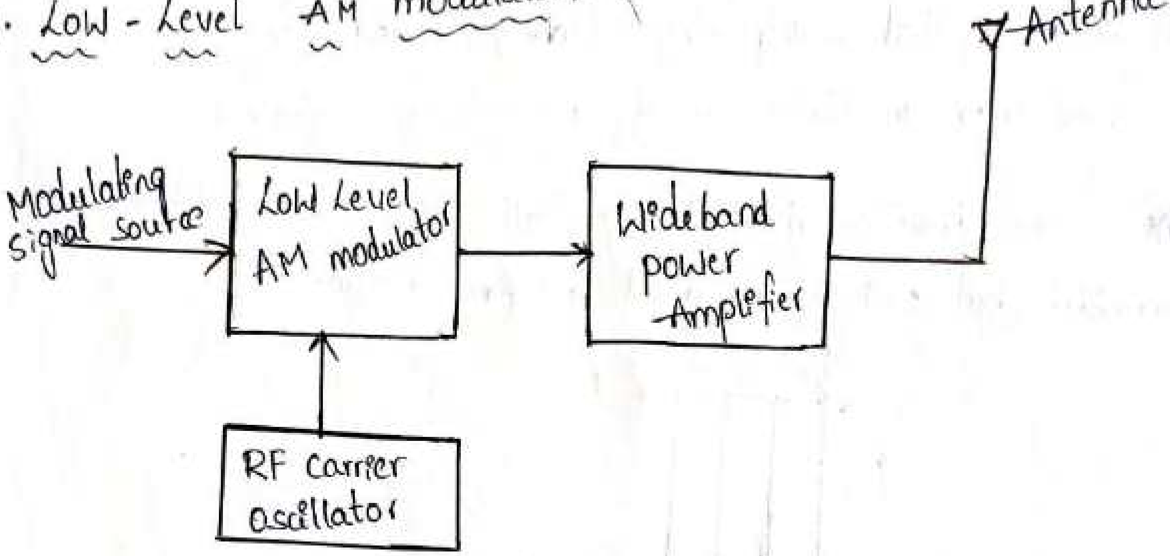
* In TDM, all the signals operate with same frequency at differnt time slots.

Generation of AM Waves

* The device which is used to generate amplitude modulated wave is called Amplitude Modulator.

* The AM modulators is classified into two types
    1. Low Level AM modulator.
    2. High Level AM modulator.

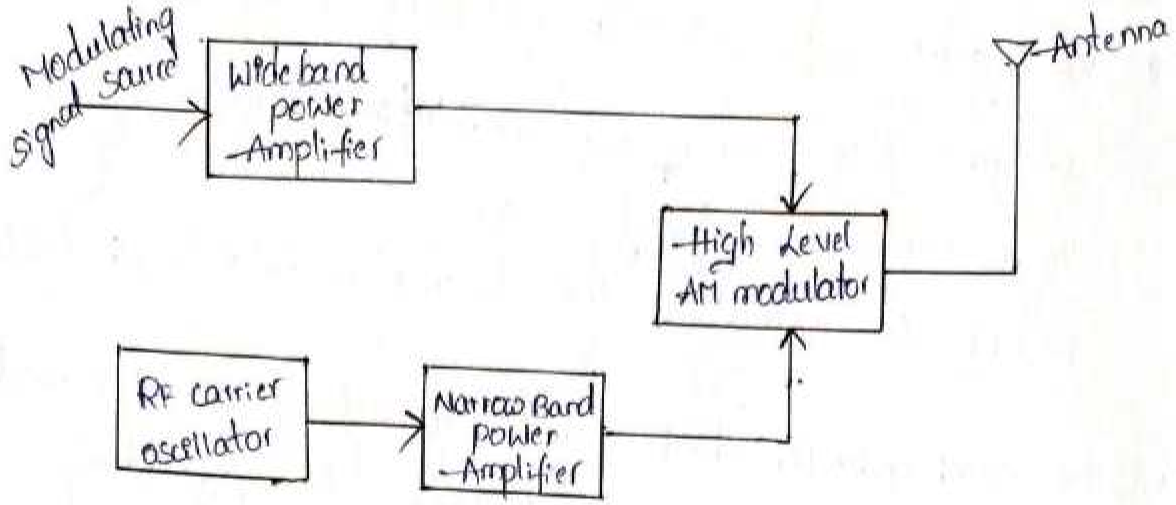
1. Low - Level AM modulator:-



* In this case, the modulation is done at low power level.
* The low level AM modulator has two inputs, one is modulating signal and other is carrier signal. carrier signal is generated by the RF carrier oscillator.
* The output at this low level AM modulator is low, therefore the power amplifiers are required to boost the amplitude modulated signal upto the desired output level
* A wide band power amplifier is used to preserve the side bands of the modulated signal.

Eg:- Square Law Modulator, Switching modulator

2. High- Level AM Modulator:-



* In high level AM modulator, the modulation is done at high power levels.

* Before ~~amplif~~ applying the modulating signal and carrier signal to the high level AM modulator, first we use power amplifiers.

* In this, wide band power amplifier is used to amplify the modulating signal, where as narrow band power amplifier is used to amplify the carrier signal.

* modulating signal contains lots of frequencies, to preserve all these frequency components, we use wide band power amplifier for the modulating signal.

* while the carrier signal is fixed frequency, therefore the narrow band power amplifier is sufficient for the amplification of carrier signal.

* therefore the amplified modulating signal, and the amplified carrier is given to the High level modulator and then transmitt to the Antenna

Eg:- Square Law detector, Envelope detector (or) diode detector

Square Law Modulator

* the square law modulator consists of 3 features.

1. Summing of carrier and modulating signal.
2. A Non-Linear Element.
3. BPF for extracting the desired modulation products.

* the semiconductor diodes and transistors are the most common Non-linear elements (or) devices for implementing square law modulator.

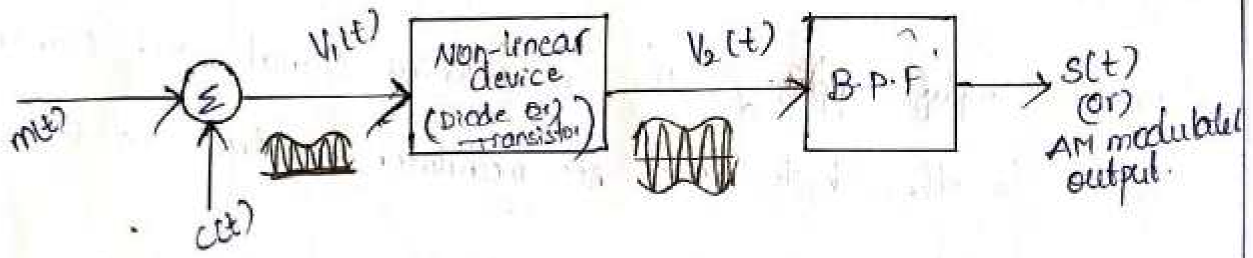
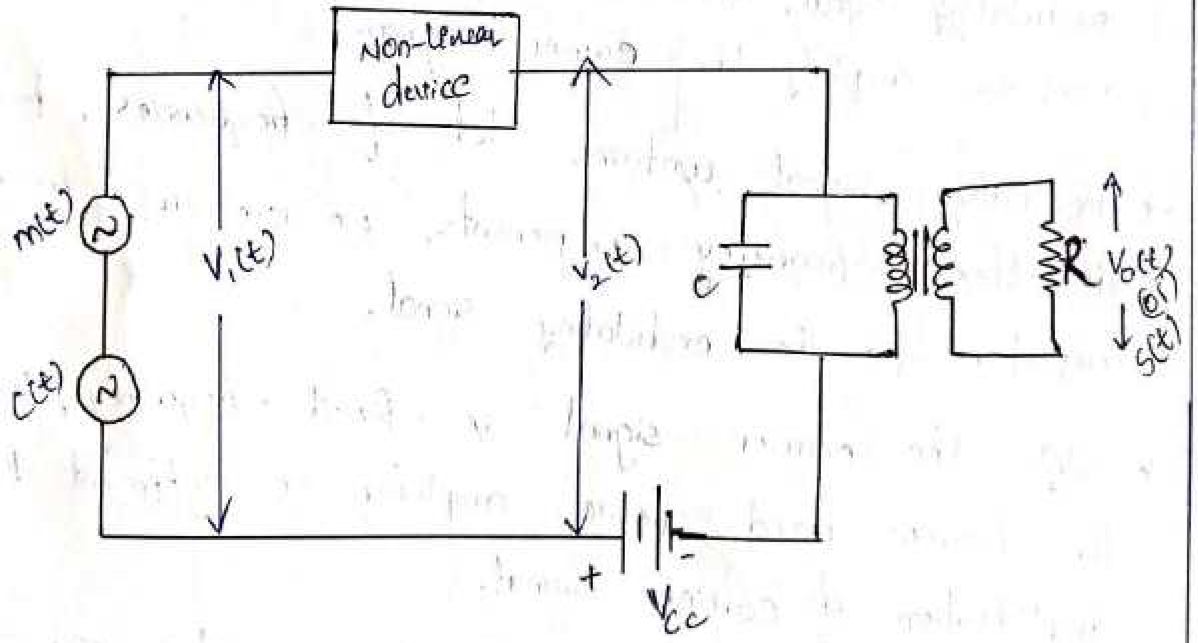


Fig: Block diagram of square law modulator.



* the semiconductor diode and transistors are the most common non-linear elements (or) devices for implementing square law modulator.

* For example, if we apply input voltage to the diode, and then passes the diode current across the load resistor R, and the output voltage across the load would be of this form.

$$\boxed{V_o = aV_{in}(t) + bV_{in}^2(t) + cV_{in}^3(t) + \cdots}$$

When the input wave is very small, then the higher order terms are neglected. In this case, consider upto 2nd order term.

$$\boxed{V_o = aV_{in}(t) + bV_{in}^2(t)}$$

* First the message signal m(t) and carrier signal c(t) are added and then applied to a Non-linear device, then it produce a modulated output and extracting desired modulated input by applying Band pass filtered.

The adder output voltage

$$V_1(t) = c(t) + m(t)$$

$$V_1(t) = m(t) + A_c \cos\omega_c t \longrightarrow ①$$

the mathematical representation of square law device (or) Non-linear device output is

$$V_2(t) = K_1 V_1(t) + K_2 V_1^2(t) + \cdots \quad ②$$

$$\therefore V_2(t) = K_1\left[m(t) + A_c\cos\omega_c t\right] + K_2\left[m(t) + A_c\cos\omega_c t\right]^2 + \cdots$$

$$= K_1 m(t) + K_1 A_c \cos\omega_c t + K_2\left[m^2(t) + A_c^2\cos^2\omega_c t + 2m(t)A_c\cos\omega_c t\right]$$

$$= K_1 m(t) + K_1 A_c \cos\omega_c t + K_2 m^2(t) + K_2 A_c^2 \cos^2\omega_c t + 2K_2 m(t) A_c \cos\omega_c t$$

$$\boxed{\cos^2 t = \frac{1+\cos 2t}{2}}$$

$$= K_1 m(t) + K_1 A_c \cos\omega_c t + K_2 m^2(t) + K_2 A_c^2\left[\frac{1+\cos 2\omega_c t}{2}\right] + 2K_2 m(t) A_c \cos\omega_c t$$

$$V_2(t) = K_1 m(t) + K_1 A_c \cos\omega_c t + K_2 m^2(t) + \frac{1}{2} K_2 A_c^2 + \frac{1}{2} K_2 A_c^2 \cos 2\omega_c t + 2K_2 m(t) A_c \cos\omega_c t$$

$$= K_1 m(t) + K_2 m^2(t) + \frac{1}{2} K_2 A_c^2 + \frac{1}{2} K_2 A_c^2 \cos 2\omega_c t + K_1 A_c \cos\omega_c t + 2K_2 A_m \cos\omega_m t \cdot A_c \cos\omega_c t.$$

$$V_2(t) = \underbrace{K_1 m(t)}_{\text{message signal}} + \underbrace{K_2 m^2(t)}_{\text{square of message signal}} + \overbrace{\frac{1}{2} K_2 A_c^2}^{\text{DC component}} + \overbrace{\frac{1}{2} K_2 A_c^2 \cos 2\omega_c t}^{\text{square of the carrier}}$$

$$+ \underbrace{K_1 A_c \left[1 + \frac{2K_2}{K_1} m(t)\right] \cos\omega_c t}_{\text{Modulated output.}} \longrightarrow (3).$$

* The eq (3) is transmitted through B.P.F, this B.P.F allows only desired frequencies i.e at center freq ω_c, U.S.B freq $\omega_c+\omega_m$, L.S.B freq $\omega_c-\omega_m$ only, remaining unwanted components are rejected by using B.P.F.

* Therefore after passing through the B.P.F, the output voltage

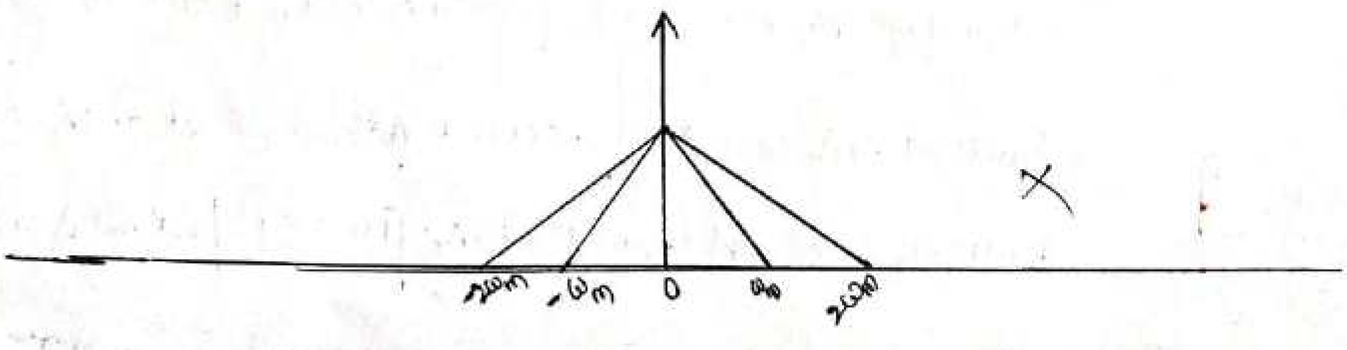
$$S(t) \text{ (or) } V_0(t) = K_1 A_c \left[1 + \frac{2K_2}{K_1} m(t)\right] \cos\omega_c t \longrightarrow (4)$$

* The general mathematical equation is

$$\boxed{S(t) = A_c [1 + K_a m(t)] \cos\omega_c t} \longrightarrow (5)$$

comparing eq (4) & (5)

where $K_a = \frac{2K_2}{K_1}$



$V_2(t) = K_1 A_m \cos\omega_m t + K_2[m(t)\cdot m(t)] + \frac{1}{2}K_2 A_c^2 + \frac{1}{2}K_2 A_c^2 \cos 2\omega_c t + K_1 A_c \cos\omega_c t + 2K_2 A_c A_m \cos\omega_c t \cdot \cos\omega_m t$

Apply Fourier Transform on $V_2(t)$

$$\therefore V_2(\omega) = K_1 A_m\left[\frac{\delta(\omega-\omega_m)+\delta(\omega+\omega_m)}{2}\right] + K_2\left[M(\omega) * M(\omega)\right] + K_2 A_c^2 \delta(\omega) + \frac{1}{2}K_2 A_c^2\left[\frac{\delta(\omega-2\omega_c)+\delta(\omega+2\omega_c)}{2}\right] + K_1 A_c\left[\frac{\delta(\omega-\omega_c)+\delta(\omega+\omega_c)}{2}\right]$$

$$+ \frac{2K_2 A_c A_m}{2} 2\cos\omega_c t \cdot \cos\omega_m t.$$

$K_2 A_m^2 \cos^2\omega_m t$

$K_2 A_m^2\left(\frac{1+\cos 2\omega_m t}{2}\right)$

$\frac{1}{2}K_2 A_m^2 + \frac{1}{2}K_2 A_m^2\left[\frac{\delta(\omega-2\omega_m)+\delta(\omega+2\omega_m)}{2}\right]$

$K_2 A_m^2\delta(\omega) + \frac{1}{4}K_2 A_m^2\left[\delta(\omega-2\omega_m)+\delta(\omega+2\omega_m)\right]$

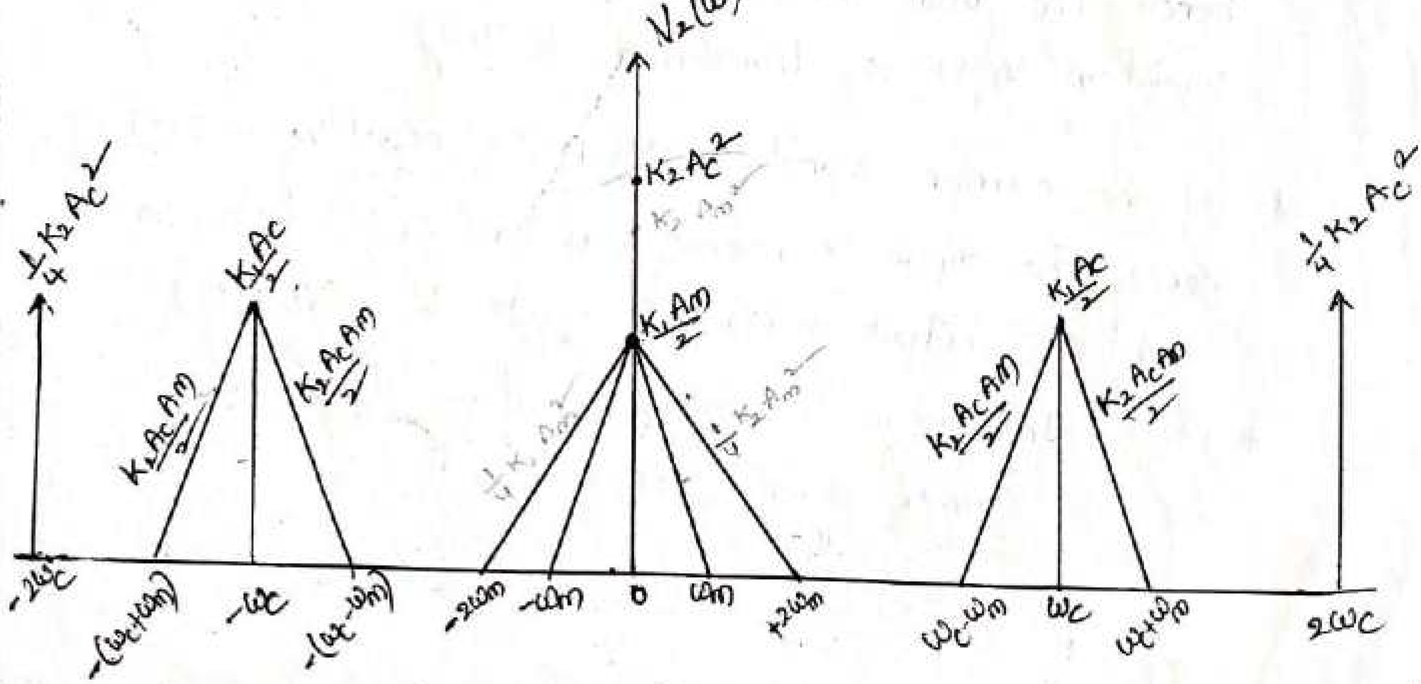
$$V_2(\omega) = \frac{K_1 A_m}{2}\left[\delta(\omega-\omega_m)+\delta(\omega+\omega_m)\right] + K_2\left[M(\omega)*M(\omega)\right] + K_2 A_c^2\delta(\omega) + \frac{1}{4}K_2 A_c^2\left[\delta(\omega-2\omega_c)+\delta(\omega+2\omega_c)\right] + \frac{K_1 A_c}{2}\left[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)\right]$$

$$+ K_2 A_c A_m\left[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t\right]$$

$$V_2(\omega) = \frac{K_1 A_m}{2}\left[\delta(\omega-\omega_m)+\delta(\omega+\omega_m)\right] + K_2\left[M(\omega)*M(\omega)\right] + K_2 A_c^2\delta(\omega) + \frac{1}{4}K_2 A_c^2\left[\delta(\omega-2\omega_c)+\delta(\omega+2\omega_c)\right] + \frac{K_1 A_c}{2}\left[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)\right]$$

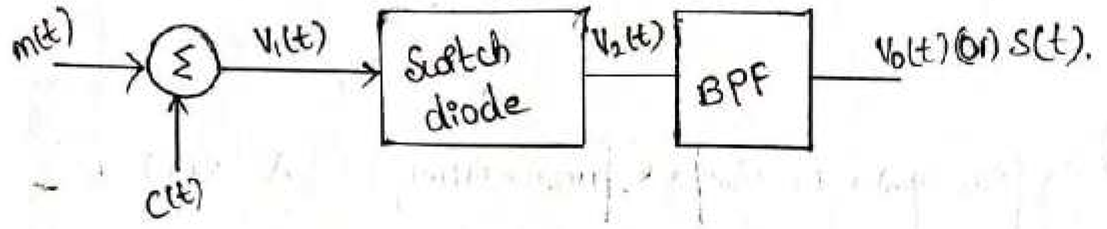
$$+ \frac{K_2 A_c A_m}{2}\left[\delta(\omega-(\omega_c+\omega_m)) + \delta(\omega+(\omega_c+\omega_m))\right] + \frac{K_2 A_c A_m}{2}\left[\delta(\omega-(\omega_c-\omega_m)) + \delta(\omega+(\omega_c-\omega_m))\right]$$

Frequency Spectrum



Switching Modulator :-

* The switching modulator is same as square law modulator but the difference is the square law modulator operates in non-linear mode and switching modulator operates in linear mode
* The Block diagram of switching modulator as show in below fig.



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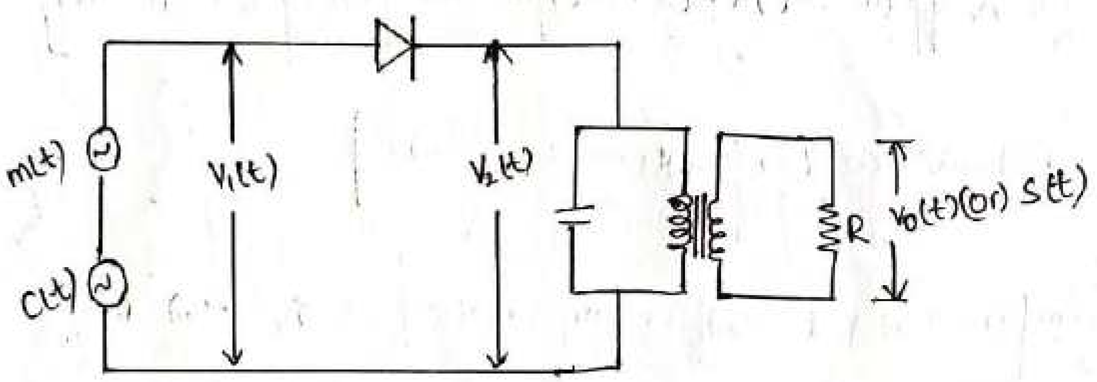


* Here $V_1(t) = m(t) + c(t)$

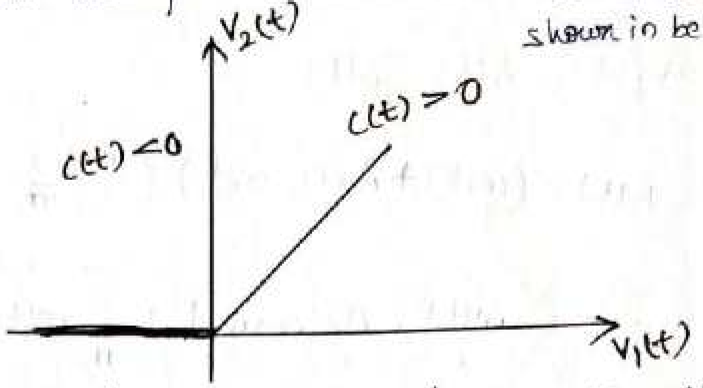
$$V_1(t) = m(t) + A_c \cos\omega_c t \longrightarrow (1)$$

* If the carrier signal amplitude is positive i.e $c(t) > 0$, hence the diode is forward bias (or) "ON" state. In this condition $V_1(t)$ is transferred to $V_2(t)$.
* If the carrier signal amplitude is negative ie $c(t) < 0$, hence the diode is reverse bias (or) "OFF" state. In this condition output $V_2(t)$ is equal to zero ("0")
* Hence the diode is ON/OFF condition, it is controlled by the carrier signal c(t).

* The diode input and output voltage characteristics $V_1(t)$ & $V_2(t)$ is shown in below fig.

* The diode output voltage is

$$V_2(t) = \begin{cases} V_1(t) ; \text{ if } c(t) > 0 \\ 0 \quad : \text{ if } c(t) < 0 \end{cases}$$



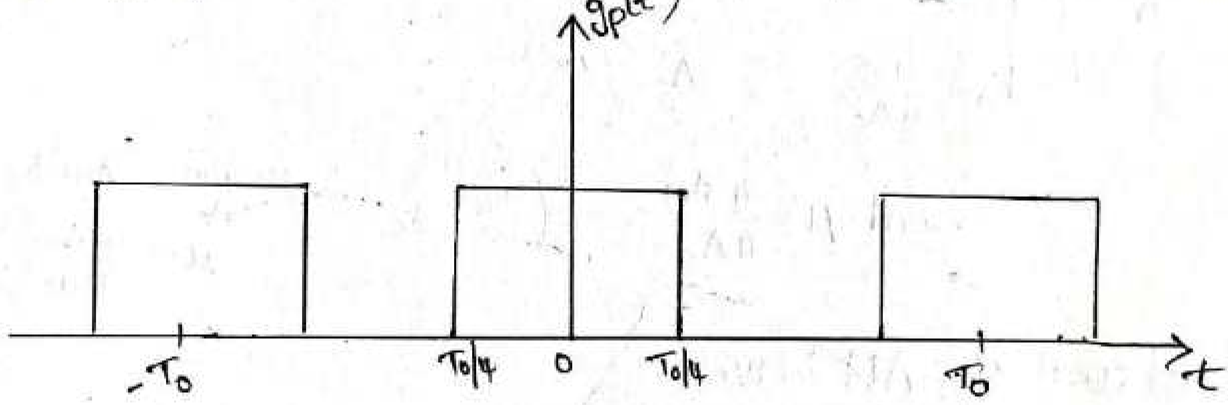
* If the input signal is represent in periodic nature, then the calculation of parameters is very simple..

* the output voltage of diode i.e $V_2(t)$ varies periodically between the values of "$V_1(t)$" and "0". Therefore the diode output voltage is expressed as

$$V_2(t) = V_1(t) \cdot g_p(t) \longrightarrow (2)$$

Where $g_p(t)$ is periodic pulse train.

* The periodic pulse train $g_p(t)$ graphical view as shown in below fig.



* $g_p(t)$ is expressed in trigonometric mathematical equation of fourier series is given by.

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos((2n-1)\omega_c t)$$

$$= \frac{1}{2} + \frac{2}{\pi}\cos\omega_c t - \underbrace{\frac{2}{3\pi}\cos 3\omega_c t + - - - -}_{\text{odd harmonics}}$$

$$\therefore g_p(t) = \frac{1}{2} + \frac{2}{\pi}\cos\omega_c t \longrightarrow (3)$$

From eq ①, ② and ③

$$V_2(t) = V_1(t) \cdot g_p(t)$$

$$V_2(t) = (m(t) + A_c \cos\omega_c t)\left(\frac{1}{2} + \frac{2}{\pi}\cos\omega_c t\right)$$

$$V_2(t) = \frac{m(t)}{2} + \frac{A_c}{2}\cos\omega_c t + \frac{2}{\pi}m(t)\cos\omega_c t + \frac{2A_c}{\pi}\cos^2\omega_c t \longrightarrow ④$$

$\therefore V_2(t)$ is passed through Band pass filter (B.P.F).

$$\therefore V_0(t) \text{ (or) } s(t) = \frac{A_c}{2}\cos\omega_c t + \frac{2}{\pi}m(t)\cos\omega_c t$$

$$\boxed{s(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c}m(t)\right]\cos\omega_c t} \longrightarrow ⑤$$

Mathematical eq for AM is

$$s(t) = A_c'\left[1 + K_a m(t)\right]\cos\omega_c t \longrightarrow ⑥$$

By comparing eq ⑤ & eq ⑥.

$$K_a = \frac{4}{\pi A_c}, \quad A_c' = \frac{A_c}{2}$$

$$\text{and } \mu = \frac{4A_m}{\pi A_c} \qquad \left(K_a = \frac{1}{A_c} \Rightarrow \mu = \frac{A_m}{A_c} \Rightarrow A_m \cdot K_a \quad \mu = \frac{4A_m}{\pi A_c}\right)$$

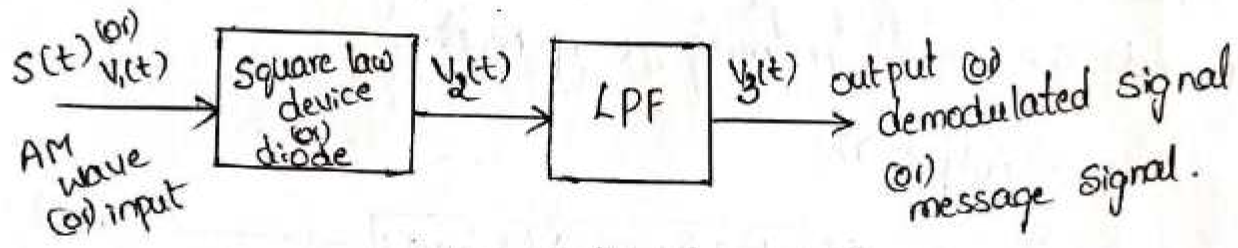
Detection of AM Waves

* The process of extracting original message signal (or) Base band signal from the modulated wave is called Demodulation (or) Detection.
* Demodulation methods are three types.
  1. Square Law detector
  2. Envelope detector (or) diode detector.
  3. Synchronous detector (or) coherent detector (or) homogenous detector.

1. Square Law detector:

* The square law demodulator device operates non-linear characteristics of diode and the modulation index is $\mu \le 1$. Hence the square law demodulator operates only on low level modulated signal.

* The block diagram of square law demodulator is shown in below:



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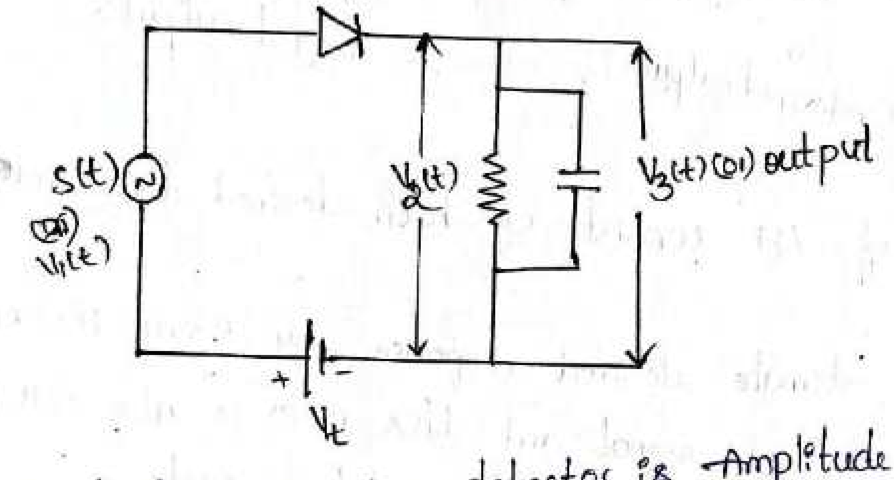


* The input of square law detector is Amplitude modulated wave.

ie $s(t) = A_c\left[1+K_a m(t)\right]\cos\omega_c t$

$V_1(t)$ (or) $S(t) = A_c\cos\omega_c t + A_c K_a m(t)\cos\omega_c t \longrightarrow (1)$

* The eq (1) i.e $s(t)$ is applied to square law device, the output of a square law device is

$$V_2(t) = K_1 V_1(t) + K_2 V_1^2(t)$$
$$= K_1\left[A_c\cos\omega_c t + A_c K_a m(t)\cos\omega_c t\right] + K_2\left[A_c\cos\omega_c t + A_c K_a m(t)\cos\omega_c t\right]^2$$
$$= K_1 A_c\cos\omega_c t + K_1 A_c K_a m(t)\cos\omega_c t + K_2 A_c^2\cos^2\omega_c t + K_2 A_c^2 K_a^2 m^2(t)\cos^2\omega_c t + 2K_2 A_c^2 K_a m(t)\cos^2\omega_c t.$$

$$V_2(t) = K_1 A_c \cos\omega_c t + K_1 A_c K_a m(t)\cos\omega_c t + K_2 A_c^2\left[\frac{1+\cos 2\omega_c t}{2}\right] + K_2 A_c^2 K_a^2 m^2(t)\left[\frac{1+\cos 2\omega_c t}{2}\right]$$

$$+ 2K_2 A_c^2 K_a m(t)\left[\frac{1+\cos 2\omega_c t}{2}\right]$$

$$V_2(t) = K_1 A_c \cos\omega_c t + K_1 A_c K_a m(t)\cos\omega_c t + \frac{1}{2}K_2 A_c^2 + \frac{1}{2}K_2 A_c^2 \cos 2\omega_c t + \frac{1}{2}K_2 A_c^2 K_a^2 m^2(t)$$

$$+ \frac{1}{2}K_2 A_c^2 K_a^2 m^2(t)\cos 2\omega_c t + K_2 A_c^2 K_a m(t) + \frac{1}{2}K_2 A_c^2 K_a m(t)\cos 2\omega_c t \longrightarrow (2)$$

eq (2) is passed through Low pass filter (L.P.F), Hence we can neglect the high frequency components.

∴ output is

$$\boxed{V_3(t) = \underbrace{\frac{1}{2}K_2 A_c^2 K_a^2 m^2(t)}_{\text{Noise signal (or) undesired output}} + \underbrace{K_2 A_c^2 K_a m(t)}_{\text{message signal (or) desired output.}}} \longrightarrow (3)$$

* The output of LPF consists of both desired and undesired outputs.
* Therefore, to estimate desired response by using the ratio of desired to undesired signal and this ratio is also called as signal to Noise ratio (S/N) in communication systems.

*. Therefore the signal to Noise ratio is

$$\frac{S}{N} = \frac{\text{Desired output}}{\text{Undesired output}} = \frac{K_2 A_c^2 K_a m(t)}{\frac{1}{2}K_2 A_c^2 K_a^2 m^2(t)} = \frac{2}{K_a m(t)} \longrightarrow (4)$$

$$\therefore \frac{S}{N} = \frac{2}{K_a m(t)} = \frac{2}{K_a A_m \cos\omega_m t} = \frac{2}{\mu \cos\omega_m t} \longrightarrow (5)$$

$$\boxed{\begin{array}{l}\mu = \frac{A_m}{A_c}\\ \mu = K_a \cdot A_m\end{array}}$$

* If $\cos\omega_m t = 1$, then the signal to Noise ratio is

$\boxed{\frac{S}{N} = \frac{2}{\mu}}$, we know that $\boxed{\eta = \frac{\mu^2}{2+\mu^2}}$

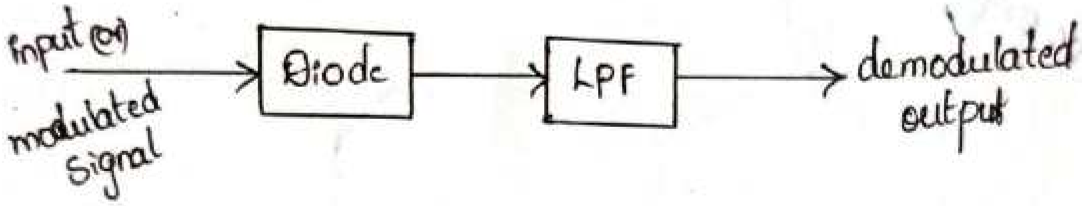
If $\frac{S}{N} = 10$, then $\mu = 0.2$ and $\eta = 2\%$

If $\frac{S}{N} = 100$, then $\mu = 0.02$ and $\eta = 0.2\%$

* From eq ④ & ⑤, the signal to noise (S/N) ratio is maximum, hence the distortion of the demodulated signal (or desired output) and modulation index (μ) is very low.

2. Envelope Detector (or) Diode Detector

* The diode detector is a very simple technique to produce demodulation.
* The block diagram of diode detector is shown in below fig.



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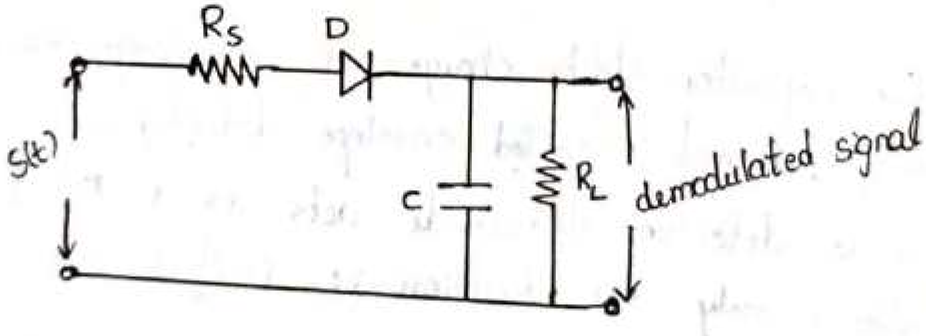
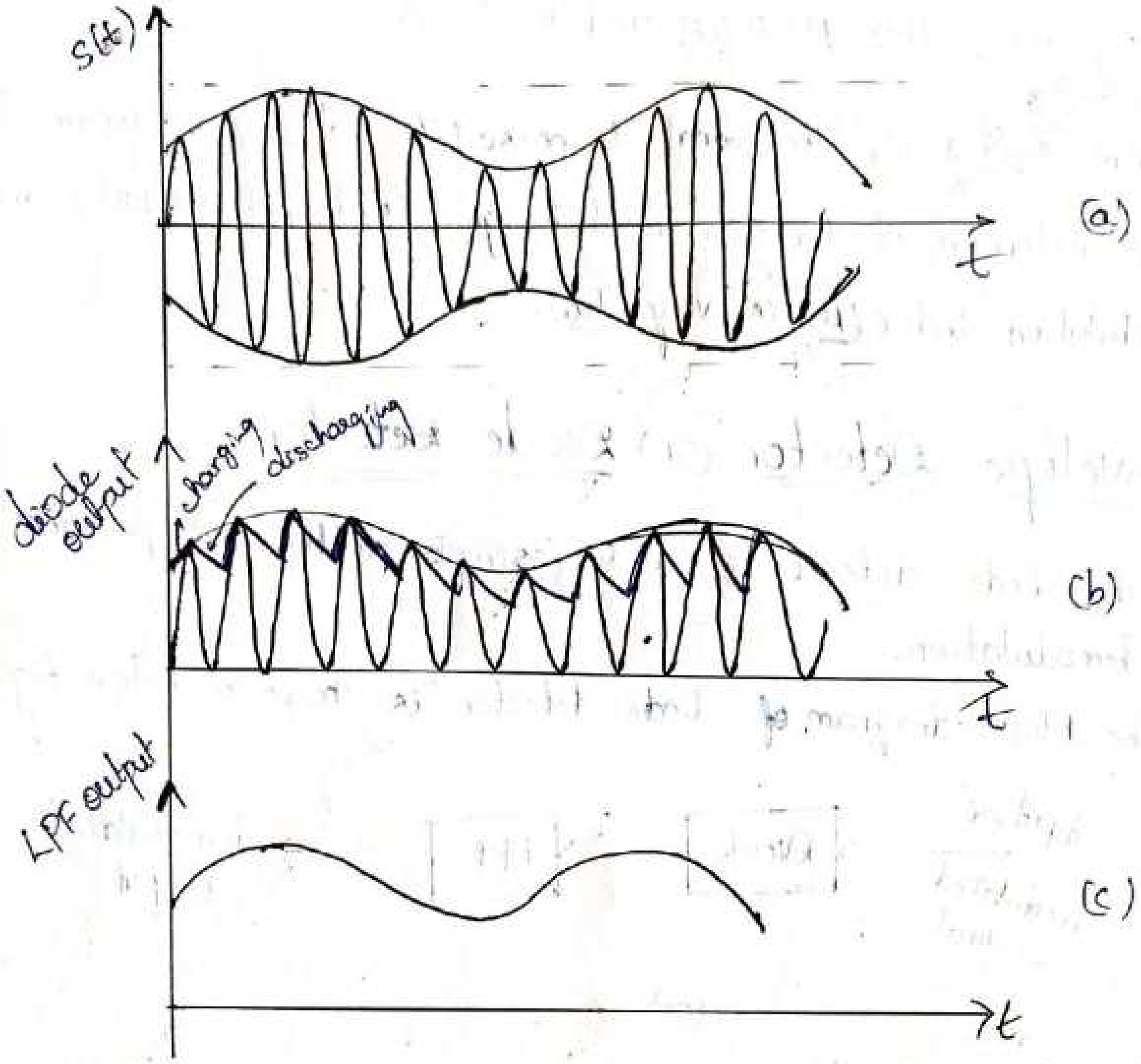


* In diode detector, the positive half cycle of the input signal (or) AM wave, the diode is forward bias (or) ON state and then the capacitor starts charging through the source resistance R_s upto the peak value of AM wave.

* The input signal is negative (or) decreasing voltage, hence the diode is reverse bias (or) OFF state and then the capacitor starts discharging through $R_L C$ until the next peak value of AM wave will come.

The graphical view of envelope detector as shown in below fig.



* In fig(b), the capacitor starts charge and discharge periods, its getting envelope, then it is called envelope detector.
* In this diode detector, the diode acts as a rectifier, rectifier allows only one direction i.e positive cycle. It clips the negative cycle.
* During charging period, $R_S C$ time constant is important

$$\boxed{R_S C << \frac{1}{f_c}}$$ → other than this, envelope cannot create properly

* During discharging period, $R_L C$ time constant is important

$$\boxed{\frac{1}{f_c} << R_L C < \frac{1}{f_m}}$$

Advantages:-

* It is very simple circuit & low cost

Disadvantages:-

1. The discharging period of the device is high, then the response of the device does not support linearity
2. The diode detector produces poor linearity, hence the distortion also more.

system

UNIT-2

DSB & SSB Modulation

Introduction:-

* When the carrier is amplitude modulated by a single sine wave, the resulting signal consists of three frequencies i.e original carrier and two side bands (USB, LSB). this system is known as Double side band Full carrier (DSBFC)

* Generally we know that, the carrier signal does not convey any information. the real information is conveyed by two side bands.

* In Amplitude modulation, transmission is insufficient because $\frac{2}{3}$rd of the power is wasted in carrier signal, by the carrier signal does not contain any information.

* In 100% modulation ($\mu = 1$), 66.66% of the power is wasted ($2/3$rd) for transmitting the carrier, and only 33.33% of the power is utilized.

* To save the $\frac{2}{3}$rd power, so we can suppress the carrier by do not affecting the message signal characteristics is called Double side Band suppressed carrier (DSB-SC) modulation

* In DSB-SC, carrier is suppressed and saved power, therefore this power distribution present only in two side bands.

DSB-SC (Double side Band- suppressed carrier) system.

* In DSB-SC, the carrier signal is completely suppressed from the modulated signal and the information is transmitted only in two side bands.

* DSB-SC is obtained by simply multiplying the modulating signal m(t) and carrier signal c(t), this is achieved by product modulator.

* ~~the block diagram of DSB-SC as shown in below fig.~~

Time domain Description:

* the mathematical eq for AM is given by

$$s(t) = A_c\,[1 + K_a m(t)]\cos\omega_c t$$

$$s(t) = A_c \cos\omega_c t + A_c K_a m(t)\cos\omega_c t \longrightarrow (1)$$

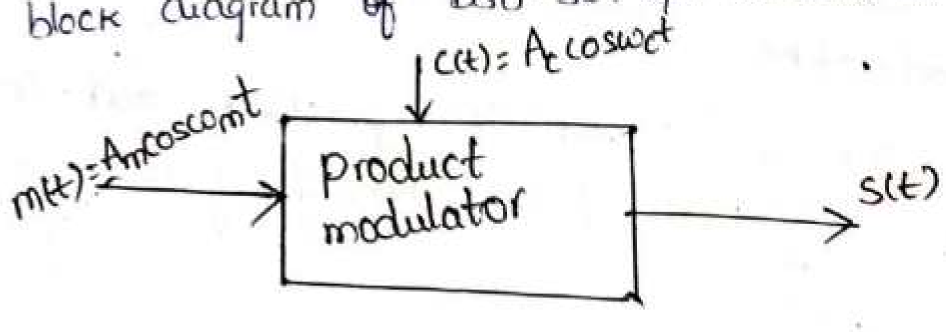
* In DSB-SC, carrier is suppressed. Therefore the modulated wave

$$s(t) \approx A_c K_a m(t)\cos\omega_c t$$

$$\boxed{s(t) \propto m(t)\,c(t)} \longrightarrow (2)$$

* we obtain a modulated wave i.e proportional to the product of the carrier wave and the base band signal (or) message signal.

* The block diagram of DSB-SC as shown in below fig.



The mathematical eq for DSB-SC is given by.

$s(t) = m(t) \cdot c(t)$ where $m(t) = A_m \cos \omega_m t$, $c(t) = A_c \cos \omega_c t$

$s(t) = (A_m \cos \omega_m t)(A_c \cos \omega_c t)$

$s(t) = A_m A_c \cos \omega_c t \cdot \cos \omega_m t$

$s(t) = \frac{A_m A_c}{2} \left[2 \cos \omega_c t \cdot \cos \omega_m t \right]$

$s(t) = \frac{A_m A_c}{2} \left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t \right]$

$$\boxed{s(t) = \frac{A_m A_c}{2} \cos 2\pi (f_c + f_m)t + \frac{A_m A_c}{2} \cos 2\pi (f_c - f_m)t} \longrightarrow (3)$$

Frequency domain Description

Time domain eq for DSB-SC is given by

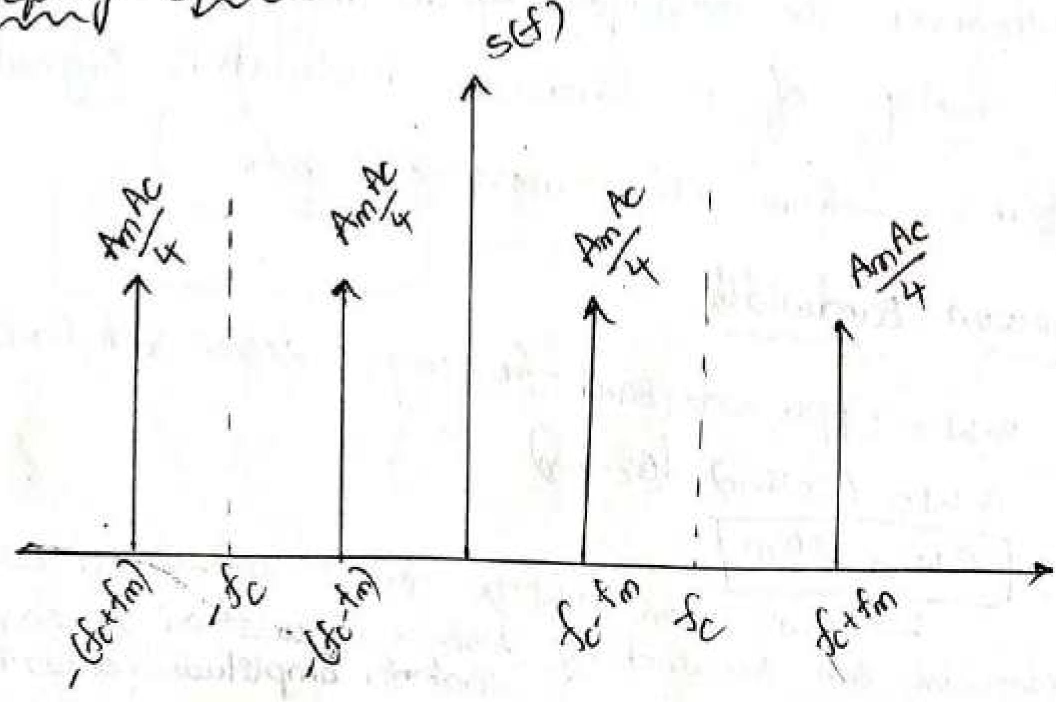
$s(t) = \frac{A_m A_c}{2} \cos 2\pi (f_c + f_m) t + \frac{A_m A_c}{2} \cos 2\pi (f_c - f_m)t$

Apply Fourier Transform on $s(t)$

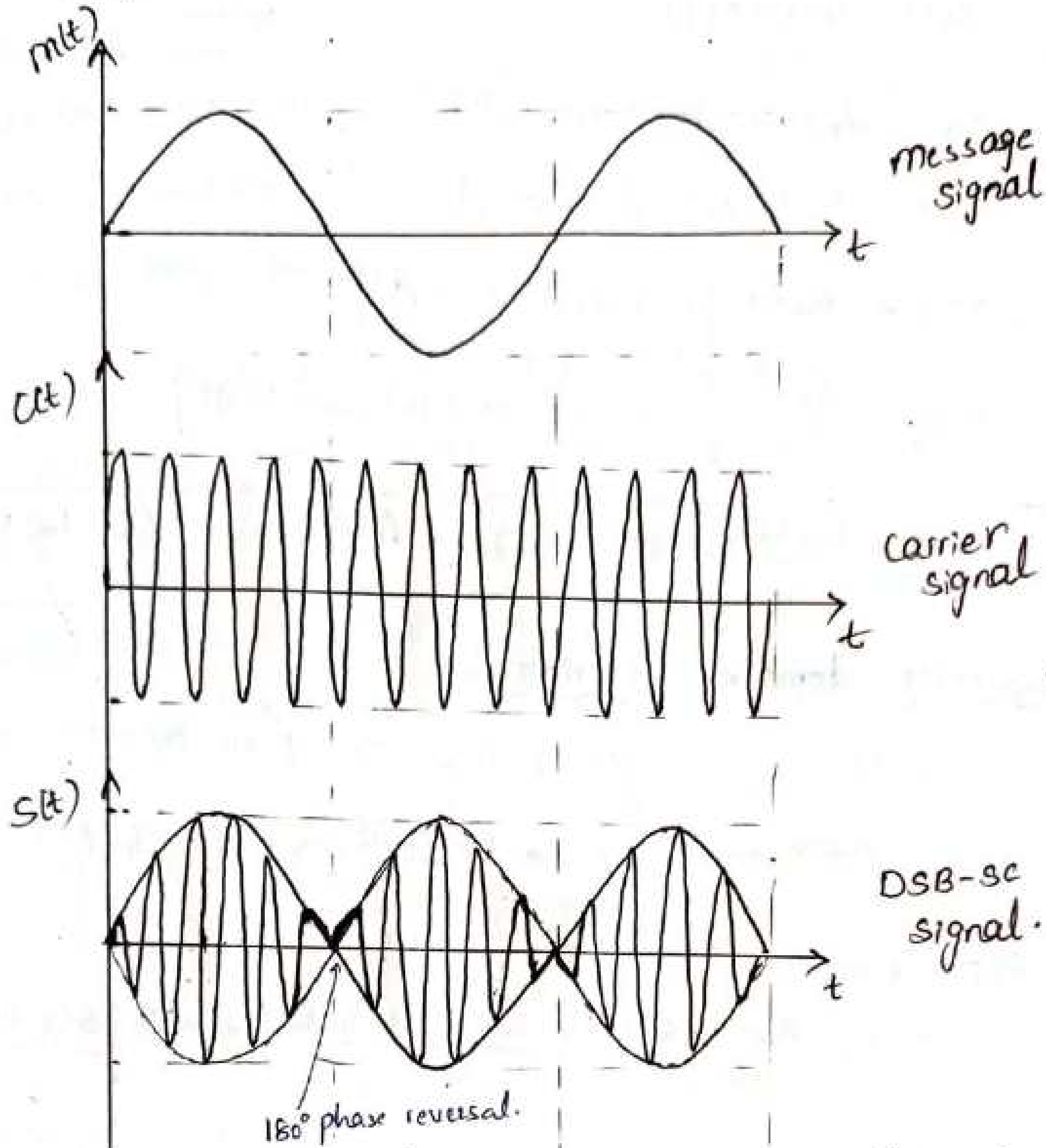
$\therefore S(f) = \frac{A_m A_c}{2} \left[\frac{\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))}{2} \right] + \frac{A_m A_c}{2} \left[\frac{\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))}{2} \right]$

$S(f) = \frac{A_m A_c}{4} \left[\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m)) \right] + \frac{A_m A_c}{4} \left[\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m)) \right] \longrightarrow (4)$

Frequency Spectrum



DSB-SC Waveforms



* From the above waveforms, we observe that the DSB-SC signal exhibits phase reversal at zero crossings. i.e whenever the message signal m(t) crosses zero, the envelope of a DSB-SC modulated signal is different from the message signal.

Transmission Bandwidth

B.W = Upper side Band frequency − Lower side band frequency

$B.W = (\omega_c + \omega_m) - (\omega_c - \omega_m)$

$\boxed{B.W = 2\omega_m}$

↳ Transmission B.W is twice the freq of message signal.

* Transmission B.W required by DSB-SC modulation is same as that for amplitude modulation i.e $2W_m$

Power calculations in DSB-SC system

The DSB-SC total power is given by.

$$P_t = P_{U.S.B} + P_{L.S.B}$$

$$= \frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R}$$

$$= \frac{\mu^2 A_c^2}{4R}$$

$$= \frac{A_c^2}{2R}\left[\frac{\mu^2}{2}\right]$$

$$\boxed{P_t = P_c\left[\frac{\mu^2}{2}\right]}$$

$$\text{power saving} = \frac{P_T - P_T'}{P_T}$$

P_T = AM power

P_T' = DSB-SC "

$$= \frac{P_c\left[1+\frac{\mu^2}{2}\right] - \frac{P_c\mu^2}{2}}{P_c\left[1+\frac{\mu^2}{2}\right]}$$

$$= \frac{P_c\left[1+\frac{\mu^2}{2} - \frac{\mu^2}{2}\right]}{P_c\left[1+\frac{\mu^2}{2}\right]} \Rightarrow \frac{P_c}{P_c\left[1+\frac{\mu^2}{2}\right]} = \frac{P_c}{P_T}$$

$$= \frac{1}{1+\frac{\mu^2}{2}}$$

$$\boxed{\text{saved power} = \frac{2}{2+\mu^2}}$$

if $\mu = 1$, saved power $= \frac{2}{3} \times 100 = 66.66\%$

if $\mu = 0.5$, saved power $= \frac{2}{2+(0.5)^2} \times 100 = 88.88\%$

Problem

Calculate the power in DSBSC when the modulation index is 60% with a carrier power is 600 watts. Find the DSB-SC transmitted power.

Sol

$$P_t = P_c\left[\frac{\mu^2}{2}\right]$$

Here $P_c = 600$ watts

$\mu = 0.6$

$$\therefore P_t = 600\left[\frac{(0.6)^2}{2}\right]$$

$$\boxed{P_t = 108 \text{ watts}}$$

Generation of DSB-SC Waves

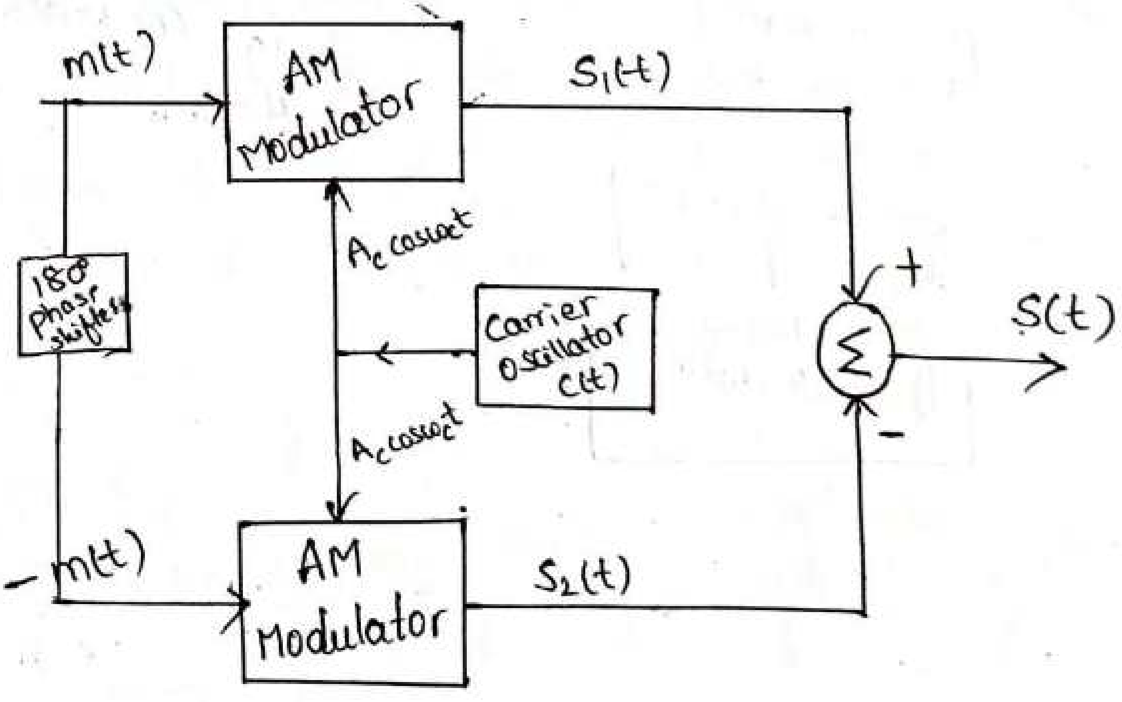
* A double side band suppressed carrier modulated wave consists simply of the product of the base band signal and the carrier signal. A device for achieving this requirement is called a product modulator.

* In general, DSB-SC modulators are two types.

1. Balanced Modulator (or) Non-linear modulator (or) Square law modulator
2. Ring Modulator (or) chopper Modulator.

1. Balanced Modulator using AM

* one possible scheme for generating a DSBSC wave is to use two AM modulators arranged in a balanced configuration, so as to suppress the carrier wave as shown in the block diagram below.



* We assume that the two AM modulators are identical except for the sign reversal of the modulating wave applied to the input of one of the AM modulator.
* The carrier signal c(t) is applied to both modulators as one of the input and message signal m(t) is another input for upper modulator and opposite polarity of m(t) (or) 180° phase shift producing message signal -m(t) is another input to lower modulator.
* Therefore the upper modulator output is

$$S_1(t) = A_c[1+K_a m(t)]\cos\omega_c t \longrightarrow (1)$$

and Lower modulator output is

$$S_2(t) = A_c[1-K_a m(t)]\cos\omega_c t \longrightarrow (2)$$

* The adder output is by substracting $S_2(t)$ from $S_1(t)$ i.e

$$s(t) = S_1(t) - S_2(t)$$

$$s(t) = A_c[1+K_a m(t)]\cos\omega_c t - A_c[1-K_a m(t)]\cos\omega_c t$$

$$s(t) = A_c\cos\omega_c t + A_c K_a m(t)\cos\omega_c t - A_c\cos\omega_c t + A_c K_a m(t)\cos\omega_c t$$

$$s(t) = 2A_c K_a m(t)\cos\omega_c t \longrightarrow (3)$$

$$\boxed{s(t) = 2K_a m(t) A_c\cos\omega_c t}$$

$K_a = \frac{1}{A_c}$

$$s(t) = 2A_c \cdot \frac{1}{A_c} m(t)\cos\omega_c t$$

$$s(t) = 2m(t)\cos\omega_c t$$

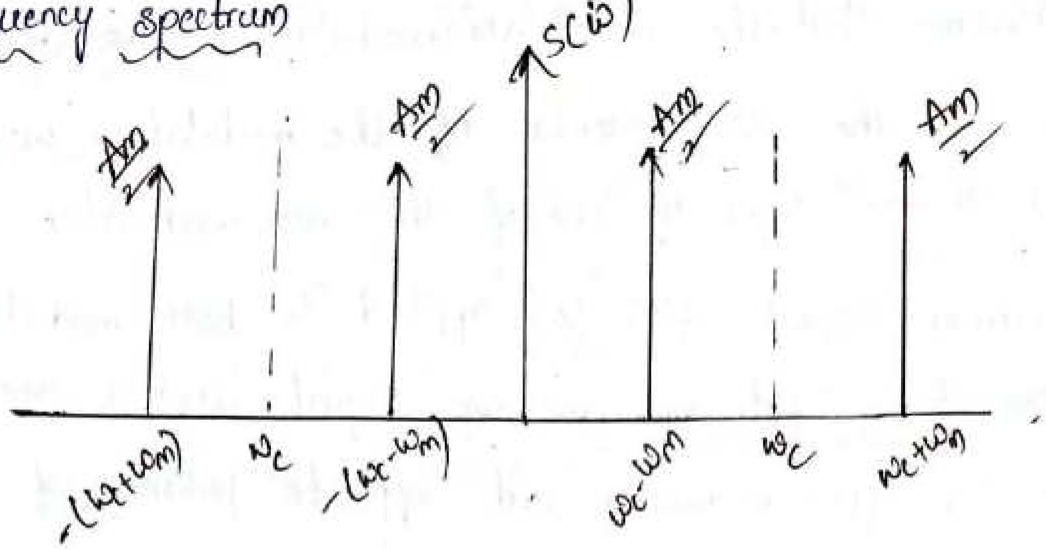
$$s(t) = 2A_m\cos\omega_m t \cdot \cos\omega_c t$$

$$s(t) = A_m[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t]$$

Apply F.T on s(t)

$$S(\omega) = \frac{A_m}{2}[\delta(\omega-(\omega_c+\omega_m)) + \delta(\omega+(\omega_c+\omega_m))] + \frac{A_m}{2}[\delta(\omega-(\omega_c-\omega_m)) + \delta(\omega+(\omega_c-\omega_m))] \longrightarrow (4)$$

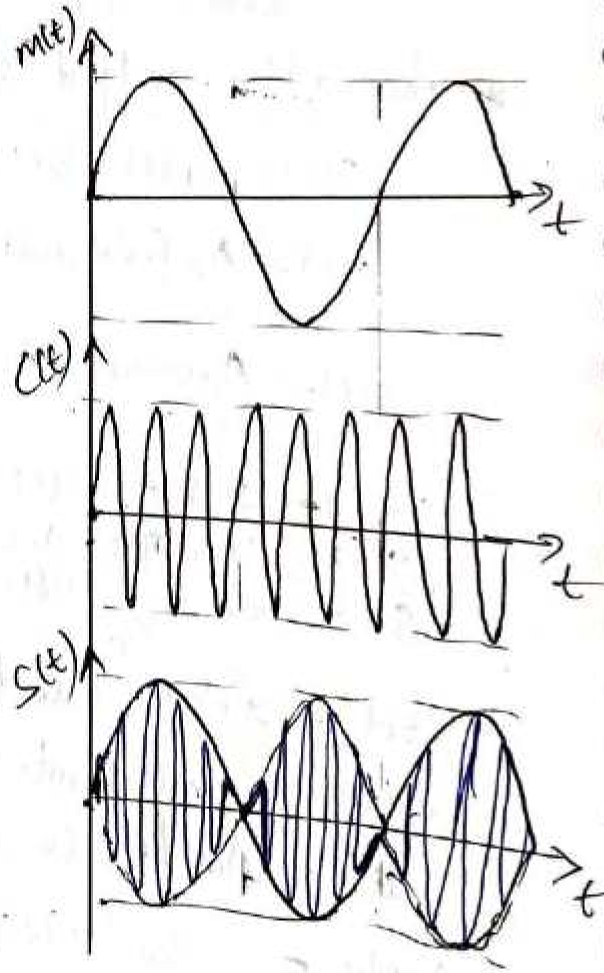
Frequency spectrum



From eq ③

* In DSB SC balanced modulator, modulation (or) amplitude sensitivity is scaling by a factor of '2' i.e $2K_a$.

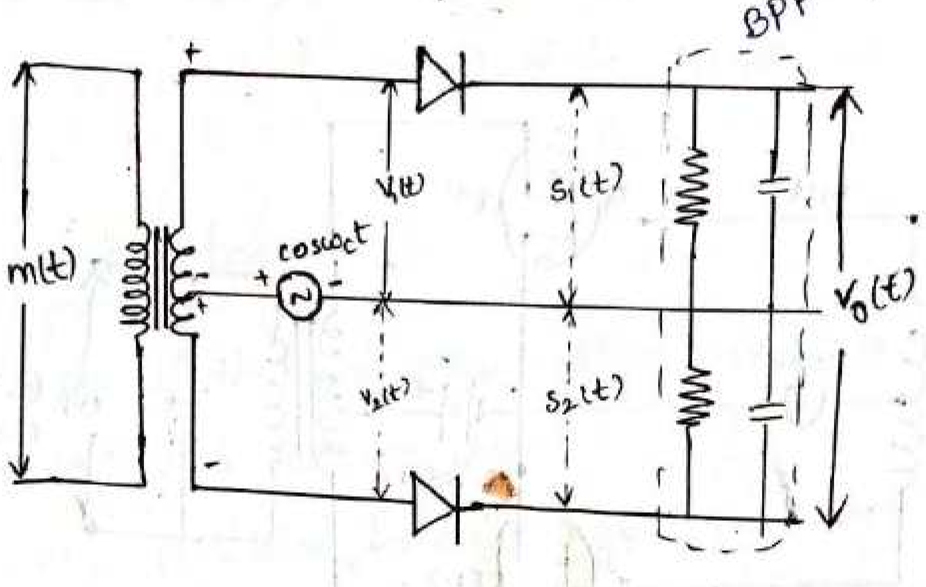
* Except for the scaling factor $2K_a$, the balanced modulation output is equal to the product of the message signal and the carrier signal, which is nothing but the DSB-sc signal.



Note :- * the DSB signals are used in FM and TV broadcasting to transmitt two channel stero signals.

* they are also used in same types of phase shift keying which is used for transmitting binary data.

Balanced modulator using Diodes



From the above diagram.

$$V_1(t) = \cos\omega_c t + m(t) \longrightarrow (1)$$

$$V_2(t) = \cos\omega_c t - m(t) \longrightarrow (2)$$

output of the Non-linear device is expressed as

$$S_1(t) = K_1 V_1(t) + K_2 V_1^2(t)$$

$$S_1(t) = K_1 \cos\omega_c t + K_1 m(t) + K_2 (\cos\omega_c t + m(t))^2$$

$$S_1(t) = K_1 \cos\omega_c t + K_1 m(t) + K_2 \cos^2\omega_c t + K_2 m^2(t) + 2K_2 m(t)\cos\omega_c t$$

and $S_2(t) = K_1 V_2(t) + K_2 V_2^2(t)$

$$S_2(t) = K_1 \cos\omega_c t - K_1 m(t) + K_2 (\cos\omega_c t - m(t))^2$$

$$S_2(t) = K_1 \cos\omega_c t - K_1 m(t) + K_2 \cos^2\omega_c t + K_2 m^2(t) - 2K_2 m(t)\cos\omega_c t$$

$$\therefore S(t) = S_1(t) - S_2(t)$$

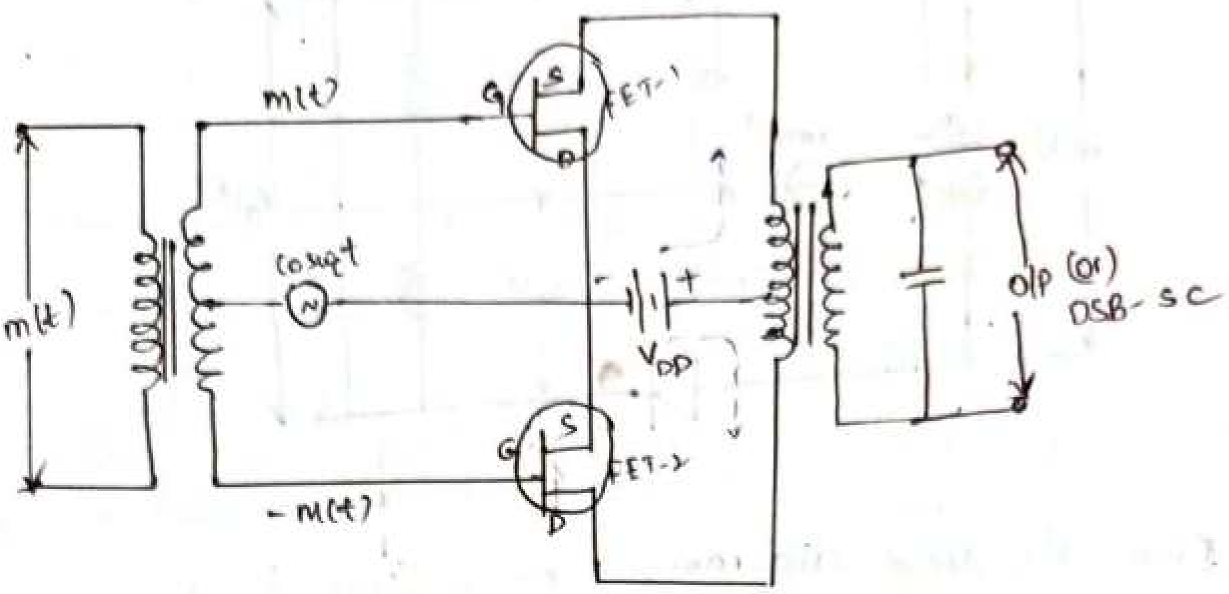
$$= K_1 \cos\omega_c t + K_1 m(t) + K_2 \cos^2\omega_c t + K_2 m^2(t) + 2K_2 m(t)\cos\omega_c t$$
$$- K_1 \cos\omega_c t + K_1 m(t) - K_2 \cos^2\omega_c t - K_2 m^2(t) + 2K_2 m(t)\cos\omega_c t$$

$$\therefore S(t) = \underbrace{2K_1 m(t)}_{\text{message signal}} + \underbrace{4K_2 m(t)\cos\omega_c t}_{\text{DSB-SC}} \longrightarrow (3)$$

Therefore $S(t)$ is passed through BPF

$$\boxed{\therefore V_0(t) = 4K_2 m(t)\cos\omega_c t}$$

Balanced modulator using FET's



* The operation of this Balanced modulator is divided into two modes.
* mode 1: absence of modulating signal.

In this mode, m(t) = 0, therefore (c(t)+m(t) = c(t)) at FET1 and also carrier signal (c(t)-m(t) = c(t)) is present at FET2. Both FETs conduct drain current, both are equal in magnitude ~~and~~ but opposite in directions. Due to this negative fields cancel each other, inducing "zero" secondary voltage i.e. output is zero. The carrier is suppressed

* mode-2: Presence of message signal and carrier signal

when the message signal is applied, the drain currents of the two FET's flow due to the combined effect of m(t) & c(t).

The FET current due to carrier are equal and opposite and hence cancel each other.

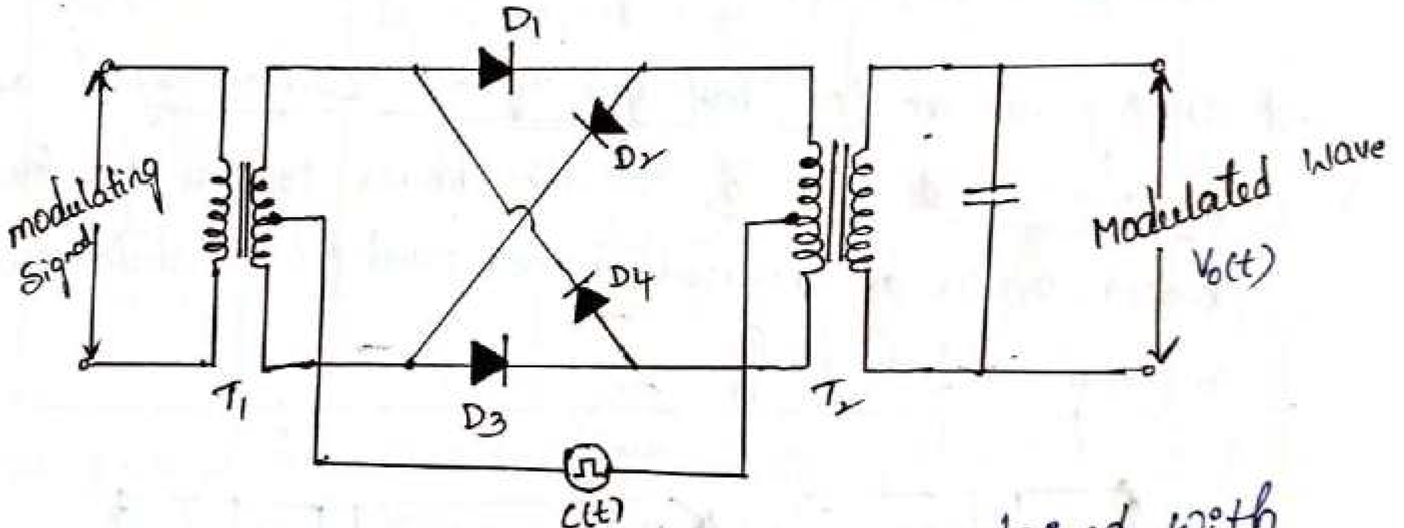
However FET currents due to the message signals are equal but not in opposite. so they do not cancel out.

this is because of the modulating signal is applied 180° out of phase to the FET-2.

Hence at the output of the circuit we get a DSB-SC signal..

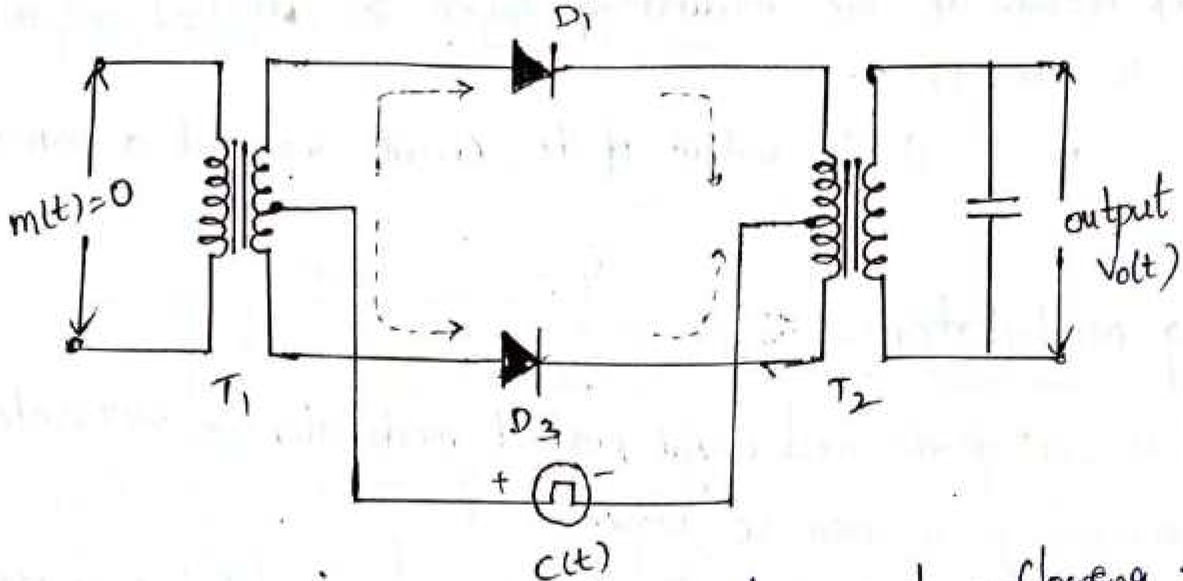
2. Ring Modulator:-

* It is one of the most useful product modulator i.e well suited for generating a DSB-SC wave.
* In Ring modulator, the 4 diodes are connected in a ring manner, hence this modulator is called "Ring Modulator"
* The block diagram of Ring modulator as shown in fig.



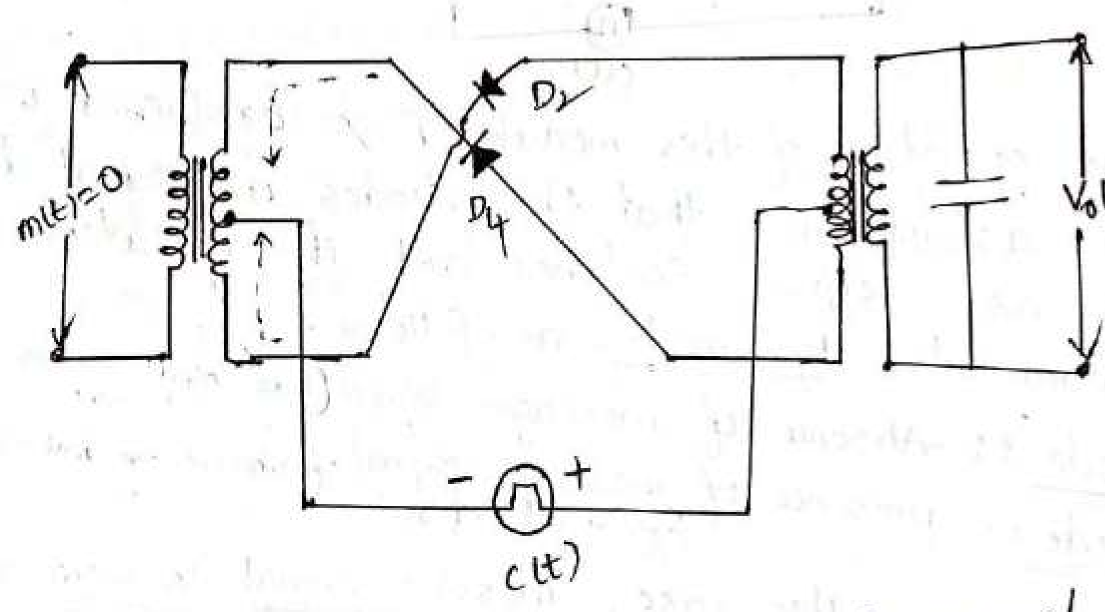
* The operation of this modulator is explained with the assumptions that the diodes are ideal diodes, acts as perfect switches and its operation can be divided into two modes as follows.
* mode 1: Absence of message signal (also observe +ve half cycle of c(t), -ve " " ")
* mode 2:- presence of message signal. (also observe +ve half cycle of c(t), -ve " " ") (+ve, -ve half cycle of m(t))

mode 1:- In this case, message signal is zero and positive half cycle of carrier signal is present, so the diode D_1, & D_3 are forward biased, D_2 and D_4 are reverse biased and its construction as shown in below figure.



* In this case, the directions of currents flowing through the diodes are opposite in direction and its magnitude is equal. therefore these currents are cancelled at transformer T_2, then the output voltage $V_0(t) = 0$

* During the negative half cycle of the carrier signal and message signal $m(t) = 0$, so the diodes D_2 & D_4 are forward biased, D_1 & D_3 are reversed biased and its construction as shown in below fig.



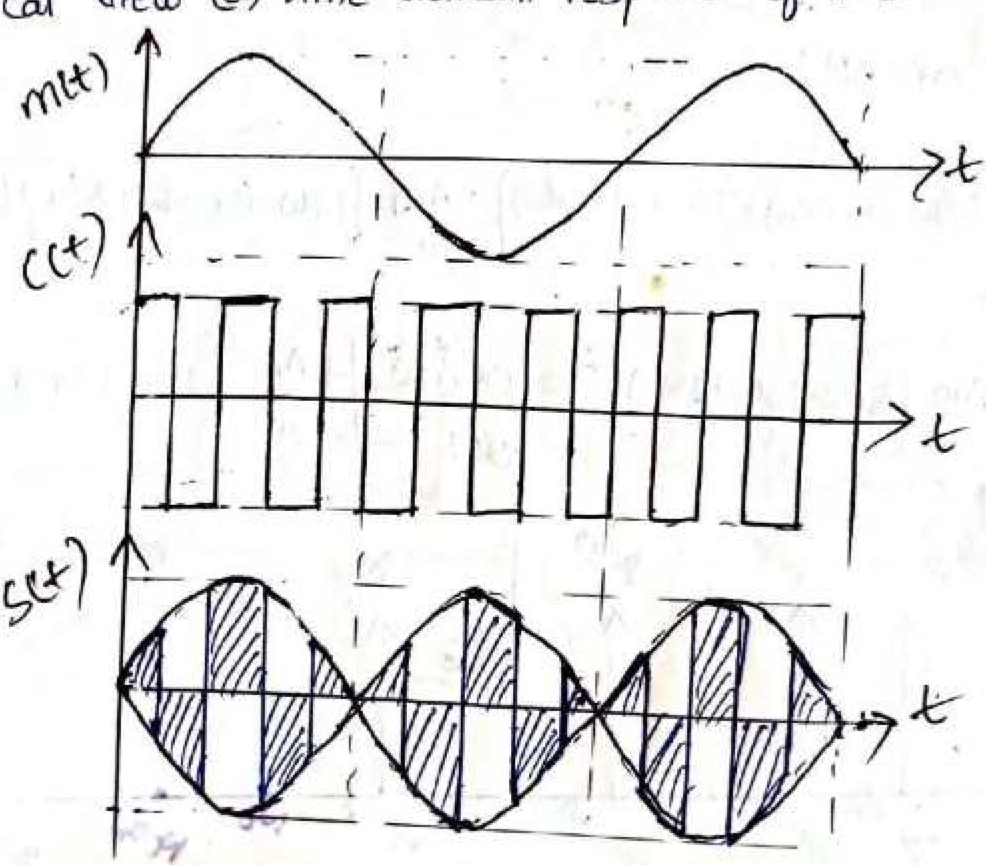
* Here also the diode currents are same in magnitude but are opposite in directions. Hence the output of secondary transformer $(V_0(t)=0)$ is zero

Mode 2:- presence of modulating signal.

In this mode 2, we have to observe the following conditions.

Input m(t)	Input c(t)	Characteristics of Diodes	output of circuits
+ve halfcycle	+ve halfcycle	D_1, D_3 are Forward biased(ON) D_2, D_4 are Reverse biased(OFF)	positive response
+ve halfcycle	-ve halfcycle	D_2, D_4 are forward biased D_1, D_3 are Reverse biased	Negative response
-ve halfcycle	+ve half cycle	D_1, D_3 are Forward biased D_2, D_4 are Reverse biased	Negative response
-ve half cycle	-ve halfcycle	D_1, D_3 are reverse biased D_2, D_4 are forward biased	positive response

* The graphical view (or) time domain response of DSB SC is



* the square signal (or) carrier signal is a periodic representation by using trigonometric fourier series, the mathematical eq. is

$$c(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos((2n-1)\omega_c t)$$

In DSB SC

$$s(t) = m(t) \cdot c(t)$$

$$= m(t) \left[\frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos((2n-1)\omega_c t) \right]$$

$$= m(t) \left[\frac{4}{\pi} \cos\omega_c t - \frac{4}{3\pi} \cos 3\omega_c t + \cdots \right]$$

$$= \frac{4}{\pi} m(t) \cos\omega_c t - \frac{4}{3\pi} m(t) \cos 3\omega_c t + \cdots$$

$$= \frac{4}{\pi} A_m \cos\omega_m t \cdot \cos\omega_c t - \frac{4}{3\pi} A_m \cos\omega_m t \cos 3\omega_c t + \cdots$$

$$= \frac{4A_m}{2\pi} \left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t \right] - \frac{4A_m}{6\pi} \left[\cos(3\omega_c + \omega_m)t + \cos(3\omega_c - \omega_m) \right]$$

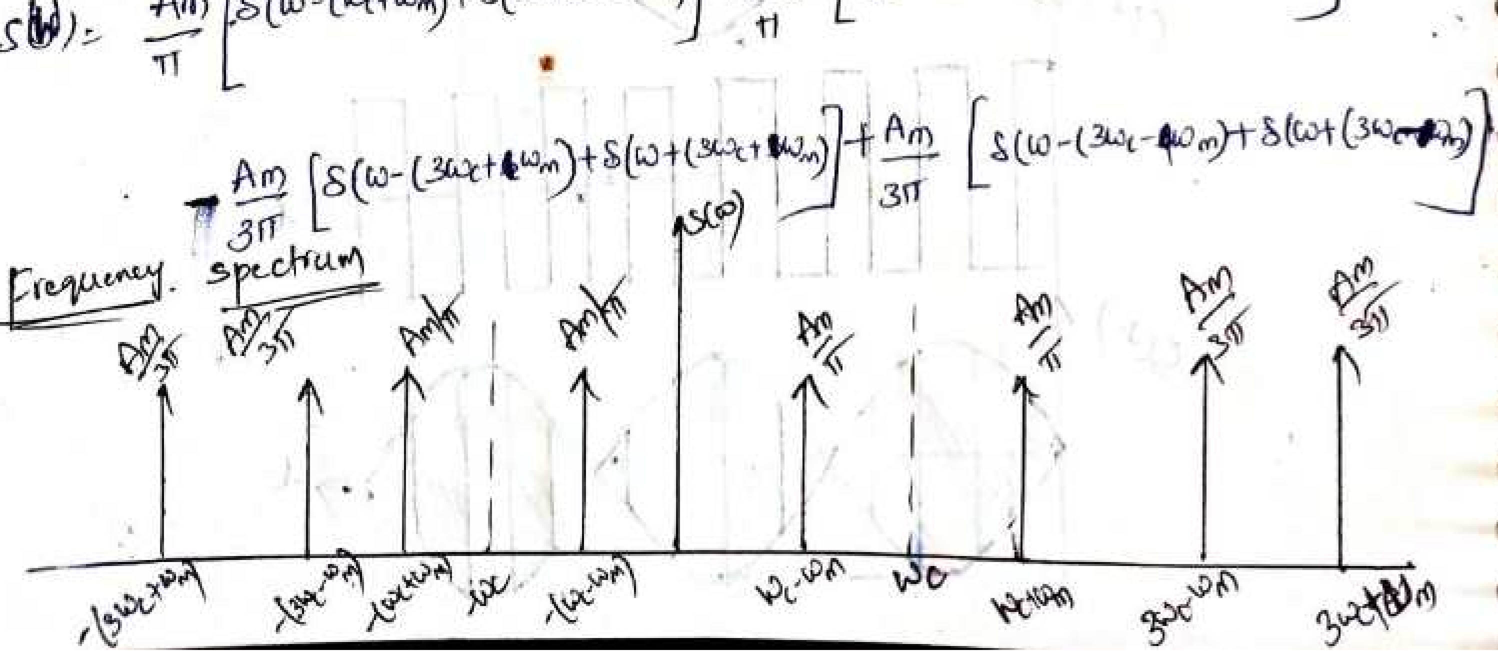
$$s(t) = \frac{2A_m}{\pi} \left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t \right] - \frac{2A_m}{3\pi} \left[\cos(3\omega_c + \omega_m)t + \cos(3\omega_c - \omega_m) \right]$$

Apply F.T on s(t)

$$S(\omega) = \frac{A_m}{\pi} \left[\delta(\omega - (\omega_c + \omega_m)) + \delta(\omega + (\omega_c + \omega_m)) \right] + \frac{A_m}{\pi} \left[\delta(\omega - (\omega_c - \omega_m)) + \delta(\omega + (\omega_c - \omega_m)) \right]$$

$$- \frac{A_m}{3\pi} \left[\delta(\omega - (3\omega_c + \omega_m)) + \delta(\omega + (3\omega_c + \omega_m)) \right] + \frac{A_m}{3\pi} \left[\delta(\omega - (3\omega_c - \omega_m)) + \delta(\omega + (3\omega_c - \omega_m)) \right]$$

Frequency spectrum

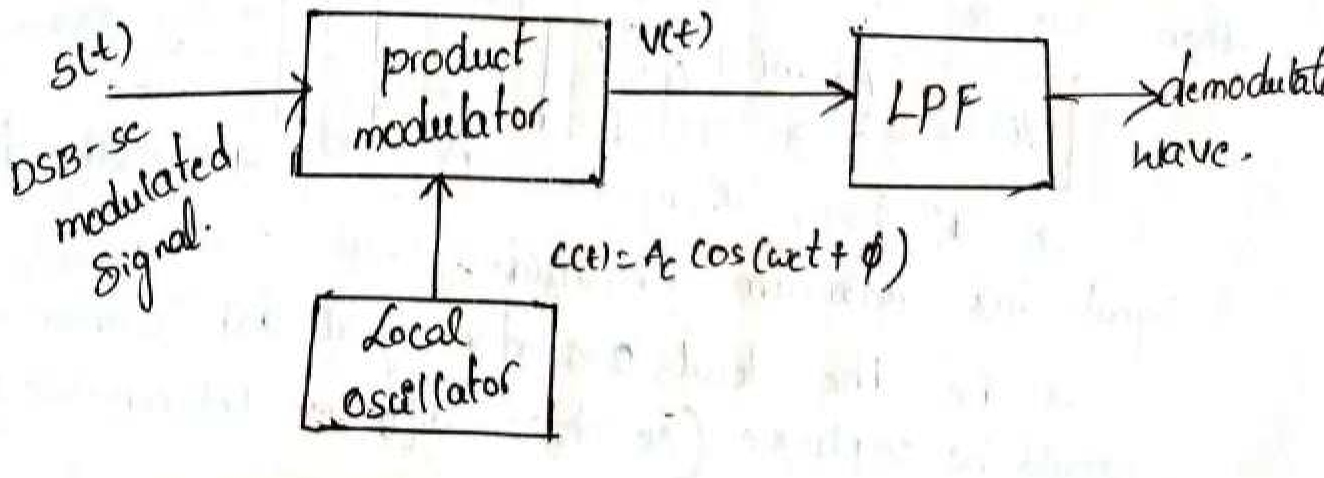


Detection of DSB-SC Waves

* It is a process of extracting original message signal from DSB-sc modulated wave is called detection (or) demodulation.
* There are two types.
  1. Synchronous detection (or) coherent detection.
  2. COSTAS Loop detection (or) Feedback management detection system

1. Synchronous (or) coherent detection:

* In this method, the same carrier signal (which is used for generating DSB-sc signal) is used to detect the message signal. Hence this process of detection is called coherent (or) synchronous detection.
* In this process, the message signal can be extracted from DSB-sc wave by multiplying it with a carrier, having the same frequency and the phase of the carrier used in DSBSC modulation.
* the resulting signal is then passed through a Low pass filter
* then the Low pass filter produce the desired output signal
* The block diagram of synchronous detector is shown in fig



The DSB-SC modulated wave output is

$$S(t) = m(t) \cdot c(t) \longrightarrow (1)$$

$$s(t) = m(t) \cdot A_c \cos\omega_c t$$

the local carrier generator (o/p of the local oscillator) is

$$c(t) = A_c \cos(\omega_c t + \phi)$$

where 'ϕ' is the phase difference between local oscillator generating signal and the carrier signal, which is used in DSB-SC modulator.

The output of product modulator as

$$V(t) = S(t) \cdot c(t) \longrightarrow (2)$$

$$\therefore V(t) = (m(t) A_c \cos\omega_c t)(A_c \cos(\omega_c t + \phi))$$

$$= A_c^2 m(t) \cos\omega_c t \cdot \cos(\omega_c t + \phi)$$

$$= \frac{A_c^2 m(t)}{2}\left[2\cos\omega_c t \cdot \cos(\omega_c t + \phi)\right]$$

$$= \frac{A_c^2 m(t)}{2}\left[\cos(\omega_c t + \omega_c t + \phi) + \cos(\omega_c t - \omega_c t - \phi)\right]$$

$$V(t) = \underbrace{\frac{A_c^2 m(t)}{2}\left[\cos(2\omega_c t + \phi)\right]}_{\text{undesired response}} + \underbrace{\frac{A_c^2 m(t)}{2}\cos\phi}_{\text{desired response}} \longrightarrow (3)$$

the product modulator output i.e $v(t)$ is applied to LPF then we get

$$\boxed{V(t) = \frac{A_c^2 m(t)}{2}\cos\phi} \longrightarrow (4)$$

* If $\phi = 0$, then $v(t) = \frac{A_c^2 m(t)}{2}$. Hence the demodulated signal has maximum amplitude.

* i.e the local oscillator signal and carrier signal should be in phase (No phase difference between two signals)

* If $\phi = \pm 90°$, then $v(t) = 0$. i.e the demodulated signal is minimum i.e zero. This effect is called as Quadrature Null Effect.
* the generated carrier signal should maintain same phase and frequency at transmitter and Receiver, therefore the detector output produce low distorted signal.
* In coherent detection, which carrier signal is used for the generation of DSB-SC, that same carrier signal is used for the Detection purpose also.

  But if there is any phase difference w.r.t carrier signal c(t), then it called as phase error
* similarly the local oscillator has identical phase, but the frequency is difference w.r.t carrier signal c(t) is called Freq error
* In general, the communication channel parameters varies randomly. hence at the receiver side i.e at the demodulated circuit the local oscillator parameters requires random variations.
* The random variations of local carrier generator is not possible, hence the demodulated output is distorted output.
* To overcome the frequency and phase erros at receiver side, therefore the local oscillator should maintain perfect synchronization by using Feedback Mechanism
* the feedback mechanism circuitary is also called as COSTAS Loop Recovery system

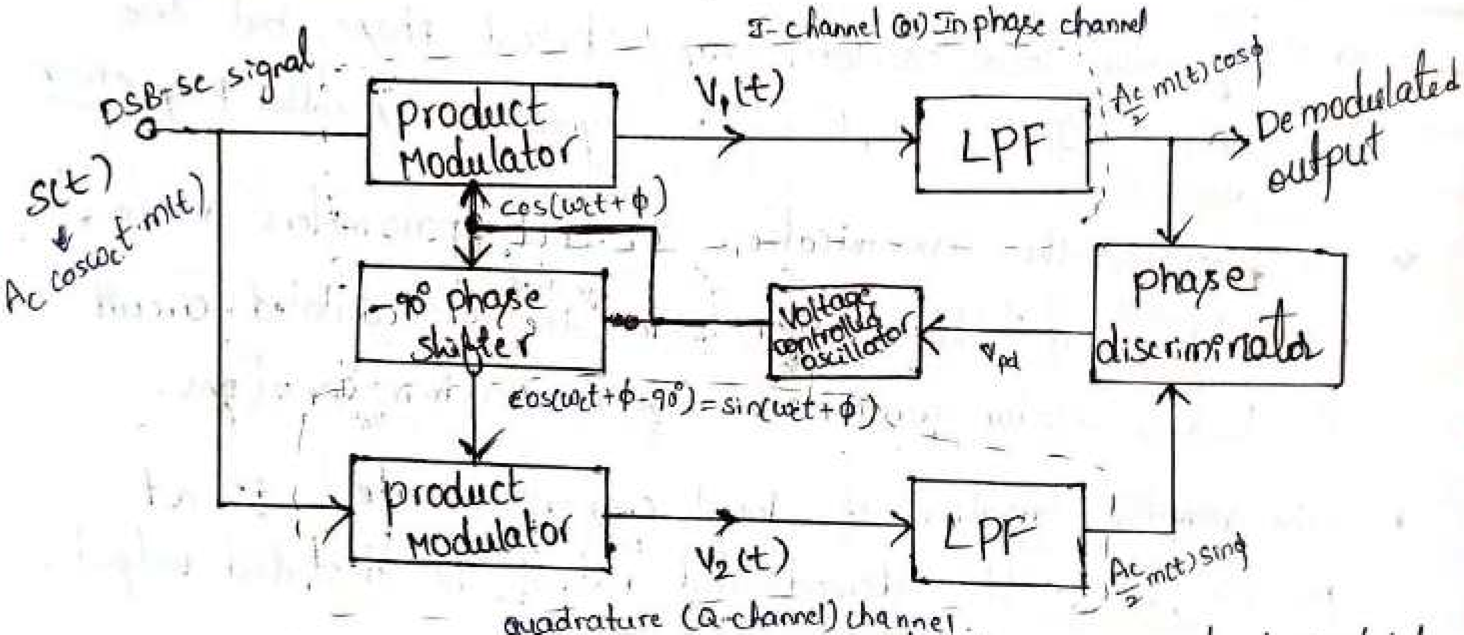
2. COSTAS Loop Detection

The COSTAS Loop is used to reduce both frequency error and phase errors in the synchronous detector. There are two channels. I-channel and Q-channel.

⇒ COSTAS Loop to reduce phase error.

It is used to make both carrier signal (used for DSBSC modulation) and local oscillated generated signal is in phase.

The block diagram of COSTAS Loop consists of two product modulators and two LPF's with common DSBSC modulated wave input as shown in below fig.



* The COSTAS Loop recovery circuit at upper product modulator input is

$$S(t) = A_c \cos\omega_c t \; m(t) \longrightarrow (1)$$

output of VCO (Another input given to product modulator) is

$$c(t) = \cos(\omega_c t+\phi) \longrightarrow (2)$$

The upper modulator output is

$$V_1(t) = S(t) \cdot c(t)$$

$$V_1(t) = [A_c \cos\omega_c t \; m(t)] [\cos(\omega_c t+\phi)]$$

$$= A_c m(t) \cos\omega_c t \cdot \cos(\omega_c t+\phi)$$

$$= \frac{A_c m(t)}{2} [2\cos\omega_c t \cdot \cos(\omega_c t+\phi)]$$

$$V_1(t) = \frac{A_c m(t)}{2}\left[\cos(\omega_c t+\omega_c t+\phi) + \cos(\omega_c t-\omega_c t+\phi)\right]$$

$$= \frac{A_c m(t)}{2}\left[\cos(2\omega_c t+\phi) + \cos\phi\right]$$

$$V_1(t) = \underbrace{\frac{A_c}{2} m(t)\cos(2\omega_c t+\phi)}_{\text{undesired signal}} + \underbrace{\frac{A_c}{2} m(t)\cos\phi}_{\text{desired signal}} \longrightarrow (3)$$

∴ this $V_1(t)$ is passed through LPF, therefore the Lowpass filter o/p is

$$\boxed{V_{01} = \frac{A_c}{2} m(t)\cos\phi} \longrightarrow (4)$$

The Lower product modulator input is

$$S(t) = A_c \cos\omega_c t\, m(t) \longrightarrow (5)$$

Another input is

$$C(t) = \cos(\omega_c t+\phi-90^\circ)$$

$$C(t) = \sin(\omega_c t+\phi) \longrightarrow (6)$$

the Lower product modulator output is

$$V_2(t) = S(t)\cdot C(t)$$

$\boxed{2\sin A\cdot\cos B = \sin(A+B)+\sin(A-B)}$

$$= A_c \cos\omega_c t\cdot m(t)\cdot\sin(\omega_c t+\phi)$$

$$= A_c m(t)\sin(\omega_c t+\phi)\cos\omega_c t$$

$$= \frac{A_c}{2} m(t)\left[2\sin(\omega_c t+\phi)\cos\omega_c t\right]$$

$$= \frac{A_c}{2} m(t)\left[\sin(\omega_c t+\phi+\omega_c t) + \sin(\omega_c t+\phi-\omega_c t)\right]$$

$$= \frac{A_c}{2} m(t)\left[\sin(2\omega_c t+\phi) + \sin\phi\right]$$

$$V_2(t) = \frac{A_c}{2} m(t)\sin(2\omega_c t+\phi) + \frac{A_c}{2} m(t)\sin\phi \longrightarrow (7)$$

∴ The $V_2(t)$ is passed through LPF, therefore the LPF o/p is

$$\boxed{V_{02} = \frac{A_c}{2} m(t)\sin\phi} \longrightarrow (8)$$

eq(4) & eq(8) is applied to phase discriminator. Based on the phase difference between these two signals, the phase discriminator produces a <u>DC control signal</u>

VCO

$\cos(\omega t+\phi)$

if $\phi = 50^\circ$

After 1 iteration $\phi = 40$

After 2nd iteration $\phi = 30^\circ$

This process continues until its value equal to zero.

$$V_{pd} = V_{o1} \cdot V_{o2}$$

$$= \frac{A_c}{2} m(t) \cos\phi \cdot \frac{A_c}{2} m(t) \sin\phi$$

$$= \frac{A_c^2}{4} m^2(t) \frac{1}{2} [2\cos\phi \cdot \sin\phi]$$

$$= \frac{A_c^2}{8} m^2(t) [\sin 2\phi]$$

$$\boxed{V_{pd} = \frac{A_c^2}{8} m^2(t) \sin 2\phi} \longrightarrow (9)$$

* this is o/p of phase discriminator, this phase discriminator proportional to phase difference 'ϕ'. (DC control signal)
* The output of phase discriminator is a input of VCO. this VCO produces the carrier signal with a certain frequency and phase, which depends on the input voltage (depending on the input voltage, it will produce oscillations)
* The VCO is used to lock frequency and phase values of signal.

<u>Case 1</u>: Ideal case. If $\phi = 0$, $V_{o1}(t) = \frac{A_c}{2} m(t)$, $V_{o2}(t) = 0$

<u>Case 2</u>: If $\phi = 20^\circ$, $V_{o1}(t) \neq 0$, then it produces certain amplitude
$V_{o2}(t) \neq 0$, then it produces certain amplitude.

therefore both channels will produce certain amplitude. Based on the voltages, phase discriminator will produce different voltage. Therefore VCO will change certain frequency and phase.

This process will continue until the ϕ values will becomes zero. (VCO correct the phase errors in the o/p signal repeatedly)

COSTAS Loop to reduce frequency error

* the local oscillator has identical phase values but the frequency difference measured w.r.t carrier signal is called frequency error.

* the local oscillated carrier generator is not a same frequency of the carrier signal. i.e

$$\omega_c + \Delta\omega_c$$

$\Delta\omega_c \rightarrow$ changing frequency
$\omega_c \rightarrow$ original frequency

* The DSBSC modulated signal mathematical eq. is

$$s(t) = m(t) \cdot c(t)$$

$$s(t) = m(t)\, A_c \cos\omega_c t \longrightarrow (1)$$

* local oscillator generated signal is

$$c(t) = A_c \cos\omega_c t$$

$$c(t) = A_c \cos(\omega_c + \Delta\omega_c)t \longrightarrow (2)$$

* the product modulator output is

$$V_1(t) = s(t) \cdot c(t)$$

$$V_1(t) = [m(t)\, A_c \cos\omega_c t]\,[A_c \cos(\omega_c + \Delta\omega_c)t]$$

$$= A_c^2 m(t) \cos\omega_c t \cdot \cos(\omega_c + \Delta\omega)t$$

$$= \frac{A_c^2}{2} m(t)\,[2\cos\omega_c t \cdot \cos(\omega_c + \Delta\omega_c)t]$$

$$= \frac{A_c^2}{2} m(t)\,[\cos(\omega_c + \omega_c + \Delta\omega_c)t + \cos(\omega_c - \omega_c + \Delta\omega_c)t]$$

$$= \frac{A_c^2}{2} m(t)\,[\cos(2\omega_c + \Delta\omega_c)t + \cos\Delta\omega_c t]$$

$$V_1(t) = \frac{A_c^2}{2} m(t) \cos(2\omega_c + \Delta\omega_c)t + \frac{A_c^2}{2} m(t) \cos\Delta\omega_c t \longrightarrow (3)$$

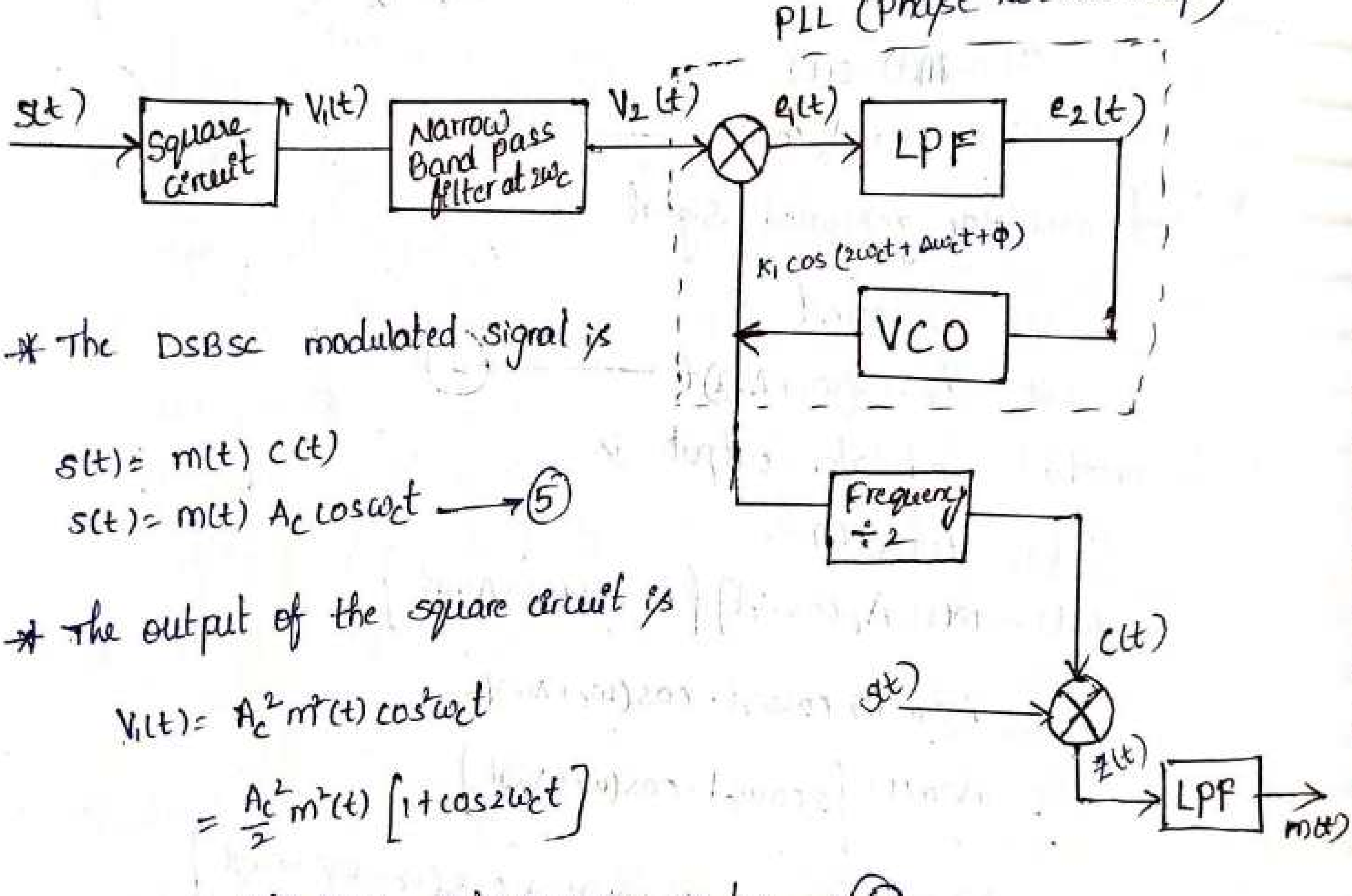
this $V_1(t)$ is passed through LPF, therefore LPF output is

$$V_{01}(t) = \frac{A_c^2}{2} m(t) \cos(\Delta\omega_c)t \longrightarrow (4)$$

* If $w = \pm \Delta w_c$, hence the carrier frequency of the local oscillator does not match with the modulated signal carrier frequency. Hence the synchronous detector produce distortion at received signal.

* To avoid the distortion by using squaring device with narrow band pass filter of general feedback mechanism technique.

* The block diagram of frequency synchronization of a costas loop is shown in below figure.



* The DSBSC modulated signal is

$$s(t) = m(t)\, c(t)$$

$$s(t) = m(t)\, A_c \cos w_c t \longrightarrow (5)$$

* The output of the square circuit is

$$V_1(t) = A_c^2 m^2(t) \cos^2 w_c t$$

$$= \frac{A_c^2}{2} m^2(t) \left[1 + \cos 2w_c t\right]$$

$$V_1(t) = \frac{A_c^2}{2} m^2(t) + \frac{A_c^2}{2} m^2(t) \cos 2w_c t \longrightarrow (6)$$

$V_1(t)$ is passed through narrow band pass filter

$$\therefore V_2(t) = \frac{A_c^2}{2} m^2(t) \cos 2w_c t \longrightarrow (7)$$

Let $\frac{A_c^2}{2} m^2(t) = K_1$

$$\therefore V_2(t) = K_1 \cos 2w_c t \longrightarrow (8)$$

$\therefore e_1(t)$ is the product of $V_2(t)$ and output of VCO

$$V_2(t) = k_1 \cos 2\omega_c t$$

o/p of VCO = $K_1 \cos(2\omega_c t + \Delta\omega_c t + \phi)$

$$\therefore e_1(t) = (K_1 \cos 2\omega_c t)(K_1 \cos(2\omega_c t + \Delta\omega_c t + \phi))$$

$$= \frac{K_1^2}{2}\left[2\cos 2\omega_c t \cdot \cos(2\omega_c t + \Delta\omega_c t + \phi)\right]$$

$$= \frac{K_1^2}{2}\left[\cos(2\omega_c t + 2\omega_c t + \Delta\omega_c t + \phi) + \cos(2\omega_c t - 2\omega_c t + \Delta\omega_c t + \phi)\right]$$

$$e_1(t) = \frac{K_1^2}{2}\left[\cos(4\omega_c t + \Delta\omega_c t + \phi) + \cos(\Delta\omega_c t + \phi)\right] \longrightarrow (9)$$

$\therefore e_1(t)$ is passed through LPF, therefore LPF output is

$$e_2(t) = \frac{K_1^2}{2}\cos(\Delta\omega_c t + \phi) \longrightarrow (10)$$

From eq (10)

If $\Delta\omega_c = 0$, and $\phi = 0$ then $e_2(t)$ value will becomes maximum.

$$\therefore e_2(t) = \frac{K_1^2}{2}$$

$\therefore$ phase difference and frequency difference is zero, therefore at the input of VCO phase and frequency is locked

$\therefore$ At the output of VCO is

$$\text{o/p of VCO} = K_1(\cos 2\omega_c t + \overset{\Delta\omega_c}{0} + \overset{\phi}{0})$$

$$= K_1 \cos 2\omega_c t \longrightarrow (11)$$

Finally output of VCO is applied to frequency divider by "2".

$$\therefore c(t) = K_1 \cos\omega_c t$$

product modulator o/p is $(x(t)) = s(t) \cdot c(t)$

$$= m(t) A_c \cos\omega_c t \cdot K_1 \cos\omega_c t$$

$$= A_c K_1 \cos^2\omega_c t \cdot m(t)$$

$$= \cancel{\frac{A_c K_1}{2}\left[2\cos^2\omega_c t \cdot m(t)\right]}$$

$$= \frac{A_c K_1}{2}\left[1 + \cos 2\omega_c t\right] m(t)$$

$$Z(t) = \frac{A_c k_1}{2} m(t) + \frac{A_c k_1}{2} m(t) \cos 2\omega_c t$$

Let $\frac{A_c k_1}{2} = k$

$$\therefore Z(t) = k m(t) + k m(t) \cos 2\omega_c t \longrightarrow (12)$$

$Z(t)$ is passed through LPF, LPF output is

$$\boxed{\text{demodulated signal} = k\, m(t)}$$

Problem

A ~~sm~~ message signal m(t) is recovered from DSBSC signal $s(t) = m(t) \cos \omega_c t$ by multiplying s(t) with local carrier generator signal $\cos(\omega_c t + \phi)$, the product signal is passed through LPF which rejects double freq signal.

(i) what is the maximum allowable value for the phase angle if the recovered signal to be 95% of maximum possible output.

(ii) If ~~the~~ base band signal is band limited to 10KHZ what is minimum value for which message signal can be recovered by filtering.

Sol

$$s(t) = m(t) \cos \omega_c t$$

$$c(t) = \cos(\omega_c t + \phi)$$

product signal output is

$$v(t) = s(t) \cdot c(t)$$

$$= m(t) \cos \omega_c t \cdot \cos(\omega_c t + \phi)$$

$$= \frac{m(t)}{2}\left[2 \cos \omega_c t \cdot \cos(\omega_c t + \phi)\right]$$

$$= \frac{m(t)}{2}\left[\cos(\omega_c t + \omega_c t + \phi) + \cos(\omega_c t - \omega_c t + \phi)\right]$$

$$= \frac{m(t)}{2}\left[\cos(2\omega_c t + \phi) + \cos\phi\right]$$

$$= \frac{m(t)}{2}\cos(2\omega_c t + \phi) + \frac{m(t)}{2}\cos\phi$$

when it is passed through LPF.

$$V_0(t) = \frac{m(t)}{2}\cos\phi$$

(i) the LPF overall o/p is

$$\frac{m(t)}{2}\cos\phi = 95\% \left[\max\left[\frac{m(t)}{2}.\cos\phi\right] \right]$$

$$\frac{\frac{m(t)}{2}.\cos\phi}{\max\left[\frac{m(t)}{2}.\cos\phi\right]} = 0.95$$

$$\phi = \cos(0.95) = 18.19$$

(ii) message signal is band limited to 10KHZ

$$\therefore V(t) = \frac{m(t)}{2}\cos(2\omega_c t + \phi) + \frac{m(t)}{2}.\cos\phi$$

* To recover message signal from the product device output signal by filtering method.

* It is necessary that the lowest frequency contained in the first part of the product device output must be greater than highest frequency contained in the second part of the product device o/p.

$$\text{i.e } 2f_c - 10KHZ > 10KHZ$$

$$f_c > 10KHZ$$

$$f_c = 10KHZ.$$

Carrier Signal re-insert technique in DSB-SC detection

* The carrier signal re-insert method to produce demodulated signal at the receiver by using local oscillator, the phase and frequency must be a proper synchronization at transmitter side to avoid distortion in the demodulated signal.

* For example:-

In general mobile communication, the carrier re-insertion is not possible in real-time, because both transmitter and receiver are not at stationary devices in all times.

* The transmitted signal of DSB-SC wave is

$$s(t) = A_c m(t)\cos\omega_c t \longrightarrow (1)$$

* If the transmitter and receiver distance is changing continuously, then the propagation delay time from transmitter to receiver

$$\tau = \frac{d}{c}$$

where
$d \to$ distance b/w Txtter to Rxer
$c \to$ velocity of light
$\tau \to$ delay time.

* Hence the receiver side i/p signal is

$s(t) = A_c m(t-\tau)$

$s(t) = A_c m(t) \cos\omega_c t$

If $t = t-\tau$

$\therefore s(t) = A_c m(t-\tau) \cos(\omega_c t - \omega_c \tau)$

$$s(t) = A_c m(t-\tau) \cos\left[2\pi f_c t - 2\pi f_c \frac{d}{c}\right]$$

$$\boxed{\therefore \phi = 2\pi f_c \frac{d}{c}}$$

* The distance is changes from transmitter to receiver in the communication system.

* The changing phase response is

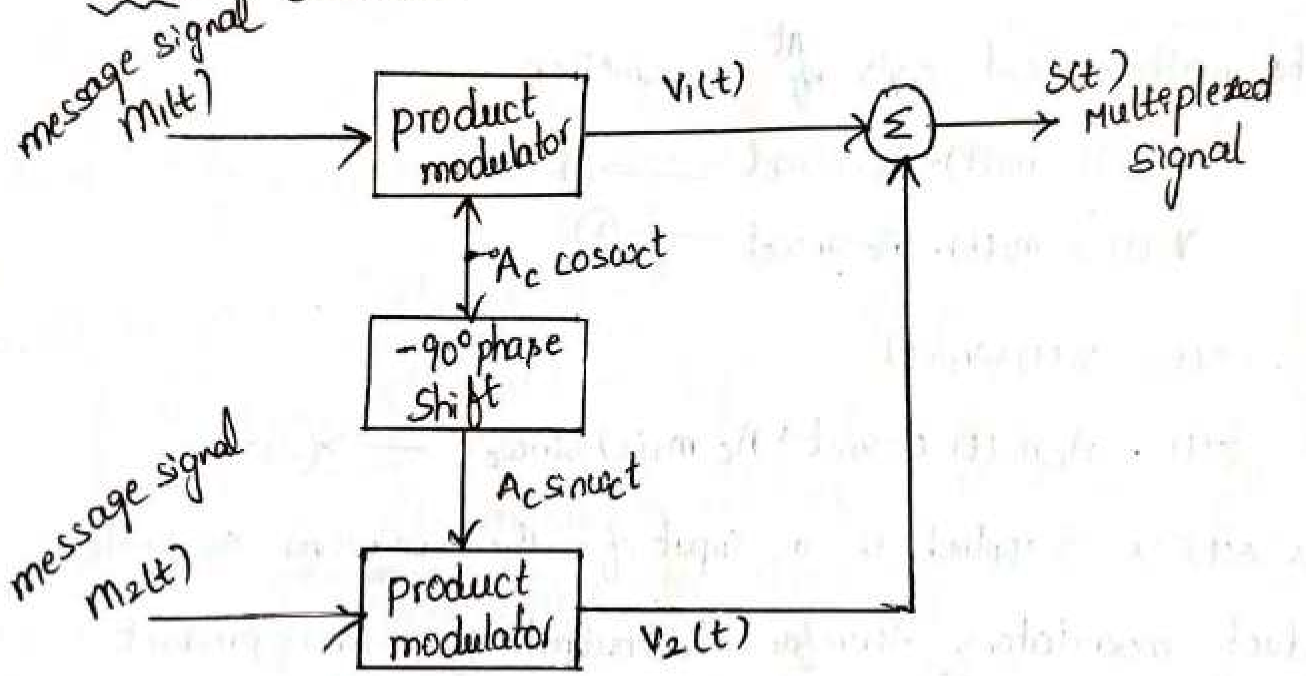
$$\boxed{\frac{\partial\phi}{\partial t} = \frac{2\pi f_c}{c} \cdot \frac{\partial d}{\partial t}}$$

Quadrature Amplitude Modulation (QAM)

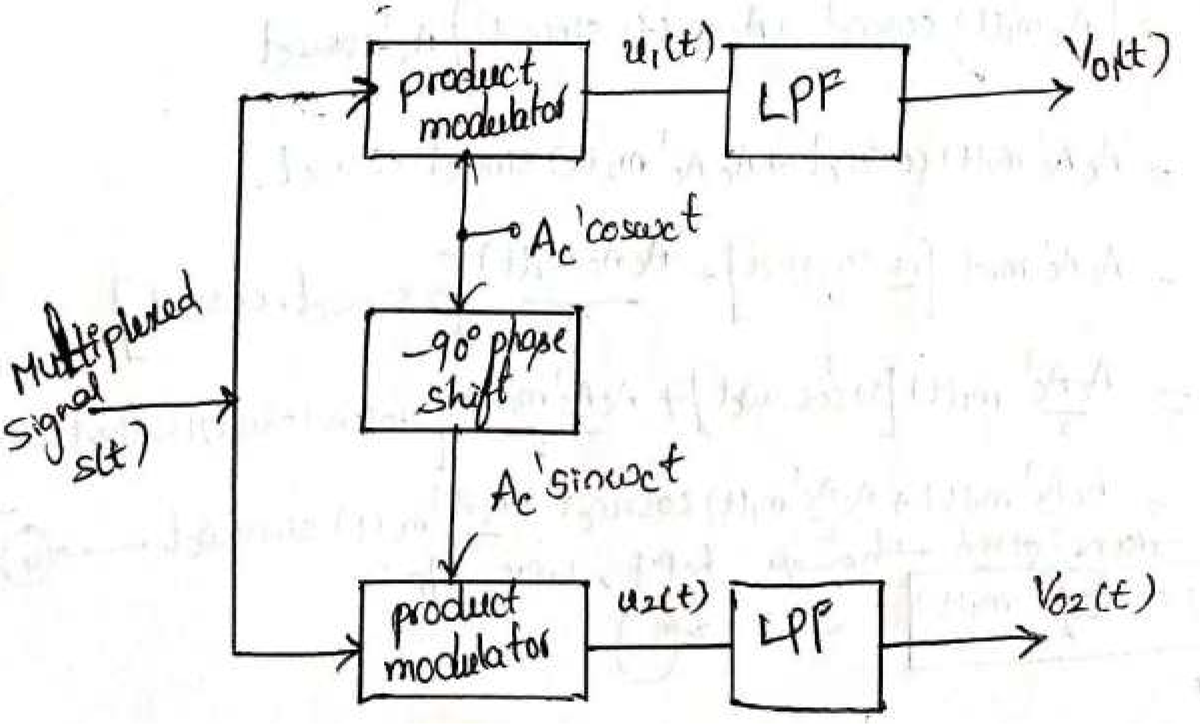
* In QAM, the two carrier waves are out of phase by 90°, therefore this scheme is known as QAM, these carriers are known as quadrature carriers.

* In Analog modulation, QAM is the combination of AM & PM
* In Digital modulation, QAM is the combination of ASK & PSK.

QAM Transmitter



QAM Receiver



* this QAM scheme enables two DSB-SC modulated waves (resulting from the application of two physically independent message signals) to occupy the same channel B.W, and allows the separation of these two message signals at the receiver output.

therefore it is also called as Bandwidth-conservation scheme. It is also called as Quadrature Carrier Multiplexing.

* the mathematical eq's of transmitter

$$v_1(t) = m_1(t) \cdot A_c \cos\omega_c t \longrightarrow (1)$$

$$v_2(t) = m_2(t) \cdot A_c \sin\omega_c t \longrightarrow (2)$$

$$\therefore s(t) = v_1(t) + v_2(t)$$

$$s(t) = A_c m_1(t) \cos\omega_c t + A_c m_2(t) \sin\omega_c t \longrightarrow (3)$$

- this s(t) is applied to a input of the receiver to both product modulators, therefore the output of first product modulator is

$$u_1(t) = s(t) \cdot c(t)$$

$$= \left[A_c m_1(t) \cos\omega_c t + A_c m_2(t) \sin\omega_c t\right] A_c' \cos\omega_c t$$

$$= A_c A_c' m_1(t) \cos^2\omega_c t + A_c A_c' m_2(t) \sin\omega_c t \cdot \cos\omega_c t.$$

$$= A_c A_c' m_1(t) \left[\frac{1+\cos 2\omega_c t}{2}\right] + \frac{A_c A_c' m_2(t)}{2}\left[2\sin\omega_c t \cdot \cos\omega_c t\right]$$

$$= \frac{A_c A_c'}{2} m_1(t)\left[1+\cos 2\omega_c t\right] + \frac{A_c A_c' m_2(t)}{2}\left[\sin(\omega_c t-\omega_c t)+\sin(\omega_c t+\omega_c t)\right]$$

$$= \frac{A_c A_c'}{2} m_1(t) + \frac{A_c A_c'}{2} m_1(t)\cos 2\omega_c t + \frac{A_c A_c'}{2} m_2(t) \sin 2\omega_c t \longrightarrow (4)$$

$u_1(t)$ is transmitted through L.P.F, L.P.F o/p is

$$\boxed{\therefore v_1(t) = \frac{A_c A_c'}{2} m_1(t)} \longrightarrow (5)$$

* o/p of second modulator is

$$u_2(t) = S(t) \cdot C(t)$$

$$u_2(t) = [A_c m_1(t) \cos\omega_c t + A_c m_2(t) \sin\omega_c t] A_c' \sin\omega_c t$$

$$= A_c A_c' m_1(t) \cos\omega_c t \cdot \sin\omega_c t + A_c A_c' m_2(t) \sin^2\omega_c t$$

$$= \frac{A_c A_c'}{2} m_1(t) [2\cos\omega_c t \cdot \sin\omega_c t] + \frac{A_c A_c'}{2} m_2(t) [1 + \sin 2\omega_c t]$$

$$= \frac{A_c A_c^*}{2} m_1(t) [\sin(\omega_c t - \omega_c t) + \sin(\omega_c t + \omega_c t)] + \frac{A_c A_c'}{2} m_2(t) + \frac{A_c A_c'}{2} m_2(t) \sin 2\omega_c t$$

$$u_2(t) = \frac{A_c A_c'}{2} m_1(t) \sin 2\omega_c t + \frac{A_c A_c'}{2} m_2(t) + \frac{A_c A_c'}{2} m_2(t) \sin 2\omega_c t \longrightarrow (6)$$

$u_2(t)$ is passed through LPF, L.P.F o/p is

$$\boxed{\therefore V_{02}(t) = \frac{A_c A_c'}{2} m_2(t)}$$

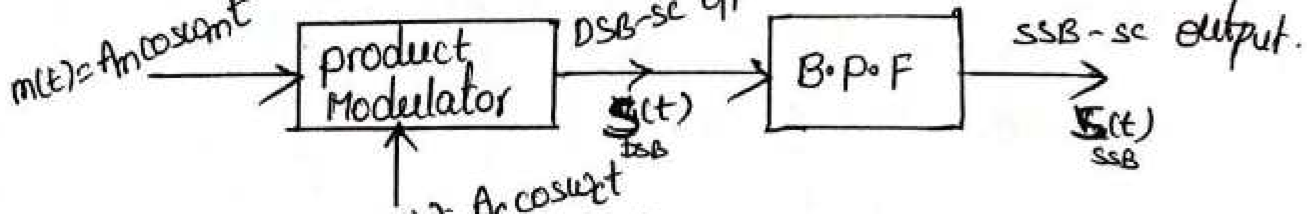
* the quadrature amplitude modulation received input signal occupies only '$2\omega_m$' bandwidth, when the input signals are more than one signal.

Single Side Band Suppressed Carrier (SSB-SC) Modulation

* In SSB-SC modulation, only one side band is transmitted, it means that the carrier wave and one side band is suppressed
* No information is contained by the carrier signal, so it is suppressed.
* The U.S.B and L.S.B are uniquely related to each other by their symmetry about the carrier frequency. It means, if amplitude and phase spectra of either sideband is given, we can uniquely find the other so only one side band is enough to transmitt the complete information. Therefore no information is lost on suppressing one sideband and the carrier
* Since in conventional AM and DSB-SC, half of the transmission Bandwidth is occupied by the U.S.B and the other half by the LSB, $(BW = 2f_m)$, so here in SSB-SC, only one side band is transmitted, therefore the transmission Bandwidth is reduced to f_m.

SSB Modulation in Single Tone Signals (or) Time domain Description for SSB-SC modulation

* The block diagram of SSB-SC as shown in fig.



$$S_{DSB-SC}(t) = m(t) \cdot c(t) \longrightarrow (1)$$

$$= A_m \cos \omega_m t \cdot A_c \cos \omega_c t \cdot$$

$S_{DSB-SC}(t) = A_m A_c \cos\omega_c t \cdot \cos\omega_m t$

$$S_{DSB-SC}(t) = \frac{A_m A_c}{2}\left[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t\right]$$

$$S_{DSB-SC}(t) = \frac{A_m A_c}{2}\cos(\omega_c+\omega_m)t + \frac{A_m A_c}{2}\cos(\omega_c-\omega_m)t \longrightarrow (2)$$

This eq (2) is passed through B.P.F, B.P.F o/p is (when we suppress the lower side band frequency) → SSB with USB Transmitted.

USSB = Upper single side Band
LSSB = Lower " " "

$$\therefore S_{USSB}(t) = \frac{A_m A_c}{2}\cos(\omega_c+\omega_m)t$$

$\cos(A+B) = \cos A\cdot\cos B - \sin A\cdot\sin B$

$$\boxed{S_{USSB}(t) = \frac{1}{2}A_m A_c \cos\omega_c t\cdot\cos\omega_m t - \frac{1}{2}A_m A_c \sin\omega_c t\cdot\sin\omega_m t} \longrightarrow (3)$$

When we suppress the upper side band frequency. (SSB with LSB Transmitted)

$$\therefore S_{LSSB}(t) = \frac{A_m A_c}{2}\cos(\omega_c-\omega_m)t$$

$\cos(A-B) = \cos A\cdot\cos B + \sin A\cdot\sin B$

$$\boxed{S_{LSSB}(t) = \frac{1}{2}A_m A_c \cos\omega_c t\cdot\cos\omega_m t + \frac{1}{2}A_m A_c \sin\omega_c t\cdot\sin\omega_m t} \longrightarrow (4)$$

General SSB wave is

$$\boxed{S_{SSB}(t) = \frac{1}{2}A_m A_c \cos\omega_c t\cdot\cos\omega_m t \overset{USSB}{\underset{LSSB}{\mp}} \frac{1}{2}A_m A_c \sin\omega_c t\cdot\sin\omega_m t} \longrightarrow (5)$$

Now consider a periodic message signal, defined by Fourier series.

$$m(t) = \sum_n a_n \cos\omega_m t$$

$$\text{also } \hat{m}(t) = \sum_n a_n \sin\omega_m t$$

mixture of sinusoidal waves with harmonically related frequencies.

sinusoidal component of m(t) (or) $\hat{m}$

* carrier c(t) is common to all the

* Therefore eq (5) can be written as

$$S_{SSB}(t) = \frac{1}{2}A_c \cos\omega_c t \sum_n a_n \cos\omega_m t \mp \frac{1}{2}A_c \sin\omega_c t \sum_n a_n \sin\omega_m t$$

$$\boxed{\quad = \frac{A_c}{2} m(t)\cos\omega_c t \mp \frac{A_c}{2}\hat{m}(t)\sin\omega_c t} \longrightarrow (6)$$

* periodic signal $\hat{m}(t)$ can be derived from the periodic modulating signal $m(t)$, simply by shifting the phase of each cosine term in $m(t)$ by $-90°$ by a method called <u>Hilbert Transform</u>

* Hilbert transformer is a wide band phase shifter whose frequency response is characterized in two parts.
  1. The magnitude response is unity for all frequencies, both +ve & -ve
  2. The phase response is $+90°$ for -ve frequencies & $-90°$ for +ve freq's.

* Hilbert Transform of $m(t)$ denoted by $\hat{m}(t)$

$$\text{thus } \hat{m}(t) = m(t) * \frac{1}{\pi t}$$

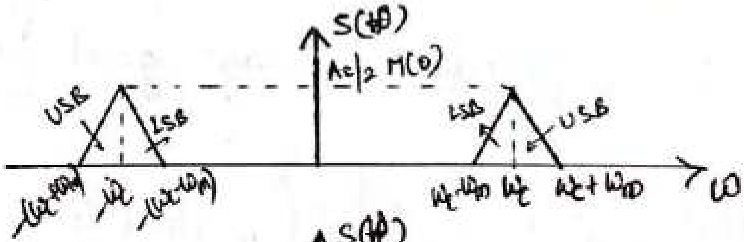
$$= \frac{1}{\pi}\int_{-\infty}^{\infty} m(\tau)\cdot\frac{1}{t-\tau}\,d\tau \quad (\text{or}) \quad \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{m(\tau)}{t-\tau}\,d\tau$$

$$m(t) \longrightarrow \boxed{\begin{array}{c}\text{phase shifter}\\ -\frac{\pi}{2}\text{ rad}\end{array}} \longrightarrow \hat{m}(t)$$

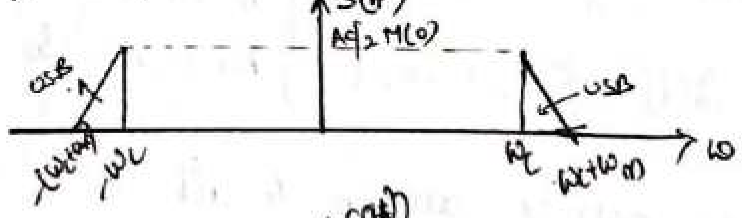
$$H\cdot T\left[A_m\cos\omega_m t\right] = A_m \sin\omega_m t.$$

Frequency domain Description for SSB-SC modulation

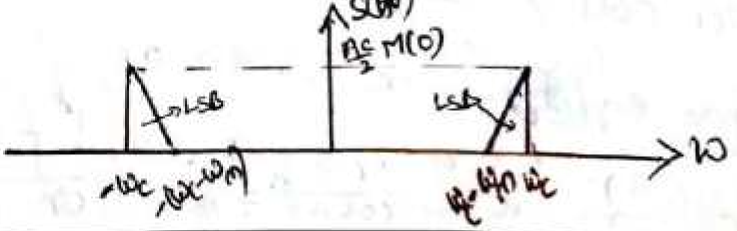
(a) spectrum of m(t)



(b) spectrum of DSB-SC



(c) spectrum of USSB



(d) spectrum of LSSB

* Basically a Hilbert transformer is a system whose transfer function is defined by

$$\boxed{H(f) = -j\,\mathrm{sgn}(f)}$$ where sgn(f) = Signum function.

* Properties of Hilbert Transform

1. A signal $f(t)$ and its Hilbert Transform $f_h(t)$ have the same energy density spectrum
2. A signal $f(t)$ and its Hilbert transform $f_h(t)$ have the same autocorrelation function.
3. A signal $f(t)$ and its Hilbert Transform $f_h(t)$ are mutually orthogonal i.e.

$$\boxed{\int_{-\infty}^{\infty} f(t) f_h(t)\, dt = 0}$$

4. If $f_h(t)$ is a Hilbert Transform of $f(t)$, then the Hilbert Transform of $f_h(t)$ is $-f(t)$ i.e

If $\boxed{H.T[f(t)] = f_h(t)}$ then $\boxed{H.T[f_h(t)] = -f(t)}$

Advantages of SSB-SC

1. It requires very less bandwidth compared to AM and DSBSC
2. The transmitted signal power is very high.
3. The efficiency also increases.
4. Power is saved, ⑤ Less amount of noise is present ⑥ signal fading is less.

Disadvantages :

1. To design SSB-SC device with sharp cut-off frequencies of the filter is more complex. Hence the transmission quality is poor.
2. cost of SSB-SC is more because the transmitter and receiver requires frequency stability.

Applications:-

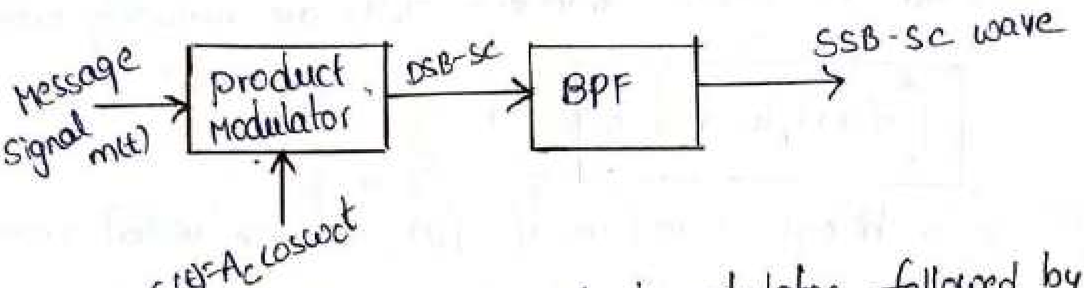
* Mobile communications
* Telemetry
* TV broadcasting
* Radar
* Military applications
* Point-to-point communication
* [illegible]

Generation of SSB-SC Modulator:-

there are 2 Types

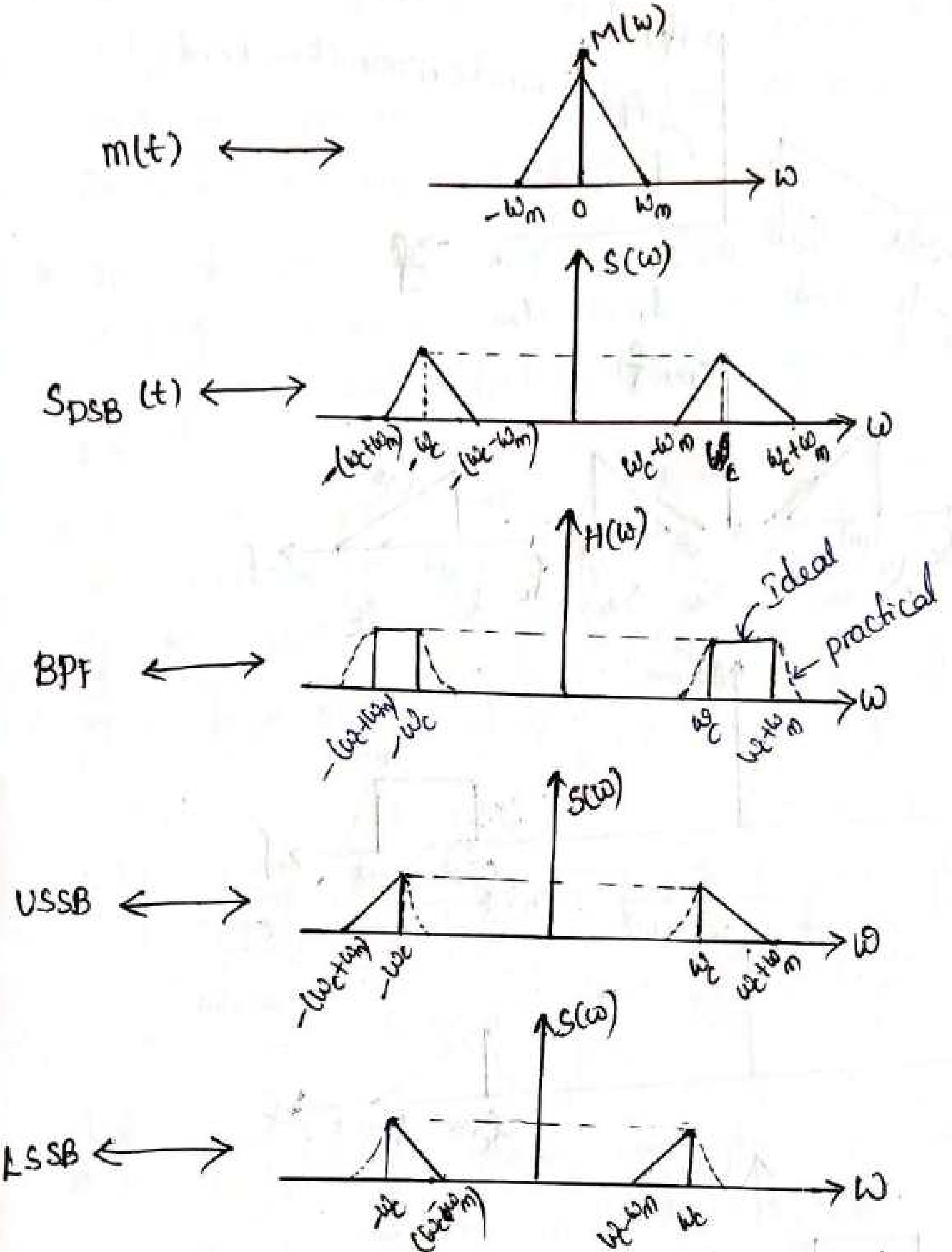
1. Frequency Discrimination Method (or) Filter Method.
2. phase Discremination Method (or) phase shift Method.

1. Frequency Discrimination Method:



* It basically consists of a product modulator followed by BPF. output of product modulator is DSB-sc modulated wave (contains two sidebands)
* BPF is designed to pass the desired side band of DSB-SC and reject the other side band, depending on whether the USSB (or) LSSB is the desired modulation.
* Design of BPF:- there must be a certain separation between the two side bands that is wide enough to accommodate the transition band of the band pass filter.
* Band pass filter should have very sharp cut-off frequency. But practically it would be difficult to separate the L.S.B and U.S.B cut-off frequency.
* It means that if we want to transmitt the U.S.B, during the transmission of U.S.B, some amount of power propagated into L.S.B also, i.e both are overlapping with each other.
* This overlapping representation as shown in below fig.

Drawback of Frequency Discrimination method

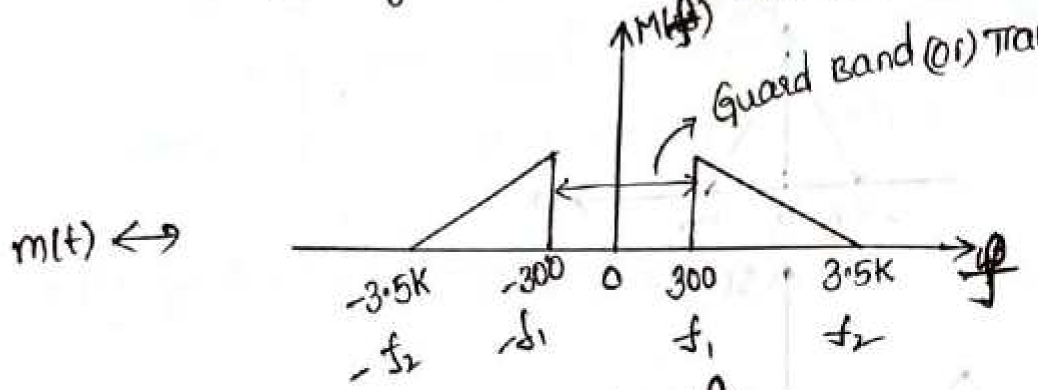


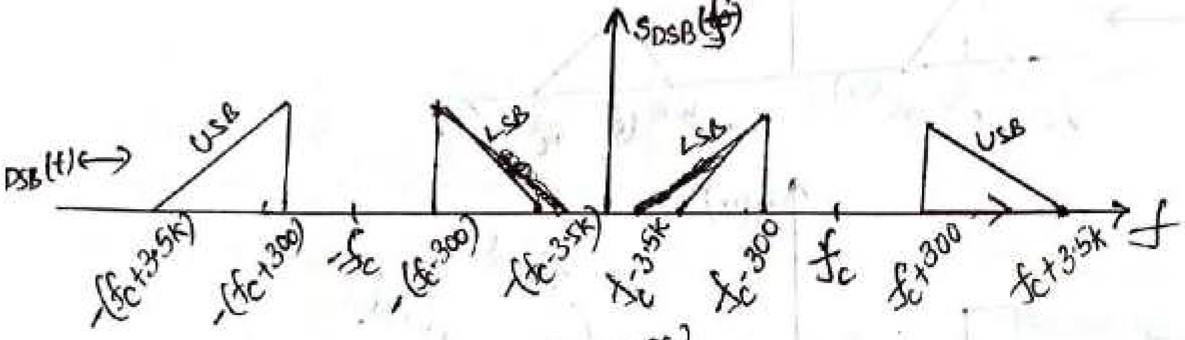
* Since, BPF is not ideal, resulting SSB signal contains additional frequency component from other side band. Because of above drawback, SSB is limited only for voice Transmission (or) speech Transmission.

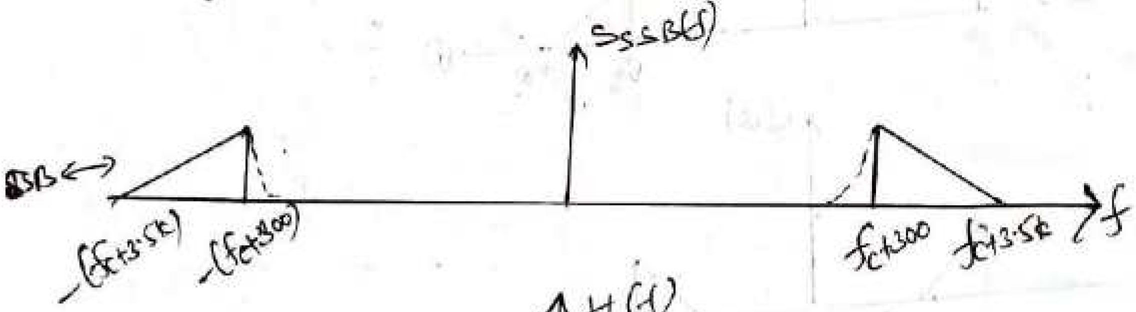
To avoid this drawback, BPF must be a certain separation between the two side bands ... $2f_1$ where f_1 is lowest freq component ... message signal.

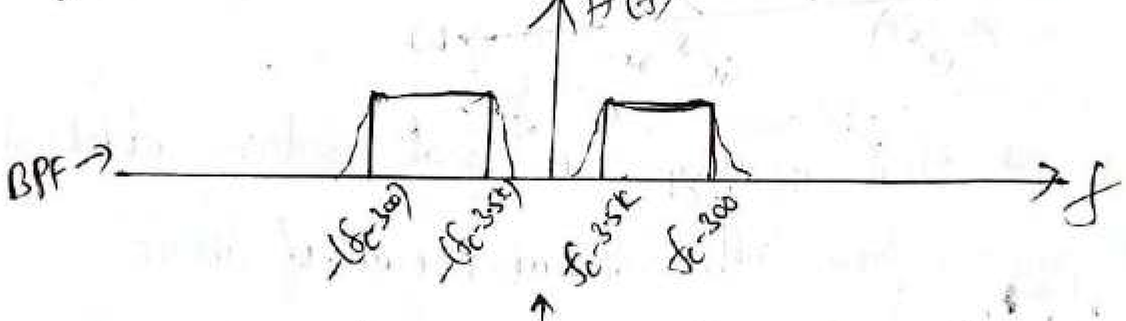
Example: speech signal (or) Voice Signal

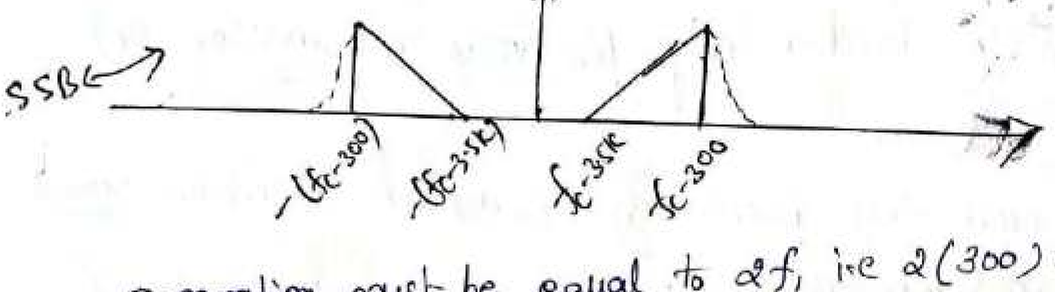
speech signal range from 300 Hz to 3.5 KHz
0.3 KHz to 3.5 KHz







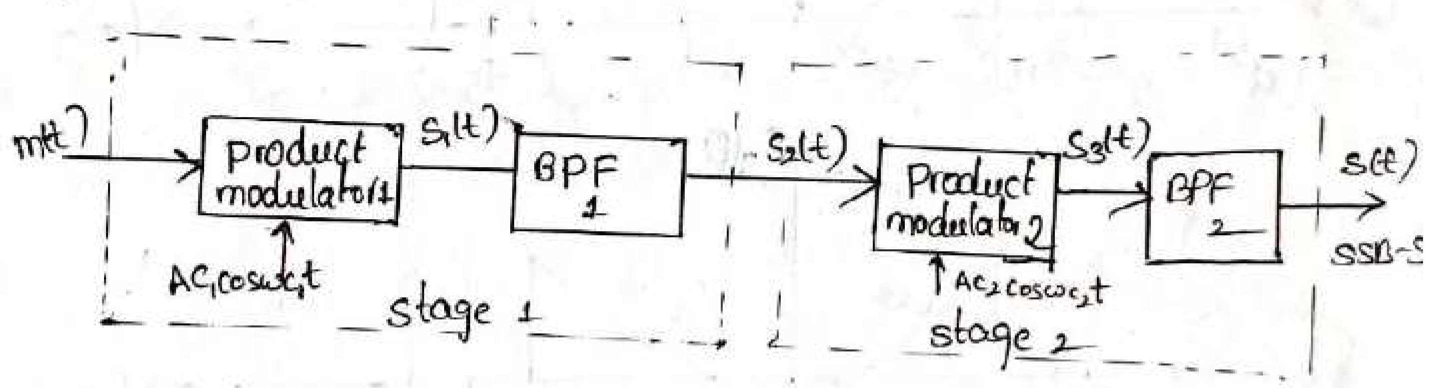




separation must be equal to $2f_1$ i.e $2(300) = 600 Hz$, this separation is called Gaurd Band (or) Transition Band.

* It will not interfere with each other (USB & LSB) by using Gaurd Band.
* To transmitt a message signal (i.e voice signal) by SSB, the bandgap of 600HZ is properly adjust between sideband, so that voice signal can be comfatably transmitted. But not suitable for video signal and audio signal.
* In this SSB-SC modulation, Design of BPF is difficult i.e very complex, when we design a B.P.F, Q-factor should be 1000 to 2000 i.e very high Quality factor. Therefore the cost of the circuit is very high.
* This SSB-SC modulation is single stage SSB-SC modulation. To reduce the cost and complexity of the circuit, we need to go for Two stage SSB-SC modulation

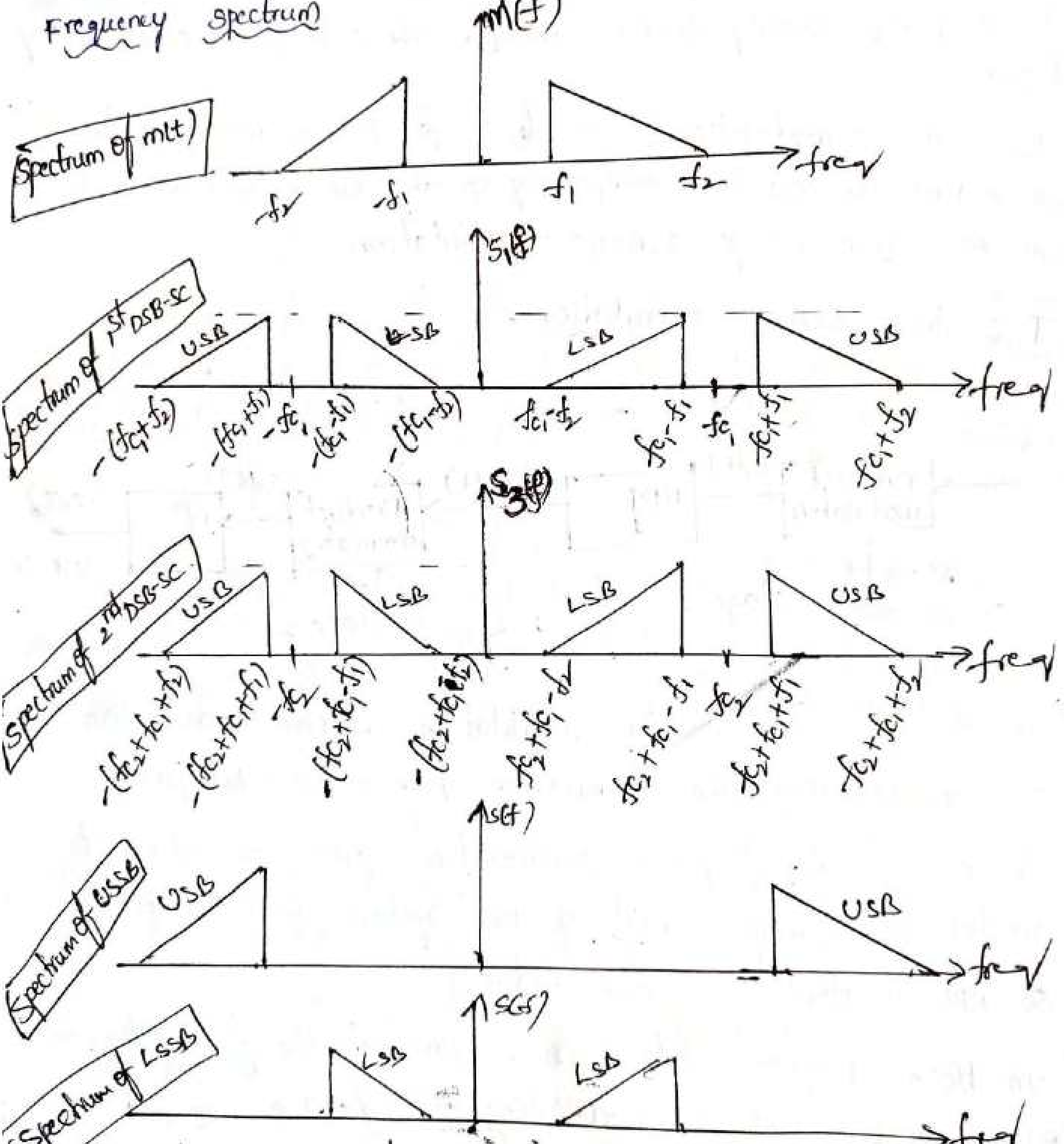
Two stage SSB-SC Modulation



* In the two stage SSB-SC modulation, we can reduce the cost by bisecting the modulation process into two parts.
* It means that frequency translation from one stage to another stage, therefore cost of the system will reduce, so BPF complexity will also reduce.
* In block diagram, the SSB-SC wave at the first filter output is used as the modulating wave for the second balanced (Product) modulator, which produces DSB-SC wave with a spectrum i.e symmetrically spaced around the 2nd carrier f_2.

* The frequency separation between the two side bands of DSB-SC wave is effectively twice the first carrier frequency ($f_{c_1} < f_{c_2}$). This enables the easy removal of the unwanted side bands.
* In two stage SSB-SC modulation, translate low frequency modulating signal into high frequency modulated signal.

Frequency spectrum



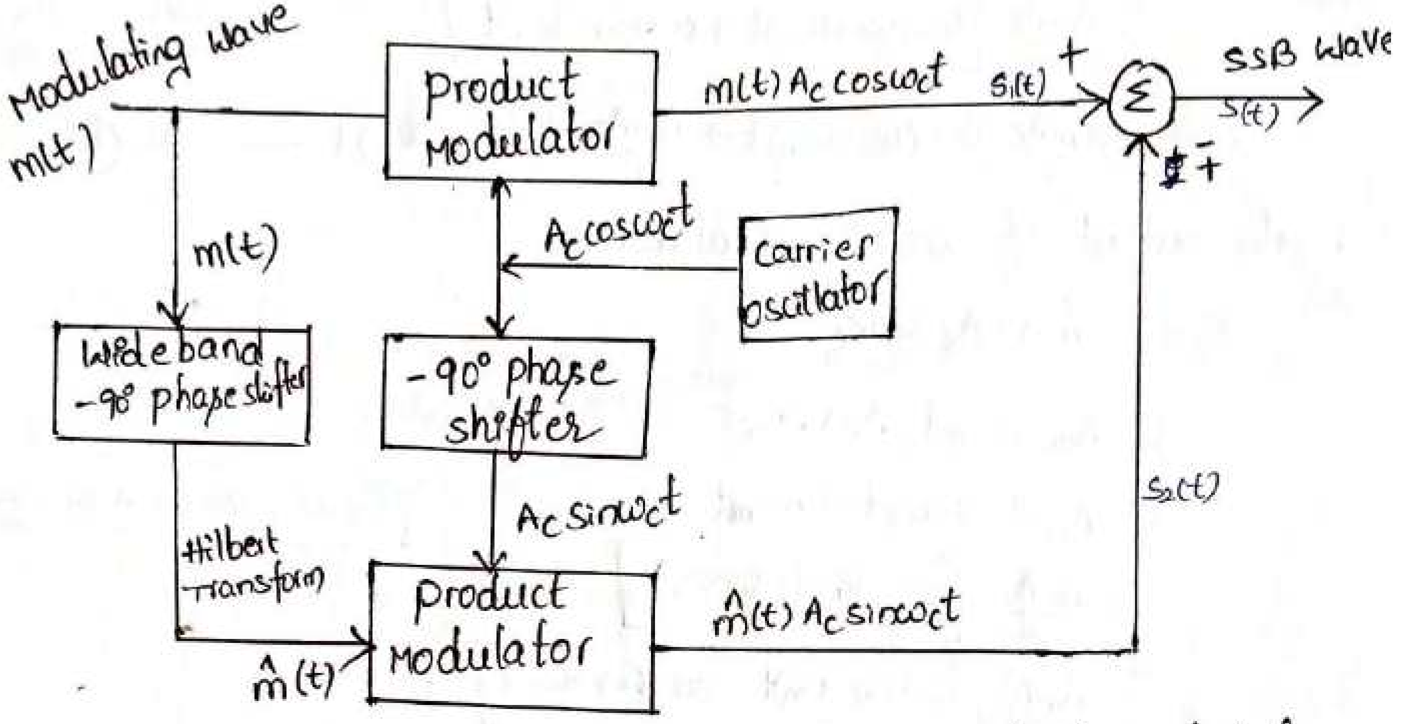
<u>For example</u>:- Take one radio receiver

↳ If we translate the freq i.e voice freq into 600MHz by having SSB-SC (cost = 10,000 Rs)

1st stage → 400MHz (cost = 2000/-) | 2nd stage → 400-600MHz (cost = 3000/-) ⟹ Therefore Total cost = 5000/-

2. phase Discrimination Method (or) phase shift method (or) Hilbert Modulator (or) Hartely Modulator

The block diagram of phase discrimination method as shown in fig.



* The phase discriminator contains two product modulators and two phase shifters with local oscillator.
* m(t) is the input of inphase product modulator and $\hat{m}(t)$ is the input of quadrature product modulator
* The role of the quadrature path including wideband phase shifter is basically to interfere with the in-phase path so as to eliminate power in one of the two sidebands (USSB (or) LSSB) depending on requirements.
* The adder (Σ) produces the output either sum of two inputs or difference of two inputs based on the polarities of the input signal.

* The output of upper modulator is

$$S_1(t) = m(t)\, A_c \cos\omega_c t$$
$$= A_m A_c \cos\omega_c t \cdot \cos\omega_m t$$
$$= \frac{A_m A_c}{2}\left[2\cos\omega_c t \cdot \cos\omega_m t\right]$$
$$= \frac{A_m A_c}{2}\left[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t\right]$$
$$S_1(t) = \frac{A_m A_c}{2}\cos(\omega_c+\omega_m)t + \frac{A_m A_c}{2}\cos(\omega_c-\omega_m)t \longrightarrow (1)$$

* The output of lower modulator is

$$S_2(t) = \hat{m}(t)\, A_c \sin\omega_c t$$
$$= A_m \sin\omega_m t \cdot A_c \sin\omega_c t$$
$$= A_m A_c \sin\omega_c t \cdot \sin\omega_m t$$

$\boxed{2\sin A \cdot \sin B = \cos(A-B) - \cos(A+B)}$

$$= \frac{A_m A_c}{2}\left[2\sin\omega_c t \cdot \sin\omega_m t\right]$$
$$= \frac{A_m A_c}{2}\left[\cos(\omega_c-\omega_m)t - \cos(\omega_c+\omega_m)t\right]$$
$$S_2(t) = \frac{A_m A_c}{2}\cos(\omega_c-\omega_m)t - \frac{A_m A_c}{2}\cos(\omega_c+\omega_m)t \longrightarrow (2)$$

$$\therefore\ S(t) = S_1(t) + S_2(t)$$
$$= \frac{A_m A_c}{2}\cos(\omega_c+\omega_m)t + \frac{A_m A_c}{2}\cos(\omega_c-\omega_m)t + \frac{A_m A_c}{2}\cos(\omega_c-\omega_m)t - \frac{A_m A_c}{2}\cos(\omega_c+\omega_m)t$$
$$= 2\left[\frac{A_m A_c}{2}\cos(\omega_c-\omega_m)t\right]$$
$$S(t) = A_m A_c \cos(\omega_c-\omega_m)t$$
$$S(t) = A_m A_c\left[\cos\omega_c t \cdot \cos\omega_m t + \sin\omega_c t \cdot \sin\omega_m t\right]$$
$$\therefore\ \boxed{S(t) = A_m A_c \cos\omega_c t \cdot \cos\omega_m t + A_m A_c \sin\omega_c t \cdot \sin\omega_m t}$$

$$S(t) = S_1(t) - S_2(t)$$
$$= \frac{A_m A_c}{2}\cos(\omega_c+\omega_m)t + \frac{A_m A_c}{2}\cos(\omega_c-\omega_m)t - \frac{A_m A_c}{2}\cos(\omega_c-\omega_m)t + \frac{A_m A_c}{2}\cos(\omega_c+\omega_m)t$$
$$S(t) = 2\left[\frac{A_m A_c}{2}\cos(\omega_c+\omega_m)t\right]$$
$$S(t) = A_m A_c \cos(\omega_c+\omega_m)t$$
$$\boxed{S(t) = A_m A_c \cos\omega_c t \cdot \cos\omega_m t - A_m A_c \sin\omega_c t \cdot \sin\omega_m t}$$

$\therefore$ output $s(t)$ is SSB-SC wave.

$$\boxed{s(t) = A_m A_c \cos\omega_c t \cdot \cos\omega_m t \underset{\text{LSSB}}{\overset{\text{USSB}}{\mp}} A_m A_c \sin\omega_c t \cdot \sin\omega_m t}$$

Advantages of phase Discrimination method:
1) It can generate the SSB at any frequency ② It can use the low audio frequencies as modulating signals ③ It is easy to switch from one sideband to the other

Power calculation in SSB-SC system

In DSB-SC, total power is

$$P_t = P_{U.S.B} + P_{L.S.B}$$

$$P_t = \frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R}$$

But in SSB-SC, either we can transmit USB or LSB, therefore total power is given by

$$P_t = P_{USB} \text{ (or) } P_{LSB}$$

$$P_t = \frac{\mu^2 A_c^2}{8R} \qquad\qquad P_c = \frac{A_c^2}{2R}$$

$$= \frac{A_c^2}{2R}\left[\frac{\mu^2}{4}\right]$$

$$\boxed{P_t = P_c\left[\frac{\mu^2}{4}\right]}$$

$$\text{Power saving} = \frac{P_T - P_T'}{P_T} \qquad \text{where } P_T = \text{AM Power}, \quad P_T' = \text{SSB-SC power}$$

$$= \frac{P_c\left[1+\frac{\mu^2}{2}\right] - P_c\left[\frac{\mu^2}{4}\right]}{P_c\left[1+\frac{\mu^2}{2}\right]}$$

$$= \frac{P_c\left[1+\frac{\mu^2}{2}-\frac{\mu^2}{4}\right]}{P_c\left[1+\frac{\mu^2}{2}\right]}$$

$$= \frac{P_c\left[1+\frac{\mu^2}{4}\right]}{P_c\left[1+\frac{\mu^2}{2}\right]} \Rightarrow \frac{4+\mu^2}{4}\times\frac{2}{2+\mu^2} = \frac{4+\mu^2}{4+2\mu^2} = \boxed{\frac{\mu^2+4}{2\mu^2+4}}$$

If $\mu = 1$, saved power $= \frac{5}{6} = 83.33\%$

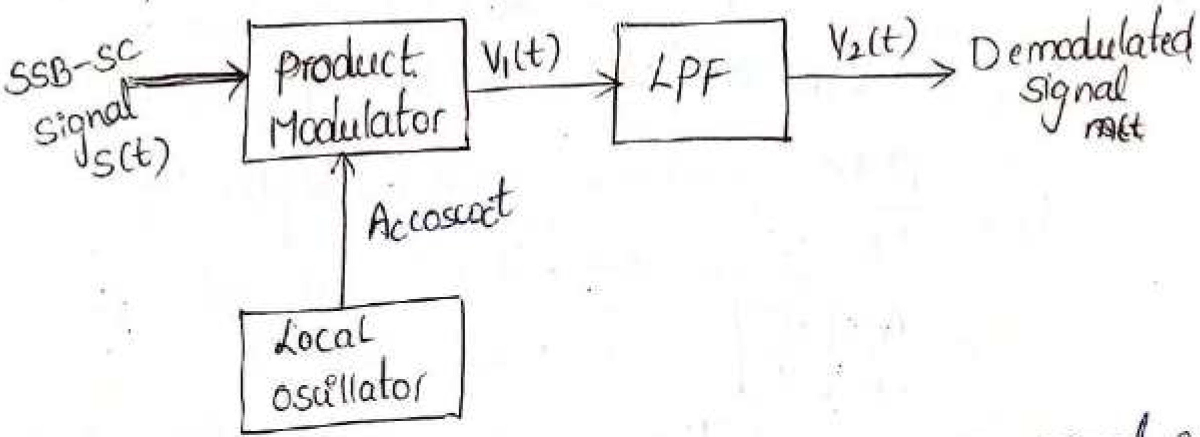
If $\mu = 0.5$, saved power $= 4.25/4.5 = 94.44\%$

Demodulation (or) Detection of SSB-SC waves

* It is a process to extract the original signal from modulated signal is called demodulation (or) detection.
* The demodulation methods in SSB-SC are.
  1. Synchronous detection (or) coherent detection
  2. Envelope detection
  3. carrier re-insertion Technique.

Synchronous (or) coherent Detection:

The block diagram of coherent detector as shown in below fig.



* In the synchronous detector, the same carrier signal is used at the time of modulated signal generation, that is are used to detect the original message signal by synchronizing same frequency and phase values.
* SSB-SC signal $s(t)$ is applied to the input of product modulator and also another input of product modulator is $A_c \cos\omega_c t$.

* $$\boxed{s(t) = \frac{A_c}{2} m(t)\cos\omega_c t \mp \frac{A_c}{2} \hat{m}(t)\sin\omega_c t} \longrightarrow (1)$$

This $s(t)$ eq is SSB-SC wave, it is present in Time domain description of SSB-SC (eq (6)) output of product modulator is $V_1(t)$

$$V_1(t) = S(t) \cdot A_c \cos\omega_c t$$

$$V_1(t) = \left[\frac{A_c}{2} m(t)\cos\omega_c t \mp \frac{A_c}{2}\hat{m}(t)\sin\omega_c t\right] A_c \cos\omega_c t$$

$$= \frac{A_c^2}{2} m(t)\cos^2\omega_c t \mp \frac{A_c^2}{2}\hat{m}(t)\cos\omega_c t \cdot \sin\omega_c t$$

$\cos A \cdot \sin B = \frac{1}{2}\left(\sin(A+B) - \sin(A-B)\right)$

$$= \frac{A_c^2}{2} m(t)\left[\frac{1+\cos 2\omega_c t}{2}\right] \mp \frac{A_c^2}{2}\hat{m}(t)\left[\frac{1}{2}\left(\sin 2\omega_c t - \sin(0)\right)\right]$$

$$= \frac{A_c^2}{2} m(t)\left[\frac{1+\cos 2\omega_c t}{2}\right] \mp \frac{A_c^2}{4}\hat{m}(t)\left[\sin 2\omega_c t\right]$$

$$V_1(t) = \frac{A_c^2}{4} m(t) + \frac{A_c^2}{2} m(t)\cos 2\omega_c t \mp \frac{A_c^2}{4}\hat{m}(t)\sin 2\omega_c t \longrightarrow (2)$$

this $V_1(t)$ is passed through LPF, LPF o/p is

$$\boxed{V_2(t) = \frac{A_c^2}{4} m(t)} \longrightarrow (3)$$

* In the demodulation of SSB-SC by using synchronous detector produce message signal with scaling factor of $\frac{A_c^2}{2}$.
* In SSB-SC demodulation, synchronous detection of local oscillator signal with modulated ~~sig~~ carrier signal is not possible in real time, hence the synchronous detection produce their output response is always distortion.
* The synchronous detection is very much useful in radar signal processing to synchronize unknown frequencies.
* If there is any frequency and phase difference, then

$$\text{Locally carrier generator} = A_c \cos[(\omega_c + \Delta\omega)t + \phi] \longrightarrow (4)$$

and SSB wave $S(t) = \frac{A_c}{2} m(t)\cos\omega_c t \mp \frac{A_c}{2}\hat{m}(t)\sin\omega_c t$

$$\therefore V_1(t) = S(t) \cdot A_c \cos[(\omega_c + \Delta\omega_c)t + \phi] \longrightarrow (5)$$

* ~~If there~~

$$V_1(t) = \left[\frac{A_c}{2} m(t)\cos\omega_c t \mp \frac{A_c}{2}\hat{m}(t)\sin\omega_c t\right] A_c\cos\left[(\omega_c+\Delta\omega_c)t+\phi\right]$$

$$= \frac{A_c^2}{2} m(t)\cos\omega_c t\cdot\cos(\omega_c t+\Delta\omega_c t+\phi) \mp \frac{A_c^2}{2}\hat{m}(t)\sin\omega_c t\cdot\cos(\omega_c t+\Delta\omega_c t+\phi)$$

$$= \frac{A_c^2}{4}\left[m(t)\cdot\cos(2\omega_c t+\Delta\omega_c t+\phi) + m(t)\cos\Delta\omega_c t+\phi\right]$$

$$\mp \frac{A_c^2}{4}\left[\hat{m}(t)\cdot\sin(2\omega_c t+\Delta\omega_c t+\phi) - \hat{m}(t)\sin\Delta\omega_c t+\phi\right]$$

$$= \frac{A_c^2}{4} m(t)\cos(2\omega_c t+\Delta\omega_c t+\phi) + \frac{A_c^2}{4} m(t)\cos\Delta\omega_c t+\phi$$

$$\mp \frac{A_c^2}{4}\hat{m}(t)\sin(2\omega_c t+\Delta\omega_c t+\phi) \pm \frac{A_c^2}{4}\hat{m}(t)\sin\Delta\omega_c t+\phi$$

this $V_1(t)$ is passed through L.P.F, LPF o/p is

$$\boxed{V_2(t) = \frac{A_c^2}{4} m(t)\cos(\Delta\omega_c t+\phi) \pm \frac{A_c^2}{4}\hat{m}(t)\sin(\Delta\omega_c t+\phi)} \rightarrow (6)$$

case (1):- $\Delta\omega_c = 0;\ \phi = 0$

$$V_2(t) = \frac{A_c^2}{4} m(t) \pm 0$$

$\boxed{V_2(t) = \frac{A_c^2}{4} m(t)}$ → this is perfect message signal.

case (2):- $\Delta\omega_c = 0;\ \phi \neq 0$

$$V_2(t) = \frac{A_c^2}{4} m(t)\cos\phi \pm \frac{A_c^2}{4}\hat{m}(t)\sin\phi$$

* Here $\hat{m}(t)$ is Hilbert transform of $m(t)$. i.e phase distortion component.
* The ~~phase~~ voice signal can tolerate this phase distortion, this effect is called Donald duck effect.
  But this phase distortion not suitable for the transmission of picture and video's.

case (3):- $\Delta\omega_c \neq 0,\ \phi = 0$

$$V_2(t) = \frac{A_c^2}{4} m(t)\cos\Delta\omega_c t \pm \frac{A_c^2}{4}\hat{m}(t)\sin\Delta\omega_c t$$

* In case (3), if causes large errors, these errors cannot be compensated, because errors depend upon time at every instant of time, there would be different error. this type of distortion cannot be used to transmitt any type of signal.

* there is a limit i.e error $\leq 30dB$

$$\boxed{\text{warble} \leq 30dB}$$

Case (4):- $\Delta\omega_c \neq 0$; $\phi \neq 0$

$$v_2(t) = \frac{A_c^2}{4}\left[m(t)\cos(\Delta\omega_c t + \phi) \pm \hat{m}(t)\sin(\Delta\omega_c t + \phi)\right]$$

In this case both frequency and phase distortion is there.

Frequency descripency in SSB-SC demodulation

From eq ⑥

$$v_2(t) = \frac{A_c^2}{4}m(t)\cos(\Delta\omega_c t + \phi) \pm \frac{A_c^2}{4}\hat{m}(t)\sin(\Delta\omega_c t + \phi)$$

$\Delta\omega_c \neq 0$, $\phi = 0$

$$V_2(t) = \frac{A_c^2}{4}m(t)\cos\Delta\omega_c t \pm \frac{A_c^2}{4}\hat{m}(t)\sin\Delta\omega_c t$$

By applying Fourier Transform

$$V_2(W) = \frac{A_c^2}{4}\left[\frac{1}{2}\left[M(\omega + \Delta\omega_c) + M(\omega - \Delta\omega_c)\right] + \frac{J}{2}\left[\hat{M}(\omega + \Delta\omega_c) - \hat{M}(\omega - \Delta\omega_c)\right]\right]$$

In Hilbert Transform, frequency domain is

$$\hat{M}(\omega) = -j\,\mathrm{sgn}(\omega)M(\omega)$$

$$\text{If } \hat{M}(\omega) = \begin{cases} -jM(\omega) & , \omega > 0 \\ J\,M(\omega) & , \omega < 0 \end{cases}$$

$$V_2(\omega) = \frac{A_c^2}{8}M(\omega + \Delta\omega_c) + \frac{A_c^2}{8}M(\omega - \Delta\omega_c) \pm J\frac{A_c^2}{8}\hat{M}(\omega + \Delta\omega_c) - J\frac{A_c^2}{8}\hat{M}(\omega - \Delta\omega_c)$$

If $\omega > 0$

$$V_2(\omega) = \frac{A_c^2}{8}M(\omega + \Delta\omega_c) + \frac{A_c^2}{8}(\omega - \Delta\omega_c) + \frac{A_c^2}{8}M(\omega + \Delta\omega_c) + \frac{A_c^2}{8}M(\omega - \Delta\omega_c)$$

$$\therefore V_2(\omega) = \boxed{\frac{A_c^2}{4} M(\omega + \Delta\omega_c)}$$

If $\omega_0 < 0$

$$V_2(\omega) = \frac{A_c^2}{8} M(\omega + \Delta\omega_c) + \frac{A_c^2}{8} M(\omega - \Delta\omega_c) - \frac{A_c^2}{8} M(\omega + \Delta\omega_c) + \frac{A_c^2}{8} M(\omega - \Delta\omega_c)$$

$$\boxed{V_2(\omega) = \frac{A_c^2}{4} M(\omega - \Delta\omega_c)}$$

$$\therefore V_2(\omega) = \begin{cases} \frac{A_c^2}{4} M(\omega + \Delta\omega_c) & : \omega > 0 \\ \frac{A_c^2}{4} M(\omega - \Delta\omega_c) & : \omega < 0 \end{cases}$$

* In SSB-SC demodulation, incoming signal is U.S.B and the input signal is causal signal i.e $\Delta\omega_c > 0$ hence the freq components of demodulation is shifted inward by $\Delta\omega_c$ freq w.r.t message signal freq. this freq shifting is also called as freq advancing
* $\Delta\omega_c < 0 \rightarrow$ Frequency delaying (USB, anticausal, shifted outward)
* If the S.S.B signal is L.S.B and causal signal $\Delta\omega_c > 0$, the freq component shifted outward by an amount of $\Delta\omega_c$ w.r.t message signal, hence it is also called as freq delaying
* If the incoming signal is L.S.B and anticausal $\Delta\omega_c < 0$, the freq component shifted inward by an amount of $\Delta\omega_c$ w.r.t message signal, hence it is also called as freq advancing
* The synchronous detection possible only a one signal i.e voice signal.

phase discripency in SSB-SC demodulation

From eq (6)

$$V_2(t) = \frac{A_c^2}{4} m(t) \cos(\Delta\omega_c t + \phi) \pm \frac{A_c^2}{4} \hat{m}(t) \sin(\Delta\omega_c t + \phi)$$

$\Delta\omega_c = 0$, $\phi \neq 0$

$$\boxed{\therefore V_2(t) = \frac{A_c^2}{4} m(t) \cos\phi \pm \frac{A_c^2}{4} \hat{m}(t) \sin\phi}$$

Apply F.T.

$$V_2(\omega) = \frac{A_c^2}{4} \left[M(\omega) \cos\phi + \hat{M}(\omega) \sin\phi \right]$$

Hilbert transform

$$\hat{M}(-\omega) = -j\,sgn(\omega) M(\omega)$$

$$\hat{M}(\omega) = \begin{cases} -jM(\omega) & ; \omega > 0 \\ jM(\omega) & : \omega < 0 \end{cases}$$

$$\therefore V_2(\omega) = \frac{A_c^2}{4} \left[M(\omega) \cos\phi - j\,sgn(\omega) M(\omega) \sin\phi \right]$$

$$= \frac{A_c^2}{4} M(\omega) \left[\cos\phi - j\,sgn(\omega) \sin\phi \right]$$

If $\omega > 0$

$$V_2(\omega) = \frac{A_c^2}{4} M(\omega)\, e^{-j\phi}$$

If $\omega < 0$

$$V_2(\omega) = \frac{A_c^2}{4} M(\omega)\, e^{j\phi}$$

* In audio signals, the phase difference is not serious, because the human ear is relatively in sensitive to the phase changes.
* Hence the txion of voice (or) music systems are not considering phase difference

* In SSB-SC demodulated signal, +ve freq's of base band signal (or) message signal produces -ve phase angles i.e '$-\phi$'
* For negative freq's of message signal produces positive phase angles i.e 'ϕ'

Vestigial Sideband Modulation (VSB):

* SSB Modulation works easily for a speech signal with an energy gap centered arround zero frequency.

  But the spetra of wideband signals (like TV videosignals & computer data) contains significant low frequencies, so it is impractical to use SSB Modulation.

* Also, the spectra of the wideband data can easily use DSB-SC modulation. However, DSB-SC requires a transmission BW equal to twice the message signal, which violates the B.W conservation requirement.

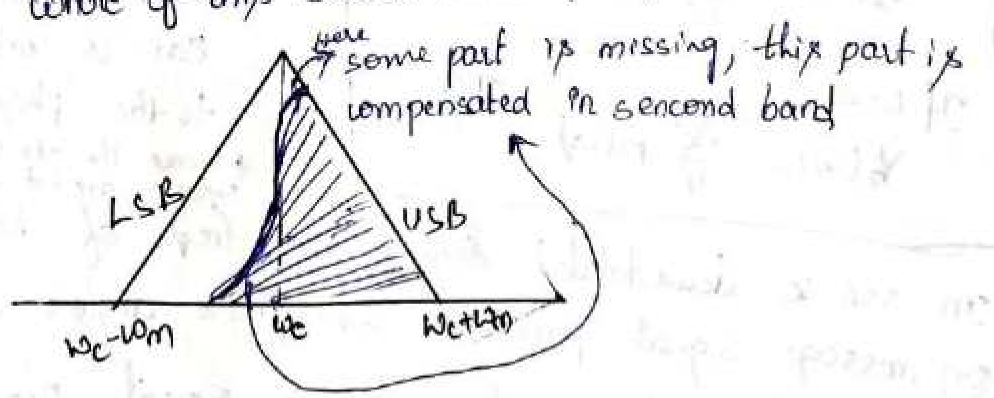
* In SSB-SC, Ideal filter cannot be generated in practical scenario

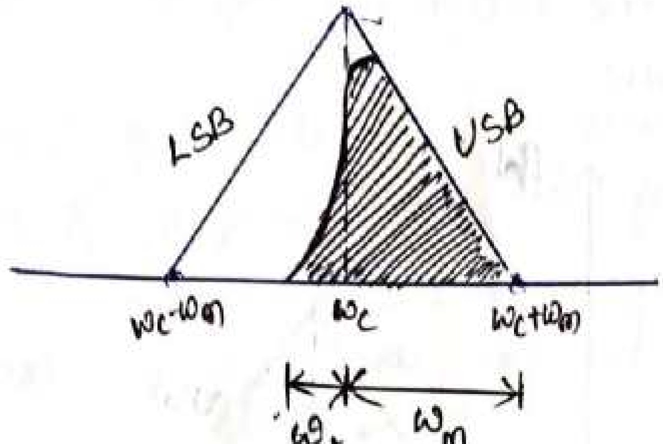
* To overcome this practical limitations, we need a compromise method of modulation that lies between SSB & DSB-SC in its spectral characteristics.

  VSB is compromise scheme

* In VSB modulation, Instead of completely removing a sideband, a trace (or) vestige of that sideband (either USB (or) LSB) is transmitted; hence the name is called "Vestigial Sideband"

* Also instead of transmitting the other side band in full, almost the whole of this second band is also transmitted.





$$\boxed{B_T = \omega_v + \omega_m}$$

$\omega_v \rightarrow$ Vestige B.W

$\omega_m \rightarrow$ Message B.W.

Typically

ω_v = 25% of message B.W

ω_v = 25% of ω_m.

* VSB B.W lies between the SSB B.W (ω_m) and DSB-SC B.W $2\omega_m$.

* In VSB-sc transmission, upper side band (USB) is added with a part of L.S.B similarly to transmit a L.S.B with a part of U.S.B is added.
* In VSB-sc, each side band contains a small width of gaurd bands to avoid interference between one side band to other sideband.

Generation of VSB-SC

Side band shaping filter

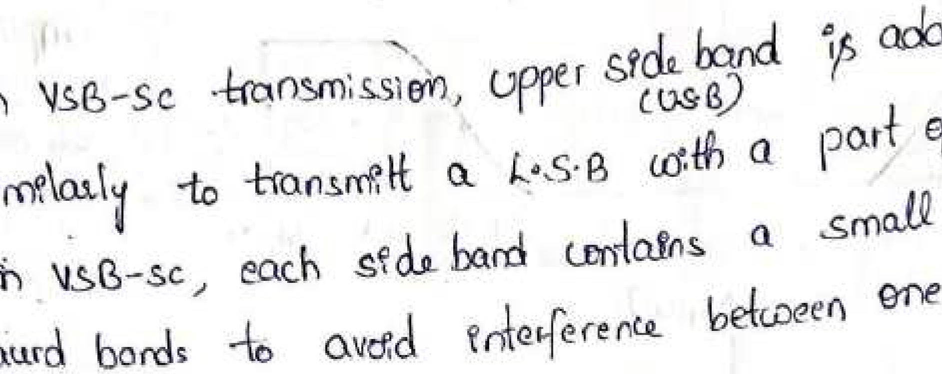
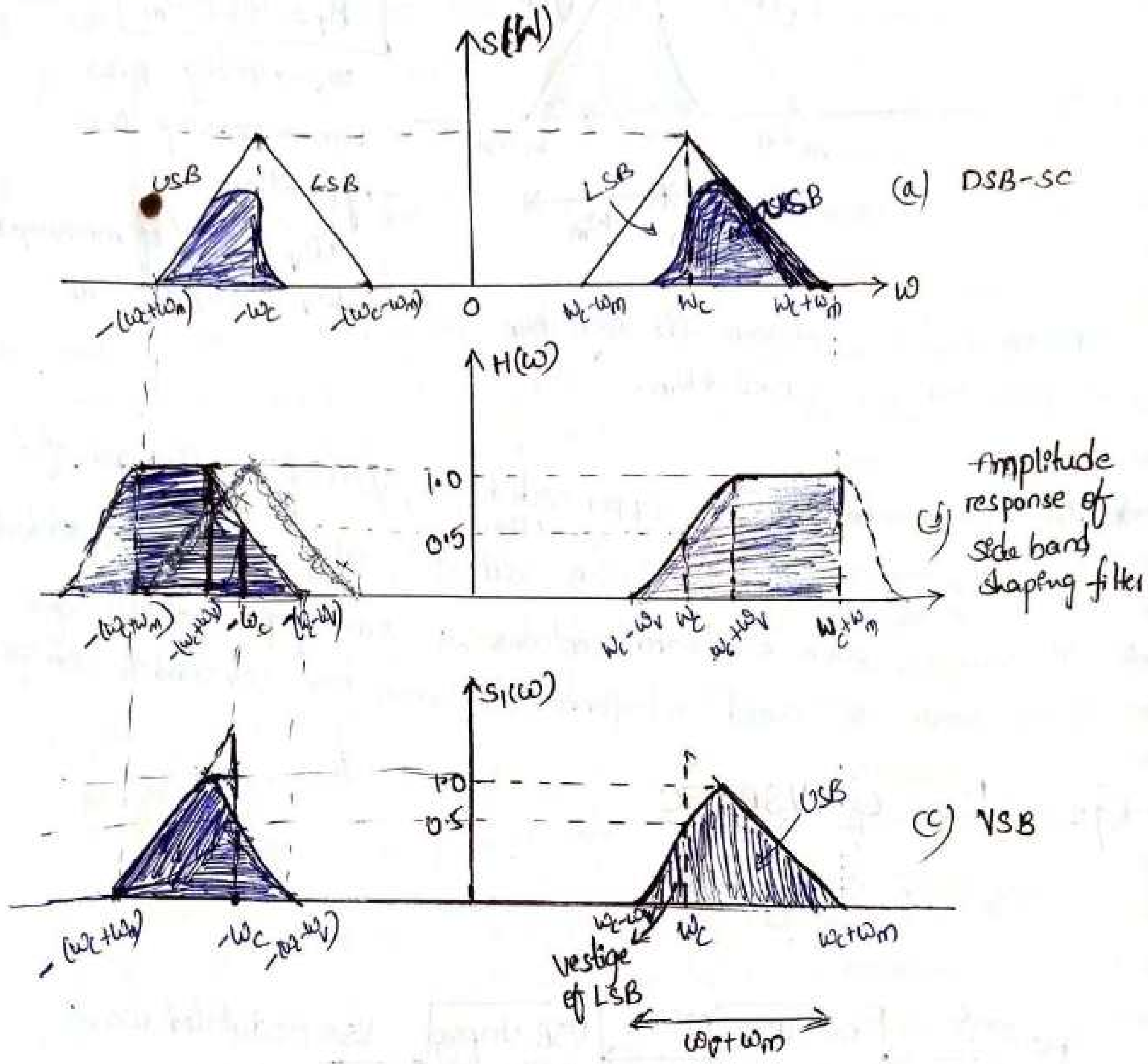


Fig: VSB modulator using Frequency discrimination.

* Transfer function of side band shaping filter is H(f)
* product modulator o/p is the product of m(t) and c(t), i.e DSB-sc, it is passed through VSB shaping filter then the output is VSB-SC

Assume, the vestige of the VSB lies in the lower side band of the DSB-SC modulated wave.



* Product modulator output is

$$s(t) = m(t) \cdot c(t)$$

$$\boxed{s(t) = A_c m(t) \cos w_c t} \longrightarrow (1)$$

Apply F.T to s(t) i.e DSB-SC.

$$\boxed{S(w) = \frac{A_c}{2}\left[M(w-w_c) + M(w+w_c)\right]} \longrightarrow (2)$$

Filter o/p

$$\boxed{S_1(w) = \frac{A_c}{2}\left[M(w-w_c) + M(w+w_c)\right] H(w)} \longrightarrow (3)$$

Advantages of VSB-SC

* It is most efficient technique because it does not required sharp cut-off frequencies to design filter and it is a compromisation technique in between DSB-SC and SSB-SC
* The B.W of VSB-SC is less compared to DSB-SC and A.M.
* The transmission of low frequency components is possible because it is having transition band in the filter.

Disadvantages

* It occupies more BW compared to SSB-SC.
* The demodulation of VSB-SC signal is more complex because one side band is mixed with other side band part.

Applications:-

* It is more suitable to use in T.V transmission because it is having audio and video combinations.
* The general video signal occupied frequency band is 4.2 MHZ.

Therefore if we use DSB-SC, then the occupied Bandwidth is

$$B.W = 2(4.2) + \text{Gaurd Band frequency} + \text{carrier frequency}$$
$$= 8.4 + 0.5 + 0.25$$
$$= 9.15 \text{ MHz}$$
$$\therefore B.W \approx 9 \text{ MHZ}.$$

In TV

fer: 250KHz (or) 0.25 MHz

Sound carrier

Gaurd Band $= 2(f_m)$
$= 2(250)$
$= 500 \text{ KHz}$
(or) 0.5 MHz

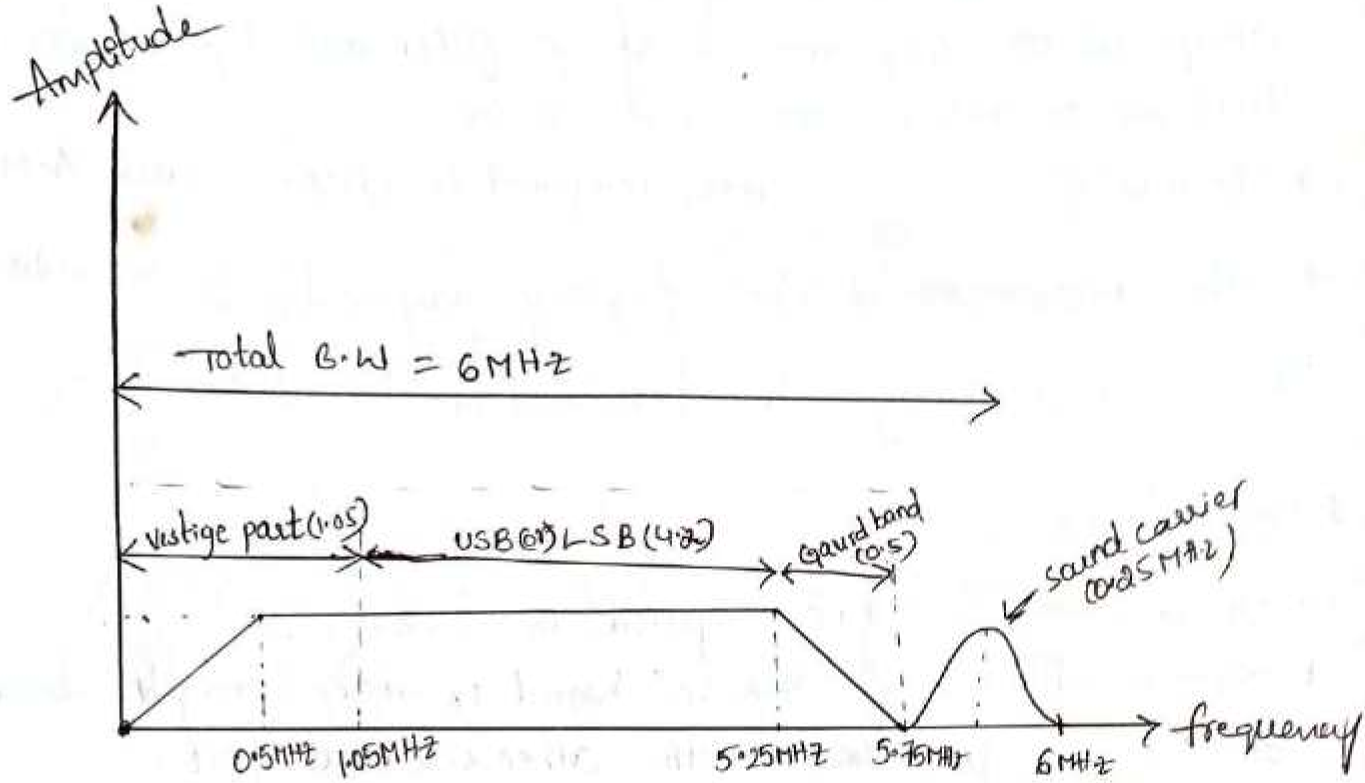
* But If we use VSB-SC, then the occupied Bandwidth is

$$B.W = 4.2 + \text{Gaurd Band frequency} + \text{carrier frequency}.$$
$$= 4.2 + 0.5 + 0.25$$
$$\approx 5 \text{ MHZ} \rightarrow \text{USB}$$

LSB: 25% of $f_m = 0.25 \times 4.2 = 1.05$
$\therefore B.W = 5 + 1.05 = 6.105$ MH

* By using VSB-SC, We are saving approximately 3 MHz compared to DSB-SC

* In real time, the television transmission frequency allocation in graphical view by using VSB as shown in below figure.

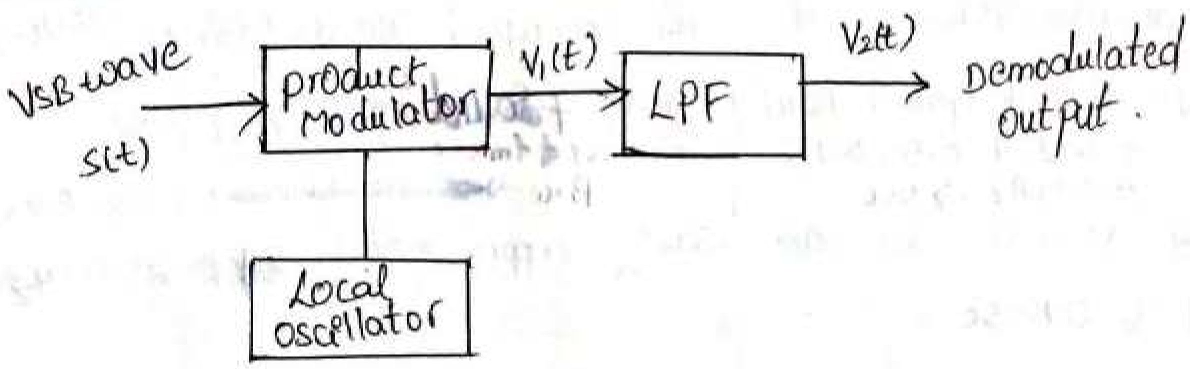


Demodulation (or) Detection of VSB-SC

* It is a process to extract the original signal from modulated signal is called demodulation (or) detection.
* The demodulation methods in VSB-SC are.
    1. Synchronous (or) coherent detection.
    2. Envelope detection.

1. Synchronous detection:

* The demodulation of VSB-SC by using synchronous detector block diagram is



product modulator output $V_1(t)$ is

$$V_1(t) = S(t)\, c(t)$$

$$V_1(t) = S(t) \cdot A_c \cos w_c t \longrightarrow ①$$

Apply F.T on $V_1(t)$

$$V_1(\omega) = \frac{A_c}{2}\left[S(\omega-\omega_c) + S(\omega+\omega_c)\right] \longrightarrow ②$$

we know that

$$S(\omega) = \frac{A_c}{2}\left[M(\omega-\omega_c) + M(\omega+\omega_c)\right] H(\omega) \longrightarrow ③$$

If $w = \omega - \omega_c$ then eq ③ is modified as

$$S(\omega-\omega_c) = \frac{A_c}{2}\left[M(\omega-\omega_c-\omega_c) + M(\omega-\omega_c+\omega_c)\right] H(\omega-\omega_c)$$

$$= \frac{A_c}{2}\left[M(\omega-2\omega_c) + M(\omega)\right] H(\omega-\omega_c) \longrightarrow ④$$

If $W = \omega + \omega_c$ then eq ③ is modified as

$$S(\omega+\omega_c) = \frac{A_c}{2}\left[M(\omega+\omega_c-\omega_c) + M(\omega+\omega_c+\omega_c)\right] H(\omega+\omega_c)$$

$$= \frac{A_c}{2}\left[M(\omega) + M(\omega+2\omega_c)\right] H(\omega+\omega_c) \longrightarrow 5$$

substitute eq ④ & eq ⑤ in eq ②

$$\therefore V_1(\omega) = \frac{A_c}{2}\left\{\left[\frac{A_c}{2}\left[M(\omega-2\omega_c) + M(\omega)\right] H(\omega-\omega_c)\right] + \frac{A_c}{2}\left[M(\omega) + M(\omega+2\omega_c)\right] H(\omega+\omega_c)\right]\right\}$$

$$= \frac{A_c^2}{4} M(\omega-2\omega_c) \cdot H(\omega-\omega_c) + \frac{A_c^2}{4} M(\omega) H(\omega-\omega_c) + \frac{A_c^2}{4} M(\omega) H(\omega+\omega_c) + \frac{A_c^2}{4} M(\omega+2\omega_c) H(\omega+\omega_c)$$

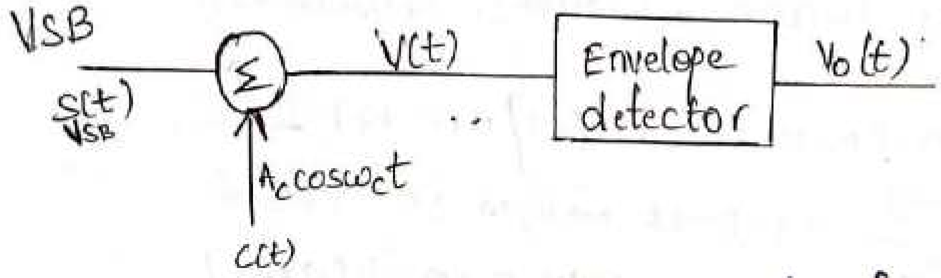
$$V_1(\omega) = M(\omega)\frac{A_c^2}{4}\left[H(\omega-\omega_c) + H(\omega+\omega_c)\right] + \frac{A_c^2}{4} M(\omega-2\omega_c) H(\omega-\omega_c) + \frac{A_c^2}{4} M(\omega+2\omega_c) H(\omega+\omega_c) \longrightarrow 6$$

this $V_1(\omega)$ is passed through LPF, L.P.F output is

$$V_2(\omega) = \underbrace{\frac{A_c^2}{4} M(\omega)}_{\text{original signal}} \underbrace{\left[H(\omega-\omega_c) + H(\omega+\omega_c)\right]}_{\text{Filter allow extra frequency vestige (or) part of the other side.}}$$

2. Envelope detection of a VSB wave pulse carrier

* In commercial television broadcasting, a sizable carrier is transmitted together with the modulated wave i.e VSB wave.
* This makes it possible to demodulate the incoming modulated wave by an envelope detector in the receiver.
* The block diagram of envelope detector of a VSB pulse carrier is shown in below figure.



* We know that, the VSB modulated with full USB and a vestige of LSB is given by.

$$\boxed{S_{VSB}(t) = \frac{A_c}{2} m(t) \cos\omega_c t - \frac{A_c}{2} \hat{m}(t) \sin\omega_c t} \longrightarrow (1)$$

The above eq. is present in time domain description of VSB.

* Adding carrier component $A_c \cos\omega_c t$ to eq (1) scaled by a factor 'K_a', therefore the modulated wave applied to the envelope detector input is.

$$V(t) = A_c \cos\omega_c t + K_a S_{VSB}(t)$$

$$= \frac{A_c}{2} K_a \left[m(t) \cos\omega_c t - \hat{m}(t) \sin\omega_c t\right] + A_c \cos\omega_c t$$

$$= \frac{A_c}{2} K_a m(t) \cos\omega_c t - \frac{A_c}{2} K_a \hat{m}(t) \sin\omega_c t + A_c \cos\omega_c t$$

$$\boxed{V(t) = \underbrace{A_c \cos\omega_c t \left[1 + \frac{K_a}{2} m(t)\right]}_{\text{Inphase}} - \underbrace{\frac{K_a A_c}{2} \hat{m}(t) \sin\omega_c t}_{\text{Quadrature}}} \longrightarrow (2)$$

* $V(t)$ is applied to the input of envelope detector, therefore the output of envelope detector is

$$V_0(t) = \sqrt{(\text{Inphase component})^2 + (\text{Quadrature component})^2}$$

$$= \sqrt{A_c^2\left[1+\frac{K_a}{2}m(t)\right]^2 + A_c^2\left[\frac{K_a}{2}\hat{m}(t)\right]^2}$$

$$= \sqrt{A_c^2\left[1+\frac{K_a}{2}m(t)\right]^2 + \frac{\left[1+\frac{K_a}{2}m(t)\right]^2}{\left[1+\frac{K_a}{2}m(t)\right]^2}\cdot A_c^2\left[\frac{K_a}{2}\hat{m}(t)\right]^2}$$

$$= \sqrt{A_c^2\left[1+\frac{K_a}{2}m(t)\right]^2\left\{1+\frac{\left[\frac{K_a}{2}\hat{m}(t)\right]^2}{\left[1+\frac{K_a}{2}m(t)\right]^2}\right\}}$$

$$\boxed{V_0(t) = A_c\left[1+\frac{K_a}{2}m(t)\right]\sqrt{1+\frac{\left[\frac{K_a}{2}\hat{m}(t)\right]^2}{\left[1+\frac{K_a}{2}m(t)\right]^2}}} \longrightarrow (3)$$

The eq (3) indicates that the distortion is contributed by the quadrature component $\hat{m}(t)$ of the incoming VSB wave.

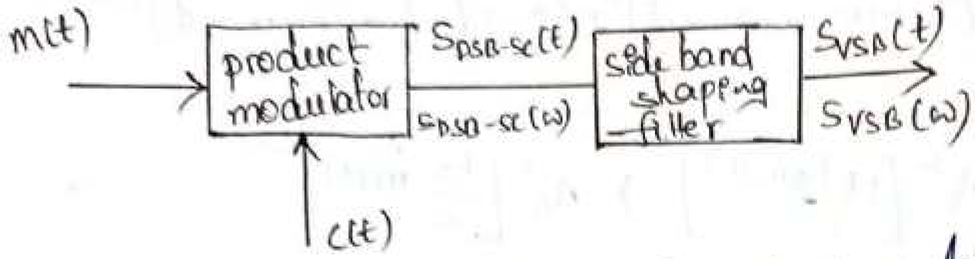
* The distortion can be reduced by using two methods.

1. By reducing percentage modulation (μ) to reduce K_a

$$K_a = \frac{1}{A_c}, \quad \mu = \frac{A_m}{A_c} \Rightarrow \mu = K_a \cdot A_m$$

2. By increasing the width of the vestigial sideband to reduce $\hat{m}(t)$.

Time domain description of VSB signal



* $H(\omega)$ is the transfer function of side band filter.

$$\boxed{H(\omega) = \frac{S_{VSB}(\omega)}{S_{DSB-SC}(\omega)}} \longrightarrow (1)$$

$$\boxed{S_{VSB}(\omega) = H(\omega) \cdot S_{DSB-SC}(\omega)} \longrightarrow (2)$$

Apply inverse F.T on $S_{VSB}(\omega)$

$$S_{VSB}(t) = F^{-1}[S_{VSB}(\omega)]$$
$$= F^{-1}[H(\omega)] \otimes F^{-1}[S_{DSB-SC}(\omega)]$$
$$= h(t) \otimes S_{DSB-SC}(t)$$

$$\boxed{S_{VSB}(t) = h(t) \otimes A_c m(t) \cos\omega_c t} \longrightarrow (3)$$

$$\hat{m}(t) = m(t) \otimes \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} m(\tau)\frac{1}{t-\tau}\,d\tau$$

Replace the first term with τ

$$S_{VSB}(t) = A_c \int_{-\infty}^{\infty} h(\tau)\, m(t-\tau) \cos\omega_c(t-\tau)\, d\tau$$
$$= A_c \int_{-\infty}^{\infty} h(\tau)\, m(t-\tau) [\cos\omega_c t \cdot \cos\omega_c\tau + \sin\omega_c t \cdot \sin\omega_c\tau]\, d\tau$$

$$\boxed{S_{VSB}(t) = A_c \cos\omega_c t \underbrace{\int_{-\infty}^{\infty} h(\tau)\, m(t-\tau) \cos\omega_c\tau\, d\tau}_{1/2\, m(t)} + A_c \sin\omega_c t \underbrace{\int_{-\infty}^{\infty} h(\tau)\, m(t-\tau) \sin\omega_c\tau\, d\tau}_{1/2\, \hat{m}(t)}} \longrightarrow (4)$$

From eq (4)

Any component multiplied by inphase carrier ($A_c \cos\omega_c t$) i.e called as Inphase component ($\frac{1}{2}m(t)$) and the another one is multiplied by quadrature carrier ($A_c \sin\omega_c t$) is called quadrature phase component ($\frac{1}{2}\hat{m}(t)$)

$\therefore$ eq (4) can be modified as

$$\boxed{S_{VSB}(t) = \frac{A_c}{2} m(t) \cos\omega_c t + \frac{A_c}{2} \hat{m}(t) \sin\omega_c t} \longrightarrow (5)$$

eq (5) is VSB modulated wave with full LSB and a vestige of USB.

Similarly.

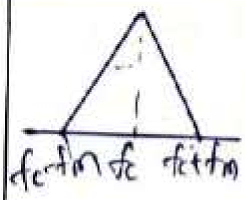
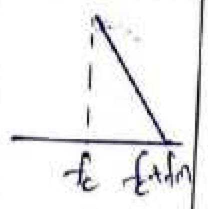
$$\boxed{s_{VSB}(t) = \frac{A_c}{2} m(t) \cos\omega_c t - \frac{A_c}{2} \hat{m}(t) \sin\omega_c t} \longrightarrow (6)$$

* eq (6) is VSB modulated wave with full USB added with vestige of LSB.

* Therefore the combined eq is given by.

$$\boxed{s_{VSB}(t) = \frac{A_c}{2} m(t) \cos\omega_c t \pm \frac{A_c}{2} \hat{m}(t) \sin\omega_c t}$$

Comparision of different AM Techniques

parameters	AM (or) DSB-FC	DSB-SC	SSB	VSB
Carrier suppression	NO	Fully	Fully	partially
Side Band suppression	NO	NO	one side band is suppressed	one sideband is partially suppressed
Bandwidth	$2f_m$	$2f_m$	f_m	$f_m + f_v$
Transmission efficiency	Less	Better than DSBFC	Best	Lower than SSB
power Transmission	$P_c + P_{USB} + P_{LSB}$	$P_{USB} + P_{LSB}$	P_{USB} (or) P_{LSB}	$P_{USB} + P_v$ (or) $P_{LSB} + P_v$
complexity	Simple	Simple	most complex	less complex than SSB
Applications	Radio Broadcasting	Radio Broadcasting	point-to-point communication	TV Transmission
spectrum				

UNIT-III

Angle Modulation

Introduction:-

* Angle Modulation is the process in which the angle of the carrier wave is varied inaccordance with the instantaneous value of the modulating signal(or) message signal.
* In angle modulation, the amplitude of the carrier wave is maintained constant

Why we need Angle Modulation (or) Advantages

* Angle modulation is more immune to channel noise and interfearence than Amplitude Modulation, because the signal characteristics changing either frequency (or) phase values.

  For example, when we send a signal with variation of amplitude by antenna, because of noise. maximum defect that will happen with amplitude only, minimum defect that will happen with freq and phase (In AM, once the signal has been contaminated by noise, it could not be removed)
* Immune to Noise means reduced noise, then it improves the signal to noise ratio fidelity
* Angle modulated signals are more power efficient.
  → Amplitude of FM (or) PM wave remains constant
  → Amplitude of is independent of modulation depth
  → Low Level modulation may be used, but all subsequent amplifiers can be class 'C' and therefore more power efficient.
  → All transmitted power in FM are useful, making it more power efficient.
* standard frequency allocations provide gaurd band between FM broadcast stations, so there is less adjacent channel interference.

* Types of Angle Modulation

There are two types of angle modulation.

1. Frequency Modulation.
2. phase Modulation.

1. Frequency Modulation:- the frequency of the carrier is varied inaccordance with the instantaneous value of the message signal.

2. phase Modulation:- the phase of the carrier signal is varied inaccordance with the instantaneous value of the message signal.

* Angle modulated wave is mathematically expressed as

$$\boxed{S(t) = A_c \cos[\theta_i(t)]}$$

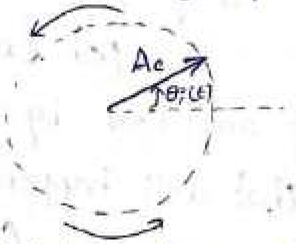
where $A_c \rightarrow$ Carrier Amplitude

$\theta_i \rightarrow$ angle at time 't'

→ complete oscillations occur whenever angle $\theta_i(t)$ changes by 2π radians

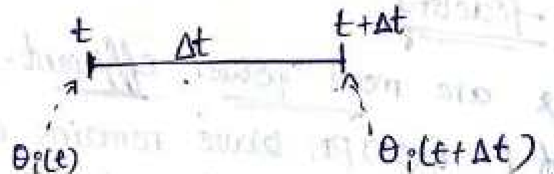
→ Rotating phasor with angular velocity (rad/sec)

$\frac{d\theta_i}{dt}$ w.r.t t



Rotating phasor with angular velocity. (rad/sec)

→ consider, small interval Δt



* Average frequency (Hz) :- It is over a small interval from t to $t+\Delta t$ is

$$\boxed{f_{\Delta t}(t) = \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi\Delta t}}$$

We know that

$S(t) = A\cos\theta$

(or) $= A\cos\omega t$

(or) $= A\cos 2\pi f t$

so, $\theta = 2\pi f t$

(or) $f = \frac{\theta}{2\pi t}$

* With the help of Average freq (Hz), we can determine the instantaneous frequency ($f_i(t)$)

* Instantaneous frequency $f_i(t)$: As $\Delta t \to 0$

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t) = \lim_{\Delta t \to 0} \left[\frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi \Delta t} \right]$$

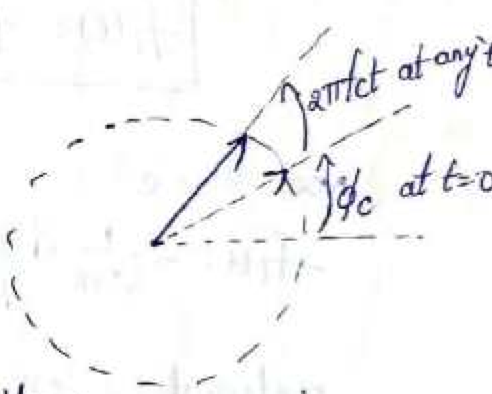
$$f_i(t) = \frac{1}{2\pi} \underbrace{\lim_{\Delta t \to 0} \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi \Delta t}}$$

$\hookrightarrow \frac{d\theta_i(t)}{dt} \to$ Angular Velocity of rotating phasor

$$\boxed{\therefore f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}}$$

* For unmodulated carrier, i.e $m(t) = 0$

$$\boxed{\theta_c(t) = \theta_i(t) = 2\pi f_c t + \phi_c}$$



where $\phi_c \to$ Angle of unmodulated carrier at time $t = 0$

$2\pi f_c t \to$ constant angular velocity (rad/sec)

* Let us consider

$c(t) = A_c \cos\theta_c t = A_c \cos(2\pi f_c t + \phi_c)$

$m(t) = A_m \cos\omega_m t$

$$\boxed{S(t) = A_c \cos\theta_i(t)}$$

phase modulation:-

Before modulation, the carrier angle is $\theta_c = 2\pi f_c t + \phi_c$

where ϕ_c is unmodulated carrier at time $t=0$

For our convienience, $\phi_c = 0 \Rightarrow$ then $\theta_c = 2\pi f_c t$

according to the definition $\boxed{\therefore \theta_i(t) = 2\pi f_c t + K_p m(t)}$

where
$K_p \rightarrow$ phase sensitivity factor of the modulator (rad/volt)

We know that $s(t) = A_c \cos(\theta_i(t))$

$\boxed{s(t) = A_c \cos[2\pi f_c t + K_p m(t)]}$ phase modulated wave.

Frequency Modulation:

$\boxed{f_i(t) = f_c + K_f m(t)}$

where
$K_f \rightarrow$ Freq sensitivity factor of the modulator (Hz/volt)

Now, since

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

Integration on both sides.

$$\text{so } \theta_i(t) = 2\pi \int_0^t f_i(\tau)\, d\tau$$

$$= 2\pi \int_0^t f_c + K_f m(\tau)\, d\tau$$

Also

$$\theta_i(t) = 2\pi f_c t + 2\pi K_f \int_0^t m(\tau)\, d\tau$$

where
$2\pi f_c t \rightarrow$ Angle of unmodulated carrier
$2\pi K_f \int_0^t m(\tau) d\tau \rightarrow$ Increase (or) decrease in the instantaneous phase $\theta_i(t)$ due to $m(t)$

$\therefore$ FM modulated wave is

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t m(\tau) d\tau\right]}$$

problem

An FM wave is defined by $s(t) = 10\cos[10\pi t + \sin(4\pi t)]$. calculate the instantaneous frequency of s(t).

sol

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$$

$$= \frac{1}{2\pi}\frac{d}{dt}\left[10\pi t + \sin 4\pi t\right]$$

$$= \frac{1}{2\pi}\left[10\pi + 4\pi\cos 4\pi t\right]$$

$$\boxed{f_i(t) = 5 + 2\cos(4\pi t)} \text{ Hz}$$

properties of Angle Modulated waves

1. consistancy of Transmitted power:

$$s(t) = A_c\cos[2\pi f_c t + k_p m(t)] \longrightarrow \text{PM}$$

$$s(t) = A_c\cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right] \longrightarrow \text{FM}.$$

Amplitude of PM & FM waves are maintained at a constant value equal to the carrier amplitude A_c for all time t, irrespective of the sensitivity factor k_p & k_f

Average transmitted power of angle modulated wave is

$$\boxed{P_{avg} = \frac{1}{2}A_c^2}$$ Assumed, the load Resistor $R_L = 1\Omega$

In PM & FM, Amplitude is constant i.e transmitted power also maintained a constant level. i.e called consistancy of transmitted power.

2. Non-linearity of the modulation process:-

* Both PM & FM waves violates the principle of super position.
* Assume message signal m(t) is made up of two different components $m_1(t)$ & $m_2(t)$ as

$$m(t) = m_1(t) + m_2(t)$$

Let $s(t)$, $s_1(t)$ & $s_2(t)$ denote the PM waves produced by $m(t)$, $m_1(t)$ & $m_2(t)$

$$s(t) = A_c \cos\left[2\pi f_c t + k_p\left[m_1(t) + m_2(t)\right]\right]$$

$$s_1(t) = A_c \cos\left[2\pi f_c t + k_p m_1(t)\right]$$

$$\& \; s_2(t) = A_c \cos\left[2\pi f_c t + k_p m_2(t)\right]$$

So, $\boxed{s(t) \neq s_1(t) + s_2(t)}$ → violate the principle of superposition.

3. Irregularity of Zero Crossing:-

* zero crossings are defined as the instants of time at which a waveform changes its amplitude from a positive to negative (or) any other way.

* In FM & PM waveforms, the zero crossing of a PM & FM wave no longer have a perfect regularity across the time scale.

4. visualization difficulty of message waveform

* there is the difficulty in visualizing the message waveform in angle modulated waves due to the non-linear character of PM & FM.

* while in case of AM, we see the message waveform as the envelope of the modulated wave provided the % modulation is less than 100% modulation.

Relationship between PM & FM

units
$k_p \rightarrow rad/volt$

$k_f \rightarrow Hz/volt.$

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)] \longrightarrow P.M \longrightarrow (1)$$

$$s(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau] \longrightarrow F.M. \longrightarrow (2)$$

So, comparing eq (1) & (2)

$$k_p m(t) \Longleftrightarrow 2\pi k_f \int_0^t m(\tau) d\tau$$

$$\left(\frac{rad}{volt}\right) m(t) \Longleftrightarrow 2\pi\left(\frac{Hz}{volt}\right)\int_0^t m(\tau) d\tau$$

$2\pi(Hz) = 2\pi f = \omega$
radians

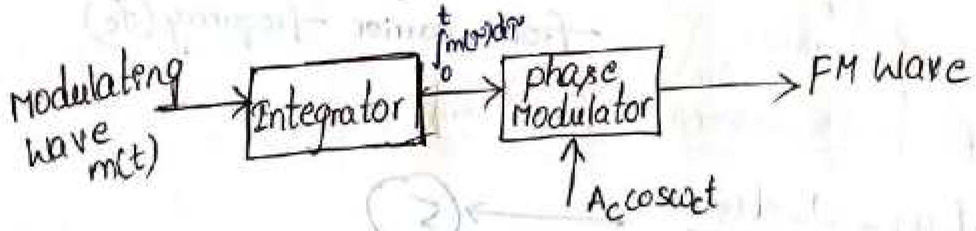
$$\left(\frac{rad}{volt}\right) m(t) \Longleftrightarrow \left(\frac{rad}{volt}\right)\int_0^t m(\tau) d\tau$$

$$\underbrace{m(t)}_{PM} \Longleftrightarrow \underbrace{\int_0^t m(\tau) d\tau}_{F.M}$$

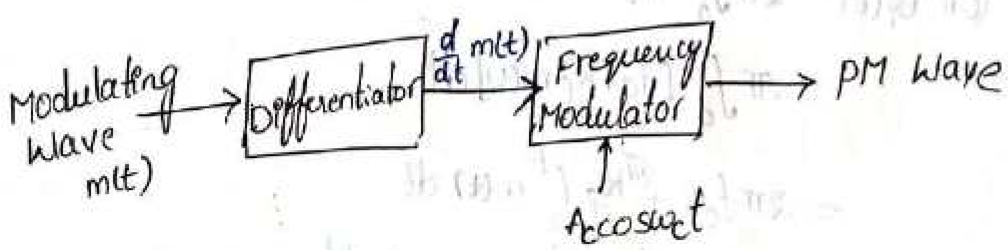
Also
$\frac{d\, m(t)}{dt} = m(t)$
PM = FM

∴ FM wave is viewed as a PM wave, produced by the modulating wave $\int_0^t m(\tau) d\tau$ inplace of $m(t)$

∴ FM wave can be generated by using phase modulator.



Also PM wave can be generated by using Frequency Modulator



Single Tone Frequency modulation

* The FM signal s(t) is a non-linear function of the modulating signal m(t), which makes frequency modulation is a non-linear modulation process.

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t m(\tau)d\tau\right]} \longrightarrow (1)$$

* consider, single Tone modulating signal

$$\boxed{m(t) = A_m \cos\omega_m t} \longrightarrow (2)$$

The instantaneous frequency of FM is given by

$$f_i(t) = f_c + K_f m(t) \longrightarrow (3)$$

$$f_i(t) = f_c + K_f A_m \cos\omega_m t \qquad \text{where } K_f A_m = \Delta f$$

$$\boxed{f_i(t) = f_c + \Delta f \cos\omega_m t} \longrightarrow (4) \qquad \Delta f \rightarrow \text{Frequency Deviation.}$$

$\Delta f = K_f A_m \rightarrow$ Frequency Deviation $\rightarrow$ It is representing the maximum departure of the instantaneous freq (f_i) from carrier frequency (f_c).

Deviation of frequency from one to another i.e freq. deviation

* we know that

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} \longrightarrow (5)$$

$$\text{(or) } \theta_i(t) = 2\pi \int_0^t f_i(t)\,dt$$

$$= 2\pi \int_0^t [f_c + K_f m(t)]\,dt$$

$$= 2\pi f_c t + 2\pi K_f \int_0^t m(t)\,dt$$

$$= 2\pi f_c t + 2\pi K_f \int_0^t A_m \cos\omega_m t\,dt$$

$$= 2\pi f_c t + 2\pi K_f \int_0^t A_m \cos 2\pi f_m t\,dt$$

$$= 2\pi f_c t + \frac{2\pi K_f A_m}{2\pi f_m} \cdot \sin(2\pi f_m t)$$

$$= 2\pi f_c t + \frac{K_f A_m}{f_m} \cdot \sin(2\pi f_m t)$$

$$\boxed{\theta_i(t) = 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)} \longrightarrow (6)$$

where $\frac{\Delta f}{f_m} = \beta$ = ~~phase~~ ~~Freq deviation ratio~~ Modulation Index

β → Modulation index

$$\boxed{\therefore \theta_i(t) = 2\pi f_c t + \beta \sin 2\pi f_m t} \longrightarrow (7)$$

~~phase~~ ~~Deviation ratio or~~ Modulation index (β) :- The ratio of frequency deviation Δf to the modulating frequency f_m is called modulation index of the FM wave. It is denoted by "β"

* "β" represents the phase deviation of the FM wave i.e maximum departure of the angle $\theta_i(t)$ from angle ($2\pi f_c t$) of unmodulated carrier, it is measured in radians

$\therefore$ $S(t)$ i.e eq (1) can be modified interms of β i.e eq (7)

$$\boxed{\therefore S(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]}$$

* Depending on the value of β, we may distinguish two cases of Frequency Modulation.

* If $\beta < 1$, ~~other~~ then the carrier frequency added in the FM modulated signal is very small, hence this technique is called Narrow Band Frequency Modulation (NBFM)

* If $\beta > 1$, then the carrier frequency added in the FM modulated signal is very large, hence this technique is called Wide band Frequency Modulation [WBFM]

Problem

1) A sinusoidal modulating signal amplitude is 5V and frequency is 2KHZ applied to F.M generator which have a frequency sensitivity 40HZ per volt and to calculate the frequency deviation & modulation index

sol

Given $A_m = 5V$

$f_m = 2KHZ$

$K_f = 40HZ/volt$

Frequency Deviation $(\Delta f) = K_f \cdot A_m$

$= 40 \times 5 = 200HZ.$

Modulation Index $(\beta) = \frac{\Delta f}{f_m}$

$= \frac{200}{2000}$

$\boxed{\beta = 0.1HZ}$

$\beta = 0.1$ i.e $\beta < 1$, hence the frequency is NBFM.

2) An FM wave $s(t) = 20\cos\left[(8\pi \times 10^6 t) + 9\sin(2\pi \times 10^3 t)\right]$, calculate frequency deviation.

sol the general mathematical expression for FM modulated wave is

$$S(t) = A_c \{\cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

$A_c = 20$

$f_c = 4 \times 10^6$

$f_m = 10^3$

$\beta = 9$

$8\pi \times 10^6$

$2 \times 4\pi \times 10^6 t$

Frequency Deviation $(\Delta f) = K_f \cdot A_m$

$$\beta = \frac{\Delta f}{f_m}$$

$$\Delta f = \beta \cdot f_m$$

$$= 9 \times 10^3$$

$\boxed{\Delta f = 9 KHZ}$ Hence this is a wide band FM

3) A 25MHZ carrier is modulated by a 400 HZ audio sine wave. If the carrier voltage is 4V and the maximum deviation is 10KHZ Write the equation for the FM modulated wave.

Sol

FM modulated wave is

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

$f_c = 25MHZ$

$f_m = 400HZ$

$A_c = 4V$

max. deviation $(\Delta f) = 10KHZ$

Max. freq deviation $(f_c + \Delta f) = 10KHz$
$= 0.01M$

$25MHz + \Delta f = 0.01MHz$

$\Delta f = -24.99 MHz$

$$\therefore \beta = \frac{\Delta f}{f_m} = \frac{10KHZ}{400KHZ} = 25$$

$$\therefore s(t) = 4\cos[2\pi(25\times10^6)t + 25 \sin 2\pi(400)t]$$

$$s(t) = 4\cos[0.157\times10^9 t + 25 \sin(2.513\times10^3 t)]$$

4) In an FM system, when the audio frequency is 500 HZ and modulating voltage is 2.5V, the deviation produced is 5KHZ. If the modulating voltage is now increased to 7.5V, (a) calculate the new value of frequency deviation produced.

(b) If the AF voltage is raised to 10V, while the modulating freq dropped to 250HZ. what is the frequency deviation.

(c) calculate the modulation index in each case.

Sol (a) $f_m = 500HZ$, $A_m = 2.5V$, $\Delta f = 5KHZ$

A

$\cancel{\frac{\Delta f}{f_m}}$

$\Delta f = K_f \cdot A_m$

$$K_f = \frac{\Delta f}{A_m} = \frac{5KHZ}{2.5V} = 2KHZ/volt.$$

(a) $\cancel{K_f = \frac{\Delta f}{A_m} = 5KHZ}$ If A_m is increased to 7.5V, $\Delta f = ?$

$$\therefore \Delta f = K_f \cdot A_m$$

$$\Delta f = 2 \times 7.5$$

$$\Delta f = 15 KHZ$$

(b) $A_m = 10V$

$f_m = 250 Hz$

Freq Deviation $= (\Delta f) = k_f \cdot A_m$

$= [2 KHz/volt][10]$

$= 20 KHz$

(c) calculate Modulation index in each case (β)

$\beta = \frac{\Delta f}{f_m}$

$\beta_1 = \frac{5 KHz}{500 Hz}$

$\boxed{\beta_1 = 10}$

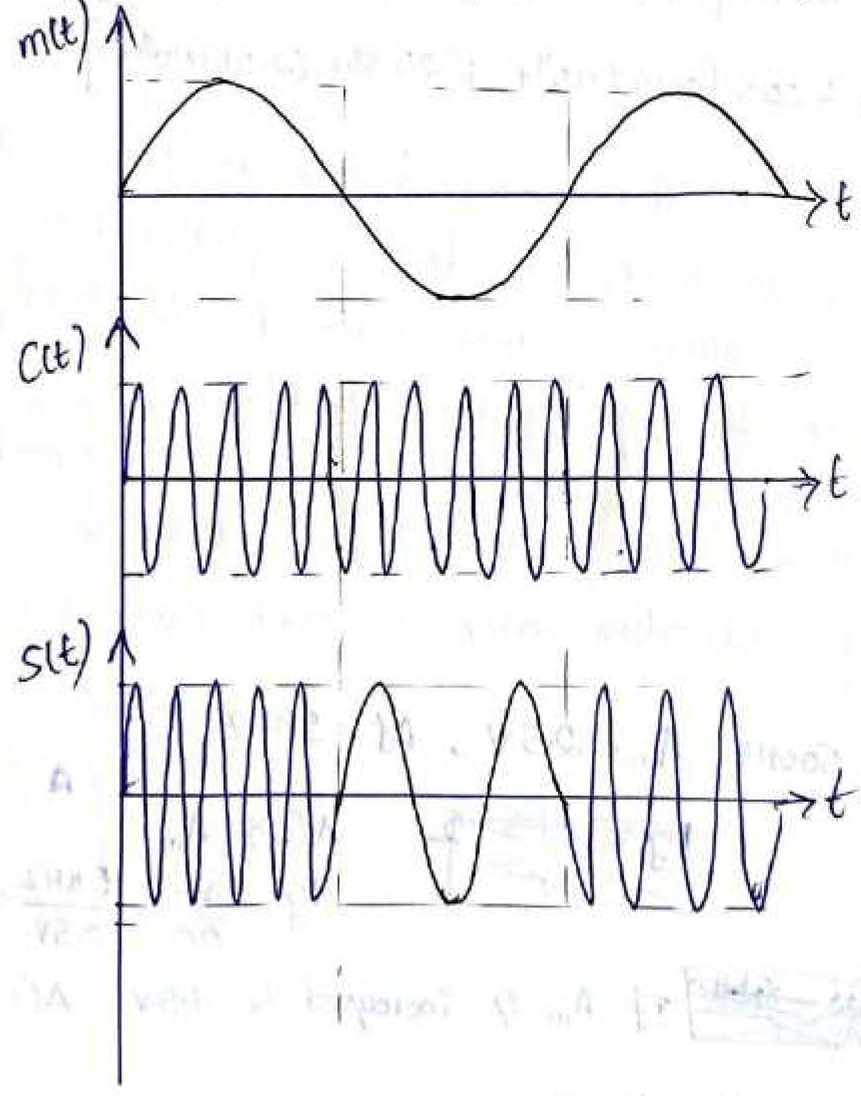
$\beta_2 = \frac{15 KHz}{500 Hz}$

$\boxed{\beta_2 = 30}$

$\beta_3 = \frac{20 KHz}{250 Hz}$

$\boxed{\beta_3 = 80}$

Graphical view of Frequency Modulation Signals in Time Domain



Deviation Ratio in FM

* while designing FM systems, it is necessary to know the what is maximum permissible value of modulation index.
* The maximum permissible value of modulation index is called as Deviation Ratio.
* $$\boxed{\text{Deviation Ratio} = \frac{\text{Maximum value of frequency Deviation}}{\text{Maximum value of modulating signal frequency}}}$$

For example, maximum frequency deviation permitted for F.M broadcast is ± 75 KHz and maximum modulating frequency = 15 KHz

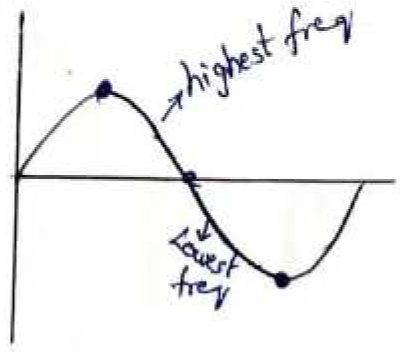
$\therefore$ Deviation Ratio $= \frac{75\text{ KHz}}{15\text{ KHz}} = 5.$

* Maximum frequency of FM signal is $(f_{max}) = f_c + \Delta f$
* The minimum frequency of FM signal is $(f_{min}) = f_c - \Delta f$

Carrier Swing:- The total variation in frequency from lowest possible frequency to the highest possible frequency is called Carrier Swing.

$$\boxed{f_{c.s} = 2\Delta f}$$

where Δf is the frequency deviation.



Problem

In a FM system, a 7KHZ message signal freq modulated 107.6MHZ carrier wave with the frequency deviation of 50KHZ find

(a) carrier swing in the FM signal and modulation index.

(b) Highest frequency and lowest frequency attained.

Sol

$f_m = 7KHZ$

$f_c = 107.6 MHZ$

$\Delta f = 50KHZ$

(a) Carrier swing $= 2\Delta f$

$= 2(50KHZ) = 100 KHZ$

Modulation Index $(\beta) = \frac{\Delta f}{f_m} = \frac{50KHZ}{7KHZ} = 7.1428$

(b) Highest Frequency $= f_c + \Delta f$

$= 107.6MHZ + 50 KHZ$

$= 107.65 MHZ$

Lowest Frequency $= f_c - \Delta f$

$= 107.6MHZ - 50K$

$= 107.55 MHZ$

Transmission Bandwidth of FM wave

It is stated carson's rule

↳ John Renshaw carson was an American transmission theorist for early communications systems.

He invented single side band modulation and develped the **carson B.W rule** for estimating frequency modulation (F.M) bandwidth.

* carson's rule stated that the bandwidth of FM wave is equal to twice that of maximum frequency deviation and maximum modulating frequency.

* $$B.W_{F.M} = 2(\Delta f + f_m)$$

$$= 2.\left[\Delta f\left[1+\frac{f_m}{\Delta f}\right]\right]$$

$$= 2\left[\Delta f\left[1+\frac{1}{\beta}\right]\right]$$

when $\beta >> 1$

$$\boxed{\therefore B.W_{WBFM} = 2\,\Delta f} \quad (\because \text{neglect } \tfrac{1}{\beta})$$

Similarly

~~when β~~

$$B.W_{F.M} = 2\left[f_m\left(\frac{\Delta f}{f_m}+1\right)\right]$$

$$= 2\left[f_m(\beta+1)\right]$$

when $\beta << 1$

$$\boxed{\therefore B.W_{NBFM} = 2\,f_m} \quad (\because \text{neglect } \beta)$$

Narrow Band Frequency Modulation (NBFM)

* If the modulation index $(\beta) < 1$, ~~then~~ that frequency modulation is called Narrow band Frequency Modulation (NBFM)

* The general mathematical equation of frequency modulation is

$$\boxed{s(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]} \longrightarrow (1)$$

$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$$s(t) = A_c \cos 2\pi f_c t \cdot \cos(\beta \sin 2\pi f_m t) - A_c \sin 2\pi f_c t \cdot \sin(\beta \sin 2\pi f_m t) \longrightarrow (2)$$

* Incase of NBFM, $\beta < 1$ rad

$\downarrow$ Let $\beta = 0$

$\therefore \cos(\beta \sin 2\pi f_m t) \simeq 1$

similarly $\sin(\beta \sin 2\pi f_m t) \simeq \beta \sin 2\pi f_m t$

$\sin(0) = 0$

$\cos(0) = 1$

therefore eq (2) can be modified as

$$s(t) = A_c \cos 2\pi f_c t - A_c \sin 2\pi f_c t \cdot [\beta \sin 2\pi f_m t]$$

$$\boxed{s(t) = A_c \cos 2\pi f_c t - \beta A_c \sin 2\pi f_c t \cdot \sin 2\pi f_m t} \longrightarrow (3)$$

$$2 \sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$$

$$s(t) = A_c \cos 2\pi f_c t - \frac{\beta A_c}{2}\left[\cos(2\pi(f_c - f_m)t) - \cos(2\pi(f_c + f_m)t)\right]$$

$$= A_c \cos 2\pi f_c t + \frac{A_c \beta}{2} \cos(2\pi(f_c + f_m))t - \frac{A_c \beta}{2} \cos(2\pi(f_c - f_m)t$$

$$\boxed{s(t)_{NBFM} = A_c \cos 2\pi f_c t + \frac{A_c \beta}{2}\left[\cos(2\pi(f_c + f_m)t - \cos(2\pi(f_c - f_m)t)\right]} \longrightarrow (4)$$

Expression for AM is

$$\boxed{s(t)_{AM} = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2}\left[\cos(2\pi(f_c + f_m)t + \cos(2\pi(f_c - f_m)t\right]} \longrightarrow (5)$$

* The basic difference between NBFM & AM is that the sign of the LSB frequency in NBFM is reversed.

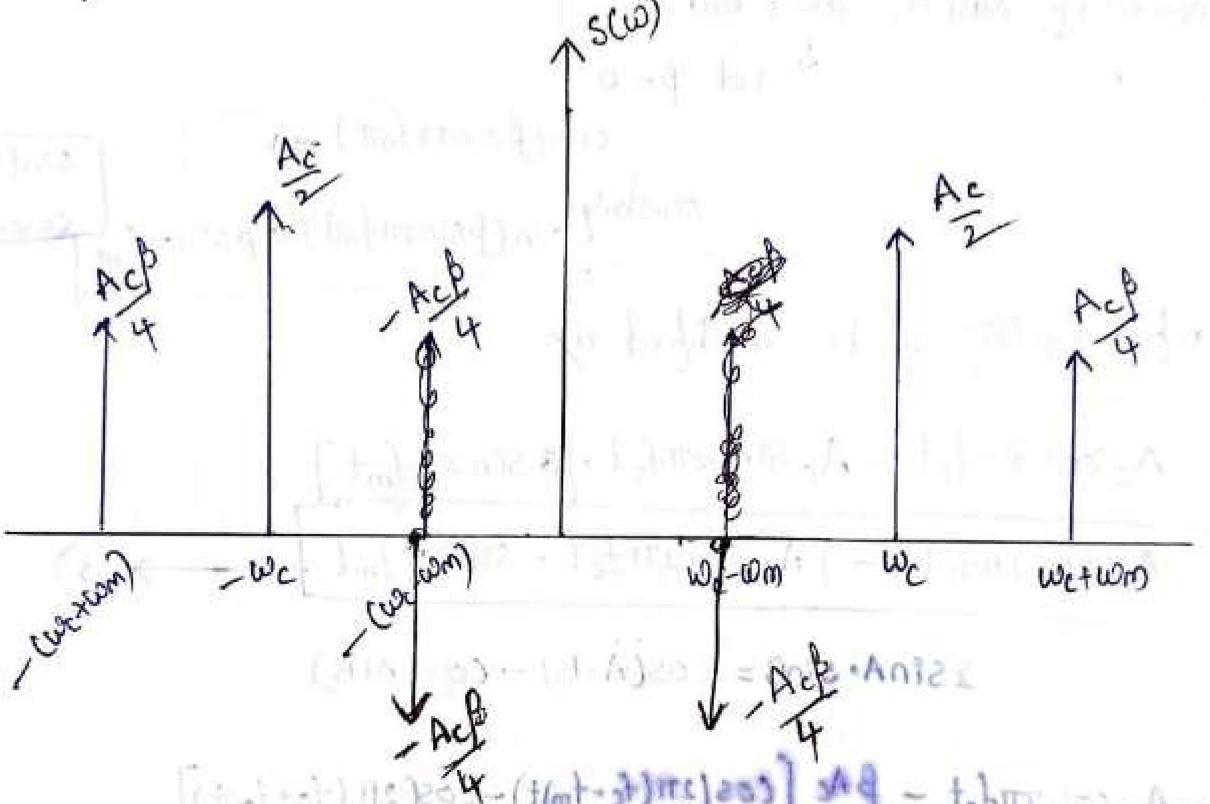
* Apply F.T on S(t)

$\therefore \cancel{S(f) = \frac{A_c}{2}\left[\delta(f-f_c)+\delta(\omega+\omega_c)\right] +}$

$$S(\omega) = \frac{A_c}{2}\left[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)\right] + \frac{A_c\beta}{4}\left[\delta(\omega-(\omega_c+\omega_m)) + \delta(\omega+(\omega_c+\omega_m))\right]$$

$$-\frac{A_c\beta}{4}\left[\delta(\omega-(\omega_c-\omega_m)) + \delta(\omega+(\omega_c-\omega_m))\right] \longrightarrow (6)$$

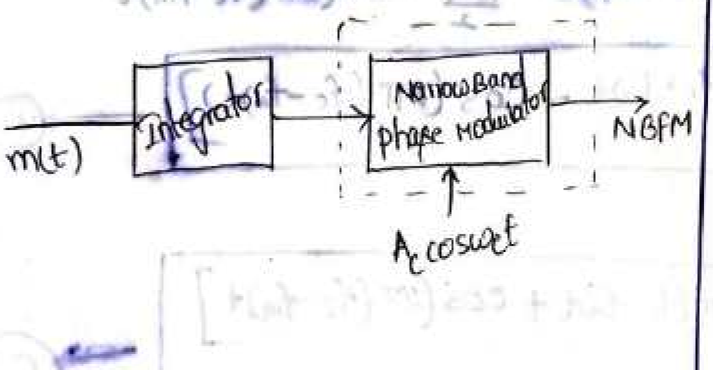
Frequency spectrum

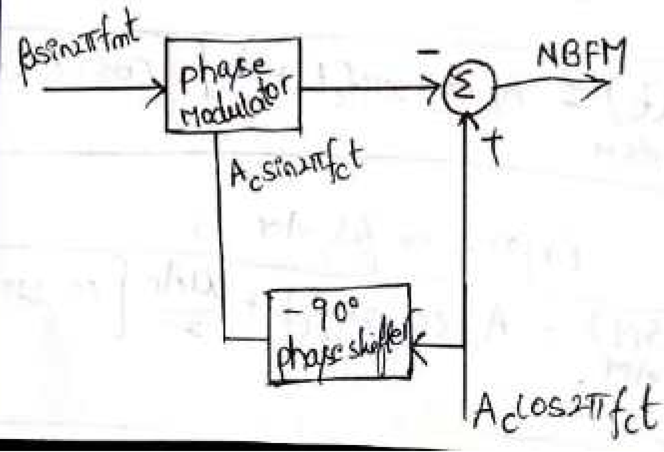


eq (3) is

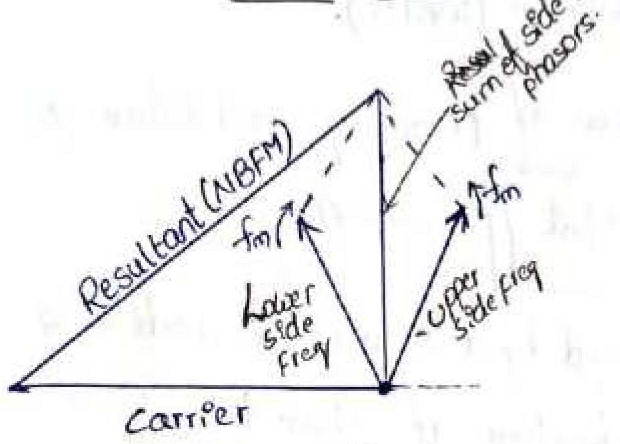
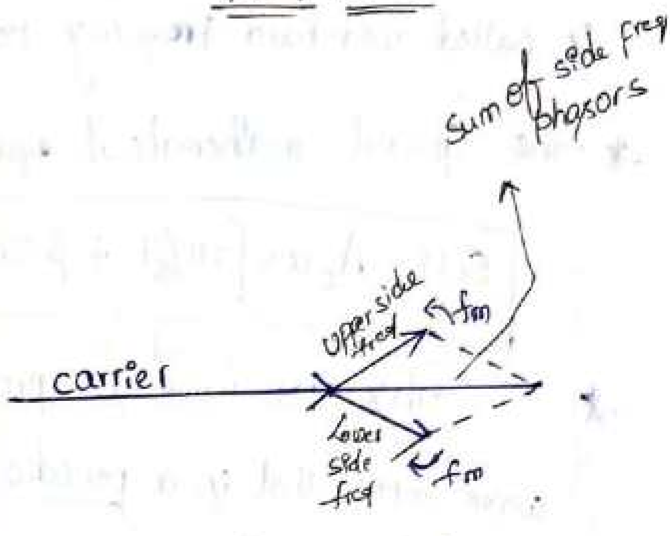
$$\boxed{S(t) = A_c\cos 2\pi f_c t - \beta A_c \sin 2\pi f_c t \cdot \sin 2\pi f_m t}$$

$\Downarrow$





Phasor comparision of NBFM & AM Waves

NBFM Wave	AM Wave
	
(1) carrier phasor as reference because carrier amplitude is constant in bot NBFM & AM (In eq (4) & (5))	(1) carrier phasor as reference.
(2) The upper side frequency and lower side frequency (sum of two side freq phasors) is always perpendular to the carrier reference phasor	(2) The resultant of the two side frequency phasors is always inphase with the carrier phase.
(3) The amplitude of the resultant phasor is approximately same as the carrier phasor amplitude, but always out of phase w.r.t reference.	(3) The resultant phasor represent the corresponding AM wave that has a different amplitude from that of the carrier phasor but always inphase w.r.t reference.
(4) The resultant phasor is 90° with the carrier, and the upper side freq and lower side freq i.e fm makes some angle w.r.t carrier phasor and side band freq phasor rotates in the direction of increasing angle w.r.t carrier.	(4) The resultant phasor is always inphase w.r.t carrier (reference) and side band frequency phasor rotates in the direction of increasing angle i.e made w.r.t carrier.

Wide Band Frequency Modulation (WBFM)

* If the modulation index (β) > 1, then that frequency modulation is called wideband Frequency Modulation (WBFM).

* the General mathematical equation of Frequency modulation is

$$\boxed{S_{FM}(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right]} \longrightarrow (1)$$

* this FM wave is produced by a sinusoidal modulating wave m(t), that is a periodic function of time 't' only when $f_c = n f_m$ (i.e integral multiple of modulating frequency)

* Here f_c is large enough compared to bandwidth of FM wave.

* To simplify matters, we use complex representation of band pass signals

$$e^{j\theta} = \underbrace{\cos\theta}_{\text{Real part}} + \underbrace{j\sin\theta}_{\text{imaginary part}}$$

Here we are considered as real parts

$$S_{FM}(t) = A_c \text{Real}\left\{e^{j(2\pi f_c t + \beta \sin 2\pi f_m t)}\right\}$$

Time domain representation of WBFM interms of exponential

$$\boxed{S_{FM}(t) = A_c \text{Real}\left\{e^{j2\pi f_c t} \cdot e^{j\beta \sin 2\pi f_m t}\right\}} \longrightarrow (2)$$

$e^{j\beta \sin 2\pi f_m t}$ → this term is a periodic with a period $(T) = \frac{1}{f_m}$

$$\boxed{\therefore g(t) = e^{j\beta \sin 2\pi f_m t}} \longrightarrow (3)$$

$\omega_0 = 2\pi f_m$

$T = \frac{1}{f_m}$

* g(t) is represented in complex Fourier series

$$g(t) = e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} C_n e^{j2\pi f_m n t} \longrightarrow (4)$$

where

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} g(t)\, e^{-jn\omega_0 t}\, dt$$

$$\boxed{c_n = \frac{1}{1/f_m}\int_{-1/2f_m}^{1/2f_m} e^{j\beta\sin 2\pi f_m t}\cdot e^{-j2\pi f_m n t}\, dt} \longrightarrow (5)$$

if $2\pi f_m t = \theta \Rightarrow 2\pi f_m dt = d\theta$

$dt = \frac{d\theta}{2\pi f_m}$

$$\therefore c_n = f_m\int_{-\pi}^{\pi} e^{j\beta\sin\theta}\cdot e^{-jn\theta}\cdot\frac{d\theta}{2\pi f_m}$$

if $t = \frac{-1}{2f_m}$, then $\theta = -\pi$

$2\pi f_m\left(\frac{-1}{2f_m}\right) = -\pi$

if $t = \frac{1}{2f_m}$, then $\theta = \pi$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\beta\sin\theta}\cdot e^{-jn\theta}\, d\theta$$

$$\boxed{c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta\sin\theta - n\theta)}\, d\theta} \longrightarrow (6)$$

eq (6) is a $J_n(\beta) \Rightarrow$ Bessel function of 1st kind of nth order and of argument β

$\therefore$ eq (4) can be modified as

$$g(t) = e^{j\beta\sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} c_n\, e^{j2\pi f_m n t}$$

$$\boxed{g(t) = e^{j\beta\sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi f_m n t}} \longrightarrow (7)$$

eq (7) is substituted in eq (2)

$$s_{WBFM}(t) = A_c\,\mathrm{Real}\left[\sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi f_m n t}\cdot e^{j2\pi f_c t}\right]$$

$$= A_c\,\mathrm{Real}\left[\sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi(f_c + nf_m)t}\right]$$

$$\boxed{s_{WBFM}(t) = A_c\sum_{n=-\infty}^{\infty} J_n(\beta)\cos\left[2\pi(f_c + nf_m)t\right]} \longrightarrow (8)$$

$\therefore$ eq (8) is a general expression for single-tone W.B. F.M.

Apply fourier transform on $S_{WBFM}(t)$

$$\boxed{S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f-(f_c+nf_m)) + \delta(f+(f_c+nf_m)) \right]} \rightarrow (9)$$

$$\boxed{S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f-f_c-nf_m) + \delta(f+f_c+nf_m) \right]} \rightarrow (10)$$

It shows contains a infinite number of delta functions Spaced at $f = f_c \pm nf_m$ for $n = 0, \pm 1, \pm 2 \ldots$

* FM signal is composed of carrier with an amplitude $\frac{A_c}{2} J_0(\beta)$ and a set of side frequency spaced symmetrically on either side of the carrier at a freq separation of $f_m, 2f_m, 3f_m \ldots$ etc.

Bessel function

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin\theta - n\theta)} \, d\theta.$$

Properties of Bessel functions

1. $J_n(\beta)$ decreases when 'n' increases

$$J_0(\beta) > J_1(\beta) > J_2(\beta) \ldots$$

2. $J_n(\beta) = (-1)^n J_n(\beta)$

$$J_{-n}(\beta) = J_n(\beta) : \text{when } n \text{ is even}$$
$$J_{-n}(\beta) = -J_n(\beta) : \text{when } n \text{ is odd.}$$

3. $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$

4. $J_n(\beta)$ is always results in real quantity

5. For small values of β

$$J_0(\beta) \approx 1$$
$$J_1(\beta) \approx \frac{\beta}{2}$$
$$J_n(\beta) \approx 0 \text{ for } n > 1$$

$$\boxed{J_n(\beta) = \frac{\beta^n}{2^n n!}}$$

eq (8) is

$$S_{WBFM}(t) = A_C \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + nf_m)t]$$

Using approximations for Bessel function $J_n(\beta)$

$$S_{WBFM}(t) = A_C \cos 2\pi f_c t + A_C \cdot \overset{J_1(\beta)}{\frac{\beta}{2}} \cos(2\pi(f_c + f_m)t) + A_C \cdot J_{-1}(\beta) \cos(2\pi(f_c - f_m)$$

$$= A_C \cos 2\pi f_c t + \frac{A_C \beta}{2} \cos(2\pi(f_c + f_m)t - \frac{A_C \beta}{2} \cos(2\pi(f_c - f_m)t$$

$$\boxed{S_{WBFM}(t) = A_C \cos 2\pi f_c t + \frac{\beta A_c}{2} \left[\cos(2\pi(f_c + f_m)t - \cos(2\pi(f_c - f_m)t\right]}$$

The above equation is also same as NBFM, only difference is $\beta < 1$; it is a NBFM, if $\beta > 1$, then it is a WBFM.

Graphical Representation of Bessel function is.

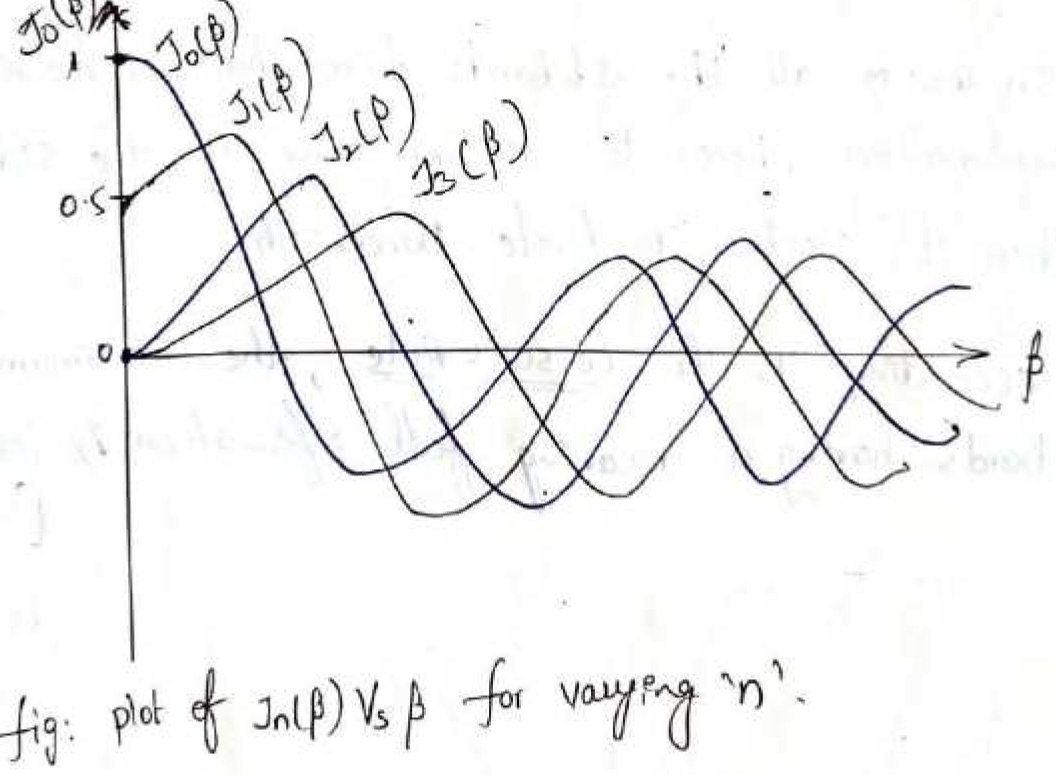
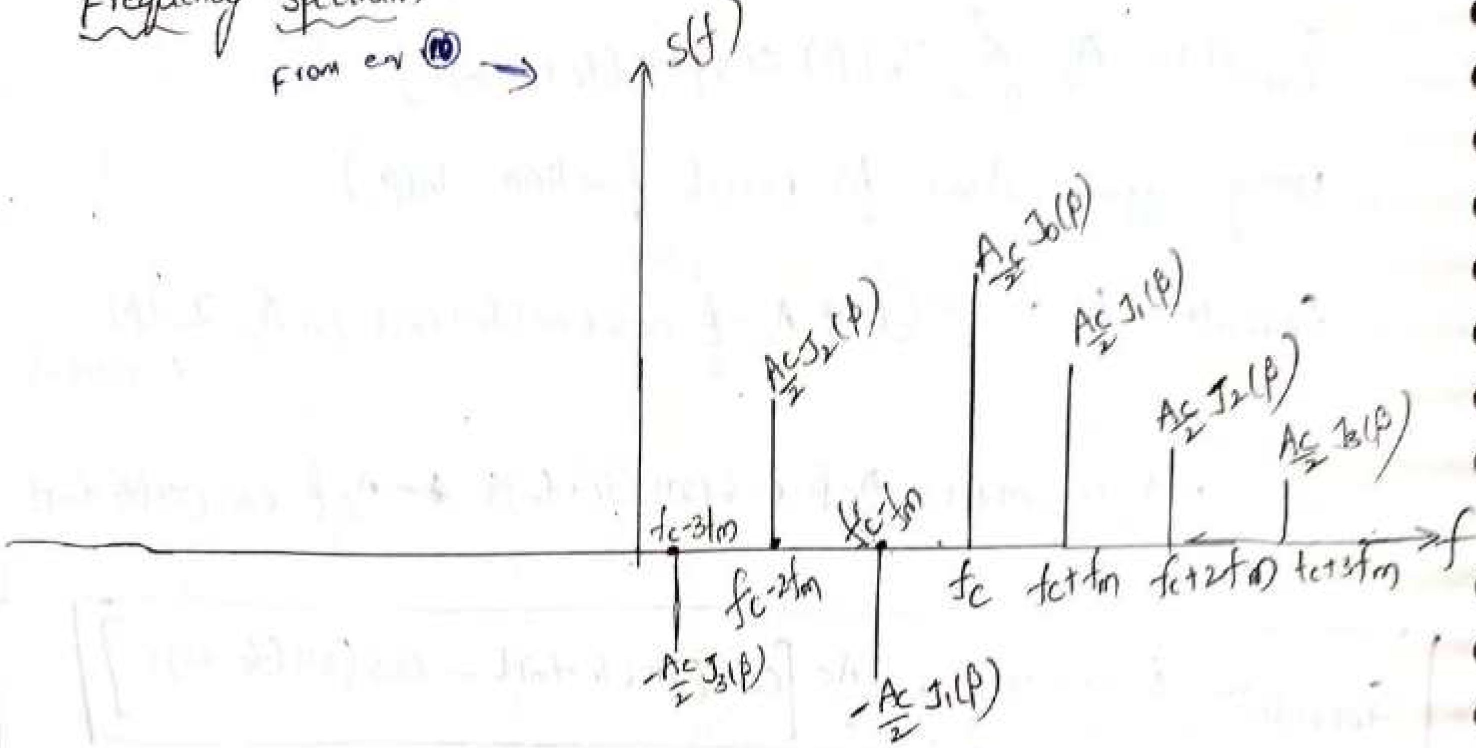


fig: plot of $J_n(\beta)$ Vs β for varying 'n'.

Frequency spectrum

From eq (10) →



* In WBFM, the Bandwidth is infinite, i.e for one U.S.B, one L.S.B having B.W is $2f_m$, similarly two U.SB, two L.S.B having a B.W is $4f_m$.

* therefore the Wide band F.M bandwidth is '∞'

* In WBFM, all the sidebands do not have a meaning full information, hence to discard some of the sidebands, then it produce a finite bandwidth.

* According to the Carson's Rule, the maximum side band's having a meaning full information is $[\beta+1]$ sidebands.

ie $[\beta+1]\,2f_m$

Transmission Bandwidth of FM wave

In theory, the F.M wave contains an infinite number of side frequencies, so Bandwidth is required to transmit F.M is approximately "∞". (infinite)

In practice, the F.M wave is effectively limited to a finite number of significant side frequencies with a specified amount of distortion.

Carson's Rule stated that the bandwidth of F.M wave is equal to twice that of maximum frequency deviation and maximum modulating frequency.

* B.W of WBFM is limited by considering only the lower freq significant S.B's & rejecting higher freq insignificant S.B's.

$$B_T = 2(\Delta f + f_m)$$

$$= 2\Delta f + 2f_m$$

$$= 2\Delta f\left(1 + \frac{f_m}{\Delta f}\right)$$

$$B_T = 2\Delta f\left(1 + \frac{1}{\beta}\right)$$

$$B.W = (\beta+1)2f_m$$

$$= \left(\frac{\Delta f}{f_m}+1\right)2f_m$$

$$= (\Delta f + f_m)2$$

$$\boxed{B.W = 2(\Delta f + f_m)}$$

case (1)

If $\beta >> 1$

$$\boxed{\therefore B_{T(WBFM)} = 2\Delta f}$$

similarly

$$B_T = 2f_m\left(\frac{\Delta f}{f_m}+1\right)$$

$$B_T = 2f_m(\beta+1)$$

case (2)

If $\beta << 1$

$$\boxed{\therefore B_{T(NBFM)} = 2f_m}$$

~~Problem~~

Transmitted power in NBFM

$$P_t = P_c + P_{U.SB} + P_{L.SB}$$

$$P_c = \frac{V_{r.m.s}^2}{R} = \frac{\left(\frac{A_c}{\sqrt{2}}\right)^2}{R} = \frac{A_c^2}{2R}$$

$$P_{U.SB} = \frac{\left(\frac{\frac{\beta A_c}{2}}{\sqrt{2}}\right)^2}{R} = \frac{A_c^2 \beta^2}{8R}.$$

$$P_{L.S.B} = \frac{\left(\frac{-\frac{\beta A_c}{2}}{\sqrt{2}}\right)^2}{R} = \frac{A_c^2 \beta^2}{8R}$$

$$\therefore P_t = \frac{A_c^2}{2R} + \frac{A_c^2\beta^2}{8R} + \frac{A_c^2\beta^2}{8R}$$

$$P_t = \frac{A_c^2}{2R} + \frac{A_c^2\beta^2}{4R}$$

$$P_t = \frac{A_c^2}{2R}\left[1 + \frac{\beta^2}{2}\right]$$

$$\boxed{\therefore P_t = P_c\left[1 + \frac{\beta^2}{2}\right]}$$

Transmitted power in WBFM

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + nf_m)t)$$

Average power of an WBFM may also be determined by

$$P = \frac{1}{2} A_c^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

$$\therefore P = \frac{1}{2} A_c^2 (1)$$

$$\boxed{\therefore P = \frac{1}{2} A_c^2}$$

$$P = \frac{V_{r.m.s}^2}{R}$$

$$V_{r.m.s} = \frac{A_c \sum_{n=-\infty}^{\infty} J_n(\beta)}{\sqrt{2}}$$

Bessels property

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

$$P = \frac{A_c^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta)}{2R}$$

$R = 1\,\Omega$

$$P = \frac{A_c^2}{2} \cdot \sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

$$P = \frac{A_c^2}{2}(1)$$

$$\boxed{P = \frac{A_c^2}{2}}$$

problem

A Frequency modulated wave $s(t) = 20\cos(8\pi 10^6 t + 9\sin(2\pi \times 10^3 t))$

Calculate frequency deviation, Bandwidth and power value.

sol

$s(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$

Here $\beta = 9 \Rightarrow$ W.B.FM

$A_c = 20$

$f_m = 10^3$

$\beta = 9$

$\therefore$ Frequency deviation $(\Delta f) = ?$

$\beta = \frac{\Delta f}{f_m}$

$\Delta f = \beta \cdot f_m \Rightarrow (9)(10^3)$.

Bandwidth = ?

$B_T(WBFM) = 2\Delta f$

$= 2(9 \times 10^3)$

$\boxed{B_T = 18 \times 10^3 \text{ Hz}}$

power $= \frac{A_c^2}{2} = \frac{(20)^2}{2} = \frac{400}{2} = 200$

According to carson's Rule

$B.W = 2\Delta f\left[1 + \frac{1}{\beta}\right] \Rightarrow 2\Delta f\left[1 + \frac{1}{9}\right]$

$\Rightarrow 2(9 \times 10^3)[1.11]$

$\boxed{B.W = 20 \text{ KHz}}$

2) Given the angle modulated signal

$s(t) = 10\cos[2\pi 10^8 t + 200\sin 2\pi 10^3 t]$ determine

a) carrier frequency (f_c), modulating frequency (f_m), modulation index (β)

maximum deviation (Δf), and the B.W

b) what power will this FM wave dissipate in 1Ω resistor

sol $s(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$

(a) $\boxed{f_c = 10^8 \text{ Hz}}$

$\boxed{f_m = 10^3 \text{ Hz}}$

$\beta = \frac{\Delta f}{f_m}$

Here $\Delta f = \beta \cdot f_m$

$= (200)(10^3)$

$\boxed{\Delta f = 2 \times 10^5 \text{ Hz}}$

$\therefore \boxed{\beta = \frac{2 \times 10^5}{10^3} = 200}$

Here $\beta = 200$
i.e $\beta > 1$

$B.W = 2\Delta f$

WBFM $= 2(2 \times 10^5) = \boxed{4 \times 10^5 \text{ Hz}}$

(b) power dissipated $P = \frac{A_c^2}{2R}$

$= \frac{(10)^2}{2(1)}$

$= \frac{100}{2}$

$\boxed{P = 50W}$

3) A 100MHz carrier has a peak voltage of 5V, the carrier is frequency modulated by a sinusoidal message signal of frequency 2KHz and the frequency deviation is 75KHz. write the expression for the modulated carrier signal.

Sol

$f_c = 100MHz$
$f_c = 100 \times 10^6$
$f_c = 10^8 Hz$

$A_c = 5V$
$f_m = 2KHz$
$= 2 \times 10^3 Hz$

$\Delta f = 75KHz$
$\Delta f = 75 \times 10^3 Hz$

$$\beta = \frac{\Delta f}{f_m} = \frac{75}{2} = 37.5$$

We know that

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

$$\boxed{s(t) = 5\cos[2\pi \times 10^8 t + 37.5 \sin 4\pi \times 10^3 t]}$$

4) Assume the maximum value of frequency deviation is 50KHz for a certain FM transmission given that maximum modulating frequency is 15KHz then Calculate the transmission Bandwidth.

$\Delta f = 50KHz$
$f_m = 15KHz$

$$\beta = \frac{\Delta f}{f_m} = \frac{50KHz}{15KHz} = 3.33$$

$\beta > 1$ (WBFM)

$$\therefore B.W = 2(\Delta f + 2f_m)$$
$$= 2\Delta f\left(1 + \frac{f_m}{\Delta f}\right)$$

$$B.W = 2\Delta f\left[1 + \frac{1}{\beta}\right] \Rightarrow 2 \times 50\left[1 + \frac{1}{3.33}\right] \Rightarrow 100[1.30]$$

$$\boxed{B.W \Rightarrow 130KHz}$$

Here $\beta > 1$ ($\because 1/\beta$ neglected)

$$\therefore B.W = 2\Delta f$$

$$\boxed{B.W = 2(50K) = 100KHz}$$

Comparision between Narrow Band Frequency modulation and wide band Frequency modulation.

S.NO	Parameter	Narrow Band Frequency modulation (NBFM)	Wide Band Frequency modulation (WBFM)
1.	Modulation Index	~~greater~~ less than "1" (or) slightly greater than "1"	greater than "1" ($\beta >> 1$)
2.	Frequency Deviation	5 KHZ	75 KHZ (15 times of N.B.FM)
3.	Range of modulating signal frequency	30 HZ - 3 KHZ	30 HZ - 15 KHZ
4.	Bandwidth	$2f_m$ (same as A.M)	$30f_m$ (15 times of A.M)
5.	Maximum modulation Index	slightly greater than "1"	5 to 2,500
6.	Applications	It is suitable for voice communications like walkitalkie, mobile, Telemedicine, F.M communication (Radio), Ambulance services.	It is suitable for entertainment broadcasting, music [can be used for high quality music transmission] (single Transmitter, many Receivers)
7.	Noise	Low	High
8.	Spectrum	Carrier + U.S.B + L.S.B	Carrier + "n" Side Bands.

Generation of Frequency Modulation

there are two methods.

1. Direct Method — (a) Reactance modulator; (b) Varactor diode modulator.
2. Indirect Method (or) Armstrong Method.

1. Direct Method :- (a) (Reactance modulator) In this method, the carrier signal is constant frequency signal, this carrier signal is generated with the help of tank circuit i.e LC oscillator circuit.

- The FM wave is generated by using VCO, An oscillator circuit whose frequency is directly controlled by applying a modulating voltage is called voltage controlled oscillator.
- A VCO is basically a sinusoidal oscillator, having a capacitor, used as a reactive element.
- In this reactance modulator, Hartley oscillator is used for the generation of F.M.

The frequency of the oscillator is changed by incremented variation in the reactive components involved in the tuned circuits.

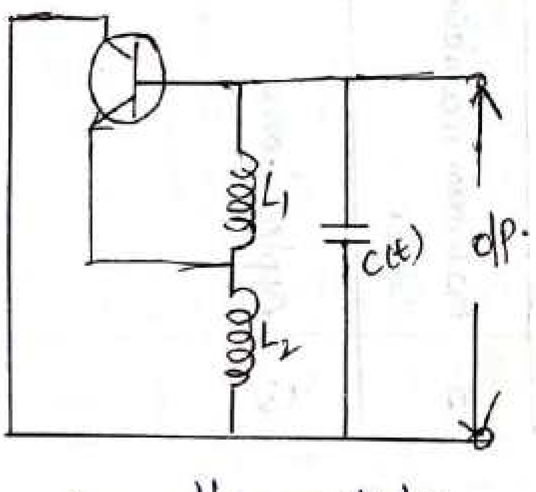
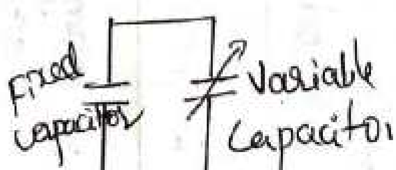


fig: Hartley oscillator.

Where C(t) is instantaneous value of the capacitance.

C(t) = Fixed capacitor + Variable capacitor



frequency of oscillation

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C(t)}}$$

where

$C_0 \rightarrow$ total capacitance in the absence of modulation

$\Delta C \rightarrow$ max change in capacitance value after modulation.

$K_s \rightarrow$ constant sensitivity of reactance modulator

Here

$$C(t) = \cancel{C_0 + \Delta C \cos(2\pi f_m t)} \quad C - k_s m(t)$$

$$\therefore f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)(C-K_s m(t))}}$$

$$= \frac{1}{2\pi\sqrt{(L_1+L_2)\,C\left[1-\frac{K_s}{C}m(t)\right]}}$$

$$= \frac{1}{2\pi\sqrt{(L_1+L_2)\,C}} \cdot \frac{1}{\sqrt{1-\frac{K_s}{C}m(t)}}$$

where

$$f_0 = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C}}$$

$$= f_0 \cdot \left[1-\frac{K_s}{C}m(t)\right]^{-1/2}$$

$$f_i(t) = f_0\left[1+\frac{K_s}{2C}m(t)\right]$$

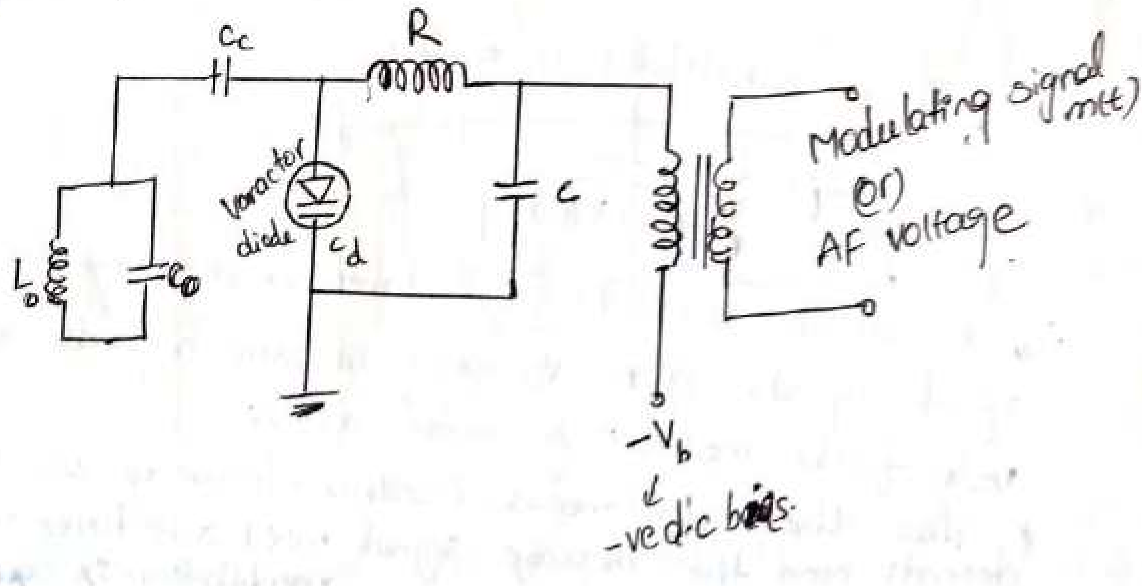
where

$$K_f = \frac{K_s}{2C}$$

$$\boxed{f_i(t) = f_0\left[1+K_f\, m(t)\right]}$$

$\therefore$ the output frequency is directly varied in accordance with the message signal m(t) with the help of oscillator.

(b) Varactor diode modulator:-



* Here AF voltage $m(t)$ is supplying input voltage to the varactor diode with the help of transformer.
* This input voltage is added to this negative d.c bias, therefore the varactor diode is reverse biased by supplying negative d.c bias connected to the positive terminal of varactor diode.
* If this varactor diode is reverse biased, then only it works as a variable capacitor.
* The varactor diode is a semiconductor diode, whose junction capacitance is varies linearly w.r.t applied voltage.

Mathematical Analysis

$$C_d = \frac{K}{\sqrt{V_D}}$$

$$C_d = K.(V_D)^{-1/2} \longrightarrow (1)$$

where
$C_d \rightarrow$ varactor diode capacitance
$V_D \rightarrow$ Varactor diode
$K \rightarrow$ constant.

where

$$V_D = V_0 + m(t)$$

$\therefore$ The oscillation frequency

$$\omega_i = \frac{1}{\sqrt{LC}}$$

where $L = L_0$
$C = C_0 + C_d$

$$\omega_i = \frac{1}{\sqrt{L_0(C_0+C_d)}} \longrightarrow (2)$$

eq (1) is substituted in eq (2)

$$\boxed{\omega_i = \frac{1}{\sqrt{L_0[C_0+K.(V_D)^{-1/2}]}}}$$

$\therefore$ we conclude that the instantaneous frequency ω_i in F.M signal depends upon V_D which in turn depends upon the value of the modulating signal $m(t)$.

* Thus the instantaneous oscillator frequency ω_i also depends upon the message signal $m(t)$ and hence frequency modulation is generated.

Advantages of Direct method

1. circuit is easy
2. cheap.

Disadvantages

1. LC oscillator is not stable. i.e. FM broadcasting cannot be done.
2. Distortion present due to non-linear varactor diode.
   (Varactor diode is nonlinear, but changes in the capacitance is linear. and also voltage and current relations are non-linear in varactor diode, because of that there is large distortion in FM signal).

Application

* It is used in high power FM transmitter. But most of the other practical applications, we use Indirect Method

2. Indirect method :-

* It is also called Armstrong modulator
* Message signal is first used to produce a NBFM, it is followed by frequency multiplication to increase the frequency deviation to the desired level, then it will generate the WBFM, hence it is called Indirect method of WBFM generation.
* The block diagram of Indirect method is

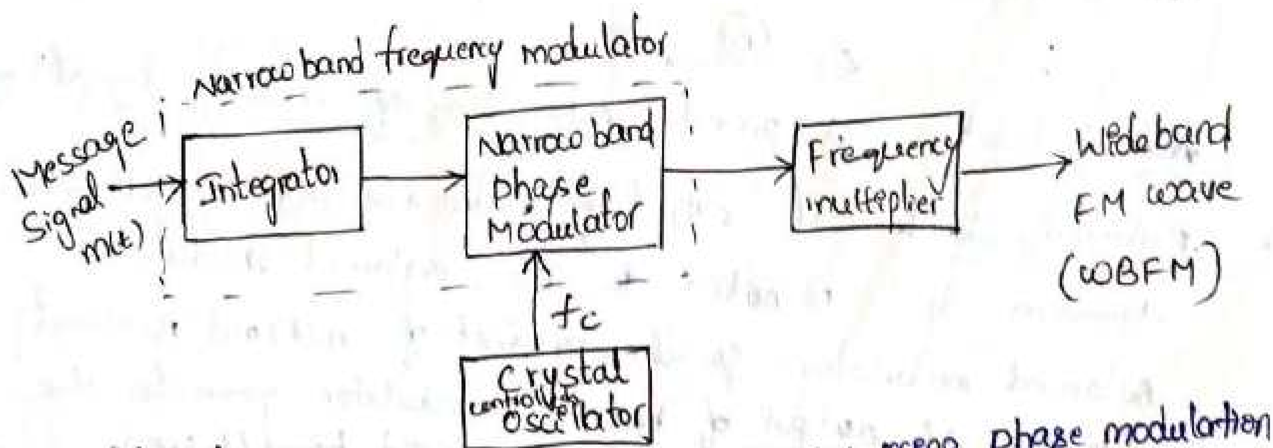
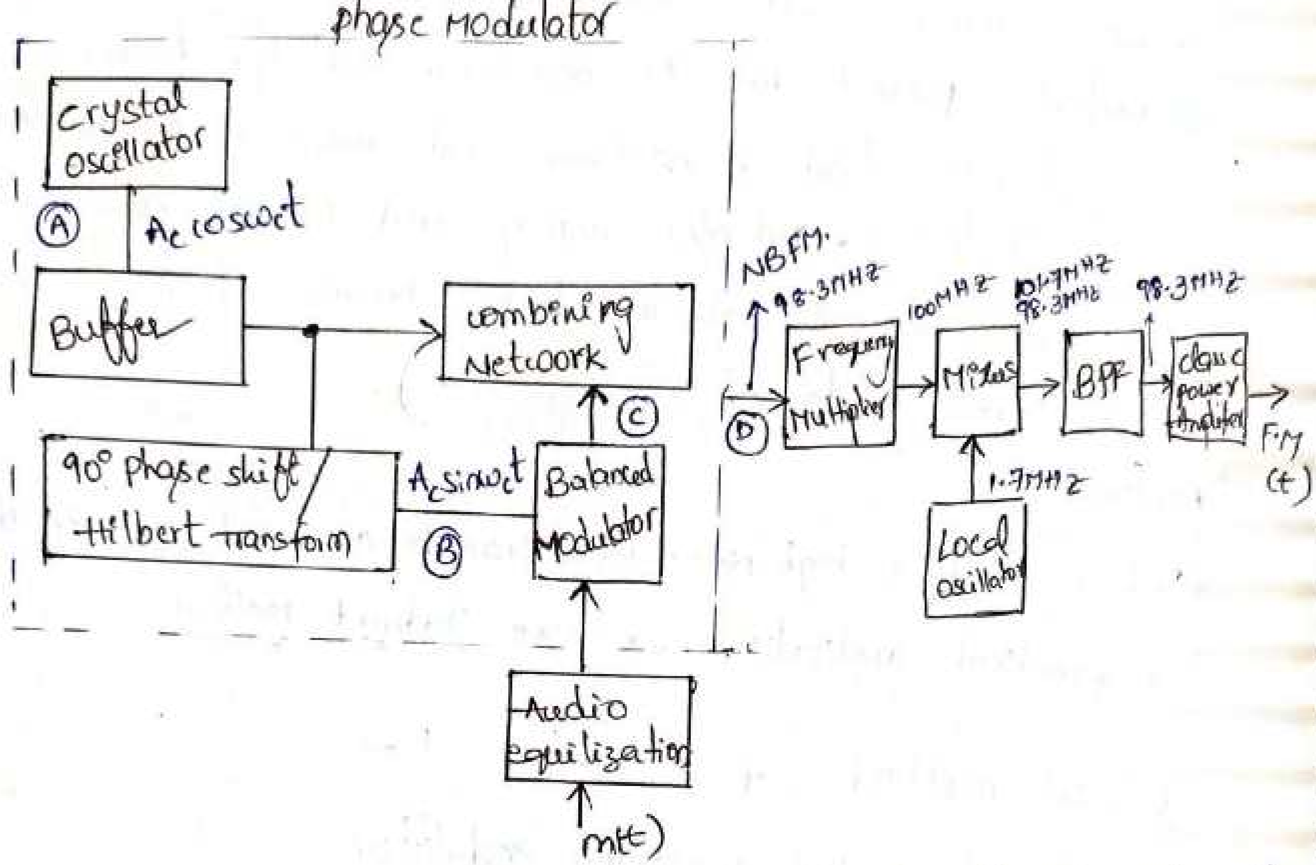


fig: Indirect method of generating a WBFM using phase modulation technique.

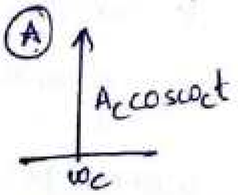
* The carrier frequency stability problem is solved by using highly stable oscillator i.e crystal oscillator in NBFM generation

Armstrong Transmitter



Crystal oscillator:- It is a stable oscillator, it generates the carrier signal

unmodulated carrier phasor diagram is



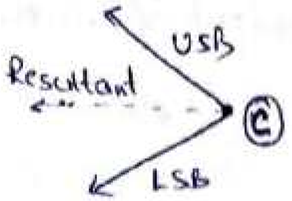
Buffer:- It holds the carrier signal at a certain period of time.

90° phase shift:- It generates 90° phase shift for the carrier signal. therefore output is $A_c \sin\omega_c t$. Its phasor diagram is

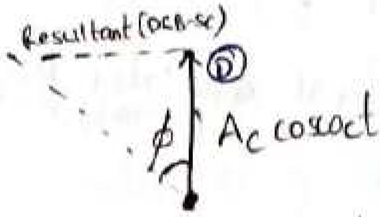
Audio equilization:- It provides integration of message signal m(t)

Balanced modulator:- the output of audio equilization and Hilbert transform output connected to the Balanced modulator. Balanced modulator is the product of m(t) and $A_c \sin\omega_c t$ therefore the output of balanced modulator generates the upper side band (U.S.B) and Lower side band (L.S.B).

therefore the phasor diagram is



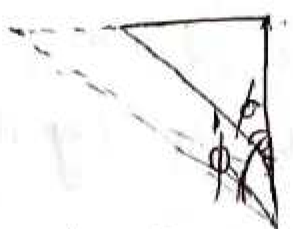
- Combining Network :- the output of balanced modulator and $A_c \cos\omega_c t$ both are combined together. therefore the phasor diagram is



the Amplitude of U.S.B and L.S.B $\propto \frac{\mu A_c}{2}$

$$\boxed{\mu = \frac{A_m}{A_c}}$$

If A_m increases, then μ increases, therefore amplitude of U.S.B and LSB also increases. Therefore the phasor diagram is



$\therefore$ the ϕ changes according to the amplitude of message signal $m(t)$.

* If ϕ increases, then A_m increases, then $m(t)$ increases.

Frequency multiplier:-

* If the input is $A_c \cos\omega_c t$, then the output of frequency multiplier is $(A_c^2 \cos^2\omega_c t) \Rightarrow A_c^2 \left[\frac{1+\cos 2\omega_c t}{2}\right]$. therefore the frequency multiplier performs frequency doublers and triplers.

* If we are using series of frequency multipliers, then we can produce the desired output.

Mixer:- output of frequency multiplier (f_c) and local oscillator frequency (f_L) is applied to the mixer, therefore the output of mixer generates the addition of two frequencies $(f_c + f_L)$ and subtraction of two signals $(f_c - f_L)$.

For example:-

* F.M signal frequency is 98.3MHz (input of frequency multiplier)
* By using frequency multiplier, we may generate 100MHz signal.
* Now, we are suppling local oscillator (1.7MHz) frequency according to this 100MHz signal.

$\therefore$ output of mixer = 100MHz + 1.7MHz
= 101.7MHz

and 100MHz − 1.7MHz
= 98.3MHz

BPF:- The output of mixer passed through BPF, this BPF allows 98.3MHz only. therefore the output is 98.3MHz.

class 'c' power amplifier :- It is used to amplify the received signal.

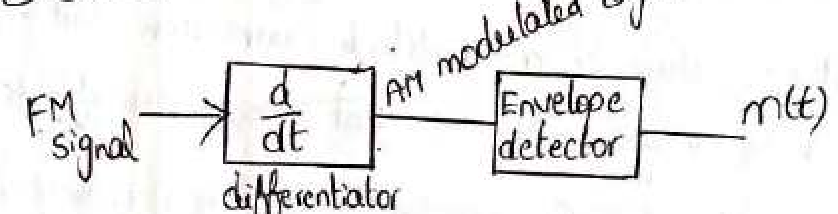
Detection of FM waves

It is used to extract the message signal from modulated signal is called demodulation of FM signal.

There are two types.

1. Frequency discrimination method
   (a) simple slope detection
   (b) Balanced slope detection
2. Phase discrimination method.
   (a) Foster seeley detection
   (b) Ratio Detector
   (c) phase Locked Loop.
   (d) Zero crossing detector.

FM Demodulation:



F.M signal

$$S_{FM}(t) = A_c \cos \omega_c t + \phi(t) \longrightarrow (1)$$

where $\phi(t) = K_f \int m(t) dt$.

$$S_{FM}(t) = A_c \cos \omega_c t + [K_f \int m(t) dt]$$

eq(1) is passed through differentiator

$$\therefore \frac{d}{dt} S_{FM}(t) = -A_c \sin(\omega_c t + \phi(t))\left(\omega_c + \frac{d}{dt}\phi(t)\right)$$

$$= A_c \sin(\omega_c t + \phi(t) + \pi)\left(\omega_c + \frac{d}{dt}\phi(t)\right)$$

$$= A_c \sin(\omega_c t + \phi(t) + \pi)(\omega_c + K_f m(t))$$

$$\boxed{\frac{d}{dt} S_{F.M}(t) = A_c \omega_c \left[1 + \frac{K_f}{\omega_c} m(t)\right] \sin(\omega_c t + \phi(t) + \pi)} \longrightarrow (2)$$

$\phi(t) = K_f \int m(t) dt$

$\frac{d}{dt}\phi(t) = K_f \frac{d}{dt} \int m(t) dt$

$\frac{d}{dt}\phi(t) = K_f\, m(t)$

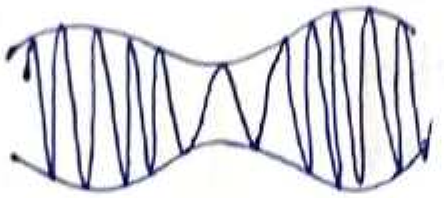
eq(1) is look like Amplitude modulated signal

$$\boxed{AM = A_c\{1+\mu m(t)\}\cos\omega_c t}$$

∴ eq(1) is passed through envelope detector, o/p of envelope detector is

envelope detector o/p = $A_c\omega_c + k_f m(t)$ ⟶ (3)

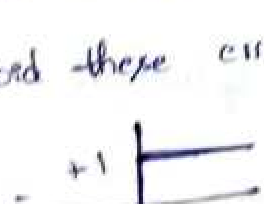
In eq(1), there is a amplitude variation and frequency (or) phase variation.



∴ this waveform is passed through envelope detector, therefore it detects the envelope i.e message signal.

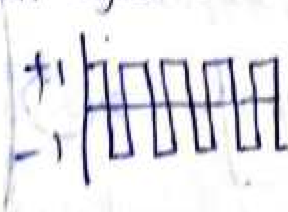
Limitations of FM Demodulation:

* In FM Demodulation, there is a amplitude variation and frequency variation. Amplitude is varying, it means that noise is affecting more.
* therefore, errors present at the receiver, envelope detector is also not able to detect the two signals.
* To avoid these errors we use limiters

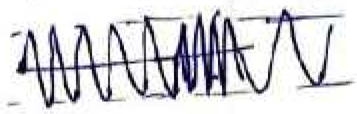


* FM signal is first passed through limiter,
  ↳ If amplitude goes above, then it is +1
  If amplitude goes below then it is -1

* therefore FM signal converted into pulses

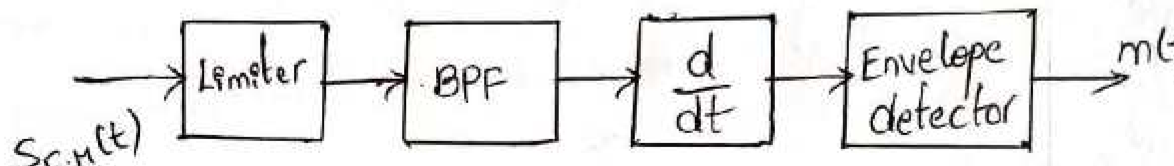


* we need to convert these pulses into sinusoidal-form, pulses contains Various harmonic components (frequency components). i.e $\omega_c, 2\omega_c, 3\omega_c$ - - -
* so that these pulses passed through BPF at ω_c, therefore the o/p is



* Above fig, amplitude is constant and frequency is varying w.r.t m(t)

∴ The modified receiver would be.



Frequency discrimination method

(a) Simple slope detection:

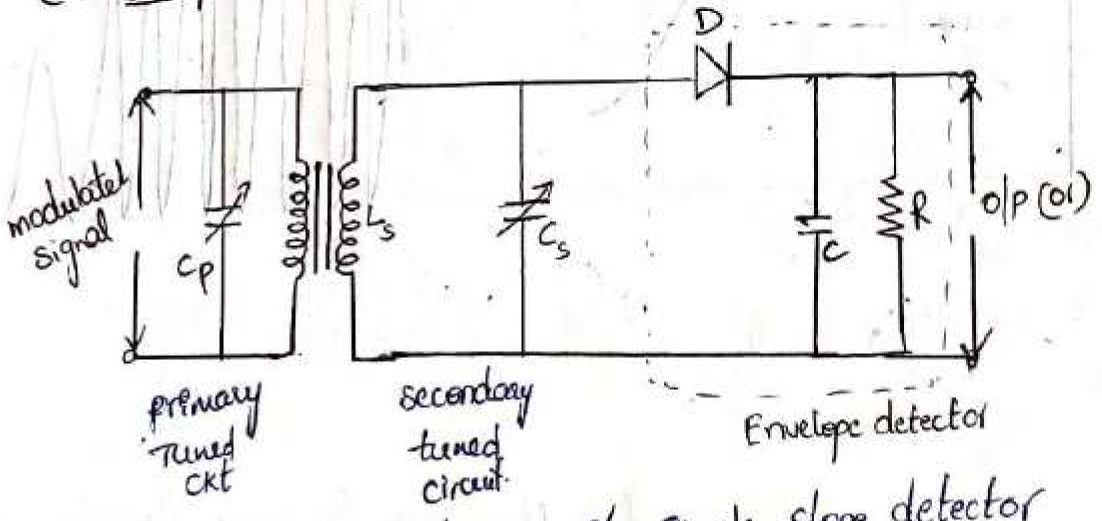
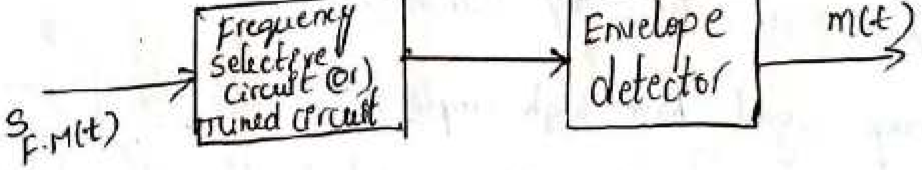


fig: ~~Bloc~~ Circuit diagram of Simple Slope detector

(or)

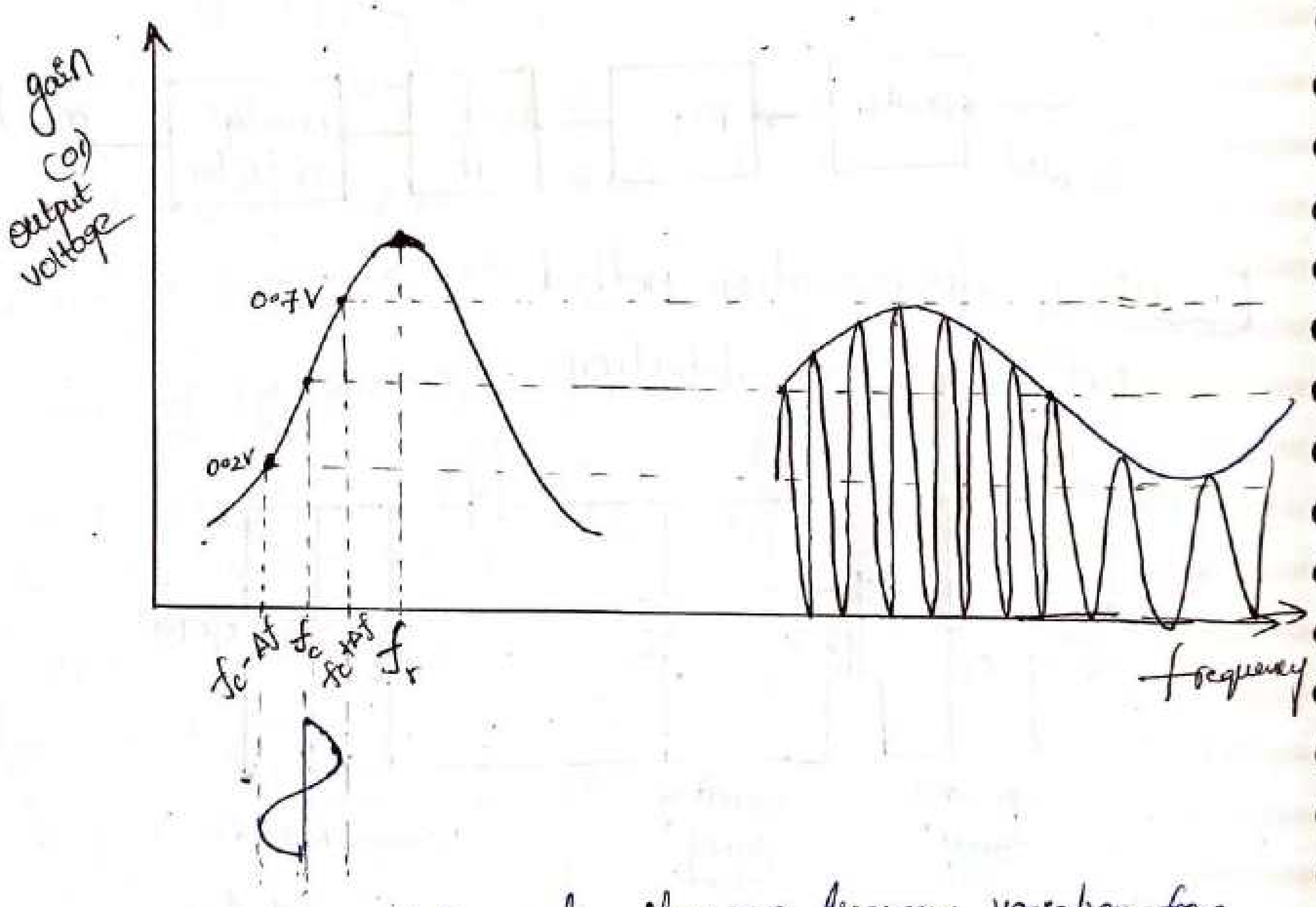


* primary tuned circuit resonant frequency is f_c and the secondary tuned circuit resonant frequency is f_r.

* condition for the tuned circuit is given by

$$\boxed{f_r >> f_c}$$

Frequency selective circuit characteristics are



* From frequency modulated signal, there is a frequency variation from $f_c-\Delta f$ ~~to~~ f_c - $f_c+\Delta f$.
* After passing through the tuned circuit, it will generate the amplitude as well as frequency variations.
* If the message signal have high amplitude, then the frequency is $f_c+\Delta f$. Similarly if the message signal have low amplitude, then the frequency is $f_c-\Delta f$.
* From the tuned circuit characteristics, if we increase the frequency

then the gain is increases, similarly if we decrease the frequency, then the gain is decreases.

* ouput of tuned circuit is given to the envelope detector. this envelope detector, detects the envelope of the modulated signal

Advantages

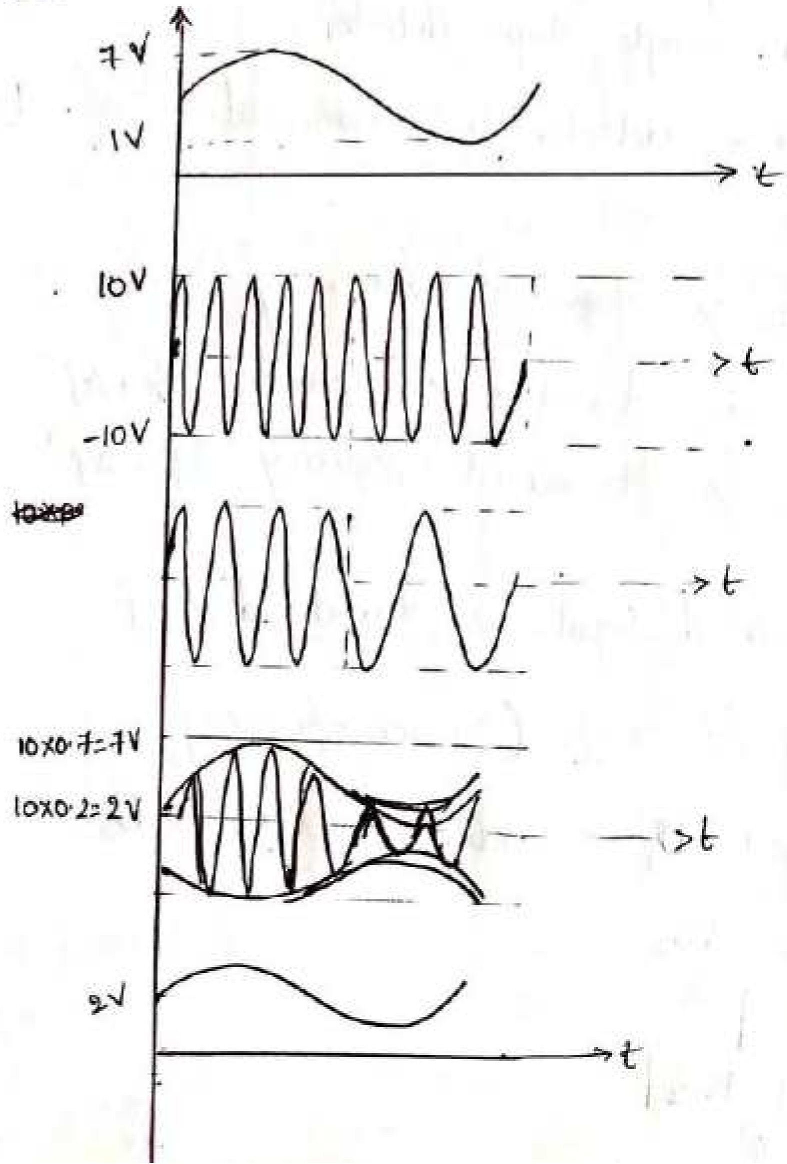
* circuit is simple.

Drawbacks

* It is linear only over a limited frequency range, it means that ~~......~~ reconstruction of message signal is not perfectly corresponds to transmitted message signal, hence it is called as slope error

* It is difficult to tune the frequencies at different frequencies.

Waveforms for simple slope detector



$f_{max} = 1400KHz$

$f_{min} = 200KHz$

(b) Balanced slope detector

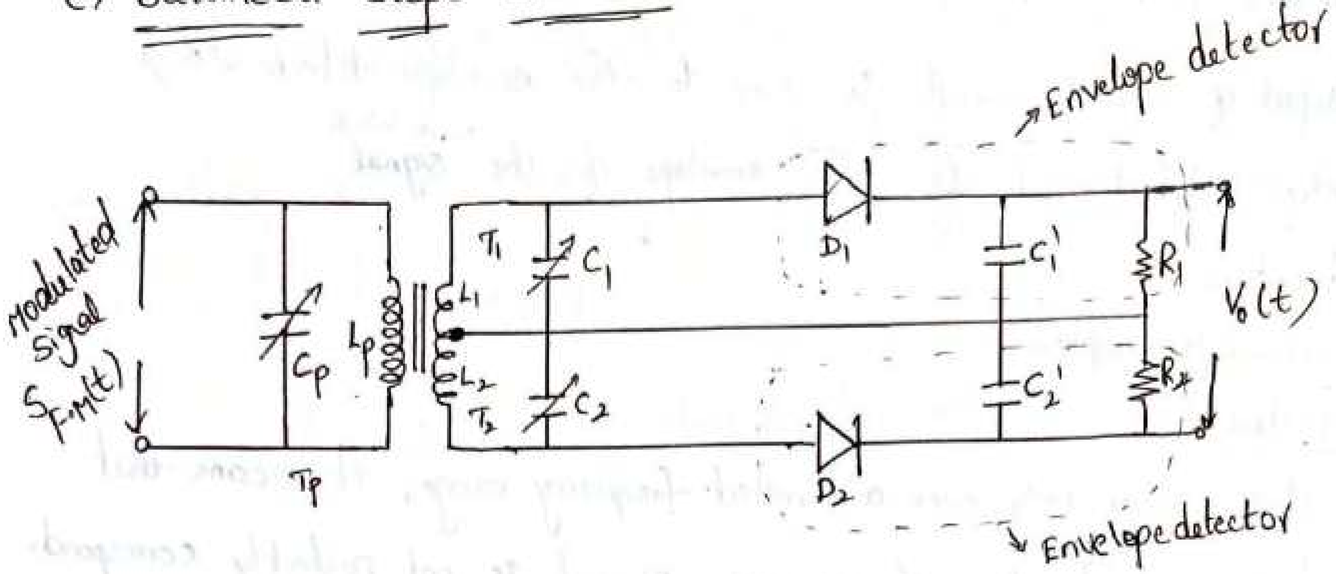


Fig: circuit diagram of Balanced slope detector

* In this circuit, there are 3 tank circuits, all these tuned circuits are tuned at three different frequencies.
* upper part is one simple slope detector
* Lower part is another simple slope detector.
* therefore Balanced slope detector is a combination of two simple slope detectors
* Primary tuned circuit is tuned at frequency f_c
  secondary transformer T_1 is tuned at frequency '$f_c + \Delta f$'
  secondary transformer T_2 is tuned at frequency '$f_c - \Delta f$'

Case (i):- If the modulated input is tuned to 'f'

If input frequency $(f) = f_c$ (carrier frequency)

$\therefore$ Induced voltage at D_1 = Induced voltage at D_2

$$V_{D_1} = V_{D_2}$$

$$|V_{R_1}| = |V_{R_2}|$$

$$\therefore V_0(t) = |V_{R_1}| - |V_{R_2}|$$

$$\boxed{\therefore V_0(t) = 0}$$

Case (ii)

If the modulated input is tuned in between $f_c < f < f_c + \Delta f$

$\therefore$ Induced voltage at D_1 > Induced voltage at D_2

$$V_{D_1} > V_{D_2}$$

$\therefore V_o(t) = V_{R_1} - V_{R_2} > 0$

$$\boxed{At\ f_c + \Delta f = V_{max} = V_o(t)}$$

Case (iii):-

If the modulated input is tuned in between $f_c - \Delta f < f < f_c$

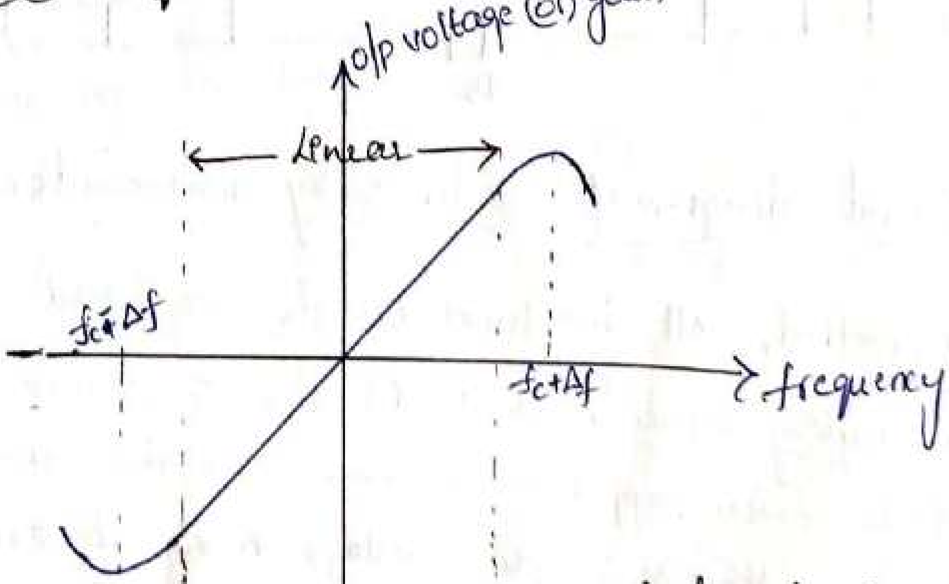
$\therefore$ Induced voltage at D_1 < Induced voltage at D_2

$\therefore V_{D_1} < V_{D_2}$

$V_o(t) = V_{R_1} - V_{R_2} < 0$

$$\boxed{At\ f_c - \Delta f = V_{min} = V_o(t)}$$

characteristics of Balanced slope detector



If $f_c - \Delta f < f < f_c$

$V_o(t) < 0$

↳ $V_o(t)$ is negative

At $f = f_c$

$V_o(t) = 0$

If $f_c < f < f_c + \Delta f$

$V_o(t) > 0$

↳ $V_o(t)$ is positive

Advantages

* The circuit is more efficient than simple slope detector.
* It has better linearity than the simple slope detector.

Drawbacks

* Eventhough linearity is good, but it is not good enough
* ~~The~~ Difficult to tune this circuit, since the three tuned circuits are to be tuned at different frequencies i.e f_c, $f_c+\Delta f$ and $f_c-\Delta f$.
* Amplitude limiting is not provided.

Phase descrimination method

(a) Faster-seeley detection:-

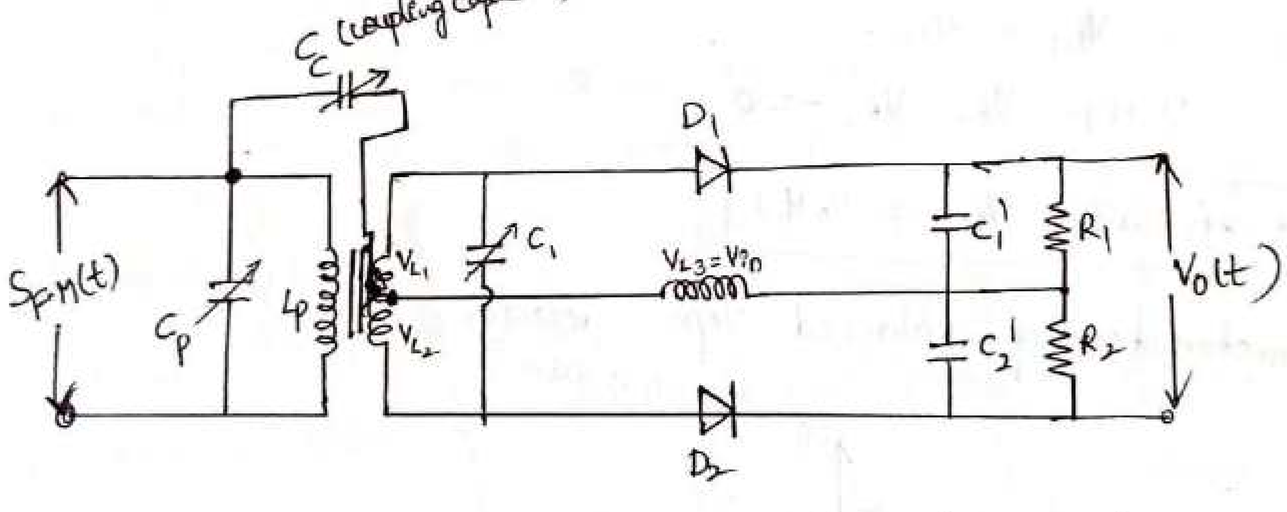


Fig:- Circuit diagram of Faster seeley discriminator

* In this method, All the tuned circuits are tuned to same frequency.
* In this method, coupling capacitor (C_c) is used, this C_c is used to couple the input to center-tapped transformer and then passed across the ~~through~~ inductor L_3, therefore input voltage appear at inductor L_3.

$\therefore \boxed{V_{L3} = V_{in}}$

* voltage across L_1 and voltage across L_2, both are $180°$ out of phase with each other
* Diode D_1, C_1, R_1 and D_2, C_2, R_2 acting as a envelope detector.
* Voltage induced in secondary is $90°$ out of phase with input voltage (V_{in})
* voltage across diode 'D_1' is vector sum of V_{L_1} and V_{L_3}

similarly voltage across diode 'D_2' is vector sum of V_{L_2} and V_{L_3}

$$V_{D_1} = V_{L_1} + V_{L_3}$$
$$V_{D_2} = V_{L_2} + V_{L_3}$$

* the output voltage $V_o(t)$ is difference between voltage across C_1 and voltage across C_2.

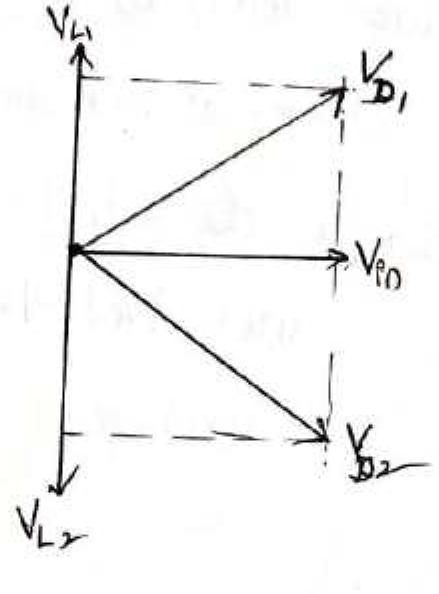
$$\therefore V_o(t) = V_{C_1} - V_{C_2}$$

Case (i):

* when $f_{in} = f_c$, then the phase between primary and secondary voltage is $90°$
* voltage at diode D_1 = voltage at Diode D_2
* Both capacitor charges at equal magnitude.

$$\therefore V_o(t) = V_{C_1} = V_{C_2}$$

$$\boxed{\therefore V_o(t) = 0}$$

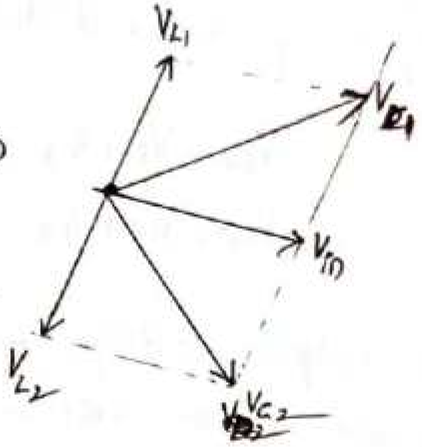


Case (ii):-

* when $f_{in} > f_c$, then the phase between primary and secondary voltage is less than 90°
* $X_L > X_C$ and secondary circuit becomes more inductive.
* secondary current lags secondary voltage by some angle ϕ
* voltage across D_1 is more than voltage across D_2
* C_1 charges at more magnitude than C_2
* Therefore the output voltage

$$V_o(t) = |V_{C1}| - |V_{C2}| > 0$$

$$V_o(t) > 0$$

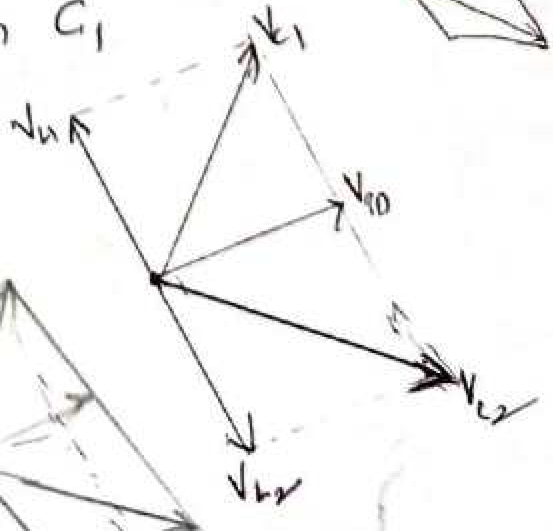


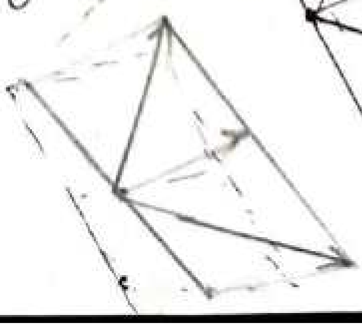
Case (iii):-

* when $f_{in} < f_c$, then the phase between primary and secondary voltage is greater than 90° secondary circuit becomes more capacitive.
* $X_C > X_L$ and secondary circuit becomes more capacitive.
* secondary current leads secondary voltage by some angle ϕ
* voltage across D_2 is more than voltage across D_1
* C_2 charges at more magnitude than C_1
* Therefore the output voltage

$$V_o(t) = |V_{C1}| - |V_{C2}| < 0$$

$$V_o(t) < 0$$





Advantages of Foster seeley detector

* Three tuned circuits tuned at same frequency.
* There would be no complex circuit design (easy to construct using discrete components ie L, C)
* Linearity is better.

Disadvantages :-

* Limiter cannot be used
* It is expensive due to higher cost of the transformer required.
* Not suitable to use IC technology.

(b) Ratio Detector

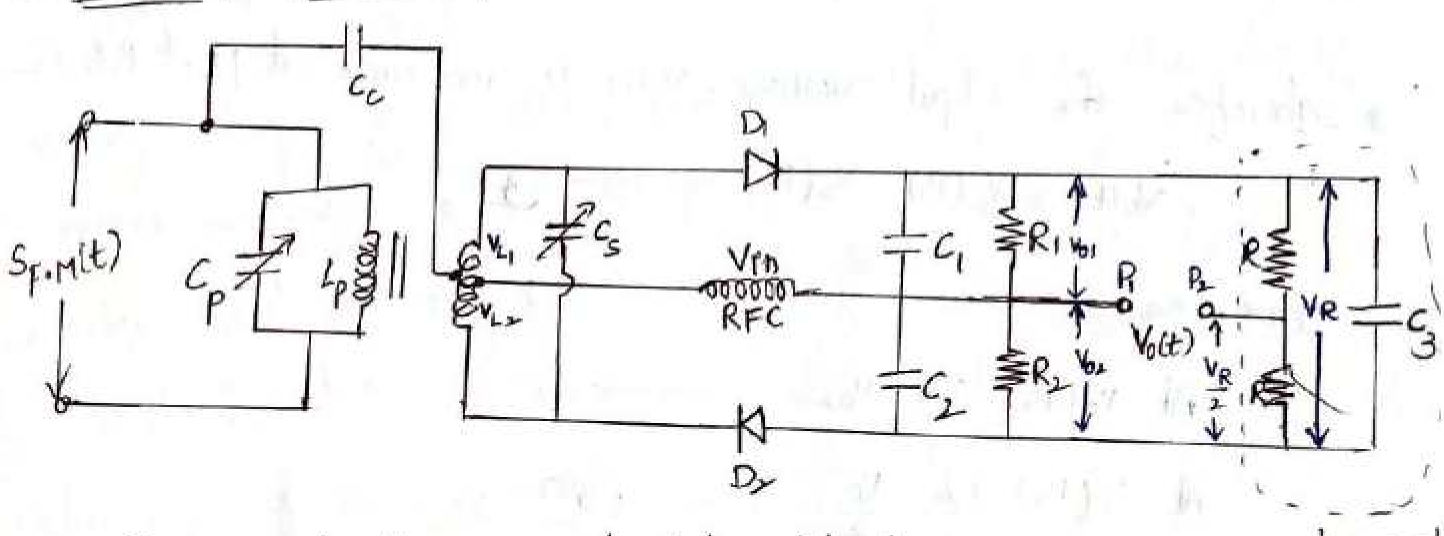


fig: Circuit diagram of Ratio detector

* Ratio detector circuit is same as Foster seeley discriminator but only difference is, direction of D_2 is reversed and large value of capacitor C_3 has been included in this circuit. $V_0(t)$ is also taken somewhere else
* coupling capacitor C_c provides input voltage to the secondary circuit.
* voltage at D_1 = $V_{L_1} + V_{in}$

  Voltage at D_2 = $V_{L_2} + V_{in}$

In circuit diagram

→ output voltage at R_1 is V_{01}

→ output voltage at R_2 is V_{02}

$\therefore V_0(t) = V_{01} - V_{02}$ → ① → eq ① is distributed across R & R, i.e $V_0(t)$ therefore output is denoted as V_R.

$\boxed{V_R = V_{01} + V_{02}}$ → ② (Because diode D_2 is reversed)

→ This $V_0(t)$ is distributed evenly at R and R i.e across upper 'R' output voltage is $V_R/2$ and across lower 'R' output voltage is $V_R/2$.

→ Therefore the output voltage $V_0(t)$ is measured at point P_1 & P_2

$$V_0(t) = V_0(P_1) - V_0(P_2) \longrightarrow ③$$

From eq ②

At $V_0(P_1)$ is V_{02} → ④

At $V_0(P_2)$ is $\frac{V_R}{2}$ → ⑤

eq ④ & ⑤ substituted in eq ③

$$\therefore V_0(t) = V_{02} - \frac{V_R}{2} \longrightarrow ⑥$$

eq ② is substituted in eq ⑥

$$\therefore V_0(t) = V_{02} - \frac{V_{01}+V_{02}}{2}$$

$$\boxed{V_0(t) = \frac{V_{02} - V_{01}}{2}}$$

Case (i)

At $f = f_c$

$V_{01} = V_{02}$

$\boxed{\therefore V_0(t) = 0}$.

Case (ii)

If $f > f_c$

$$V_o(t) = \frac{V_{o2} - V_{o1}}{2} > 0$$

$$\boxed{V_o(t) > 0}$$

Case (iii)

If $f < f_c$

then $V_o(t) = \frac{V_{o2} - V_{o1}}{2} < 0$

$$\boxed{\therefore V_o(t) < 0}$$

* $V_o(t) = \frac{V_{o2} - V_{o1}}{2}$

$$V_o(t) = \frac{V_{o2}}{2}\left[1 - \frac{V_{o1}}{V_{o2}}\right]$$

* Let us take $\frac{V_{o1}}{V_{o2}}$ (ratio), whenever $V_o(t)$ changes, this ratio $\frac{V_{o1}}{V_{o2}}$ changes according to the voltage.
* this $V_o(t)$ changes due to ratio changes, it is called Ratio Detector

Advantages of Ratio detector

1. Less affected through noise due to inbuilt limiter
2. It has simple circuit
3. High Bandwidth
4. High Linearity.

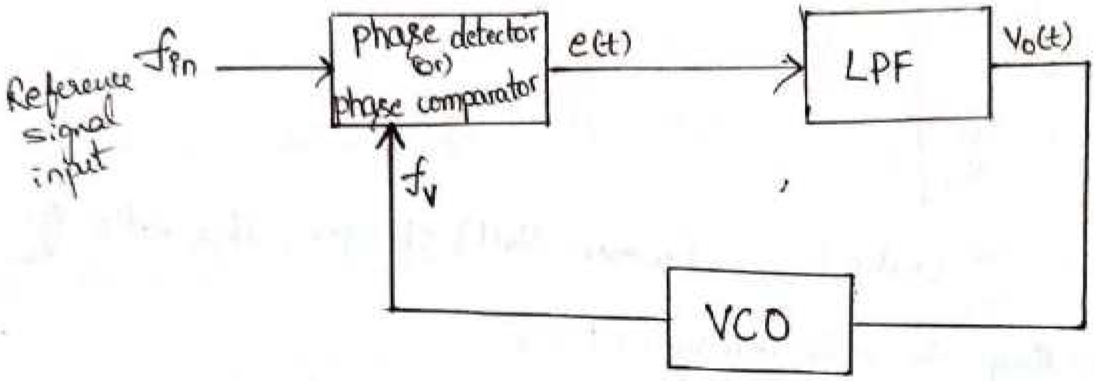
Disadvantages

1. It cannot be implemented over IC's
2. High cost.

(C) Phase Locked Loop (PLL)

* PLL :- A PLL is a non-linear feedback system that tracks the phase of the input signal and minimized phase error at local oscillator.

Block diagram of PLL:



* There are three blocks in the PLL
  1. phase detecto (or) phase comparator
  2. L.P.F (Low pass felter)
  3. VCO (voltage controlled oscillator)

* PLL operates in three modes
  (i) Free Running mode
  (ii) capture Range
  (iii) Lock Range

(i) Free Running mode:- PLL is said to be free running mode, when there is no input (f_{in}) applied to the phase detector

(ii) capture Range :- when ~~voltage~~ input is applied to the phase detector, then the VCO starts to change the frequency by appling input voltage. i.e VCO is trying to capture to the incoming signal frequency. ~~also~~ therefore PLL is said to be capture mode.

(iii) Lock Range :- The lock range is defined as the range of frequency over which the charges the input frequency and output frequency i.e VCO freq (f_v) is zero, then it referes as phase lock

$$f_{in} - f_v = 0$$
$$\therefore f_{in} = f_v$$

(or)

* The ~~out of~~ output of VCO changes continuously, until it becomes equal to the input signal frequency (f_{in}), then it is called lock mode

* If $f_{in} = f_v$, then there is no phase difference i.e $\Delta\phi = 0$ i.e loop is in the lock condition.

Operation :-

* When control voltage changes, frequency of oscillation changes.
* Whenever the loop is turned "ON", then the VCO runs at the center frequency (f_o) and this frequency is known as free running frequency

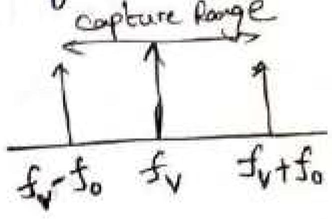
* phase detector compares the reference signal input (f_{in}) and oscillator frequency output (f_v), based on that it generates the error signal (e(t))

* LPF : This error signal e(t) is passed through LPF, LPF generates the error voltage based on the error signal.

* VCO :- Based on the error voltage, VCO can reduce (or) increase the oscillator frequency until the oscillator frequency locks to the input frequency ($f_{in} = f_v$)

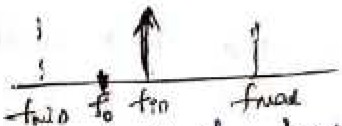
* Under the lock condition, there is no phase difference between two signals.

* Under no lock condition, the PLL can acquire the lock only if the input signal within the capture range of the PLL.

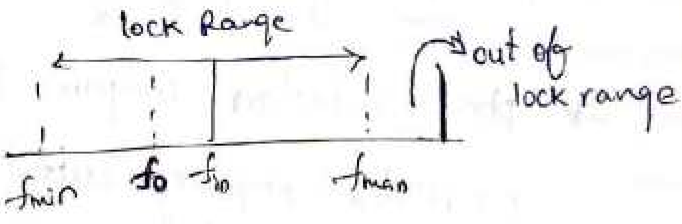


* Whenever the input signal is within the capture range, then the VCO can lock to the input signal.

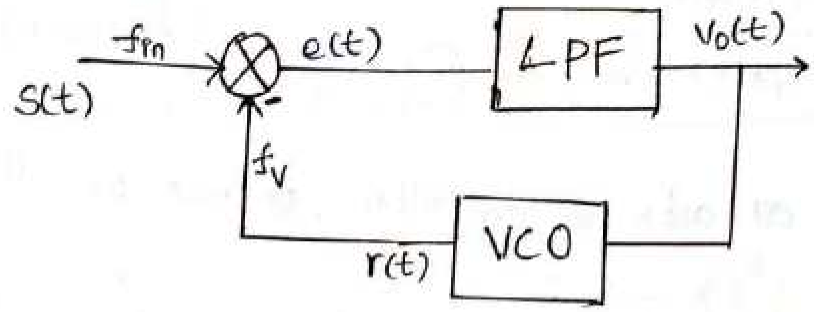
* Let's say, the VCO is already locked to the input frequency (f_{in}), now if the input frequency changes, then the VCO will follow that frequency, provided that the input frequency is within the lock range.



* But if the input frequency goes out of the lock range, the VCO starts running at the free running frequency and it won't be able to lock to the input frequency until the input frequency is within the capture range of the PLL.



Derivation of phase locked loop



* It is a negative feedback system, it is used to extract the message signal from the modulated signal ($s(t)$)

$$s(t) = A_c \cos\left[\omega_c t + k_f \int_0^t m(t)\,dt\right]$$

where
$$\phi_1(t) = k_f \int_0^t m(t)\,dt$$

$$\therefore s(t) = A_c \cos\left[\omega_c t + \phi_1(t)\right] \longrightarrow (1)$$

Now define the locally generated FM wave by VCO as

$$r(t) = A_v \,\cancel{\cos}\, \sin\left[\omega_c t + k_v \int_0^t V_o(t)\,dt\right]$$

$K_v \rightarrow$ VCO sensitivity constant

where
$$\phi_2(t) = K_v \int_0^t V_o(t)\,dt$$

$$\therefore r(t) = A_v \sin\left[\omega_c t + \phi_2(t)\right] \longrightarrow (2)$$

$~~\sin A \cdot \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]~~$

$2\cos A \cdot \sin B = \sin(A+B) - \sin(A-B)$

$$\therefore e(t) = s(t)(r(t))$$

$$= A_c \cos\left[\omega_c t + \phi_1(t)\right] \cdot A_v \sin\left[\omega_c t + \phi_2(t)\right]$$

$$= \frac{A_c A_v}{2}\left[\sin(2\omega_c t + \phi_1(t) + \phi_2(t)) - \sin(\phi_1(t) - \phi_2(t))\right] \longrightarrow (3)$$

This e(t) is passed through LPF

$$e(t) = \frac{A_c A_v}{2} \sin(\phi_1(t) - \phi_2(t))$$

$\phi_e(t) = \phi_1(t) - \phi_2(t)$

$\phi_e(t) << 1$

$\sin(\phi_e(t)) = \phi_e(t)$

$$e(t) = \frac{A_c A_v}{2} \sin \phi_e(t)$$

$$\boxed{\therefore e(t) = \frac{A_c A_v}{2} \phi_e(t)} \longrightarrow (4)$$

This error signal e(t) acts on loop filter, produce overall output $V_o(t)$.

Let h(t) = impulse response of loop filter.

$$e(t) \longrightarrow \underset{\text{}}{\overset{h(t)}{\boxed{\text{Loop filter}}}} \longrightarrow V_o(t)$$

$$V_o(t) = e(t) * h(t)$$

$$V_o(t) = \frac{A_c A_v}{2} \phi_e(t) * h(t) \longrightarrow (5)$$

Apply F.T on $V_o(t)$

$$\boxed{V_o(\omega) = \frac{A_c A_v}{2} \phi_e(\omega) H(\omega)} \longrightarrow (6)$$

We know that

$$\phi_e(t) = \phi_1(t) - \phi_2(t)$$

$$\phi_e(t) = \phi_1(t) - K_v \int_0^t V_o(t)\, dt$$

$$\phi_e(t) = \phi_1(t) - K_v \int_0^t \frac{A_c A_v}{2} \phi_e(t) * h(t)\, dt$$

Differentiate on both sides

$$\frac{d}{dt}\phi_e(t) = \frac{d}{dt}\phi_1(t) - K_v \frac{d}{dt}\int_0^t \frac{A_c A_v}{2} \phi_e(t) * h(t)$$

$$\frac{d}{dt}\phi_e(t) = \frac{d}{dt}\phi_1(t) - K_v \frac{A_c A_v}{2} \phi_e(t) * h(t) \longrightarrow (7)$$

Apply F.T on eq (7)

$$j\omega \phi_e(\omega) = j\omega \phi_1(\omega) - K_v \frac{A_c A_v}{2} \phi_e(\omega) \cdot H(\omega)$$

$$j\omega \phi_e(\omega) + K_v \frac{A_c A_v}{2} \phi_e(\omega) H(\omega) = j\omega \phi_1(\omega)$$

$$\phi_e(\omega)\left[j\omega + K_v \frac{A_c A_v}{2} H(\omega)\right] = j\omega \phi_1(\omega)$$

$$\phi_e(\omega) = \frac{j\omega\phi_1(\omega)}{j\omega + K_v \frac{A_c A_v}{2} H(\omega)}$$

$$= \frac{j\omega\phi_1(\omega)}{j\omega\left(1 + \frac{K_v A_c A_v H(\omega)}{2j\omega}\right)}$$

$$\boxed{\phi_e(\omega) = \frac{\phi_1(\omega)}{1 + \frac{K_v A_c A_v H(\omega)}{2j\omega}}} \longrightarrow (8)$$

If $H(\omega)$ increases, then $\phi_e(\omega)$ decreases,

$$\therefore \phi_1(t) \approx \phi_2(t)$$

If $H(\omega)$ is very high, then

$$\boxed{\phi_e(\omega) = \frac{\phi_1(\omega)}{\frac{K_v A_c A_v H(\omega)}{2j\omega}}} \longrightarrow (9)$$

This eq (9) is substituted in eq (6)

$$V_o(\omega) = \frac{A_c A_v}{2} \phi_e(\omega) H(\omega)$$

$$V_o(\omega) = \frac{A_c A_v}{2}\left[\frac{2j\omega\phi_1(\omega)}{K_v A_c A_v H(\omega)} \cdot H(\omega)\right]$$

$$\boxed{\therefore V_o(\omega) = \frac{j\omega\phi_1(\omega)}{K_v}} \longrightarrow (10)$$

Apply Inverse Fourier Transform

$$V_o(t) = \frac{1}{K_v} \cdot \frac{d}{dt}\phi_1(t)$$

$$\frac{d}{dt}\phi_1(t) = \frac{d}{dt}\left[K_f \int_0^t m(t)\,dt\right]$$

$$= K_f m(t)$$

$$\therefore V_o(t) = \frac{1}{K_v}(K_f m(t))$$

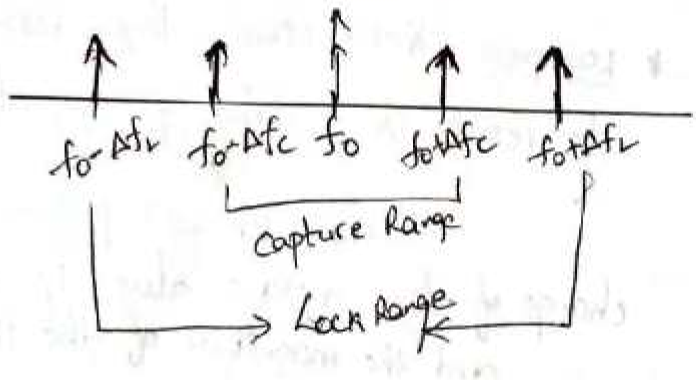
$$\boxed{V_o(t) \propto m(t)}$$

pull in Time :- the total time taken by the PLL to establish lock is called pull in Time. (or) Lock in time.

Capture Range $= 2\Delta f_c$

Lock Range $= 2\Delta f_L$

Capture Range is narrower compare to the PLL lock range



Advantages of PLL

1. Linearity
2. Insensitive to amplitude Noise
3. Ease of incorporation into IC's (PLL are very easy to implement in an Integrated ckt)
4. Low cost (no inductor is required for the vco ckt, As inductors are relatively expensive components, this can reduce the overall cost)

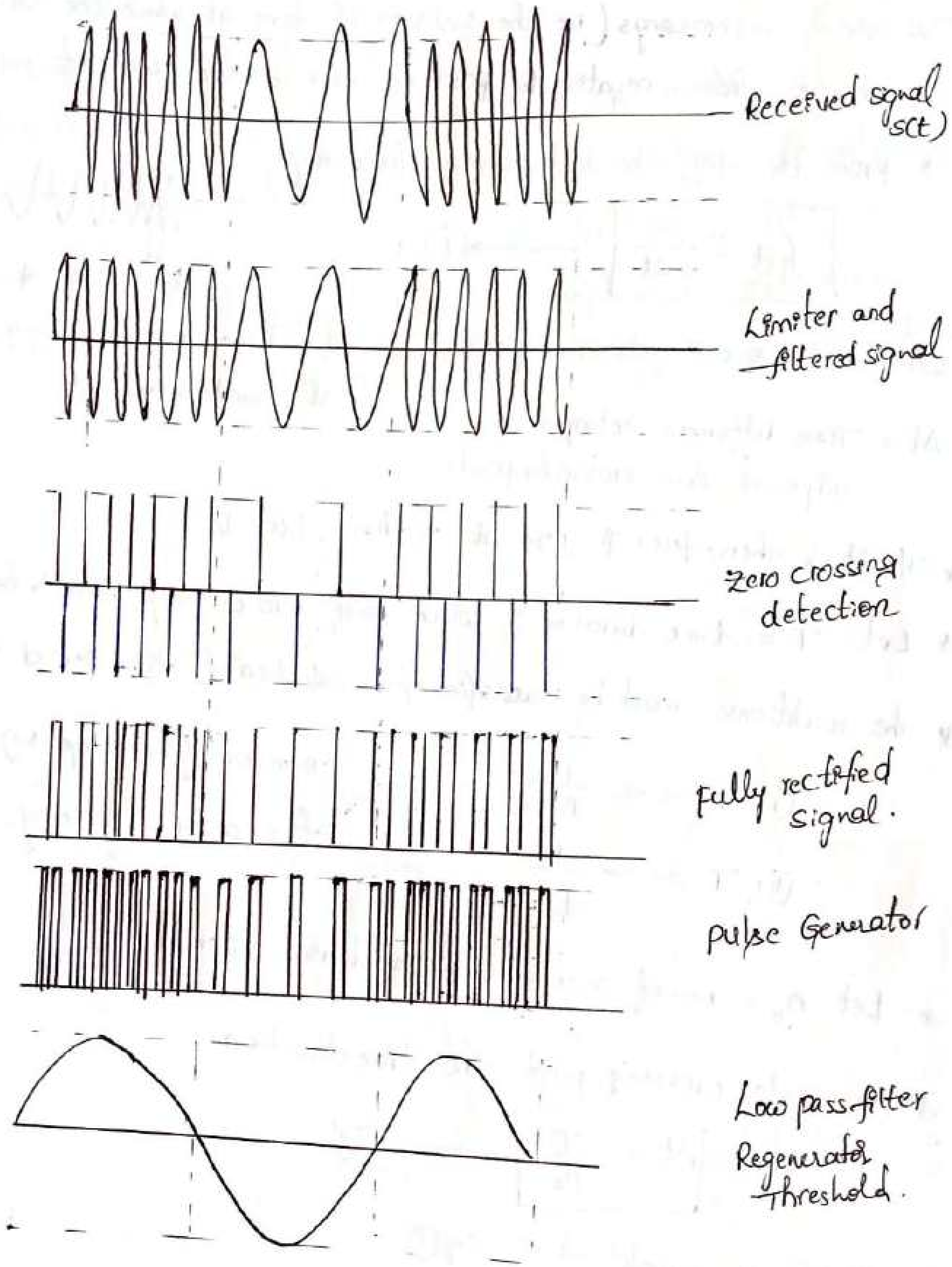
(d) Zero crossing Detector

Block diagram of zero crossing detector of FM Demodulation.

* Limiter Amplitude of a received signal can change, so limiter and filter is needed. the output of limiter is connected to zero crossing detector
* zero crossing detector:- which detects zero crossings and produce spikes
* Full wave rectification:- this peak signal is the fully rectified and so some kind of digital signal is available.
* pulse Generator:- this near digital signal is connected to pulse generator which produces pulses (the same duration 'T' for each pulse).
* Low pass filter:- Finally this PWM signal is filtered by using LPF and the result is modulating signal.

The LPF will perform an averaging operation, the rate of change of this average value is related to the message frequency, and the magnitude of the change to the depth of modulation at the generator

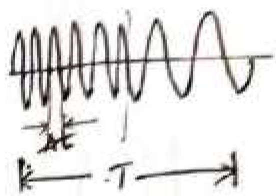
Waveforms


Received signal s(t)
Limiter and filtered signal
zero crossing detection
Fully rectified signal.
pulse Generator
Low pass filter
Regenerator
Threshold.

Mathematical expression for zero crossing detector

* The zero crossings (i.e the instants of time at which the waveform of FM wave changes from negative to positive value (or) vice versa)

* From the fig, the instantaneous frequency



$$\boxed{f_i(t) \cong \frac{1}{2\Delta t}} \longrightarrow (1)$$

where $2\Delta t$ is one cycle Time period

$\Delta t \rightarrow$ half cycle of zero crossing.

$2\Delta t \rightarrow$ complete full cycle.

Δt = Time difference between the adjacent zero crossover points.

* If $\Delta t \downarrow$ then $f_i(t) \uparrow$, If $\Delta t \uparrow$ then $f_i(t) \downarrow$

* Let T = time duration in which no. of zero crossings have to be counted.

* The conditions must be satisfied for selection of time period 'T'.

(i) $T << \frac{1}{B.W.}$

(ii) $T >> \frac{1}{f_c}$

B.W = B.W of message signal

f_c = carrier frequency.

* Let n_0 = no. of zero crossings during interval 'T'

* For single crossing point, the time duration

$$\boxed{\Delta t = \frac{T}{n_0}} \longrightarrow (2)$$

eq (2) is substituted in eq (1)

$$f_i(t) = \frac{1}{2\left(\frac{T}{n_0}\right)} = \frac{n_0}{2T} \longrightarrow (3)$$

$$\boxed{f_i(t) \propto n_0}$$

* no. of zero crossings is the replica of instantaneous freq of F.M Signal.

∴ The instantaneous frequency of FM signal is directly proportional to number of zero crossings.

* It means that, if the modulated wave is having higher frequency then the number of zero crossings will be greater.
* Since by definition of frequency modulation, the instantaneous frequency $f_i(t)$ is a linear function of message signal $m(t)$; hence if n_0 is known which is a replica of $f_i(t)$, then the message signal can be achieved by using zero crossing detector.

*

UNIT-IV

Transmitters and Receivers

Transmitter:- A transmitter is an electronic device, which modifies the incoming message signal to make it suitable for transmission over the communication channel.

Receiver:- A Receiver is an electronic device, which recovers the original message signal from the modified message signal (modulated signal)

Function of Transmitter

* Every transmitter has three basic functions as follows.

1. The transmitter must generate a signal of correct frequency at a desired point in the spectrum.
2. It must provide some form of modulation to modulate the carrier
3. It must provide sufficient power amplification in order to carry the modulated signal to a long distance.

Block diagram

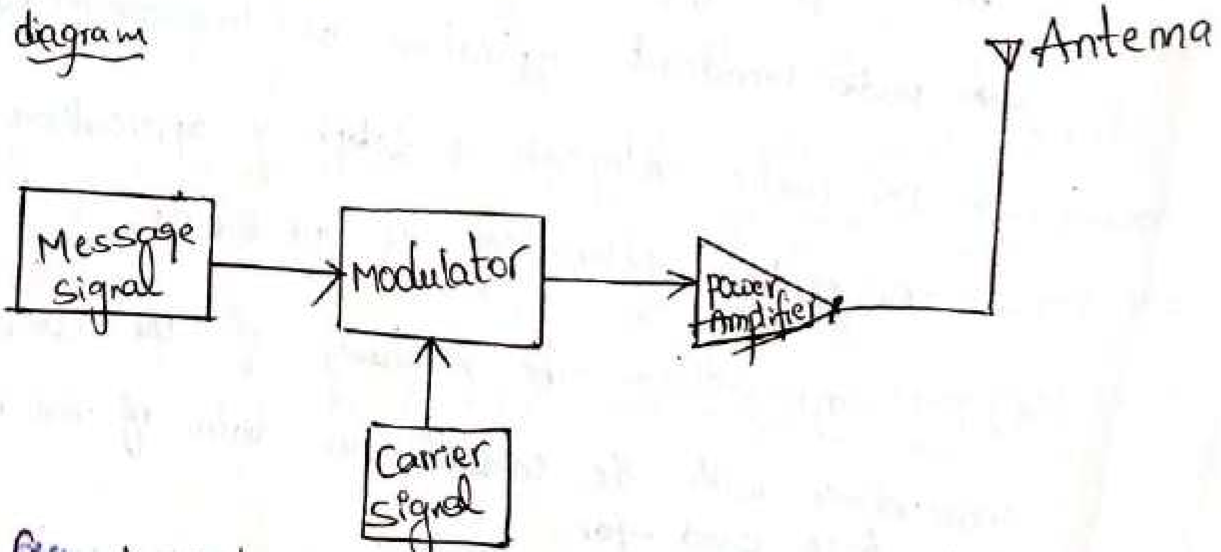


fig: Block diagram of Radio Transmitter.

Classification of Radio Transmitters

* Radio Transmitters are classified into 4 categories.
  1. According to the type of modulation used.
  2. Based on the service involved
  3. Based on the Frequency Range used
  4. Based on the power used.

1. According to the type of modulation:

According to the type of modulation used, Radio transmitters are classified as

(a) AM Transmitters
(b) FM Transmitters
(c) PM Transmitters.

(a) AM Transmitters:- In this case of transmitter, amplitude of carrier is varied inaccordance with the instantaneous value of the modulating signal.

These types of transmitters are used in

* Radio broadcast application on long wave, medium wave, shortwaves
* For audio Telegraph & telephony application on shortwave
* TV picture telecasting in VHF & UHF.

(b) FM Transmitter:- The frequency of the carrier is varied inaccordance with the instantaneous value of the modulating signal. It is used for

* Radio broadcast application in VHF & UHF range

* TV sound broadcast in VHF & UHF
* Radio Telephone communication in UHF & VHF range over short distance

(C) PM Transmitters :- The phase of the carrier signal is varied inaccordance with the instantaneous value of the modulating signal.

Application :- Telephony and Telegraphy.

2. Based on the service involved

According to the service used, It is classified as

(a) Radio Broadcast Transmitters.
(b) Radio Telephony Transmitters.
(c) Radio Telegraph Transmitters.
(d) Television Transmitters.
(e) Radar Transmitters.
(f) Navigational Transmitters.

(a) Radio Broadcast Transmitters :- These transmitters are used to transmit speech, music etc. These transmitters have high stability of carrier frequency, low distortion & Noise.

This transmitter may be AM (or) PM Transmitter operates on medium wave and short wave and radiate carrier power as low as 1 Kw and high as 1000 kw or more.

FM transmitter operate on UHF and VHF and radiate carrier power of the order of 1000 kw and more.

(b) Radio Telephony Transmitters :-

* These are designed for transmitting telephone signals over a long distance by radio frequencies.

* Radio Telephone transmitter may be AM (or) FM type.

* The AM telephone transmitter usually work on SW (short wave) have a output ~~pow~~ carrier power of few KW and can be used for point-to-point communication over long distance.

* The FM telephone transmitter works on UHF & VHF ranges and it requires very small power that is less than 1KW and it is used for short distance communication not exceeding 40km.

(C) Radio Telegraph Transmitters:-

* In this point-to-point communication is involved.

* The transmitting antenna is designed for directing electromagnetic energy into a narrow beam and directed towards distance receiving antenna.

(d) Television Transmitters:- * In TV transmitters, one is used for picture transmission, another is used for sound transmission.

* Both are operated in VHF (or) UHF range.

* The picture transmission is amplitude modulated occupying a B.W of above 5.5 MHZ. In this case VSB transmission is used.

* It is having a B.W occupied by the TV channel is about 7MHZ

(e) RADAR Transmitters:- In this case, the carrier is pulse modulated and it operates at microwave frequency typically 300 MHZ to 19000 MHZ.

There are 2 types of RADAR transmitters.

(i) pulse Radar (ii) continuous wave RADAR

(i) pulse RADAR :- It uses modulation of carrier signal, it gives o/p high output power such as 100KW and it is operated in microwave frequency typically 3GHz to 10GHz

(ii) CW RADAR :- It uses F.M of carrier voltage

(f) Navigational Transmitters :- It is used to send navigational signals over sea (or) air.

3. Based on the carrier Frequency Range used

It is classified as

(a) Long wave Transmitter

(b) Medium wave Transmitter

(c) Short wave Transmitter

(d) V.H.F and U.H.F Transmitters.

(e) Microwave Transmitters.

(a) Long Wave Transmitter :- Here wavelength is high, therefore frequency is Low. frequency is arround less than 300KHz

* It is propagated at along the ground, there-fore attenuation is large. (signal cannot travel to longer distances)
* It is used for Regional broadcast in places where disturbance are not severe.
* Not economical.
* It requires more number of repeaters.

(b) Medium wave Transmitter:- Here wavelength of carrier signal is medium, therefore the frequency range is 525KHz - 1625KHz

* Transmitter carrier power range is 1KW - 1000KW only.
* It is used for regional broadcast, there is no requirement of repeaters in the near by areas.
* It gives good, uniform coverage arround transmitter

(c) Short wave Transmitter:- Here the wavelength of carrier signal is very low, therefore the frequency is very high.

* Its frequency range is 3MHZ - 30MHZ.
* High carrier power is not required as attenuation through ionosphere is not severe (Because freq is high, it is travel upto long distances like ionosphere, attenuation also less)
* Long distance transmission using ionospheric reflections.

(d) VHF & UHF transmitters:- These transmitters operate either on VHF range (30-300MHZ) (or) UHF range (300MHZ - 3000MHZ) and these are used for FM broadcast, TV telecast, FM radio telephony etc.

(e) Microwave Transmitters:- These transmitters operate on micro ~~frequency~~ Wavelength (i.e) beyond 1000MHZ and are used in Radio, TV, microwave link between two adjoining countries.

4. Based on the power used :-

Based on the power level, it is classified as

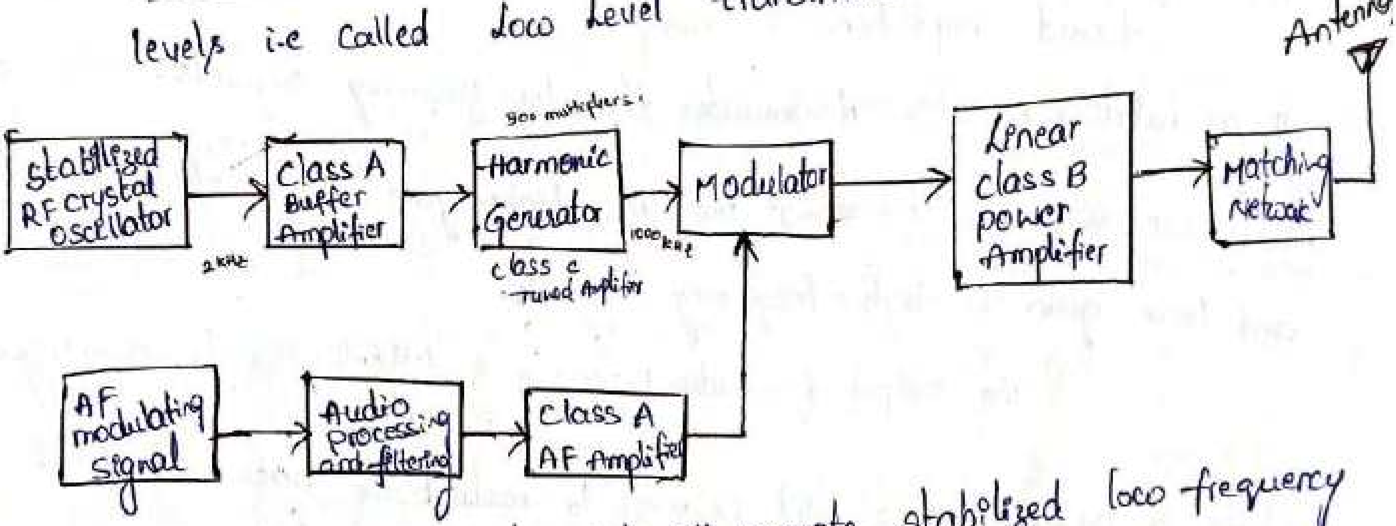
(a) Low Level Transmitter

(b) High Level Transmitter.

these low level and high level transmitters will work on medium wave & short wave transmission, do not work on long wave transmission.

AM Transmitter:- It performs not only modulation but also raises the power level of the modulated signal to the desired extent for effective Txion

(a) Low Level Transmitter :- Modulation is occurred at low power levels i.e called low level transmitter



stabilized RF crystal oscillator:- It will generate stabilized low frequency signal.

Class A Buffer Amplifier:- It is very high impedance amplifier, which protects the backward flow of current.

It means that if any load current is reflected back, then the carrier frequency will change at the output of crystal oscillator, it will become a random signal. Therefore we need to protect the signal from the back current.

Finally stabilized output i.e stable carrier frequency is protected with the help of class-A buffer amplifier.

* It matches the o/p impedance of the carrier oscillator with the i/p impedance of freq multiplier

Harmonic Generator:- the output of RF crystal oscillator is low carrier signal, but generally carrier signal must be very high frequency signal.

* Number of multipliers are used in harmonic generator.
* with the help of harmonic generator, ~~that~~ low carrier signal is multiplied to get a high frequency signal.

For ex:- stabilized RF crystal oscillator = 2KHZ

∴ Harmonic generator O/P = 1000KHZ (Here we are using 500 multipliers)

i.e tuned at 1000 KHZ, this harmonic generator is a class C tuned amplifier.

AF modulating signal:- It generates the low frequency signals.

Audio processing and Filtering:- It provides higher gain to low frequency signals and lower gain to high frequency signals.

The output of audio processing & filtering is low B.W signal.

class A AF Amplifier:- It is used to restrict the backward flow of load current, it is also a high impedance amplifier.

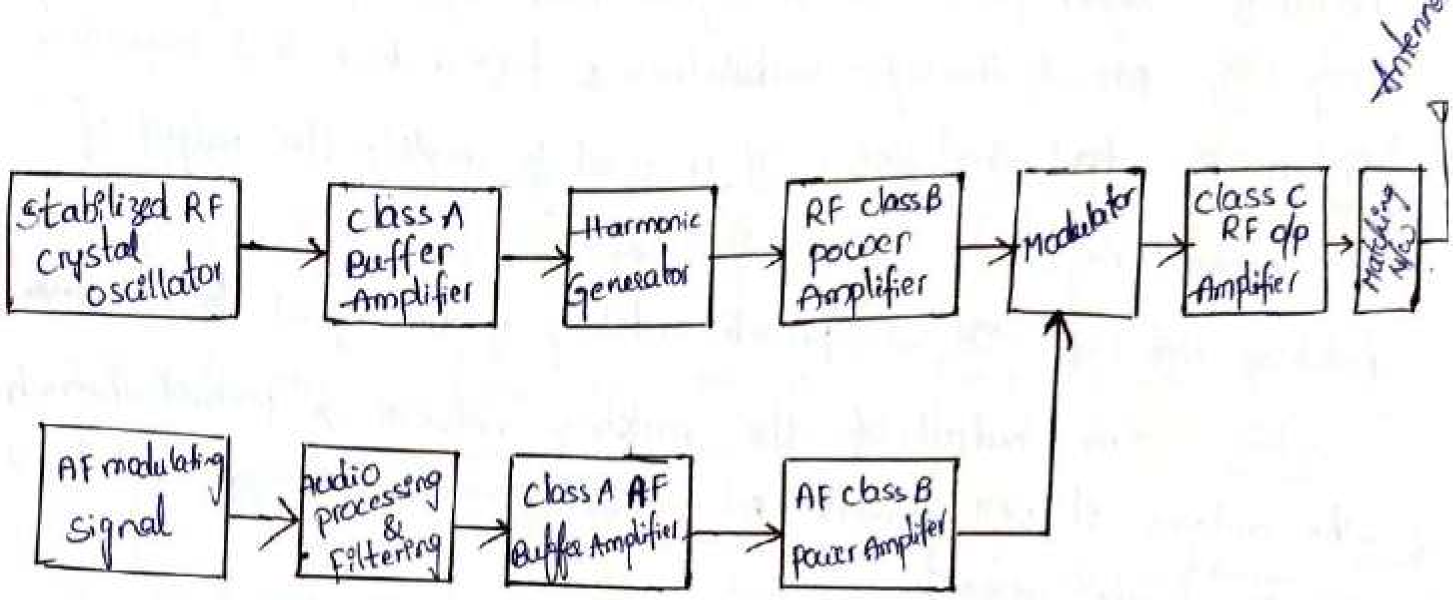
Modulator:- ~~we~~ can use any type of amplitude modulator, Here the amplitude of the carrier signal (O/P of H.G) is varied inaccordance with the message signal (O/P of Audio processing & filtering).

Linear class B power Amplifer:- At low power levels, increase the power levels by using this amplifier, and also amplify the power of the signal.

Matching network:- wastage of power is very less by using this matching network.

It will provide matching of the signal to the Antenna. So that reflections will not be there, therefore power wastage is less.

(b) High Level Transmitter:-



* Modulation is occured at high power levels i.e called High level transmitter
* Here power amplifier is used before the modulator process.

Stabilized RF crystal oscillator:- It will generate the stable low frequency signal

Class A Buffer amplifier:- It is used to ~~protect~~ reduce the backward flow of current i.e used to protect the output of RF crystal oscillator (stable frequency).

Harmonic generator:- operating freq is not sufficient, so we need to increase the fc, then we use freq multipliers. It will generate the high-frequency signal with the help of number of multipliers present in harmonic generator.

RF class B power Amplifier:- It will boost the power of the signal.
so high power carrier is propagated to the modulator.

AF modulating signal:- It will generate the audio frequency signal.

Audio processing & Filtering:- It provides higher gain to low frequency signals and lower gain to high frequency signals.

Class A RF Buffer Amplifier:- ~~It is used to amplify the output of audio processing and filtering.~~
It is used to restrict the backward flow of load current.

AF class B power amplifier:- It is used amplify the audio frequency signal.

Modulator:- higher power carrier signal and higher power of message signal is passed through modulator. so it is a high level transmitter.

Class C RF output Amplifier:- It is used to amplify the output of the modulator.

Matching Network:- It will provide matching of the signal to the Antenna.

Antenna:- The output of the matching network is passed through the antenna, it will convert into electromagnetic energy and it will be propagated into the space.

Advantage:-

* power efficiency > 80%
* Amplifiers operate at low power.

Disadvantage:-

* It requires high amplitude of input signal.
* It is complex and expensive circuit.

Advantages of Low Level Transmitter

* It is working at low voltage levels.
* Less power consumption.
* It is simple to design.

Disadvantages of Low Level Transmitter

* Low power efficiency.

Comparision between Low Level and High Level Modulation

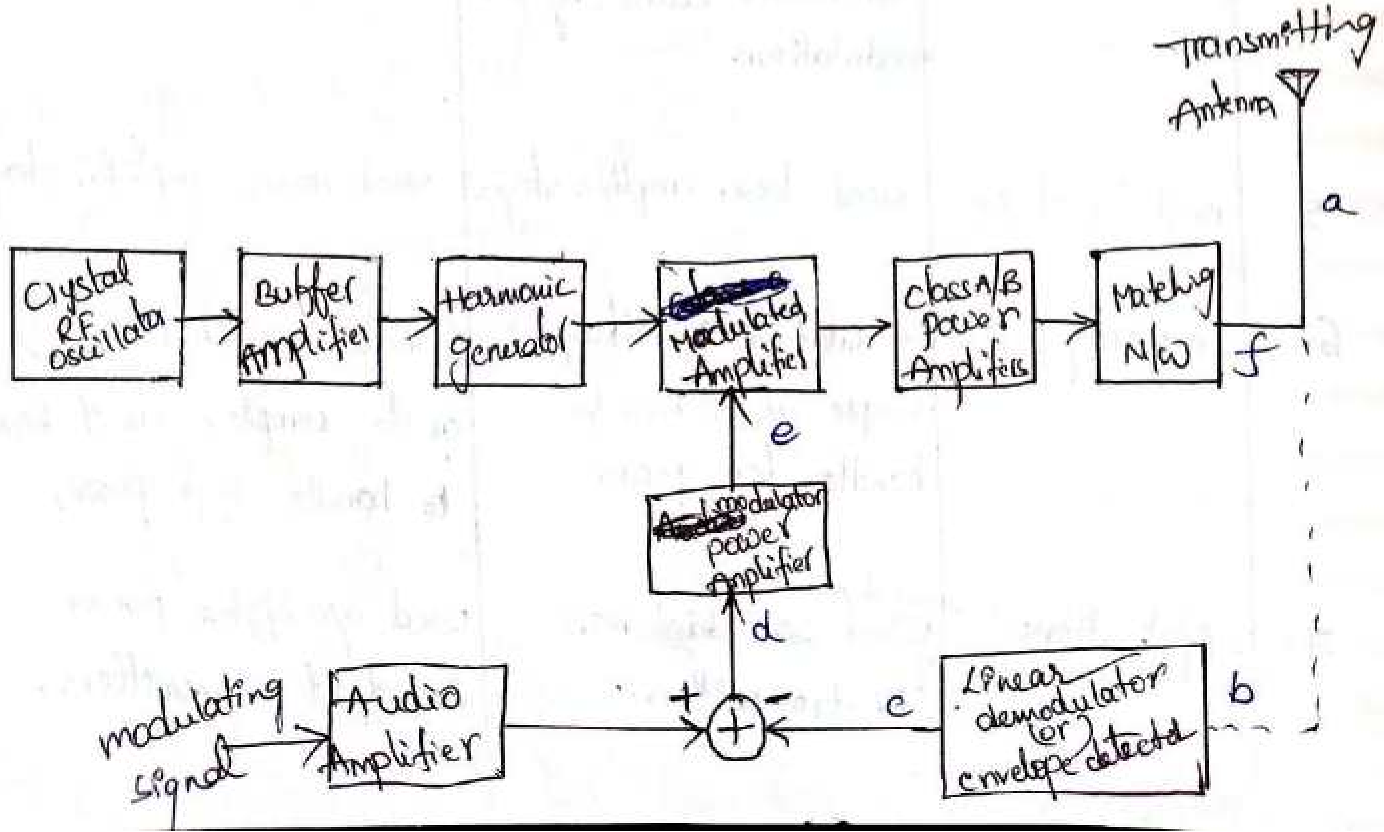
S.No	Parameter	Low Level Modulation	High Level Modulation
1.	Power Level	Modulation is carried out at low power level	Modulation is carried out at high power level.
2.	Audio power	Low audio power is required to produce modulation	High power is required to produce modulation.
3.	power efficiency	After modulation, linear amplifiers can only be used. This gives low power efficiency.	Before the modulation also amplifiers are used. This leads to high power efficiency
4.	Amplifiers used	Linear amplifiers such as class A amplifiers are used because all stages must be capable of handling amplitude variations caused by the modulation.	High efficient class 'C' amplifiers are used.
5.	Amplifier stages	Need lesser amplifier stages	Need more amplifier stages
6.	complexity.	Modulation circuitary is simple as it has to handle low power.	Modulation circuitary is quite complex as it has to handle high power.
7.	Applications.	Used ~~as higher~~ sometimes in TV transmitters.	used as higher power broadcast transmitters.

Effect of Feedback on performance of AM Transmitter

* ~~these~~ Before going to discuss the feedback on perfomance of AM Transmitter, we have to know about class 'c' amplifiers.
* Class 'c' amplifiers are having high efficienty but it has narrower B.W. Because of this narrower B.W, it will be cutting off sidebands, so there is a chance of distortion.
* class 'c' amplifiers cannot be used for the reproducing proper modulating signal
* Any amplifiers should posses the sufficient bandwidth to amplify the signals.
* To avoid ~~of~~ these drawbacks, we go for negative feedback in low level transmitters

Negative Feedback in Low level Transmitters

* These amplifiers are used in AM broadcast transmitters with view to improve the performance.



~~FM Transmitter~~

* This negative feedback reduces the distortion in a class c modulator system. Reduces the distortion means, it linearizes the output of the class c modulator.

* the AM signal fed to the antenna should ideally have its envelope.

* Extracting the message signal from the modulated signal (a) i.e the distorted envelope.

* this distorted signal is given to envelope detector, it detects the message signal with distorted output.

* this distorted output and original message signal is given to the summation.

* this distorted signal is subtracted from the original message signal.

* Therefore perfect message signal is given to the modulator power amplifier, finally it fed to the Antenna.

FM Transmitters

* The FM transmitter is a low power transmitter which uses frequency modulated waves for transmitting the sound, and this transmitter transmits the audio signals through the carrier wave by the difference of frequency

(OR)

* the FM transmitter where the frequency of the carrier signal is varied inaccordance with the modulating signal.

* These FM transmitters are used for
  → Radio broadcasting in VHF & UHF
  → TV sound broadcast

* FM frequency range is 88 to 108 MHZ

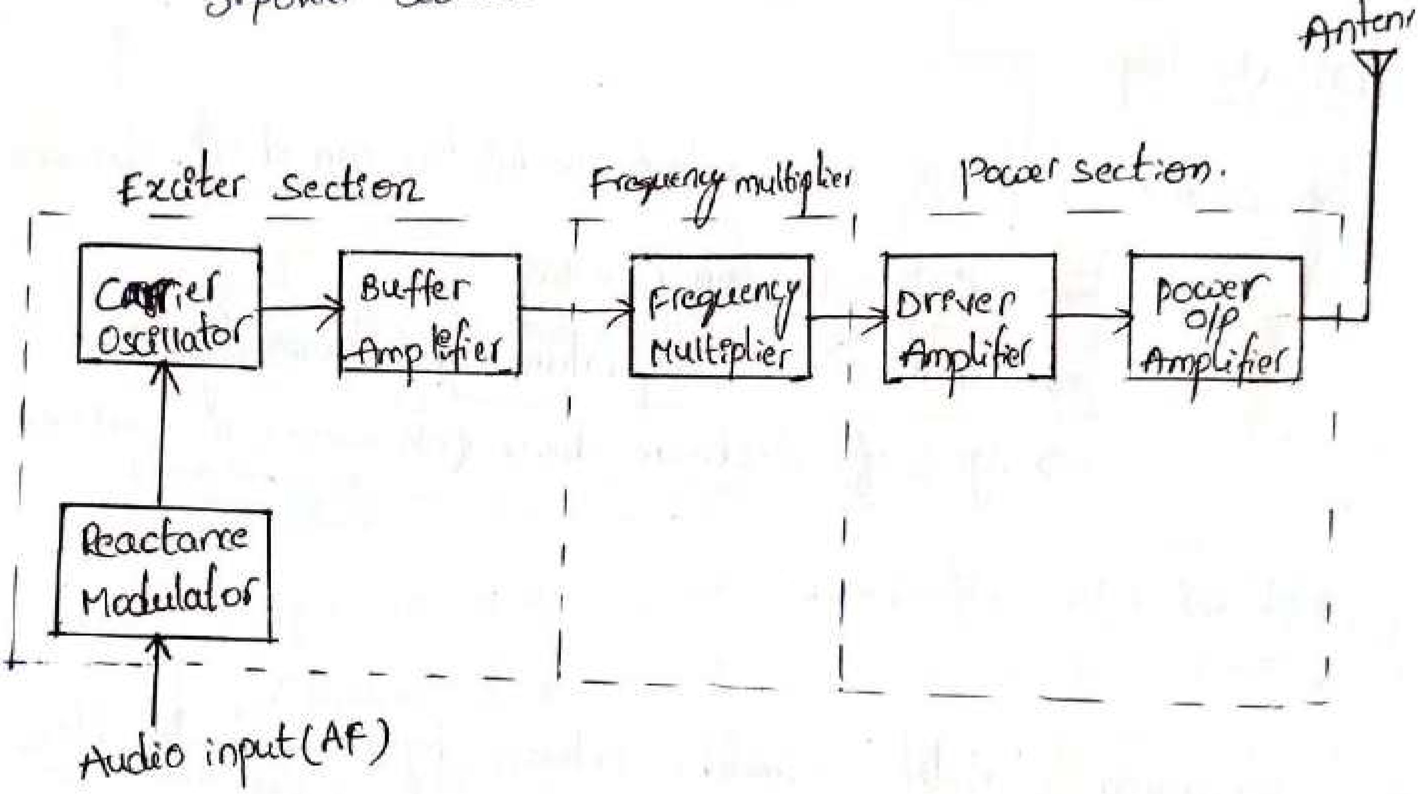
Types of FM Transmitters

* Based ~~upon~~ the modulation used, there are two types of FM transmitters.

1. Directly Modulated FM Transmitter
2. Indirectly Modulated FM Transmitter.

1. Directly Modulated FM Transmitter

* The transmitter is mainly having three blocks
    1. Exciter section
    2. Frequency multiplier.
    3. power section



* Exciter section consists of Reactance modulator, carrier oscillator, Buffer Amplifier.

Reactance Modulator:-
* Modulating signal (AF signal) is given to the reactance modulator, o/p of R.M given to carrier oscillator. In that reactance modulator, capacitance changes, then frequency changes by applying AF signal
* Carrier oscillator:- If capacitance changes in the Reactance modulator, this o/p connected to carrier oscillator, then frequency changes in the carrier oscillator.
* Buffer amplifier:- It is used to protect the backward flow of current, that is to maintain the same frequency without any variations.

Frequency multiplier:- It is used to increase the carrier frequency by using 'n' number of multipliers. But it should not be in decimal numbers (1.5, 2.5 ---), it should be in integers (2, 3, 4 - - -).

* Power section consists of Driver amplifier and power amplifier.

Driver Amplifier:- It is used to amplify the received signal.

power amplifier:- It is used to increase the power level of the modulated signal.

Disadvantage

* carrier frequency of reactance modulator can drift because of
  - ⇒ variation in supply voltage
  - ⇒ variation in temperature and humidity
  - ⇒ Aging of electronic device (old devices, it is not used for sometimes)

* It has a low efficiency.

* so required stability can be achieve by using "Automatic Frequency control" (AFC) circuit.

Frequency stability in FM Transmitter (AFC circuit):-

Master oscillator:- It is a LC oscillator, It generates the carrier frequency.

Reactance Modulator:- According to the input variations (modulating signal) we find that frequency of oscillations changes.

Buffer:- It isolates the master oscillator frequency.

Limiter:- It controls the amplitude variations of FM signal.

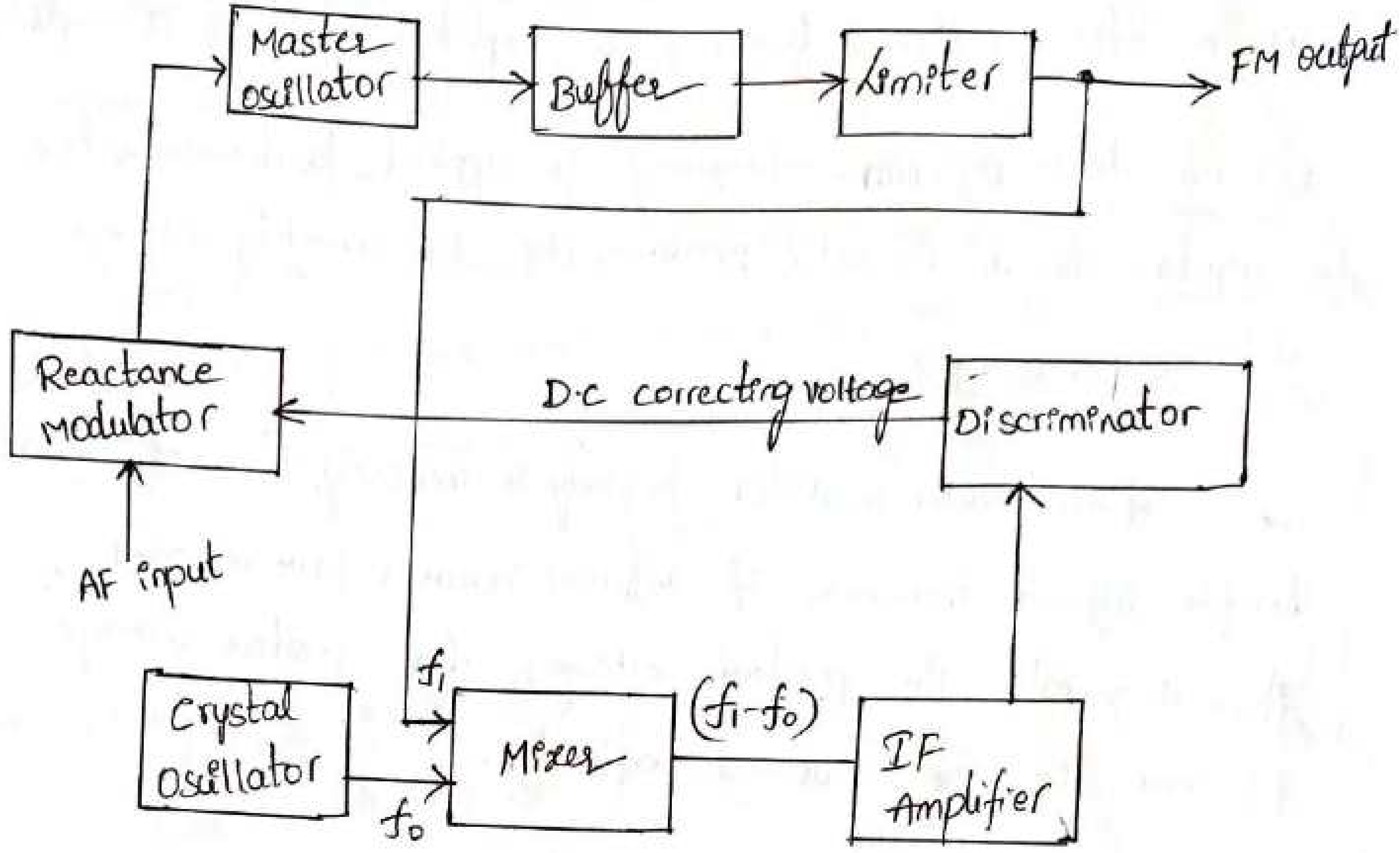


Fig:- Block diagram of AFC circuit

* When we use a LC oscillator (master oscillator), this carrier frequency can drift (sometimes it is larger, or lesser than actual frequency) due to supply variation, temperature changes etc. therefore to avoid this variation, we use AFC circuit

* crystal oscillator:- It generates the fixed frequency, this frequency is same as master oscillator frequency (f_0).

* Mixer:- It has two inputs, one is crystal oscillator and another input is FM output (f_1).

Mixer can produce summer and difference frequency. Here we are using difference frequency. therefore output of mixer is $(f_1 - f_0)$.

This difference is find out the drift in the master oscillator.

IF Amplifer :- Difference frequency is amplified by using IF amplifier

Discriminator :- Difference frequency is applied to discriminator, therefore output of the discriminator produce the DC correcting voltage.

For example

* If the master oscillator frequency is increasing, then f_1 increasing, therefore difference increases, if difference increases (+ve voltage), it will generate the positive voltage, this positive voltage is given to the Reactance modulator.

$$\boxed{C = g_m r_c}$$

$$\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$$

* During the +ve voltage, transconductance (gm) of reactance modulator increases, then capacitance (C) increases, If 'c' increases, then frequency decreases.

$$\boxed{+ve \rightarrow g_m\uparrow\ C\uparrow \rightarrow f\downarrow}$$

* similarly if the master oscillator freq is decreases, then f_1 decreases, therefore difference decreases, if difference decreases, discriminator output will generate the negative d.c voltage,

* During the -ve voltage, gm of reactance modulator decreases, then 'c' decreases, If 'c' decreases, then frequency increases.

$$\boxed{-ve \Rightarrow g_m\downarrow\ C\downarrow \rightarrow f\uparrow}$$

→ Let f_0 (actual frequency) = 90MHz

Master oscillator drift in higher i.e

$f_1 = 100MHz$

→ $f_1 - f_0 = 100MHz - 90MHz = 10MHz$

→ discriminator generates +ve d.c voltage, given to reactance modulator

$g_m \uparrow, C \uparrow \rightarrow f \downarrow$ (It means that, by using AFC circuit, it is going to reduce the freq, because master oscillator drift in higher)

→ similarly f_0 (actual frequency) = 90MHz

Master oscillator frequency drift in lower i.e

$f_1 = 70MHz$

→ $f_1 - f_0 = 70 - 90 = -20MHz$

→ discriminator generates -ve d.c voltage, given to the reactance modulator,

g_m also decreases $\downarrow$, $C \downarrow \rightarrow f \uparrow$

2. Indirectly Modulated FM Transmitter (or) phase modulated FM Transmitter (or) Armstrong FM transmitter

This topic explanation is same as generation of FM topic (Indirect method), in 3rd unit.

Radio Receiver

* A Radio Receiver is defined as the electronic equipment which picks up the desired signal, then rejects the unwanted signal and amplifies the desired signal and then demodulates the modulated signal to get the original signal.

Functions of Receiver

* Receiver separates the message signal from the carrier signal.
* It reconstructs actual signal using output transducers
* It is a device that converts electromagnetic signals into sound signals.
* It will amplify the reconstructed ~~original message~~ signal also

Classifications of Receiver

. AM Broadcast Receiver

FM Broadcast Receiver

TV Receiver

Communication Receiver

Radar Receiver

AM Broadcast Receiver:- It is mainly used for speech and music signals. It is used in long wave, short wave & medium wave bands (freq ranges)

FM Broadcast Receiver :- It is used for receiving the broadcast programmes, from FM broadcast transmitters, which will be operating at VHF & UHF's.

TV Receiver:- It is used for television broadcasting purpose. we can use at VHF & UHF's. frequency ranges.

communication Receiver:- For reception of codes, landline telephone etc

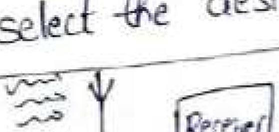
Radar Receiver:- It is used for radio detection, satellite communication, long distance communication.

Receiver characteristics

These receiver characteristics are define the performance and quality of receiver. of the receiver. There are 3 types

1. selectivity
2. sensitivity
3. Fidelity.

1. selectivity :- The ability of the receiver to select the desired signal.



For example

98.0 MHz $\boxed{98.3MHz}$ 98.6 MHz

you want to listen this frequency.

selectivity depends on tuned Circuit.

Here 98MHz and 98.6 MHz are very close to 98.3MHz. this selectivity selects the 98.3MHz and rejects the other freq's i.e 98MHz & 98.6 MHz

$Q = \frac{f_c}{BW}$

if BW ↓ Q ↑ selectivity ↑

if BW ↑ Q ↓ selectivity ↓

Sensitivity:- The ability of the receiver to detect the weakest signals and amplify it.

* It depends on gain of amplifier stages.

For example

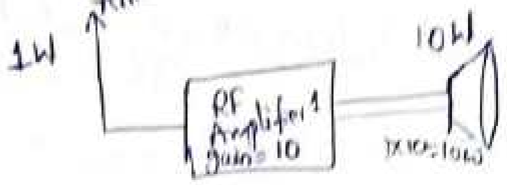


fig ①

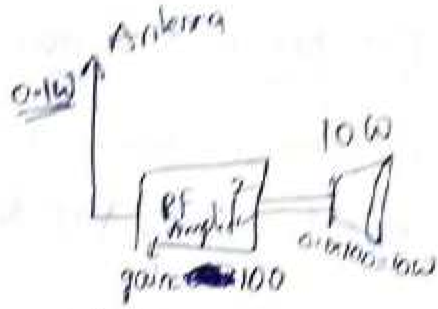


fig ②

* In both fig's, fig ② is weakest signal i.e 0.1W. therefore the receiver can able to detect this weakest signal, is called sensitivity

* similarly gain is more means sensitivity is more.

Fidelity:- The ability of the receiver to reproduce faithfully all the frequency components present in the ~~of~~ message signal (baseband signal)

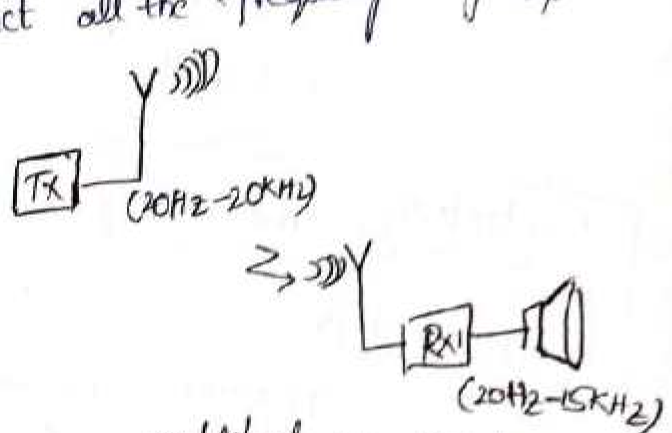
* To avoid distortion in received signal, fidelity should be high.

For example

Voice signal = 300Hz - 4.5KHz

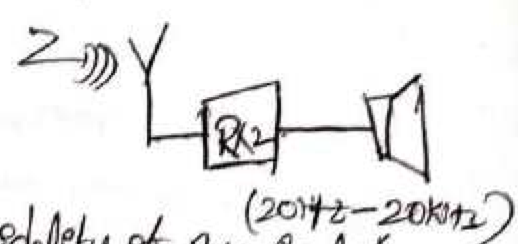
Audio signal = 20Hz - 20KHz

Receiver is able to extract all the frequency signals.



* If the fidelity of the receiver is high, then clarity of reproduced audio signal will also be high

Fidelity of Rxer1 is low



Fidelity of Rx2 is high

Classification of Radio Receiver:-

* The main function of a Receiver is to select the desired station and reject all other stations, and then demodulate it.

* Based on the construction of receiver are of two types.

1. Tuned Radio Frequency (TRF) Receiver
2. Super Hetrodyne (SHD) Receiver.

*Receivers are also classified based on modulation techniques use

(a) AM Receiver → TRF
                              → SHD

(b) FM Receiver → TRF
                              → SHD

* As per guidelines of FCC (Federal communication commission) by U.S government.

AM

Carrier frequency : 540KHz – 1650KHz

AM Bandwidth : 10KHz

Intermediate frequency (I.F) : 455 KHz

FM

Carrier Frequency : 88MHz – 108MHz

FM Bandwidth : 200KHz

Intermediate Frequency (I.F) : 10.7MHz.

AM Receivers

1. **Tuned Radio Frequency (TRF) Receiver** : The main function of a receiver is a proper selection and rejection for a TRF receiver selection and rejection is carried by RF amplifier which is a Tuned Amplifier.

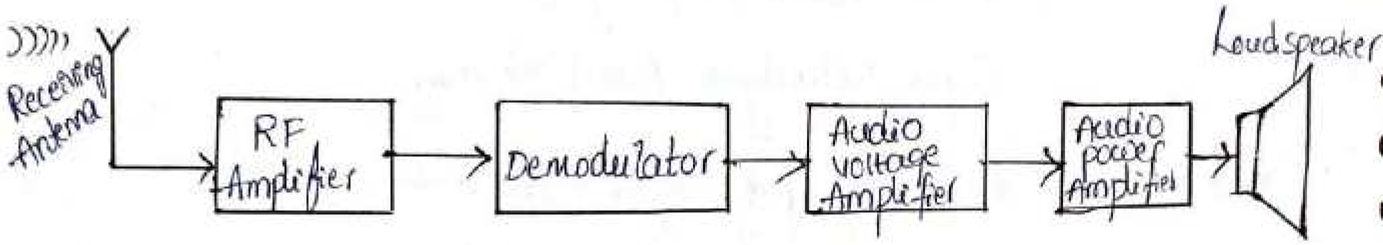
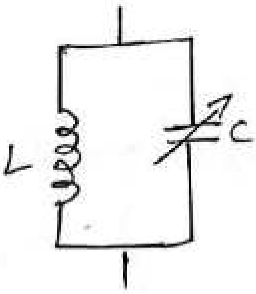


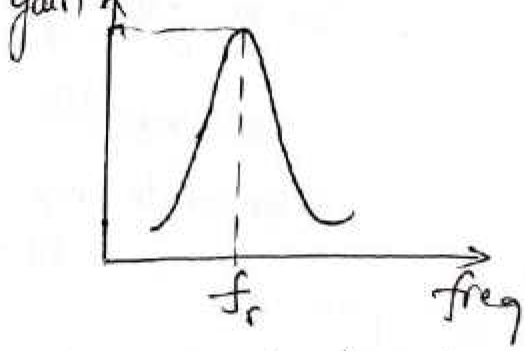
fig: Block diagram of TRF Receiver

Receiving Antenna:- It converts from EM waves into electrical signals.

RF Amplifier:- RF amplifier operation depends on Tuned ckt.

this tuned circuit is used to select the desired signal rejects the unwanted signals. Tuned circuit is a combination of capacitor and Inductor.





If varying 'C', then the frequency will come at desired frequency. At that desired frequency only, maximum gain will be occurred.

$$\boxed{f_1 = \frac{1}{2\pi\sqrt{LC}}}$$

maximum Q-factor is 80

$$\boxed{\text{Quality factor } Q = \frac{f_r}{B.W}}$$

Demodulator:- It extracts the message signal from the modulated signal.

Audio voltage Amplifier:- It increases the gain of voltage. It is a RC coupled amplifier.

Audio power Amplifier:- It increases the current, therefore the power increases. It is a class B pushpull Amplifier.

Loudspeaker:- It converts into sound waves.

Example

Case(1):- If we select the desired frequency is 600 KHz, then

Freq range: 540 KHz - 1650 KHz

AM B.W = 10 KHz

$$Q = \frac{f_r}{B.W}$$

$$Q = \frac{600\,KHz}{10\,KHz} = 60$$

$\boxed{Q = 60}$ ⇒ * This Q-factor value is less than 80 i.e within the limits only Q is occured.

* Therefore the circuit is easy.

Case(2):- If we select the desired frequency is 1600 KHz, then

$$Q = \frac{f_r}{B.W}$$

$$Q = \frac{1600\,KHz}{10\,KHz} = 160$$

$\boxed{Q = 160}$ ⇒ * This Q factor value is very much greater than 80. max. Q factor is 80 only but its Q value is 160. ~~the Q factor is~~

∴ the circuit is very complex.

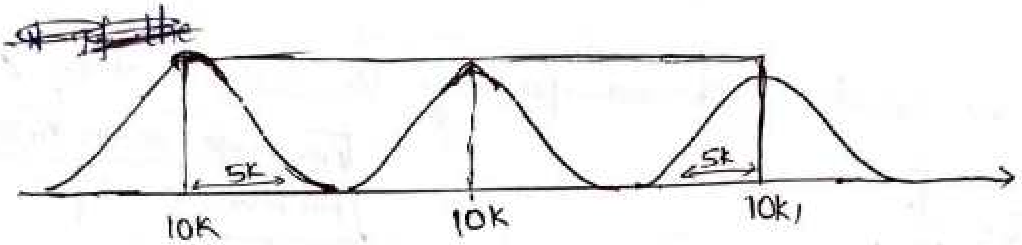
Case(3):-

If $Q=80$, $f_r = 1600^{KHz}$, then

$$B.W = \frac{f_r}{Q}$$

$$B.W = \frac{1600}{80} = 20\,KHz$$

As per the guidelines of FCC, B.W is 10KHz, but we got 20KHz..



$10+5k+5k=20k$.

* Here we are receiving three signals rather than one signal, therefore it will get crosstalk.

* If the bandwidth of tuned amplifier is $>$ 10KHz, then the undesired frequency component will be allowed i.e crosstalk.

Case (4):-

If $Q=150$, $f_r = 600\,KHz$, then

$$B.W = \frac{f_r}{Q}$$

$$B.W = \frac{600\,KHz}{150}$$

$$\boxed{B.W = 4\,KHz}$$

* As per the guidelines of FCC, B.W is 10KHz, but we got 4KHz.

* If the Bandwidth of tuned amplifier is $<$ 10KHz, then desired frequency component will be attenuated

Advantages of TRF Receiver

1. Simplest type of receiver since it does not involve mixing and IF operation.
2. very much suitable to receive single frequency.
3. It has good sensitivity.

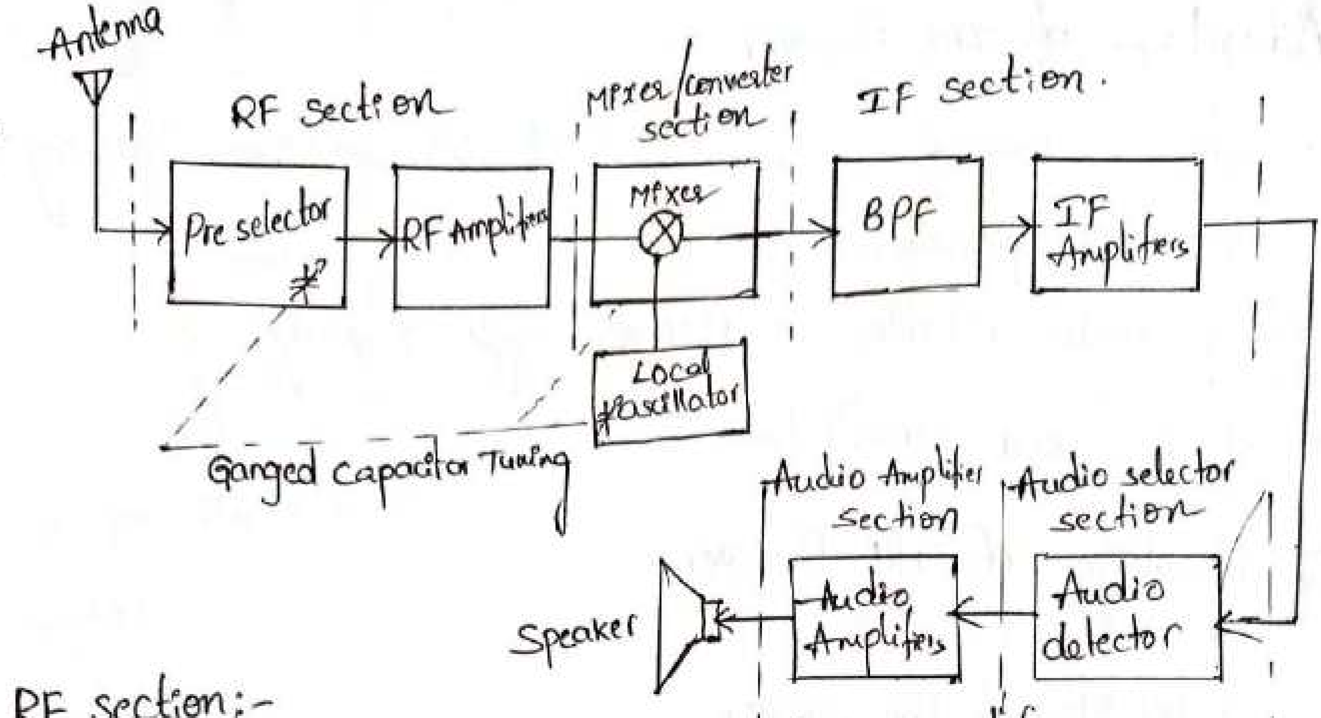
Disadvantages of TRF Receiver

1. Instability of the receiver
2. Insufficient selectivity at high frequencies and poor adjacent channel rejection. (poor selectivity)
3. Bandwidth variation over the tuning range.

To avoid drawbacks of TRF Receiver, we are using super hetrodyne Receiver.

Super Hetrodyne Receiver:-

* Hetrodyne : To mix two frequencies together in a non-linear device (or) to transmitt one frequency to another using non-linear mixing.
* It is also known as frequency conversion (or) frequency mixing, it is high frequency down converted to low frequency (I.F).
* A super hetrodyne receiver converts all incoming radio frequency (RF) signals to a lower frequency known as an intermediate frequency (I.F)



RF section:-

* It consists of a preselector and an amplifier.
* pre-selector is a broad-tuned bandpass filter with an adjustable center-frequency used to reject unwanted radio frequency i.e image frequency and to reduce the noise B.W.
* RF Amplifier determines the sensitivity of the receiver and a predominant factor in determining the noise figure for the receiver. (It raised the signal at desired level)

Mixer/converter section:-

* It consists of a radio frequency (Local oscillator) oscillator and a mixer
* choice of oscillator depends on the stability and accuracy is desired
* mixer is a non-linear device to convert radio frequency to intermediate frequencies (i.e hetrodyning process)
* The shape of the envelope, the bandwidth and the original information contained in the envelope remains unchanged although the carrier and sideband frequencies are translated from RF to IF

IF section:-

* It consists of a series of IF amplifiers and Bandpass filters to achieve most of the receiver gain and selectivity.
* The IF is always lower than the R.F because it is easier and less expensive to construct high gain, stable amplifiers for low frequency signals.
* ~~IF amplifiers are~~ IF amplifiers are less likely to oscillate (no change) than their RF counterparts.

Detector section:-

* To convert the IF signals back to the original source information.

Audio Amplifier section

* comprises several cascaded audio amplifiers and one or more speakers.

(I) How image frequency (unwanted frequency) is present in super Heterodyne Rec

Let Radio frequency

$f_{RF_1} = 600 KHz$

To get 455 KHz, ~~we~~ f_L must be ($f_{L_1} = 1055$ KHz)

$\therefore f_{I.F} = f_{L_1} - f_{RF_1}$

$= 1055 - 600$

$\boxed{f_{I.F_1} = 455 KHz}$ → Case (1).

$f_{RF_2} = 1000$ KHz

Here $f_{L_2} = 1455$ KHz

$\therefore f_{I.F_2} = 1455 - 1000$

$\boxed{f_{I.F_2} = 455 KHz}$ → Case (2)

$f_{RF_3} = 1510$ KHz

If f_L is tuned at 1055 KHz

$f_{I.F} = f_L - f_{RF}$

$= 1055 - 1510$

$f_{I.F_3} = -455$ KHz

$\boxed{f_{I.F_3} \approx 455 KHz}$ → Case (3)

$\boxed{\cos(-\theta) = \cos\theta}$

In case (3), actual f_L is 1965 KHz but for 1055 KHz also, we get 455 KHz (Intermediate freq)

* case (1) & case (3), both are ~~getting~~ having same local oscillator frequency. Case (1) is actual frequency, Case (3) is image frequency.

∴ super hetrodyne receiver is suffers from image frequency.

II) How to avoid Image frequency

* To avoid this image frequency, RF pre-selector (Gang capacitor is used) is used before mixing and also should select f_L must be greater than f_{RF}.

$$f_{I.F} = f_L - f_{R.F} \longrightarrow (1)$$

$$f_L = f_{I.F} + f_{R.F} \longrightarrow (2)$$

From eq(1)

$$f_{R.F} = f_L - f_{I.F}$$

Image frequency :- It is sum of radio frequency & twice of intermediate frequency.

$$f'_{R.F} \text{ (or) } f_{im} = f_L + f_{I.F}$$

$$f'_{RF} = (f_{I.F} + f_{R.F}) + f_{I.F}$$

$$\boxed{f'_{RF} = f_{R.F} + 2f_{I.F}}$$

why $f_L > f_{R.F}$ = ?

In TRF

Let Lower band f_{min} = 550 KHz

Higher band (max) = 1650 KHz

Ganged Tuned capacitor

$$\frac{C_{max}}{C_{min}} = \left(\frac{f_{max}}{f_{min}}\right)^2 = \left(\frac{1650}{550}\right)^2 = \boxed{9:1}.$$

In S.H.R

Case (i) : $f_L > f_{R.F}$

$f_{I.F} = f_L - f_{R.F}$

$\boxed{f_L = f_{I.F} + f_{R.F}}$

$$\therefore \frac{C_{max}}{C_{min}} = \left(\frac{(f_{RF} + f_{IF})_{max}}{(f_{RF} + f_{IF})_{min}}\right)^2$$

$$= \left[\frac{1650 + 455}{550 + 455}\right]^2 = \boxed{4.4:1} \text{ for } f_L > f_{R.F}$$

this ratio is good

Case (ii) $f_L < f_{R.F}$

$f_{I.F} = f_{RF} - f_L$

$\boxed{f_L = f_{RF} - f_{IF}}$

$$\therefore \frac{C_{max}}{C_{min}} = \left(\frac{(f_{RF} - f_{IF})_{max}}{(f_{RF} - f_{IF})_{min}}\right)^2$$

$$= \left[\frac{1650 - 455}{1650 - 455}\right]^2 = \left[\frac{1195}{95}\right]^2 = \boxed{158:1} \text{ for } f_L < f_{R.F}$$

As comparing TRF & S.H.R

TRF → 9:1 → not good as compared to S.H.R. → cost is low → ckt is simp

SHR → Case (i) → 4.4:1 → this ratio is good ⇒ cost is hig

Case (ii) → 158:1 → It is not available in market ⇒ ckt is very compl

$\boxed{\therefore f_L \text{ must be greater than } f_{R.F}}$

(III) Why 'Q' should not be very less

IFRR (Image Frequency Rejection Ratio) : It is defined as the ratio of gain at the signal frequency to the gain at the image frequency.

$$IFRR = \alpha = \frac{\text{Gain at the signal frequency}}{\text{Gain at the image frequency}}$$

α is mathematically expressed as

$$\boxed{\alpha = \sqrt{1+Q^2\rho^2}}$$

Q → quality factor

$$\boxed{\rho = \frac{f'_{im}}{f_{RF}} - \frac{f_{RF}}{f'_{im}}}$$

* If Q decreases, α decreases, then Gain at the signal frequency decreases, therefore it increases the gain at the image frequency it means that, it is difficult to reject the image frequency.

* If $Q \uparrow \Rightarrow \alpha \uparrow$, then $f_{RF} \uparrow$, therefore f'_{im} (image frequency) reduces, it means that it is easily reject the image frequency.

<u>Example</u>

If $f_{RF} = 600$, $f'_{im} = f_{RF} + 2 f_{IF}$

$= 600 + 2(455)$

$\boxed{f'_{im} = 1510 \text{ KHz}}$

AM B.W
540-1650KHz

If $f_{RF} = 800$, $f'_{im} = f_{RF} + 2 f_{IF}$

$= 800 + 2(455)$

$\boxed{f'_{im} = 1710}$

From Above examples, if $f_{RF} = 600$, image frequency is present (it is in the range of 540 to 1650). If $f_{RF} = 800$, image frequency is not present, because $f_{im} = 1710$, it is not in the range b/w 540-1650KHz

(IV) Why f_{IF} (Intermediate frequency) should not be very less

If $f_{I.F} = 50$, $f_{RF} = 800KHz$

$$f_{R.F} = f_L - f_{I.F}$$

$$f_L = f_{R.F} + f_{I.F}$$

$$f_L = 800 + 50$$

$$\boxed{f_L = 850\,KHz}$$

$$f'_{im} = f_{R.F} + 2f_{I.F}$$

$$f'_{im} = 800 + 2(50)$$

$$\boxed{f'_{im} = 900KHz}$$

If $f_{I.F} = 455$, $f_{RF} = 800KHz$

$$f_L = f_{R.F} + f_{I.F}$$

$$= 800 + 455$$

$$f_L = 1255\,KHz$$

$$f'_{im} = f_{R.F} + 2f_{I.F}$$

$$= 800 + 2(455)$$

$$\boxed{f'_{im} = 1710\,KHz}$$

+ From above examples, if I.F is very low, then image frequency will present.
+ If I.F is 455, then image frequency will not present.

(V) Q should not be very less, should not be very high

$$Q = \frac{f_r}{B.W}$$

In TRF

Let At lower band = 550KHz

At higher band = 1650KHz

$\rightarrow Q = \frac{550}{10K} = \boxed{55}$

$\rightarrow Q = \frac{1650}{10K} = \boxed{165}$

Q is in between 55 to 165

In S.H.R :- Frequency is converted into intermediate frequency (I.F) = 455KHz

$\therefore Q = \frac{455}{10K} = 45.5$

$\therefore$ Q is in between 40 to 50

$\therefore$ In SHR, Cost is lower compared to TRF. therefore selectivity also very good.

Advantages of Super Hetrodyne Receiver

* High sensitivity and selectivity
* High adjacent channel rejection
* stability is improved
* High gain
* Uniform Bandwidth is used due to fixed intermediate frequency.

Applications

All radio and TV receivers operate on the principle of superhetrodyning method.

Comparison between TRF Receiver & Super hetrodyne Receiver

S.NO	TRF Receiver	Super Hetrodyne Receiver.
1.	No frequency conversion	Frequency conversion
2.	NO I.F (Intermediate freq) frequency.	Downconvert RF signal to lower IF frequency.
3.	Instability, variation in B.W and poor selectivity due to high frequencies.	No instability, ~~no variat~~ uniform B.W is used due to fixed Intermediate frequency (I.F)
4.	Difficult to design tunable RF stages.	Main amplification takes place at IF.
5.	Rarely used	mostly used.

Problems

1. For a AM Superhetrodyne Receiver with IF & RF frequencies of 465 KHz and 950 KHz respectively. Determine local frequency, image frequency and IFRR for 'Q' of 80

Sol

(i) Local oscillator frequency $f_L = f_{R.F} + f_{I.F}$

$$= 950K + 465K$$

$$\boxed{f_L = 1415\,KHz}$$

(ii) Image frequency $f_{im} = f_{RF} + 2 f_{I.F}$

$$= 950K + 2(465)$$

$$\boxed{f_{im} = 1880\,KHz}$$

(iii) IFRR $= \alpha = \sqrt{1 + Q^2 \rho^2}$

$\boxed{Q = 80}$

$$\rho = \frac{f_{im}}{f_{RF}} - \frac{f_{RF}}{f_{im}}$$

$$\rho = \frac{1880}{950} - \frac{950}{1880}$$

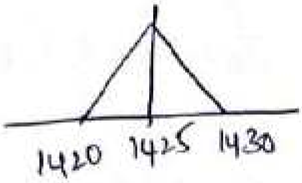
$$\boxed{\rho = 1.47}$$

$\therefore$ IFRR $= \alpha = \sqrt{1 + (80)^2 (1.47)^2}$

$$\boxed{\alpha = 117.6}$$

2. In a Super hetrodyne receiver, the input AM signal has a center frequency of 1425KHz and B.W 10KHz. The input is down converted to 455KHz. what is the image frequency

Sol Center frequency is 1425 KHz



$\therefore f_{RF} = 1425 \pm 5K.$

Image frequency $f_{im} = f_{RF} + 2f_{I.F}$

$= (1425 \pm 5K) + 2(455)$

$$\begin{array}{r} 1425 \\ +910 \\ \hline 2335 \end{array}$$

$f_{im} = (2335 \pm 5K)$

$\therefore$ The image frequency lies in the range of 2330KHz to 2340KHz.

3. when Super hetrodyne receiver is tuned to 500KHz, its local oscillator input to the mixer is 1015KHz. What is the image frequency?

Sol $f_{RF} = 500KHz$

$f_L = 1015KHz$

$f_{I.F} = f_L - f_{R.F}$

$= 1015 - 500$

$f_{IF} = 515KHz$

Image frequency

$f_{im} = f_{RF} + 2f_{I.F}$

$= 500 + 2(515)$

$\boxed{f_{im} = 1530KHz}$

4. For an AM Superhetrodyne receiver with I.F, RF & local oscillator frequencies of 410KHZ, 600KHZ and 1010KHZ respectively Determine image frequency and IFRR for a preselector 'Q' of 100.

Sol

$f_{R.F} = 600\,KHZ$

$f_{I.F} = 410\,KHZ$

$f_L = 1010\,KHZ$

$Q = 100$

Image frequency $f_{im} = f_{R.F} + 2 f_{I.F}$

$= 600 + 2(410)$

$$\boxed{f_{im} = 1420\,KHZ}$$

$$IFRR = \sqrt{1 + Q^2 \rho^2}$$

$Q = 100$

$$\rho = \frac{f_{im}}{f_{R.F}} - \frac{f_{RF}}{f_{im}}$$

$$= \frac{1420}{600} - \frac{600}{1420}$$

$$= 2.36 - 0.42$$

$$\boxed{\rho = 1.93}$$

$$\therefore IFRR = \sqrt{1 + (100)^2 (1.93)^2}$$

$$= \sqrt{1 + (10000)(3.7249)}$$

$$= \sqrt{37250}$$

$$\boxed{IFRR = 193}$$ for $Q = 100$

FM Receiver : Super hetrodyne Receiver

* FM Receiver is used for receiving frequency modulated signals.
* In FM, message signal is stored in the form of frequency variations and these frequency variations are little affected by channel noise, therefore FM transmission is very much of noise free.

* For FM : carrier frequency : 88MHz – 108MHz

  Bandwidth : 200kHz

  I.F : 10.7MHz

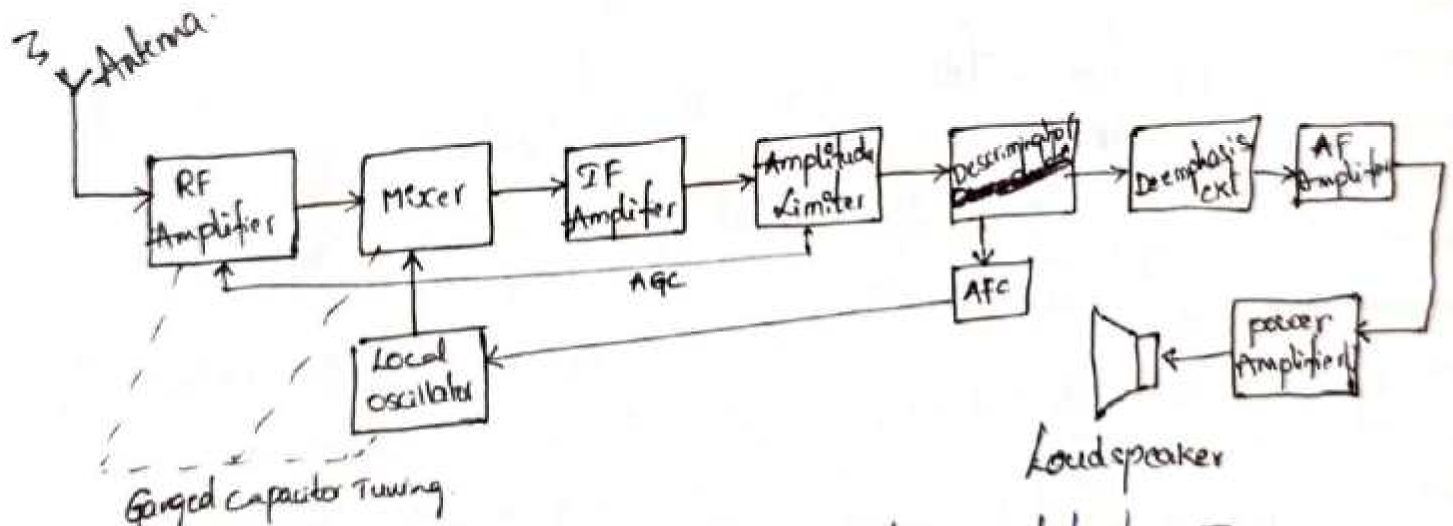


fig. Block diagram of FM Receiver of superhetrodyne Type.

* FM receiver block diagram is same as AM receiver, only difference is Amplitude limiter and De-emphasis Circuits are included in this FM receiver.

RF Amplifer:- It consists of RF tuner (parallel RLC circuit) to select a particular channel and RF amplifier stages to raise the signal level before it is fed to mixer stage.

This stage increases the sensitivity and signal to noise ratio of the signal

It is ganged with mixer and local oscillator for tuning simultaneously so that the I.F is always equal to 10.7 MHZ

Local oscillator:- Local oscillator frequency is 98.7 MHZ to 118.7 MHZ. It is always kept at high value than RF amplifier.

$$98.7 - 88 = 10.7 MHZ$$
$$118.7 - 108 = 10.7 MHZ$$

Mixer:- Hetrodyning is a process where all the high frequency components are beaten (or) brought down to an intermediate frequency level.

The output of mixer is intermediate frequency (I.F) = 10.7 MHZ.

Ganged capacitor Tuning:- Local oscillator, RF amplifier stages and mixer are having a common variable capacitor i.e Ganged capacitor If once this capacitor is tuned all the three circuits will tune in the same proportion such that the IF signal for FM is always 10.7 MHZ

IF Amplifier:- IF Amplifier is a multistage tuned class A amplifier which amplifies IF signals and there by provides

* High selectivity and stability
* overall gain of the receiver
* Better adjacent rejection ratio
* Bandwidth = 200KHZ.

Amplitude Limiter:-

* It is a Differential amplifier, it is used to remove any amplitude changes because in FM, amplitude must be kept constant.
* Amplitude variations may occur due to noise (or) interference introduced in the channel.

Discriminator:- is used for the purpose of detector.

* In FM, Discriminator is used and first converts the frequency variations into amplitude variations i.e FM to AM.
* It uses reactance modulator
* Then using detector circuit, it extracts the original signal.

Deemphasis circuit:- It is used in receiver, while the pre-emphasis circuit is used in transmitter.

* It is used to remove (or) reduce the extra boost given to high frequency audio frequency signal in the pre-emphasis circuit in transmitter before transmission.
* It is simple low pass filter with a cut-off frequency of 6.12 KHz to remove the noise

$$f = \frac{1}{2\pi RC}$$

AF and power Amplifier:- It is used to increases the strength of the AF signal, the o/p of AF amplifier cannot be given directly to the loudspeaker because of impedance and the power of the signal is low, therefore ~~it~~ amplifies the power in the power amplifier and then given to loudspeaker

Loudspeaker:- It converts from AF signal into soundwaves.

AGC (or) AVC:-

* If the received signals strength is very high then due to overload distortions occurs in detection.
* Also when the receiver is tuned from one station to another
* When ~~radio~~ radio receiver is moved from one place to another, ~~Variation in the~~ there is a variation in the signal strength, this variation causes either increase (or) decrease in the o/p signal strength of the receiver.

* In such circumstances, the gain of the receiver has to be adjusted every time. Hence AGC is necessary

→ If the received signal is high, then a large DC voltage will be generated across AGC, which is then fed to the IF, mixer and RF stages, This high voltage is inversely proportional to gain of the amplifiers, therefore decreases the overall gain of these stages. Thus AGC circuit controls the gain of the receiver automatically.

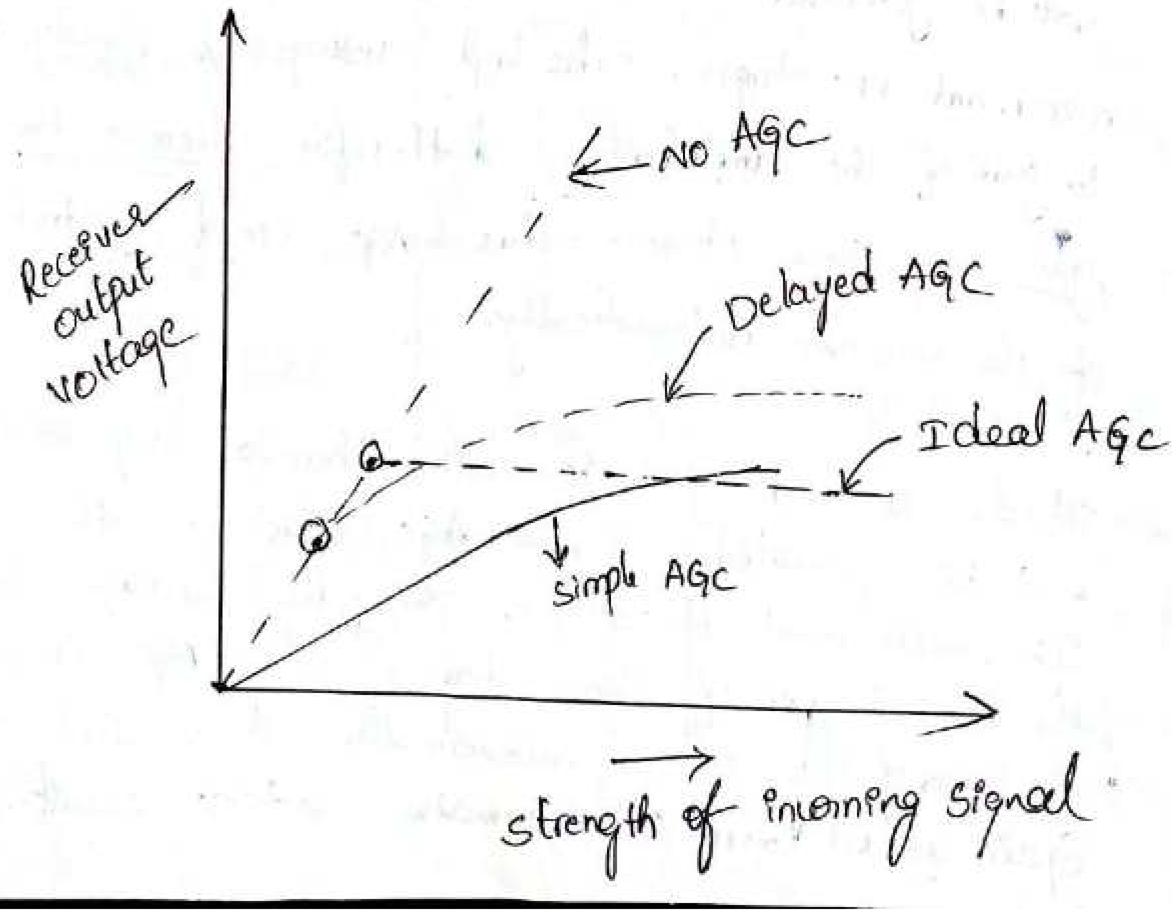
→ If the received signal is weak, then a very small d.c voltage will be generated across AGC, which is then fed to the IF, mixer and RF stages, this low voltage is increases the overall gain of these stages. Thus AGC circuit controls the gain of the receiver automatically, it is called Automatic Gain control circuit (or) Automatic voltage circuit.

Types of AGC

* signals receiving at receiver input are not of same strength. signals of strong station are strong and from weak stations are weak.
* ~~of receiver gain~~ The receiver output will fluctuate proportional to input signal strength, which is not desired.
* So AGC adjust, receiver automatically to have constant output irrespective of input signal strength.

Types

1. Simple AGC
2. Delayed AGC
3. Ideal AGC



Simple AGC :

* According to the strength of the incoming signal, there is a control on output signal.
* Irrespective of input signal, there is a some control on entire signal.
* gain of the amplifiers are proportional to the strength of the signal.
* If the signal strength is low, gain of amplifiers are used, maintain a constant output by controlling the gain

Draw back

* If the signal strength is low i.e weak signal, it is already distorted, then the gain of the amplifier increases, it means that again its weak signal extra attenuated.

Advantages

* simplicity.
* Low cost.

Applications

* Low cost domestic radio receivers

No AGC:- Receiver circuit does not have an AGC mechanisam. It is not desired one.

Ideal AGC:- It is similar to Delayed AGC. But In this case delay is not present.

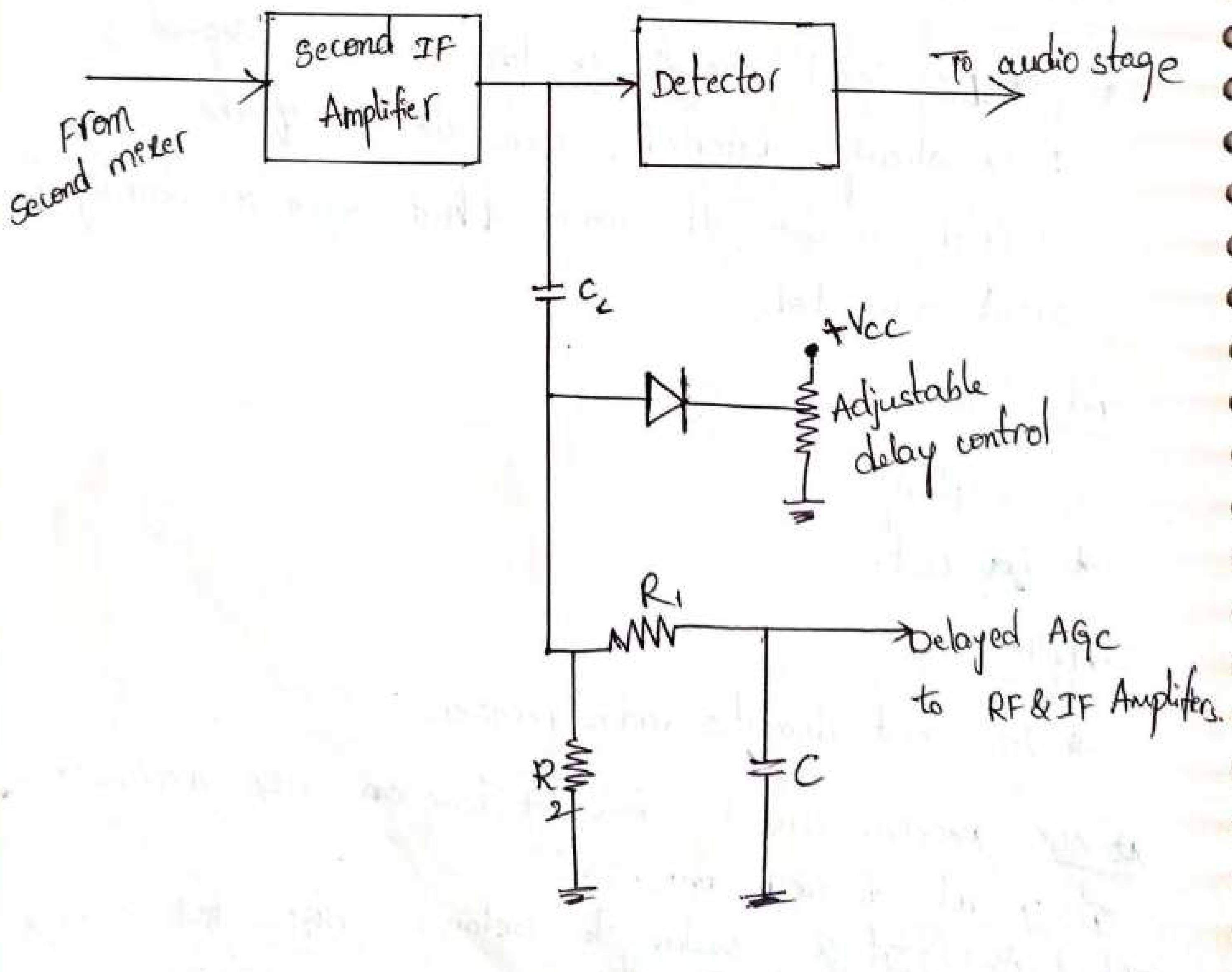
2. Delayed AGC

* In this case, AGC bias is not applied until the input signal strength reaches a predetermined level

* After this point (pre determined level) is reached, AGC bias is applied like simple AGC but more strongly.

* Thus reducing receiver gain for weak signals is avoided.

Operation of Delayed AGC



* Here an adjustable positive bias is applied to cathode of AGC diode.

* Anode of diode is connected to output of last IF Amplifier through coupling capacitor C_c.

* R_1, R_2, C are filter circuit, it is used to filter out high frequency appearing at anode of AGC diode.

* When input signal is weak, then anode of AGC diode is less potential than cathode. So it is OFF., So AGC o/p is zero

* When input signal is strong, then the AGC diode is forward biased, then the LPF remove high frequency components, only DC output is present at the output.

* Thus the delayed AGC reduces the gain for strong signal and not for weak signal.

Advantage

* weak signals are not attenuated.

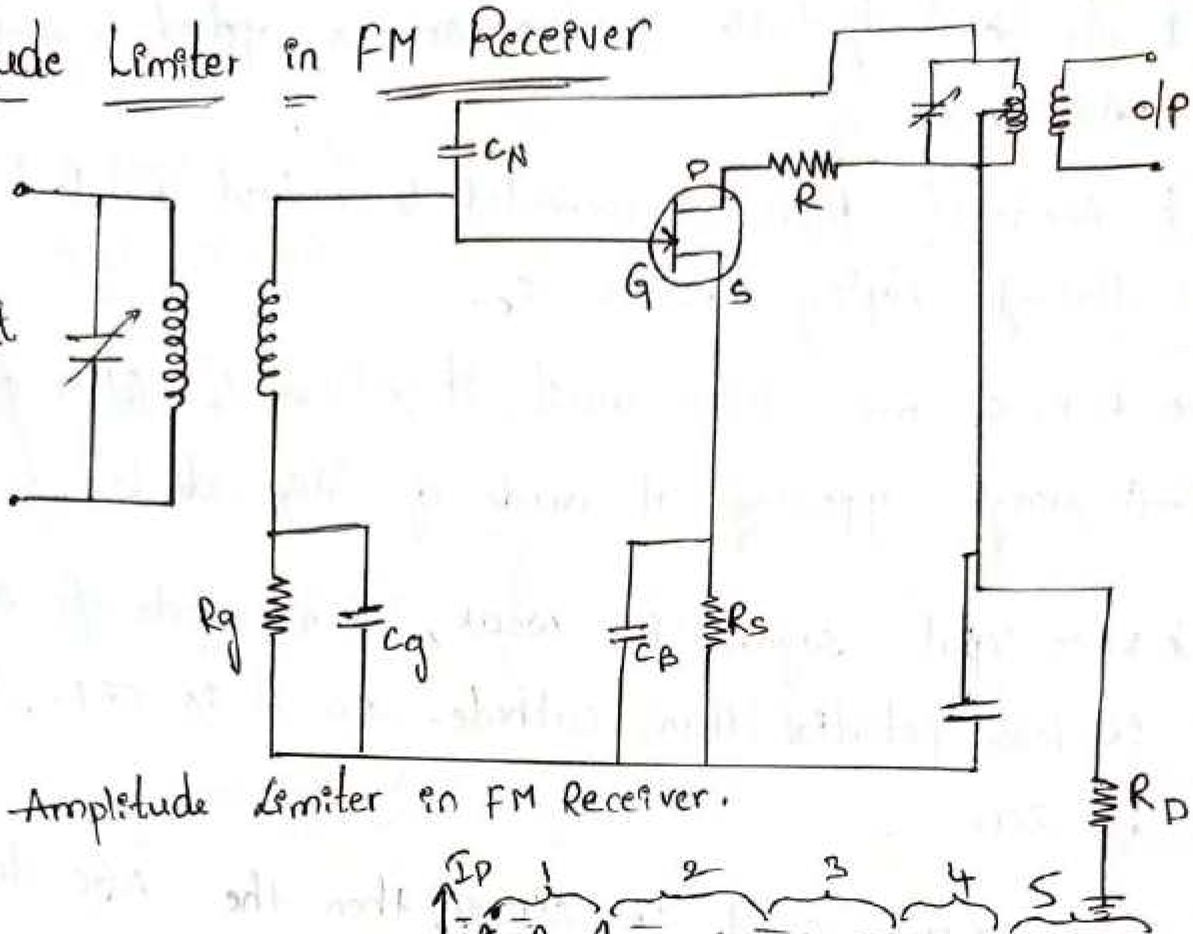
Disadvantage

* complex than simple AGC

Applications

- High quality communication receivers.

Amplitude Limiter in FM Receiver



fig(i) Amplitude Limiter in FM Receiver.

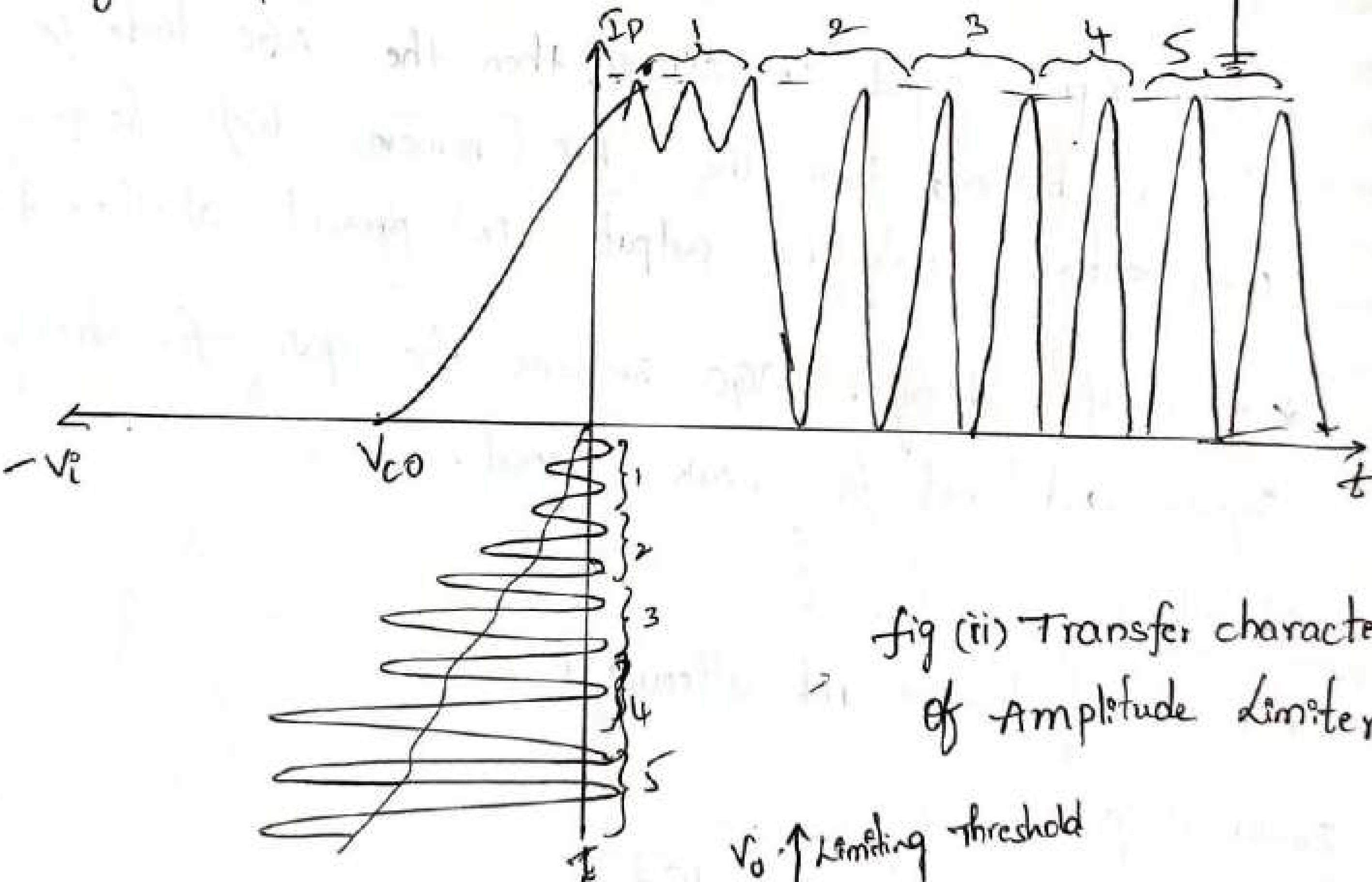


fig (ii) Transfer characteristic of Amplitude Limiter.

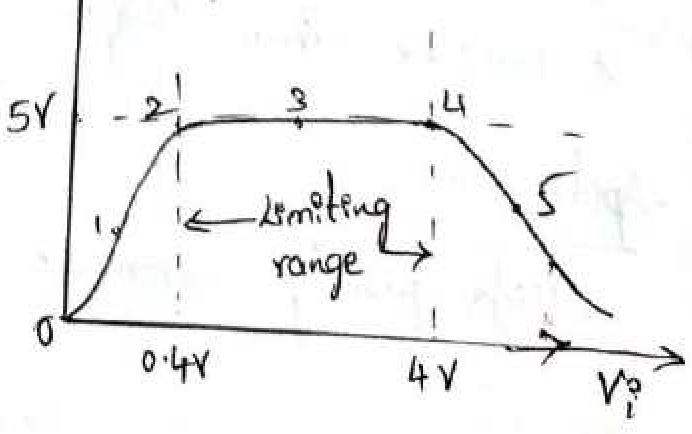


fig (iii) Input voltage versus output voltage.

* when the input voltage is very small, the output is proportional to the input.

* ~~when~~ In the circuit diagram, R_g, C_g parallel combination provides leak type bias

* R_D is the drain resistor, Drain supply voltage is dropped through this resistor R_D.

* when the input voltage is very high, leak bias current flows in the circuit, due to this leak bias, we have a negative voltage developed across the capacitor C_N, and also at the gate of the FET, therefore the output signal is clipped, i.e constant.

* Bias is increased in proportion to the size of the input voltage. It means that, if the input voltage increases, leak bias of the FET also increases.

* the amplitude limiter is achieved by leak bias process.

* If the leak bias is sufficient, then it goes to early saturation

* Early saturation of the output current is achieved by means of a low drain supply voltage. If V_{DD} is low, early saturation is occured. Due to ~~the~~ the R_D, current will be dropped. Gate to drain is forward biased under saturation condition. there is a short circuit between input and output, we have a small resistance developed across drain and tank (R)

* From fig ③, Below 0.4V and above 4 V is a non-limiting range. Below 4V, output is proportional to the input.
* Above 4V, it maintains the constant level upto 4V.
* If the limiting is not sufficient, we can use Double Limiter. Double limiter consists of two Amplitude limiters, these are connected in Cascade.
* Amplitude range between 0.4V to 20V in double limiter.

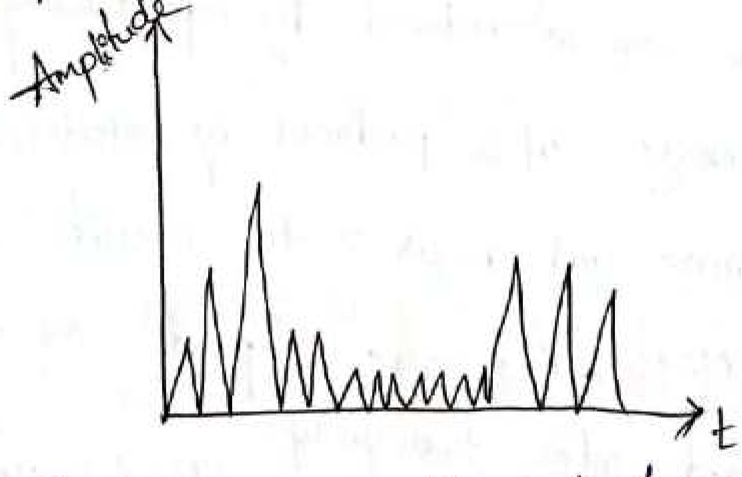
UNIT-V

Noise and pulse Modulation

Noise:-

Introduction:- In any communication system during the transmission of signal (or) receiver of signal, some unwanted signals get introduced into a communication system.

* This unwanted system degrades the quality of communicati[on] i.e unwanted system (or) meaning less quality is called Noise
* original message signal is added with noise i.e contaminated signal gives unwanted audio and video disturbances.
* In generally, the noise occurs at channel and receiver.
* the noise has no pattern and no amplitude and phase and it is a random in nature and unpredictable signal characteristics.
* the graphical view of Noise is



* Reduction of noise is possible but it can not be completely eliminat[ed]

Sources of Noise

Noise

- External Noise
- Internal Noise.

1. **External Noise**:- It is produced by external sources occurs at a stage of channels (or) medium. this external noise cannot be completely eliminated.

To avoid this noise simply by changing the location (or) place (or) to reduce some quantity of noise by using external devices like filters.

* The external noises are
  - (a) Natural (or) Atmospheric (or) static Noise
  - (b) Manmade (or) Industrial Noise
  - (c) Extra Terrestrial Noise (or) space Noise.

(a) **Natural Noise**:- It is produced by a solar flares, electronic storms and radiation in space.

* this Noise can be reduced by repositioning the antenna

(b) **Manmade Noise**:- It is produced by electrical motors. i.e it make and break in the circuit.

(C) **space Noise**:- This noise depends on the sources, it is divided into two groups.

(i) solar :- It is a electrical noise emitting from sun under the radiation of the sun *

(ii) Cosmic Noise:- It is emitting from the outer space

(iii) Black body Noise:- This noise from stars.

2. Internal Noise :- This noise is present inherently within electronic equipment. It depends on the physical nature of materal used.

Internal Noises are

(a) Thermal Noise
(b) shot Noise
(c) partition Noise
(d) Flicker Noise
(e) Transit Noise.

(a) Thermal Noise:- It is a normal noise and it is generated internal circuitary of the system. The noise generated by the system is directly proportional to temperature of the system.

Random motion of free electrons due to thermal energy received by them.

Noise power $P \propto T$

$P \propto B.W(B)$

$\boxed{P \propto K.TB}$ Where K = Boltzman's constant

(b) shot Noise:- It is produced in amplifying devices like BJT, diode.

(c) Partition Noise:- due to random fluctuations when current divides in two (or) more parts.

Eg: Transistor.

Partition Noise in transistor is more than diode.

(d) Flicker Noise:- It is appear at below KHz range due to random fluctuation in carrier density and fluctuation in conductivity.

(e) Transit noise:- Time taken by the election to travel from emitter to collector.

~~free is~~ Noise is present due to random fluctuation in output current.

Signal to Noise Ratio (SNR):-

* It is defined as the ratio of signal power to noise power.

$$\boxed{\frac{S}{N} = \frac{P_S}{P_N}}$$

where
P_S = signal power
P_N = Noise power

* Interms of dB.

$$\boxed{\left(\frac{S}{N}\right)_{dB} = 10\log\left[\frac{P_S}{P_N}\right]}$$

* power interms of voltage

$$P_S = \frac{V_S^2}{R}, \quad P_N = \frac{V_N^2}{R}$$

$\therefore$ SNR will be $\left(\frac{S}{N}\right)_{dB} = 10\log\left(\frac{V_S^2/R}{V_N^2/R}\right)$

$$\left(\frac{S}{N}\right)_{dB} = 10\log\left[\frac{V_S}{V_N}\right]^2$$

$$\boxed{\left(\frac{S}{N}\right)_{dB} = 20\log\left[\frac{V_S}{V_N}\right]}$$

* High SNR is good for transmitter and Receiver.

~~Noise~~ Figure:-

* ~~It is defined as the ratio of SNR at input to the SNR at output~~

* signal to noise ratio at transmitter.

$$(SNR)_{i/p} = \frac{\text{Average power of modulating signal}}{\text{Average power of noise at input.}} = \frac{P_i}{P_{iN}}$$

* signal to noise ratio at Receiver

$$(SNR)_{o/p} = \frac{\text{Average power of demodulated signal}}{\text{Average power of noise at output}} = \frac{P_o}{P_{oN}}$$

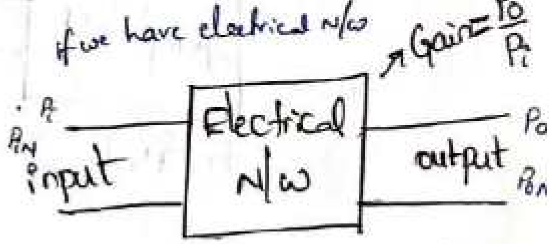
* signal to noise ratio at channel

$$(SNR)_{channel} = \frac{\text{Average power of modulated signal}}{\text{Average power of noise in message BW}}$$

Noise Figure :-

It is defined as the ratio of SNR at input to the SNR at output.

if we have electrical n/w



$$F = \frac{\text{SNR at input } \cancel{\text{output}}}{\text{SNR at output}}$$

$$F = \frac{P_i/P_{iN}}{P_o/P_{oN}} = \frac{P_i}{P_o} \cdot \frac{P_{oN}}{P_{iN}}$$

$$\text{Gain} = \frac{\text{o/p power}}{\text{i/p power}}$$

$$\boxed{F = \frac{P_{oN}}{G \cdot P_{iN}}}$$

Noise Temperature:-

* It is a temperature, which generates noise power in system.

Noise power by Amplifier is denoted by P_{Na}

$T_0 \rightarrow$ Environment Temp.

$T_{eq} \rightarrow$ Noise Temp.

* Noise power by amplifier

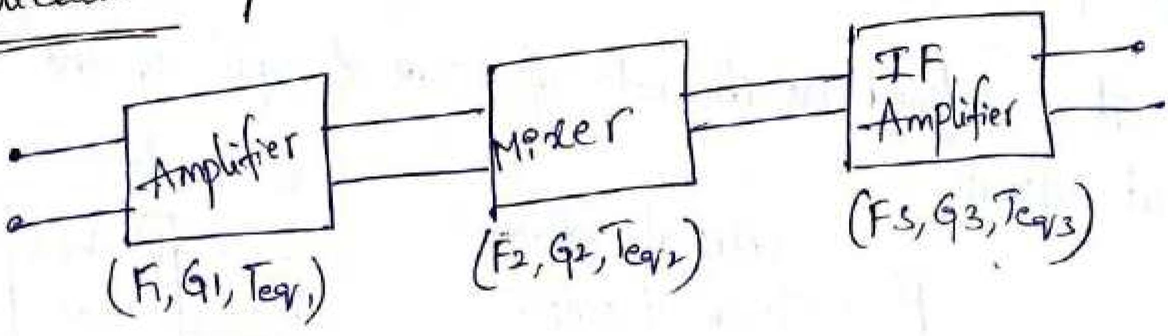
$$\boxed{P_{Na} = (F-1)\,K T_0 B.}$$

* Noise power in terms of noise temperature

$$K \cdot T_{eq} \cdot B = (F-1)\,K T_0 B$$

$$\boxed{T_{eq} = (F-1)T_0}$$

Equivalent Noise Temperature and Noise Figure in cascade communication system



Equivalent Noise Figure

$$F = F_1 + \frac{(F_2-1)}{G_1} + \frac{(F_3-1)}{G_1 G_2}$$

Noise Temperature $T_{eq} = (F-1)T_0$

Interms of T_{eq}

$$T_{eq} = T_{eq_1} + \frac{T_{eq_2}}{G_1} + \frac{T_{eq_3}}{G_1 G_2}$$

Problems

1. An Amplifier has a B.W of 4MHz with 10K as the input resistor. Calculate the RMS noise voltage at the input to this amplifier if the room temperature is 25°C

Sol:-

$B = 4MHz$
$R = 10K$
$T = 25°c = 25+273$
$= 298 kelvin$
$= 298K$
$K = 1.38 \times 10^{-23} J/K.$

R.M.S noise voltage

$$V_n = \sqrt{4KTBR}$$

$$V_n = \sqrt{4 \times 1.38 \times 10^{-23} \times 298 \times 4 \times 10^6 \times 10 \times 10^3}$$

$$\boxed{V_n = 25.64 \mu V}$$

2. Two resistors 20K, 50K are at room temperature 290K. Determine for the B.W of 100KHz, the thermal noise for the conditions.

(i) For each resistor
(ii) For two resistors in ~~parallel~~ series
(iii) For two resistors in parallel.

$R_1 = 20K$
$R_2 = 50K$
$T = 290K$
$B = 100KHz$

Sol

(ii) $R_S = R_1 + R_2 = 70k\Omega$

(iii) $R_P = \frac{R_1 R_2}{R_1 + R_2} = 14.28 k\Omega$

(i) For $\underline{20 K\Omega}$ $\Rightarrow V_n = \sqrt{4KTBR_1}$

$$V_n = \sqrt{4 \times 1.38 \times 10^{-23} \times 290 \times 100 \times 10^3 \times 20 \times 10^3}$$

$$\boxed{V_n = 5.66 \mu V}$$

For $50K\Omega$

$$V_n = \sqrt{4KTBR_2}$$

$$= \sqrt{4 \times 1.38 \times 10^{-23} \times 290 \times 100 \times 10^3 \times 50 \times 10^3} = \underline{\underline{8.95\,\mu V}}$$

→ For series $70K\Omega$

$$V_n = \sqrt{4KTBR_s}$$

$$= \sqrt{4 \times 1.38 \times 10^{-23} \times 290 \times 100 \times 10^3 \times 70 \times 10^3} = \underline{10.58\,\mu V}$$

→ For parallel ; $14.28\,K\Omega$

$$V_n = \sqrt{4KTBR_p}$$

$$= \sqrt{4 \times 1.38 \times 10^{-23} \times 290 \times 100 \times 10^3 \times 14.28 \times 10^3} = \underline{\underline{4.78\,\mu V}}$$

3. Calculate the thermal noise power available from any resistor at room temp 290K for a B.W 2MHz. Also calculate the corresponding noise voltage given that R=100

Sol

thermal noise power

$T = 290K$

$B = 2 \times 10^6$

$R = 100\Omega$

$K = 1.38 \times 10^{-23}\,J/K.$

$$P = KTB$$

$$= 1.38 \times 10^{-23} \times 290 \times 2 \times 10^6$$

$$\boxed{P = 8 \times 10^{-15}\text{ Watts}}$$

$$P = V^2/R$$

$$V = \sqrt{P \cdot R}$$

$$= \sqrt{8 \times 10^{-15} \times 100}$$

$$\boxed{V = 0.894\,\mu V}$$

$$\boxed{\begin{array}{l} V_{r.m.s} = \sqrt{4KTBR} \\ P = KTB \end{array}}$$

4. The signal power and noise power measured at input of an amplifier are 150μW and 1.5μW respectively. If the signal power at the output 1.5W and noise power is 40mW, Calculate ~~amplifier noise factor and~~ noise figure. 5

Sol.

$P_{si} = 150\mu W$

$P_{ni} = 1.5\mu W$

$P_{so} = 1.5W$

$P_{NO} = 40mW.$

$$(S/N)_{input} = \frac{P_{si}}{P_{ni}} = \frac{150\mu W}{1.5\mu W} = 100$$

$$(S/N)_o = \frac{P_{so}}{P_{no}} = \frac{1.5W}{40\times10^{-3}} = 37.5$$

$$\text{Noise } \cancel{\text{factor}}\ \text{figure} = \frac{(S/N)_i}{(S/N)_o} = \frac{100}{37.5} = 2.66$$

$$(F)_{dB} = 10\log(F) = 10\log(2.66) = \underline{4.26dB}$$

5. The signal to Noise ratio at the input of amplifier is 40dB if the noise figure of an amplifier is 20dB, calculate the signal to noise ratio at the amplifier output.

sol

$(SNR)_{i/p} = 40dB$

$(F)_{dB} = 20dB$

$(SNR)_{o/p} = ?$

$$(F)_{dB} = (SNR)_{i/p}(dB) - (S/N)_{o/p}(dB)$$

$$(SNR)_o(dB) = (SNR)_{i/p} - (F)_{dB}$$

$$= 40dB - 20dB$$

$$= 20dB$$

6) An amplifier has noise figure of 3dB. Determine its equivalent noise temperature.

Sol $(F)_{dB} = 3dB$

$(F)_{dB} = 10\log F$

$F = \text{Antilog}\left(\frac{(F)dB}{10}\right)$

$= \text{Antilog}\left(\frac{3}{10}\right)$

$= \text{Antilog}(0.3)$

$\boxed{F = 2}$ Noise factor

$T_{eq} = (F-1)T_0$

$= (2-1)T_0$

Room Temp (T_0) = 300K.

$\therefore T_{eq} = (2-1)300$

$\boxed{T_{eq} = 300K}$

7) An amplifier with 10dB noise figure and 4dB power gain is cascaded with a second amplifier which has a 10dB power gain and 10dB noise figure. What is the overall noise figure and power gain.

Sol $F_1 = 10dB = 10$

$G_1 = 4dB = 2.5$

$F_2 = 10dB$.

interms of magnitude $F_2 = 10$

$G_2 = 10dB = 10$

$F = F_1 + \frac{(F_2-1)}{G_1}$

$= 10 + \frac{9}{2.5}$

$F = 13.6$

interms of dB $\therefore F = 10\log(13.6) = 11.33\,dB$

$\therefore G = G_1 G_2$

$= 2.5 \times 10$

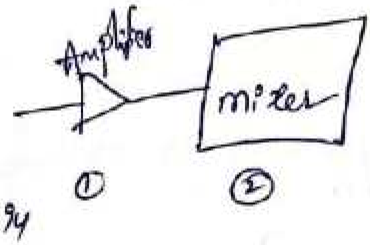
$G = 25$

$G = 10\log(25)$

$G = 13.9\,dB$.

8) A mixer stage has a noise figure of 20dB and if is preceded by an another amplifier with a noise figure of 9dB and an available power gain of 15dB. Calculate noise figure.

Sol



$F_1 = 9dB$

Antilog of (F_1) is $\frac{9}{10} = 0.9$ antilog(0.9) = 7.94

~~For~~ $F_1 = 7.94$

$G_1 = 15dB = \underline{31.62}$

$F_2 = 20dB = 100$

$$F = F_1 + \frac{(F_2 - 1)}{G_1}$$

$$= 7.94 + \frac{(100-1)}{31.62}$$

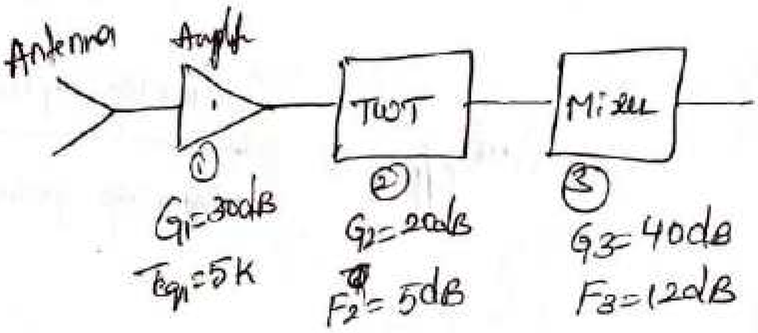
$$F = 11.67$$

In terms of dB

$$F = 10\log(11.67)$$

$$\boxed{F = 10.44\,dB}$$

9) calculate equivalent noise figure and noise temperature for given diagram



Sol $T_{eq_1} = (F_1 - 1)T_0$

$$F_1 = 1 + \frac{T_{eq1}}{T_0}$$

$$F_1 = 1 + \frac{5}{300}$$

$$F_1 = 1.02$$

$F_2 = 5\,dB$

its magnitude $F_2 = 4$

$F_3 = 12\,dB = 16$

$G_1 = 30\,dB = 1000$

$G_2 = 20\,dB = 100$

$G_3 = 40\,dB = 10^4$

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2}$$

$$F = 1.02 + \frac{(4-1)}{1000} + \frac{(16-1)}{(1000)(100)}$$

$F = 1.02315$

Interms of dB

$F = 10\log(1.02315)$

$$\boxed{F = 0.099\,dB}$$

$$T_{eq} = (F-1)T_0$$
$$= (1.02315 - 1)\,300$$

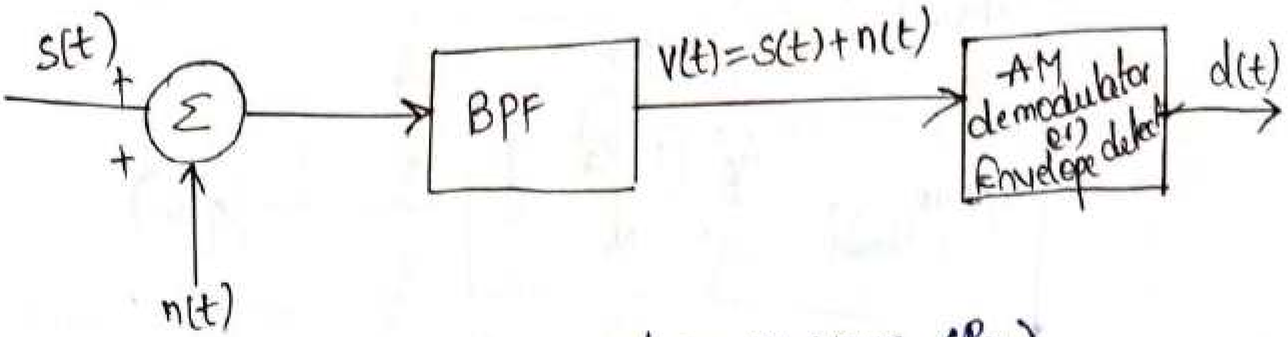
$$\boxed{T_{eq} = 6.945\ K}$$

Signal to noise ratio in Amplitude modulated (AM) Signal (or) system

$$\text{Figure of Merit} = \frac{(SNR)_{o/p}}{(SNR)_{channel}} \longrightarrow (1)$$

$$(SNR)_{channel} = \frac{\text{Average power of modulated signal } (P_{sc})}{\text{Average power of noise in message B.W } (P_{NC})} \longrightarrow (2)$$

$$(SNR)_{o/p} = \frac{\text{Average power of demodulated signal } (P_{so})}{\text{Average power of noise at o/p. } (P_{NO})} \longrightarrow (3)$$



Average power of modulated signal (or) AM wave. (P_{sc})

$$s(t) = A_c\left[1+K_a m(t)\right]\cos 2\pi f_c t$$

$$s(t) = A_c \cos 2\pi f_c t + A_c K_a m(t)\cos 2\pi f_c t$$

Average power is

(or) $P_{sc} = \left(\frac{A_c}{\sqrt{2}}\right)^2 + \left(\frac{A_c K_a m(t)}{\sqrt{2}}\right)^2$

$$= \frac{A_c^2}{2} + \frac{A_c^2 K_a^2 m^2(t)}{2}$$

$$= \frac{A_c^2}{2} + \frac{A_c^2 K_a^2 P}{2}$$

$$= \frac{A_c^2(1+K_a^2 P)}{2}$$

$$\boxed{P_{sc} = \frac{A_c^2}{2}\left[1+K_a^2 P\right]} \longrightarrow (4)$$

$P_{sc} = E\left[A_c^2(1+K_a m(t))^2 \cos^2\omega_c t\right]$

$= E\left[A_c^2(1+K_a^2 m^2(t) + 2K_a m(t))\cos^2\omega_c t\right]$

$= E\left[A_c^2\cos^2\omega_c t + A_c^2 K_a^2 m^2(t)\cos^2\omega_c t + 2K_a m(t)\cos^2\omega_c t\right]$

$= E\left[\frac{A_c^2}{2}[1+\cos 2\omega_c t]\right] + E\left[\frac{A_c^2 K_a^2 m^2(t)}{2}(1+\cos 2\omega_c t)\right]$

$+ E\left[2K_a m(t)\cos^2\omega_c t\right]$

$= \frac{A_c^2}{2} + E\left[\frac{A_c^2 K_a^2 m^2(t)}{2}\right]$

$+ 2K_a E[m(t)]E[\cos^2\omega_c t]$

$= \frac{A_c^2}{2} + \frac{A_c^2 K_a^2 E[m^2(t)]}{2}$

$= \frac{A_c^2}{2} + \frac{A_c^2 K_a^2 P}{2}$

$= \frac{A_c^2}{2}\left[1+K_a^2 P\right]$

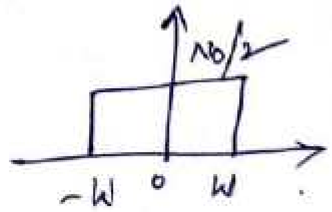
Average power of message signal is denoted as P.

$\therefore P = m^2(t)$

$E[\cos 2\omega_c t] = 0$

$E[m(t)] = 0$

Average power of noise in message B.W : (P_{NC})



$$P_{NC} = \frac{N_0}{2}\cdot(B.W)$$

$$P_{NC} = \frac{N_0}{2}\cdot(2W)$$

$$\boxed{\therefore P_{NC} = WN_0} \longrightarrow (5)$$

eq (4) & (5) substituted in eq (2)

$$(SNR)_{channel} = \frac{\frac{A_c^2}{2}\left[1+K_a^2 P\right]}{W N_0}$$

$$\boxed{(SNR)_{channel} = \frac{A_c^2\left[1+K_a^2 P\right]}{2WN_0}} \longrightarrow (6)$$

* s(t) and n(t) is passed through B.P.F

$$\therefore v(t) = s(t) + n(t) \longrightarrow (7)$$

We know that

$$s(t) = A_c\left[1+K_a m(t)\right]\cos 2\pi f_c t$$

$$n(t) = n_I(t)\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t$$

$$\therefore v(t) = A_c\left[1+K_a m(t)\right]\cos 2\pi f_c t + n_I(t)\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t$$

$$v(t) = \left[A_c + A_c K_a m(t) + n_I(t)\right]\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t$$

This v(t) is passed through envelope detector

$$\therefore d(t) = \sqrt{\left[A_c + A_c K_a m(t) + n_I(t)\right]^2 + \left[n_Q(t)\right]^2}$$

$$\therefore d(t) \approx \underset{\text{DC component}}{A_c} + \underbrace{A_c K_a m(t)}_{\text{signal}} + \underbrace{n_I(t)}_{\text{noise}} \longrightarrow (8)$$

∴ Average power of demodulated signal (P_{so}) is

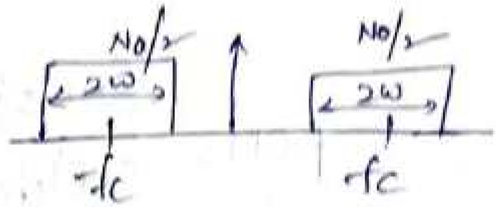
$$P_{so} = \left(\frac{A_c K_a m(t)}{\sqrt{2}}\right)^2$$

$$\boxed{P_{so} = \frac{A_c^2 K_a^2 P}{2}} \longrightarrow (9)$$

Average power of noise at output (P_{no}) is

$$P_{no} = \left(\frac{n_I(t)}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2} \cdot n_I^2(t)$$



$$= \frac{1}{2} \cdot 2\omega N_0$$

$$\boxed{P_{no} = \omega N_0} \longrightarrow (10)$$

$$\Rightarrow \frac{N_0}{2} \cdot (2\omega) + \frac{N_0}{2}(2\omega)$$

$$\Rightarrow 2 \frac{N_0}{\cancel{2}} \cancel{2}\omega$$

$$\Rightarrow 2\omega N_0$$

∴ eq (9) & (10) substituted in eq (5)

$$\therefore (SNR)_{o/p} = \frac{\frac{A_c^2 K_a^2 P}{2}}{\omega N_0}$$

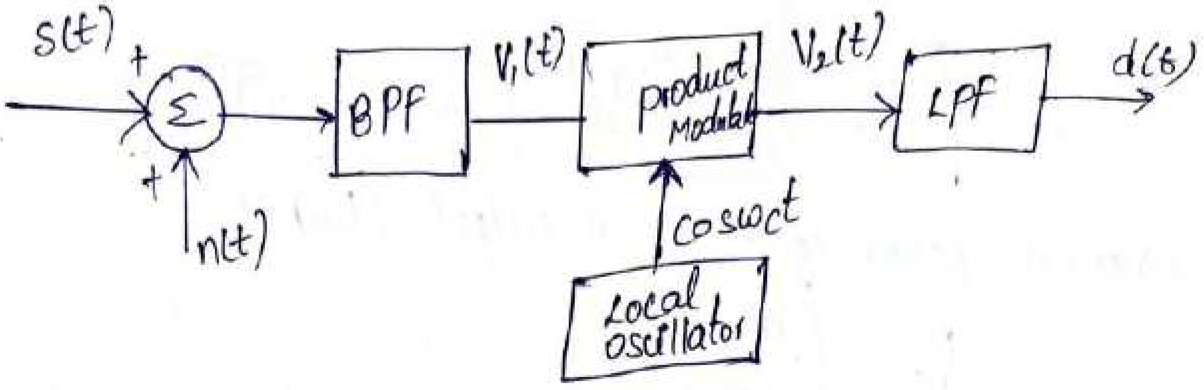
$$\boxed{(SNR)_{o/p} = \frac{A_c^2 K_a^2 P}{2\omega N_0}} \longrightarrow (11)$$

∴ eq (6) & (11) are substituted in eq (1)

$$\text{Figure of Merit (FOM)} = \frac{\frac{A_c^2 K_a^2 P}{2\omega N_0}}{\frac{A_c^2 [1 + K_a^2 P]}{2\omega N_0}}$$

$$\boxed{FOM_{AM} = \frac{K_a^2 P}{1 + K_a^2 P}}$$

Signal to Noise Ratio in DSB-SC System



$$FOM = \frac{(SNR)_{o/p}}{(SNR)_{channel}} \longrightarrow (1)$$

$$(SNR)_{channel} = \frac{P_{SC}}{P_{NC}} \longrightarrow (2)$$

$$(SNR)_{output} = \frac{P_{SO}}{P_{NO}} \longrightarrow (3)$$

Average power of DSB-SC signal (P_{sc})

$$s(t) = A_c m(t) \cos\omega_c t$$

Average power is

$$P_{sc} = \left(\frac{A_c m(t)}{\sqrt{2}}\right)^2$$

$$P_{sc} = \frac{A_c^2 m^2(t)}{2}$$

$$P_{sc} = \frac{A_c^2 P}{2} \longrightarrow (4)$$

(or) $P_{sc} = E\left[A_c^2 m^2(t) \cos^2\omega_c t\right]$

$$= E\left[A_c^2 m^2(t)\left[\frac{1+\cos 2\omega_c t}{2}\right]\right]$$

$$= \frac{A_c^2}{2} E\left[m^2(t)\right] + E\left[\cos 2\omega_c t\right]\frac{A_c^2 m^2(t)}{2}$$

$$= \frac{A_c^2}{2} E\left[m^2(t)\right]$$

$$P_{sc} = \frac{A_c^2}{2} P$$

$\boxed{E[\cos\omega_c t] = 0}$

(or) Average power of $s(t) = \frac{1}{2\pi}\int_0^{2\pi} s^2(t)\,dt$

$$= \frac{1}{2\pi}\int_0^{2\pi}\left(A_c m(t)\cos\omega_c t\right)^2 dt$$

$$= \frac{A_c^2 m^2(t)}{2\pi}\int_0^{2\pi}\cos^2\omega_c t\,dt$$

$$= \frac{A_c^2 m^2(t)}{2\pi}\int_0^{2\pi}\frac{1+\cos 2\omega_c t}{2}\,dt$$

$$\Rightarrow \frac{A_c^2 m^2(t)}{2\pi}\int_0^{2\pi}\frac{1}{2}\,dt + \int_0^{2\pi}\frac{\cos 2\omega_c t}{2}\,dt$$

$$= \frac{A_c^2 m^2(t)}{2\pi}\cdot\frac{2\pi}{2} + 0$$

$$= \frac{A_c^2 m^2(t)}{2} \Rightarrow \frac{A_c^2 P}{2}$$

Average power of noise in message B.W (P_{NC})

$$P_{NC} = \frac{N_0}{2} \cdot (B.W)$$

$$= \frac{N_0}{2}(2W)$$

$$\boxed{P_{NC} = WN_0} \longrightarrow (5)$$

eq (4) & eq (5) substituted in eq (2)

$$(SNR)_{channel} = \frac{\frac{A_c^2 P}{2}}{WN_0}$$

$$\boxed{(SNR)_{channel} = \frac{A_c^2 P}{2WN_0}} \longrightarrow (6)$$

* s(t) & n(t) is passed through B.P.F

$$\therefore V_1(t) = s(t) + n(t) \longrightarrow (7)$$

we know that

$$s(t) = A_c\, m(t) \cos\omega_c t$$

$$n(t) = n_I(t) \cos\omega_c t - n_Q(t) \sin\omega_c t$$

$$\therefore V_1(t) = A_c m(t) \cos\omega_c t + n_I(t) \cos\omega_c t - n_Q(t) \sin\omega_c t$$

$$V_1(t) = [A_c\, m(t) + n_I(t)] \cos\omega_c t - n_Q(t) \sin\omega_c t$$

$V_1(t)$ is passed through product modulator, another i/p of product modulator is $\cos\omega_c t$.

$$\therefore V_2(t) = V_1(t) \cdot c(t)$$

$$V_2(t) = V_1(t) \cdot \cos\omega_c t$$

$$= [[A_c m(t) + n_I(t)] \cos\omega_c t - n_Q(t) \sin\omega_c t] \cos\omega_c t$$

$$= [A_c m(t) + n_I(t)] \cos^2\omega_c t - n_Q(t) \sin\omega_c t \cdot \cos\omega_c t$$

$$V_2(t) = [A_c m(t) + n_I(t)] \left[\frac{1+\cos 2\omega_c t}{2}\right] - \frac{n_Q(t)}{2} \cdot \sin 2\omega_c t$$

$[A_c m(t)+n_I(t)]\cos 2\omega_c t$

$V_2(t) = \frac{1}{2}[A_c m(t) + n_I(t)] + \cancel{\cos 2\omega_c t} - \frac{1}{2} n_Q(t) \sin 2\omega_c t$

– this $V_2(t)$ is passed through L.P.F.

$$\therefore d(t) = \underbrace{\frac{1}{2} A_c m(t)}_{\text{signal}} + \underbrace{\frac{1}{2} n_I(t)}_{\text{noise}} \longrightarrow (8)$$

Average power of demodulated signal (P_{S0}) is

$$P_{S0} = \frac{1}{2}\cdot\left(\frac{A_c m(t)}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2}\cdot\frac{A_c^2 m^2(t)}{2}$$

$$= \frac{1}{2}\cdot\frac{A_c^2 P}{2}$$

$$\boxed{P_{S0} = \frac{A_c^2 P}{4}} \longrightarrow (9)$$

$$P_{S0} = E\left[\frac{A_c^2 m^2(t)}{4}\right]$$

$$= E[m^2(t)]\,\frac{A_c^2}{4}$$

$$= \frac{A_c^2}{4}\cdot P.$$

(or)

$$P_{S0} = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{A_c m(t)}{2}\right]^2 dt$$

$$P_{S0} = \frac{1}{2\pi}\int_0^{2\pi}\frac{A_c^2 m^2(t)}{4}\, dt$$

$$= \frac{A_c^2 m^2(t)}{4\cdot 2\pi}\cdot 2\pi$$

$$= \frac{A_c^2 m^2(t)}{4}$$

$$\boxed{P_{S0} = \frac{A_c^2 P}{4}}$$

Average power of noise at output (P_{n0}) is

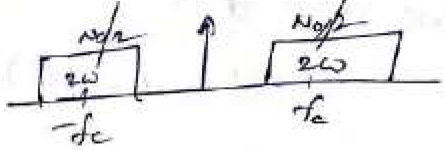
$$P_{n0} = \frac{1}{2}\cdot n_I(t)$$

$$= \frac{1}{2}\cdot\left(\frac{n_I(t)}{\sqrt{2}}\right)^2$$

$$= \frac{1}{4}\cdot n_I^2(t)$$

$$= \frac{1}{4}(2\omega N_0)$$

$$\boxed{P_{n0} = \frac{\omega N_0}{2}} \longrightarrow (10)$$



$$\frac{N_0}{2}(2\omega) + \frac{N_0}{2}(2\omega)$$

$$\Rightarrow 2\,\frac{N_0}{2}\,2\omega$$

$$\Rightarrow 2\omega N_0$$

eq (9) & (10) substituted in eq (3)

$$(SNR)_{output} = \frac{\frac{A_c^2 P}{4}}{\frac{\omega N_0}{2}}$$

$$\boxed{(SNR)_{o/p} = \frac{A_c^2 P}{2\omega N_0}} \longrightarrow (11)$$

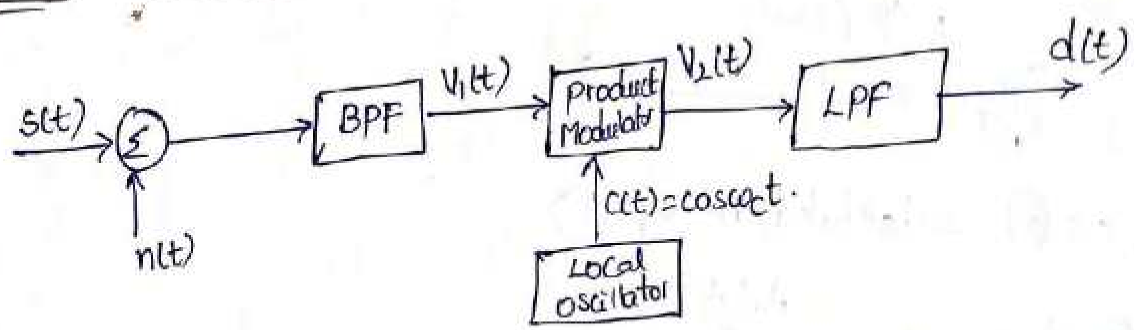
eq ⑥ & eq ⑪ substituted in eq ①

$$\text{Figure of Merit} = \frac{(SNR)_{o/p}}{(SNR)_{channel}}$$

$$= \frac{\frac{A_c^2 P}{2W N_0}}{\frac{A_c^2 P}{2W N_0}}$$

$$\boxed{(FOM)_{DSB\text{-}SC} = 1}$$

Signal to Noise Ratio in SSB-SC system



$$\text{Figure of merit (FOM)} = \frac{(SNR)_{o/p}}{(SNR)_{channel.}} \longrightarrow ①$$

$$(SNR)_{channel} = \frac{\text{Average power of modulated signal } (P_{sc})}{\text{Average power of noise in message B.W } (P_{NC})} \longrightarrow ②$$

$$(SNR)_{o/p} = \frac{\text{Average power of demodulated signal } (P_{so})}{\text{Average power of noise at o/p } (P_{No})} \longrightarrow ③$$

- The SSB-SC modulated wave having LSB is

$$S(t) = \frac{A_m A_c}{2} \cos\left[2\pi(f_c - f_m)t\right]$$

Average power

$$P_{sc} = \frac{1}{2\pi}\int_0^{2\pi} \frac{A_m^2 A_c^2}{4} + \cos^2(2\pi(f_c-f_m)t)\, dt$$

$$= \frac{A_m^2 A_c^2}{8\pi}\int_0^{2\pi} \cos^2(2\pi(f_c-f_m)t)\, dt$$

$$= \frac{A_m^2 A_c^2}{8\pi}\int_0^{2\pi} \frac{1+\cos(4\pi(f_c-f_m)t}{2}\, dt$$

$$= \frac{A_m^2 A_c^2}{8\pi}\left[\int_0^{2\pi} \frac{1}{2}\, dt + \int_0^{2\pi} \frac{\cos(4\pi(f_c-f_m)t}{2}\, dt\right.$$

$$P_{sc} = \frac{A_m^2 A_c^2}{8\pi}\left[\frac{1}{2}(2\pi - 0) + 0\right]$$ (11) p9 & (7) p8

$$\boxed{P_{sc} = \frac{A_m^2 A_c^2}{8}} \longrightarrow (4)$$

(or)

$$P_{sc} = \left(\frac{A_m A_c}{2\sqrt{2}}\right)^2$$

$$\boxed{P_{sc} = \frac{A_m^2 A_c^2}{8}}$$

Average power of noise in message B.W (P_{NC})

$$P_{NC} = \frac{N_0}{2}(B.W)$$

$$= \frac{N_0}{2}(2\omega)$$

$$\boxed{P_{NC} = \omega N_0} \longrightarrow (5)$$

eq (4) & eq (5) substituted in eq (2)

$$(SNR)_{channel} = \frac{\frac{A_m^2 A_c^2}{8}}{\omega N_0}$$

$$\boxed{(SNR)_c = \frac{A_m^2 A_c^2}{8\omega N_0}} \longrightarrow (6)$$

$$V_1(t) = s(t) + n(t) \longrightarrow (7)$$

We know that

$$s(t) = \frac{A_m A_c}{2}\cos[2\pi(f_c - f_m)t]$$

$$n(t) = n_I(t)\cos\omega_c t - n_Q(t)\sin\omega_c t$$

$$\therefore V_1(t) = \frac{A_m A_c}{2}\cos[2\pi(f_c - f_m)t] + n_I(t)\cos\omega_c t - n_Q(t)\sin\omega_c t$$

$$V_2(t) = \cancel{\frac{A_m A_c}{2}\cos}\; V_1(t)\cdot c(t)$$

$$= V_1(t)\cdot\cos\omega_c t.$$

$$\therefore \cancel{V_2(t) = \frac{A_m A_c}{2}(\cos\omega_c t - \cos\omega_m t)}\cos\omega_c t + n_I(t)\cos^2\omega_c t \cancel{- n_Q(t)\sin\omega_c t\cdot\cos\omega_c t}$$

$$V_2(t) = \cancel{\frac{A_mA_c}{2}\cos^2\omega_c t - \frac{A_mA_c}{2}\cos\omega_m t\cos\omega_c t} + n_I(t)\cos^2\omega_c t - \cancel{n_Q(t)\sin\omega_c t}\cdot\cos\omega_c t$$

$$= \cancel{\frac{A_mA_c}{2}\left[\frac{1+\cos 2\omega_c t}{2}\right] - \frac{A_mA_c}{2}\cos\omega_m t\cdot\cos}$$

$$V_2(t) = \frac{A_mA_c}{2}\cos\underbrace{[(\omega_c-\omega_m)t]}_{\cos A}\ \underbrace{\cos\omega_c t}_{\cos B} + n_I(t)\cos^2\omega_c t - n_Q(t)\sin\omega_c t\cdot\cos\omega_c t$$

$$\boxed{\frac{1}{2}\cos(A-B) + \cos(A+B) = \cos A\cdot\cos B}$$

$$= \frac{A_mA_c}{4}\left[\cos(2\omega_c-\omega_m)t + \cos\omega_m t\right] + n_I(t)\left[\frac{1+\cos 2\omega_c t}{2}\right] - \frac{n_Q(t)}{2}\cdot\sin 2\omega_c t$$

$V_2(t)$ passed through LPF

$$\therefore d(t) = \frac{A_mA_c}{4}\cos\omega_m t + \frac{n_I(t)}{2} \longrightarrow (8)$$

Average power of demodulated signal (P_{so}) is

$$P_{so} = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{A_mA_c}{4}\cos\omega_m t\right]^2 dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\frac{(A_mA_c)^2}{16}\cdot\cos^2\omega_m t\, dt$$

$$= \frac{A_m^2A_c^2}{32\pi}\int_0^{2\pi}\cos^2\omega_m t\, dt$$

$$= \frac{A_m^2A_c^2}{32\pi}\int_0^{2\pi}\frac{1+\cos 2\omega_m t}{2}\, dt$$

$$= \frac{A_m^2A_c^2}{32\pi}\left[\int_0^{2\pi}\frac{1}{2}\, dt + \int_0^{2\pi}\frac{\cos 2\omega_m t}{2}\, dt\right]$$

$$= \frac{A_m^2A_c^2}{32\pi}\left[\frac{1}{2}(2\pi-0)+0\right]$$

$$\boxed{P_{so} = \frac{A_m^2A_c^2}{32}} \longrightarrow (9)$$

Average power of noise at o/p (P_{no}) is

$$P_{no} = \frac{1}{2} \cdot n_I(t)$$

$$= \frac{1}{2} \cdot \left(\frac{n_I(t)}{\sqrt{2}}\right)^2$$

$$= \frac{n_I^2(t)}{4}$$

$$= \frac{1}{4}(\omega N_0)$$

$$\boxed{P_{no} = \frac{\omega N_0}{4}} \longrightarrow (10)$$

DSB $B \cdot \omega = 2\omega$
SS $B \cdot \omega = \omega$

$\frac{N_0}{2}(\omega) + \frac{N_0}{2}(\omega)$

$\cancel{2}\frac{N_0}{\cancel{2}} \cdot \omega$

ωN_0

eq (9) & (10) are substituted in eq (3)

$$(SNR)_{output} = \frac{\frac{A_m^2 A_c^2}{32}}{\frac{\omega N_0}{4}}$$

$$= \frac{A_m^2 A_c^2}{32} \times \frac{4}{\omega N_0}$$

$$\boxed{(SNR)_{o/p} = \frac{A_m^2 A_c^2}{8 \omega N_0}} \longrightarrow (11)$$

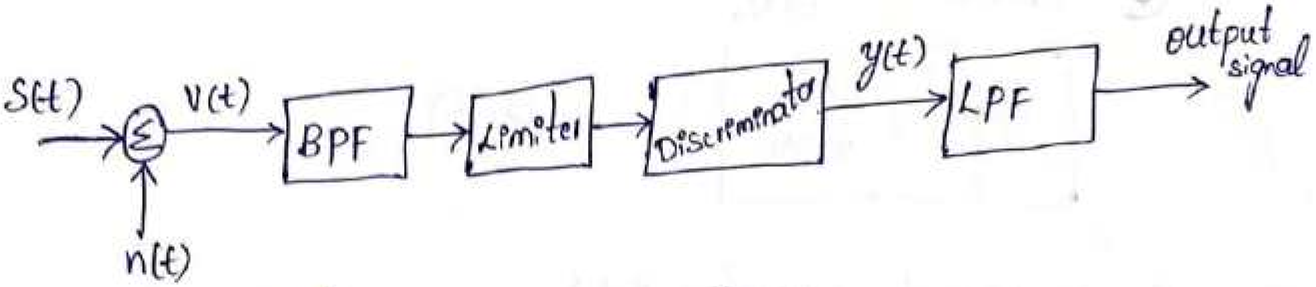
eq (6) & (11) are substituted in eq (1)

$$\text{Figure of merit} = \frac{(SNR)_{o/p}}{(SNR)_{channel}}$$

$$= \frac{\frac{A_m^2 A_c^2}{8\omega N_0}}{\frac{A_m^2 A_c^2}{8\omega N_0}}$$

$$\boxed{(FOM)_{SSB-SC} = 1}$$

Signal to Noise Ratio in FM Receiver.



$$FOM = \frac{(SNR)_{o/p}}{(SNR)_{channel}} \longrightarrow ①$$

$$(SNR)_{channel} = \frac{P_{sc}}{P_{NC}} \longrightarrow ②$$

$$(SNR)\ o/p = \frac{P_{so}}{P_{NO}} \longrightarrow ③$$

FM modulated signal

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$$

Average power of FM signal (P_{sc})

$$P_{sc} = \left(\frac{A_c}{\sqrt{2}}\right)^2$$

$$P_{sc} = \frac{A_c^2}{2} \longrightarrow ④$$

Average power of noise in message B.W (P_{NC})

$$P_{NC} = \frac{N_0}{2}(B.W)$$

$$= \frac{N_0}{2}(2W)$$

N.B.FM B.W = 2W

$$\boxed{P_{NC} = W N_0} \longrightarrow ⑤$$

eq (4) and eq (5) substituted in eq (2)

$$(SNR)_{channel} = \frac{A_c^2/2}{\omega N_0}$$

$$\boxed{(SNR)_c = \frac{A_c^2}{2\omega N_0}} \longrightarrow (6)$$

s(t) and n(t) passed through B.P.F

$$v(t) = s(t) + n(t) \longrightarrow (7)$$

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right] \longrightarrow (8)$$

$$n(t) = n_I(t)\cos\omega_c t - n_Q(t)\sin\omega_c t$$

$$n(t) = r(t)\cos(\omega_c t + \psi(t)) \longrightarrow (9)$$

Here $n(t) = r(t)\cos(\omega_c t + \psi(t))$

where $r(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$, $\psi(t) = \tan^{-1}\left[-\frac{n_Q(t)}{n_I(t)}\right]$

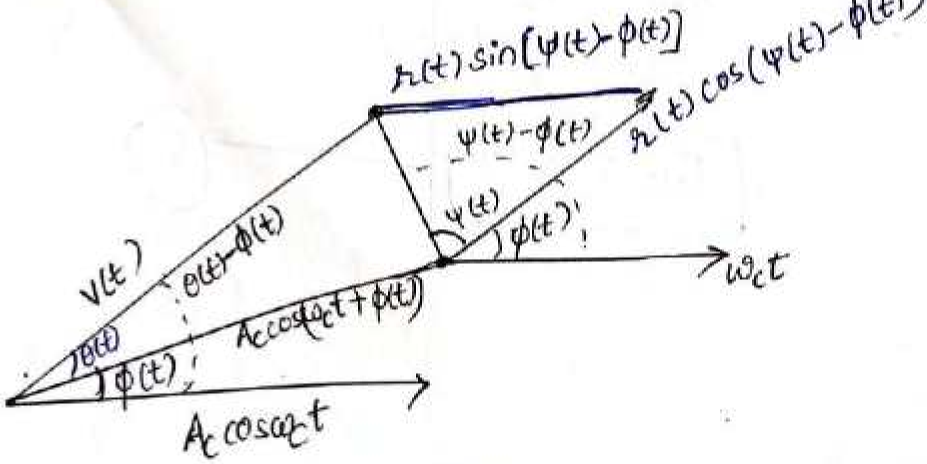
Variable Amplitude   Variable phase

eq (8) & (9) substituted in eq (7)

$$\therefore V(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right] + r(t)\cos(2\pi f_c t + \psi(t))$$

Let $\phi(t) = 2\pi k_f \int_0^t m(t)\,dt$

$$\therefore V(t) = A_c \cos\left[2\pi f_c t + \phi(t)\right] + r(t)\cos(2\pi f_c t + \psi(t)) \longrightarrow (10)$$



$$\tan[\theta(t) - \phi(t)] = \frac{r(t)\sin[\psi(t)-\phi(t)]}{A_c + r(t)\cos[\psi(t)-\phi(t)]} \quad ; r(t) << A_c$$

Here $r = \theta(t) - \phi(t)$ ∴ it is very small

$$\therefore \tan r \simeq r$$
$$\cos r \approx 1$$

$$\therefore \theta(t) - \phi(t) = \frac{r(t)\sin[\psi(t)-\phi(t)]}{A_c}$$

$$\theta(t) \approx \phi(t) + \frac{r(t)\sin[\psi(t)-\phi(t)]}{A_c} \longrightarrow (11)$$

$\phi(t) = 2\pi k_f \int_0^t m(t)dt$

$$y(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t) \longrightarrow (12)$$

$y(t) = \frac{1}{2\pi}\left[2\pi k_f \frac{d}{dt}\int_0^t m(t)\right]$

$= k_f\, m(t)$

$$y(t) = \frac{1}{2\pi}\left[\underbrace{2\pi k_f\, m(t)}_{\text{message}} + \frac{1}{A_c}\frac{d}{dt}\left[\underbrace{r(t)\sin(\psi(t)}_{\text{Noise}} - \underbrace{\phi(t)}_{\text{message}})\right]\right]$$

$$\therefore \text{Noise } n_d(t) = \frac{1}{2\pi A_c}\frac{d}{dt}\left[\underbrace{r(t)\sin\psi(t)}_{n_Q(t)}\right] \longrightarrow (13)$$

$n_Q(f) = r(f)\sin\psi f$

$n_I(f) = r(f)\cos\psi f$

In frequency domain

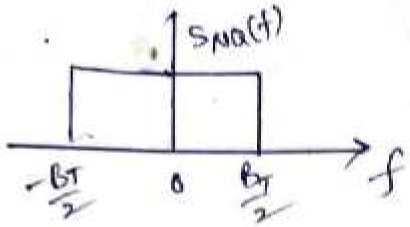
$$n_d(f) = \frac{1}{2\pi A_c} \cdot j2\pi f \cdot n_Q(f)$$

$$n_d(f) = \frac{jf}{A_c} n_Q(f)$$

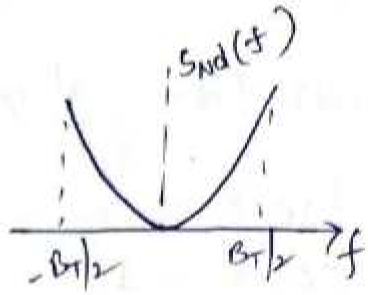
$$\frac{n_d(f)}{n_Q(f)} = H(f) = \frac{jf}{A_c} \longrightarrow (14)$$

$$S_{Nd}(f) = |H(f)|^2 S_{NQ}(f) \longrightarrow (15)$$

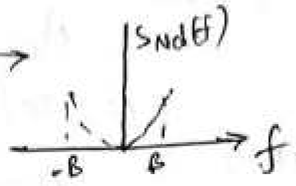
$$\therefore S_{Nd}(f) = N_0 \frac{f^2}{A_c^2} \quad ; \quad |f| \leq B_T \longrightarrow (16)$$



$\Longrightarrow$



passed through L.P.F



Average power of noise at o/p (P_{NO}) is

$$P_{NO} = \int_{-B}^{B} \frac{N_0 f^2}{A_c^2} df$$

$$P_{NO} = \frac{N_0}{A_c^2} \left. \frac{f^3}{3} \right|_{-B}^{+B}$$

$$\therefore P_{NO} = \frac{N_0 B^3}{3A_c^2} - \left(\frac{-N_0 B^3}{3A_c^2} \right)$$

$$= \frac{2N_0 B^3}{3A_c^2}$$

$$\boxed{P_{NO} = \frac{2N_0 f_m^3}{3A_c^2}} \longrightarrow (17)$$

Average power of signal at o/p (P_{SO}) is

$$P_{SO} = K_f^2 \cdot \frac{A_m^2}{2} = K_f^2 \cdot P \longrightarrow (18)$$

$$\therefore (SNR)_{o/p} = \frac{K_f^2 P}{\frac{2N_0 f_m^3}{3A_c^2}} \longrightarrow (19)$$

eq (6) & (19) substituted in eq (1)

$$FOM = \frac{K_f^2 P \cdot 3A_c^2}{2N_0 f_m^3} \cdot \frac{2 \omega N_0}{A_c^2}$$

$$FOM = \frac{3K_f^2 P}{f_m^2} \longrightarrow (20)$$

$$m(t) = A_m \cos \omega_m t$$

$$P = \frac{A_m^2}{2} \longrightarrow (21)$$

$$\beta = \frac{K_f A_m}{f_m}$$

$$K_f = \frac{\beta f_m}{A_m} \longrightarrow (22)$$

eq (21) & (22) substituted in eq (20)

$$FOM = \frac{3 \beta^2 f_m^2 \cdot A_m^2}{2 A_m^2 \cdot f_m^2}$$

$$\boxed{(FOM)_{F.M} = \frac{3}{2} \cdot \beta^2}$$

(or)

$$\boxed{(FOM)_{F.M} = 1.5 \beta^2}$$

Capture Effect

* The inherent ability of an FM system to minimize the effect of unwanted signal.
* The interference suppression in an FM receiver works well only when the interference is weaker than the desired FM input.
* This capture effect phenomenon associated with the reception of FM signal.
* If there are two signals with less difference in frequency, then only the stronger of the two signals will be demodulated.
* According to the capture effect, there is complete suppression of the weaker signal at the receiver limiter (weaker signal is attenuated)
* When both signals have same strength, the receiver may fluctuate between FM signal and noise, it is also called Picket Fencing.

Drawbacks

* When noise interference is stronger than FM signal, the FM receiver locks to interference.
* If strength of the noise interference is equal to the FM signal, then receiver locking will fluctuate between FM signal and noise.

Threshold Effect in Angle modulation System

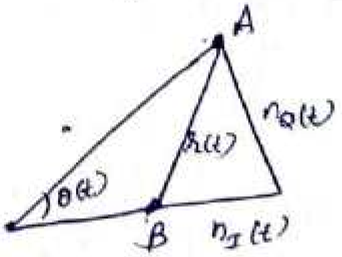
* While calculating Figure of Merit (FOM) of FM receiver, output SNR is valid only if the carrier to noise ratio (CNR) at discriminator input is high.

$$(SNR)_{o/p} = \frac{3 K_f^2 \cdot P_s A_c^2}{2 N_0 f_m^3}$$

$A_c \rightarrow$ carrier
$N_0 \rightarrow$ Noise
$\frac{A_c}{N_0} = CNR$

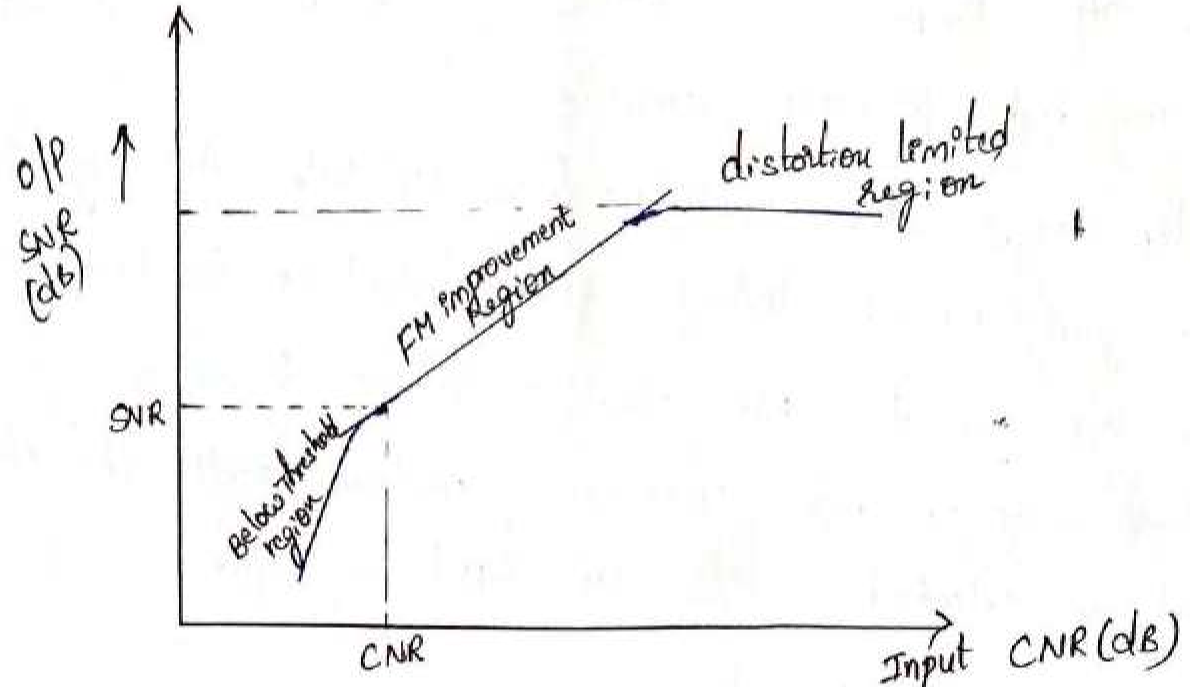
Here $SNR \propto CNR$

* If CNR decreases, then the FM receiver breaks. Therefore individual click sound is heard at receiver.
* If CNR is furthur decreases, then the click sound merge to form cracking (or) sputtering sound
* Theoretically calculated SNR value is larger but its actual value is very small. This phenomenon is called threshold Effect
* From the figure, if $\theta(t)$ changes, then point 'A' changes by $\pm 2\pi$.



Therefore amplitude and phase of $n_I(t)$ changes randomly. Therefore clicking sound will present at the receiver.

* we cannot remove this effect, but we can just extend the threshold.

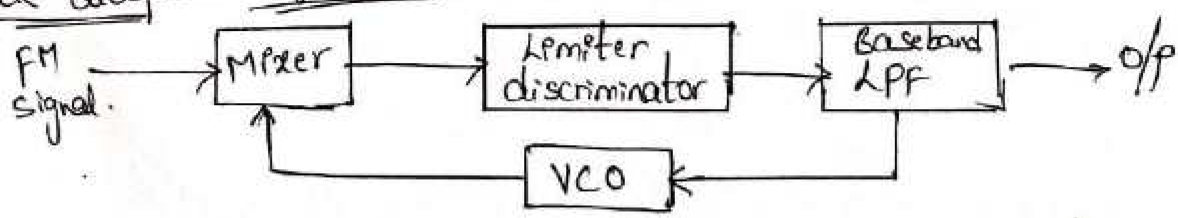


* From the above fig, Below threshold point, CNR decreases, then click sound, sputtering sound will present.
* Above threshold point i.e extended threshold, CNR increases, therefore no sounds and no sputtering sound will not be present.
* There are two methods to extend this threshold.

  1. PLL demodulator
  2. FMFB demodulator (negative feedback demodulator).

* By using FMFB, the threshold has been extended. If we have lesser CNR at input, then we get extended threshold, so there will be no sound and no sputtering sound.
* It can just minimize the effect, not completely removed.

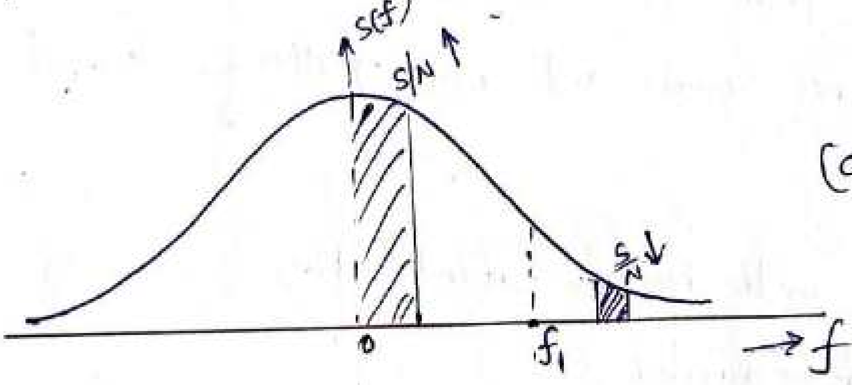
Block diagram of FM FB demodulator



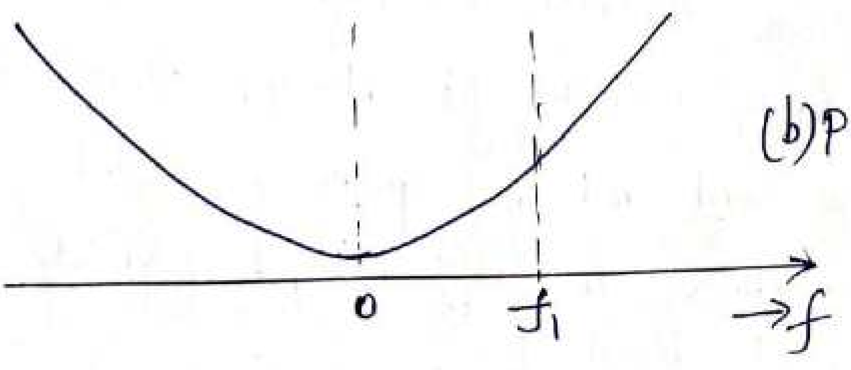
* Mixer output produces the difference, this difference will not have very high frequency variation.
* Limiter discriminator converts from FM into AM signal.
* AM signal passed through LPF. output is fed back to VCO, this VCO generates the changes in the frequency by applying voltage. This process is continue until the threshold point is extended i.e CNR input is high.

Pre-Emphasis and De-Emphasis

* pre-emphasis and De-emphasis are used to improve fidelity of FM. transmission of Audio signals.



(a) PSD of Audio signal

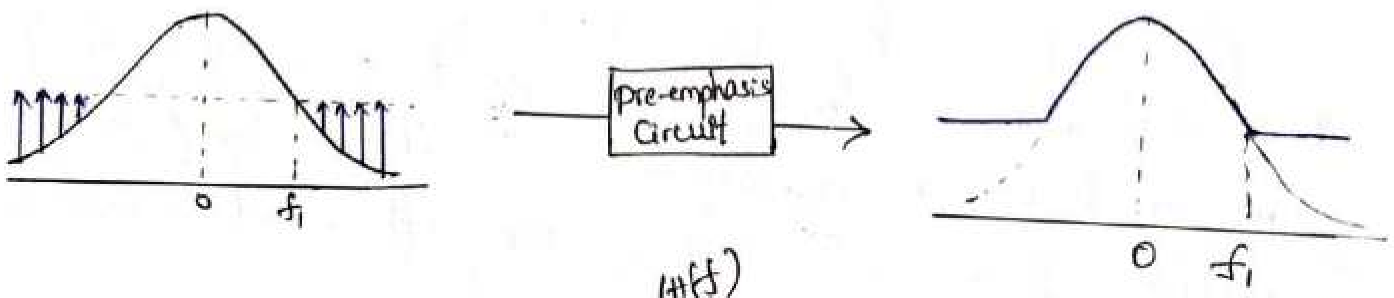


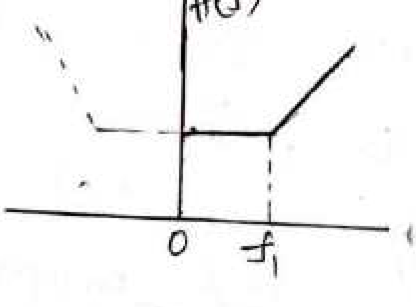
(b) PSD of Noise.

* From fig (a)&(b), Below f_1, gain is maximum and noise is minimum and above f_1 (high freq's), signal is decreases and noise is increases.
* At high frequencies, noise is more, so we need to increase the amplitude of message signal at high frequency component. this process is called pre-emphasis.
* upto frequency f_1 : $\frac{S}{N} >> 1$, so the low frequency component can be reproduced comfortably.
* Above f_1 : $\frac{S}{N} << 1$, so the high-frequency component cannot be reproduced comfortably.

Pre-emphasis :-

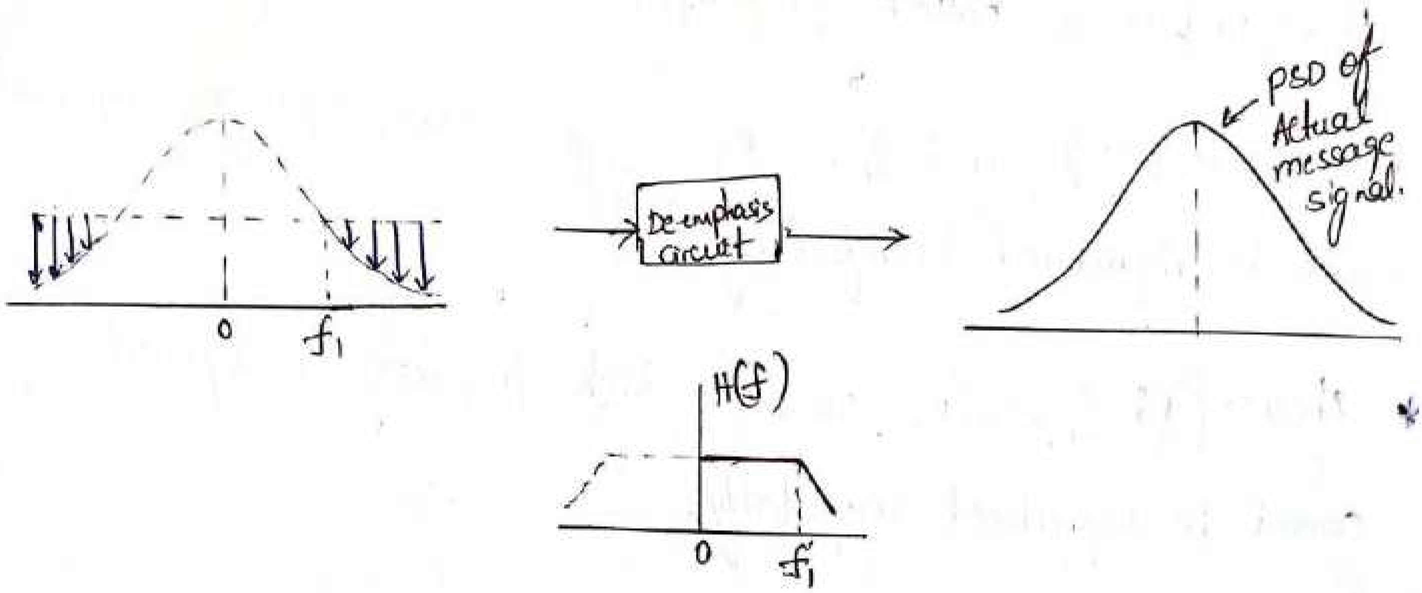
* It is the process of artificial boosting of high frequency component of message signal to increase $\frac{S}{N}$ ratio
* pre-emphasis is done in the transmitter before frequency modulation.



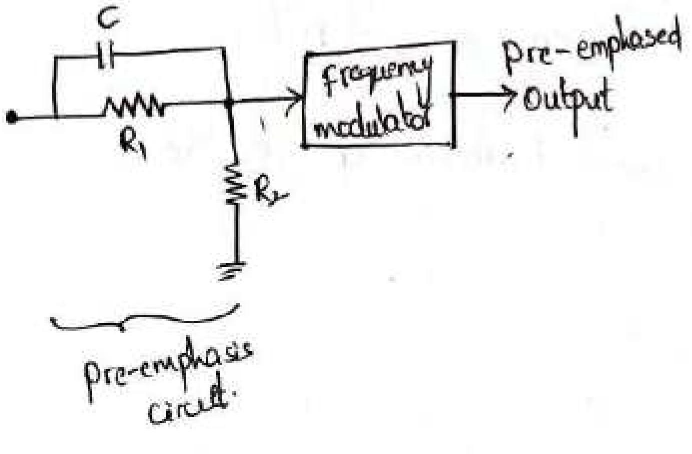


De-emphasis:-

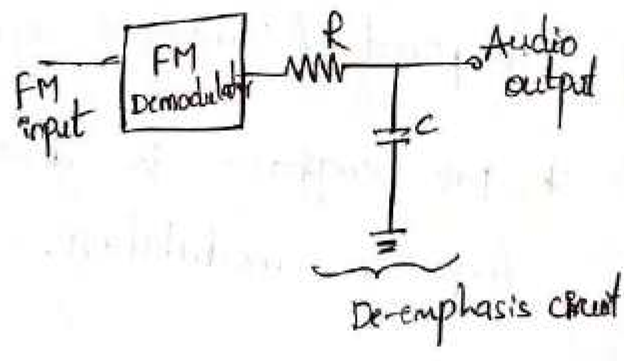
* It is the process of decreasing the strength of high-frequency component of message signal to get back the original transmitted message signal.
* De-emphasis is performed in the receiver after demodulation.



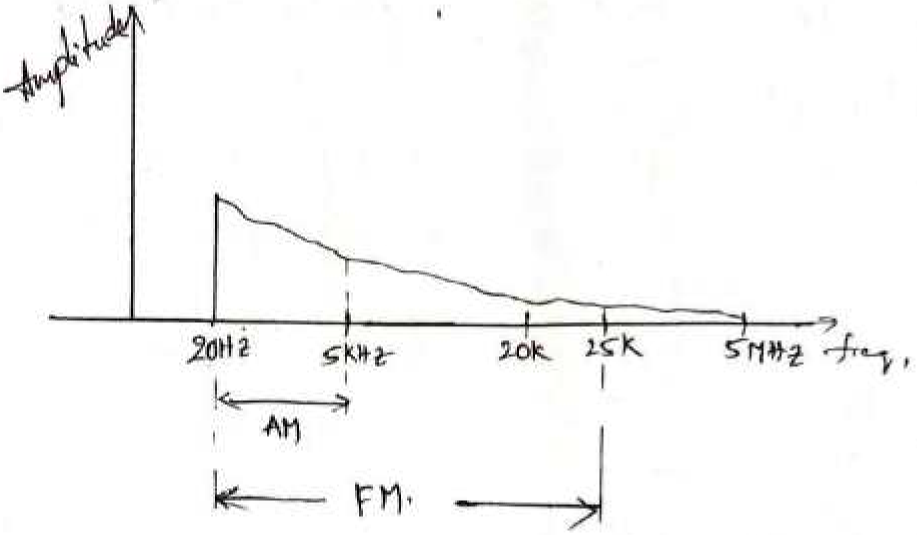
Pre-emphasis circuit



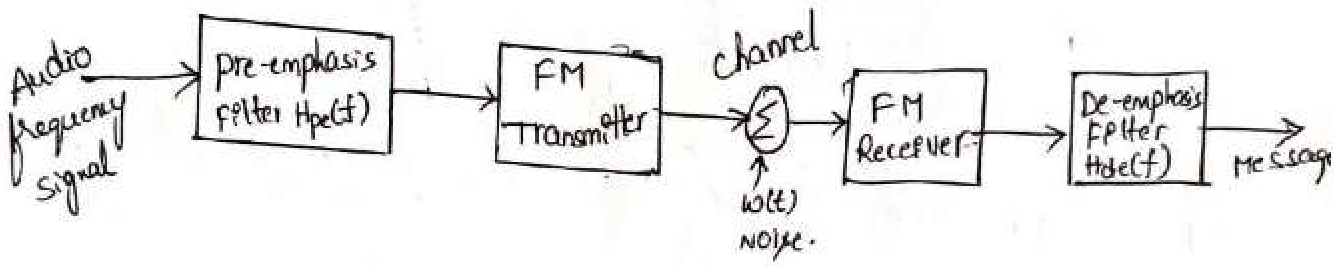
De-emphasis circuit



* In AM Transmission, low frequency component of Audio signal are considered for transmission, for these low frequency component $\frac{S}{N}$ is high, therefore pre-emphasis and De-emphasis not required for AM.
* In case of FM, high frequency component of audio signal are considered for transmission, for these high frequency component $\frac{S}{N}$ ratio is low, therefore pre-emphasis and De-emphasis are required.



Block diagram



* By using this pre-emphasis and De-emphasis, we can improve the signal and reduces the noise and also undistorted message signal will present at receiver output.

$$\therefore \; H_{de}(f) = \frac{1}{H_{pe}(f)} \quad ; \quad -W \leq f \leq W$$

Discriminator o/p $\Rightarrow$ $$S_{Nd}(f) = \begin{cases} \frac{f^2}{A_c^2} N_0 & : |f| \leq \frac{B_T}{2} \\ 0 & : \text{otherwise.} \end{cases}$$

* this discriminator output passed through de-emphasis filter ($H_{de}(f)$)

$$\therefore \; S_{No}(f) = |H_{de}(f)|^2 \, S_{Nd}(f)$$

$$\text{Average power} \cong \frac{N_0}{A_c^2} \int_{-W}^{W} f^2 \, |H_{de}(f)|^2 \, df$$

* Ideally average power of o/p noise is not affected by pre-emphasis and deemphasis process in FM system. But the noise performance

become better and can be measured by a factor is called Improvement factor (I).

$$I = \frac{\text{Average o/p noise power without pre-emphasis \& De-emphasis}}{\text{Average o/p noise power with pre-emphasis \& De-emphasis}}$$

$$I = \frac{\frac{2N_0W^3}{3A_c^2}}{\frac{N_0}{A_c^2}\int_{-W}^{W} f^2 \cdot |H_{de}(f)|^2 df}$$

$$I = \frac{2W^3}{3\int_{-W}^{W} f^2 |H_{de}(f)|^2 df}$$

*. this improvement factor (I) assume the use of High CNR at the discriminator input.

Pulse Modulation:-

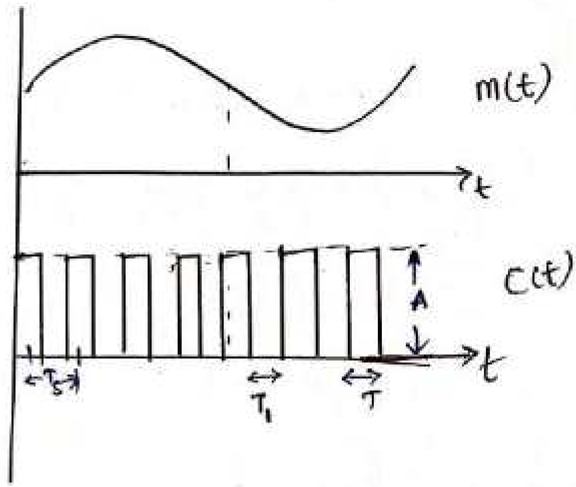
* The characteristics of the pulse is varied inaccordance with the instantaneous value of the message signal.

* pulse wave is a carrier wave.

Types:-

1. pulse Amplitude Modulation (PAM)
2. pulse Time Modulation → pulse width Modulation (PWM), pulse position Modulation (PPM)

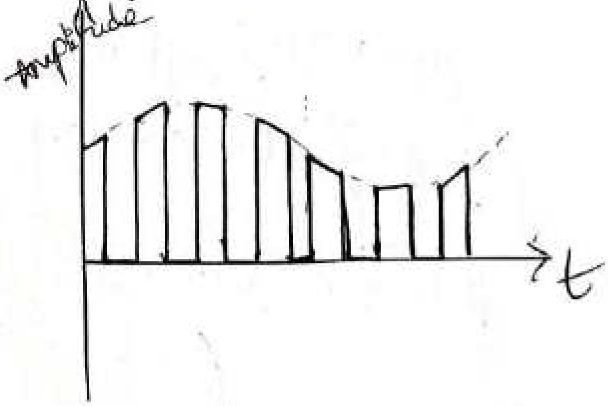
Characteristics



where
A → Amplitude
T → pulse ON Time
T_1 → Pulse OFF Time
T_s → sampling Time.

∴ These characteristics are changed inaccordance to the message signal

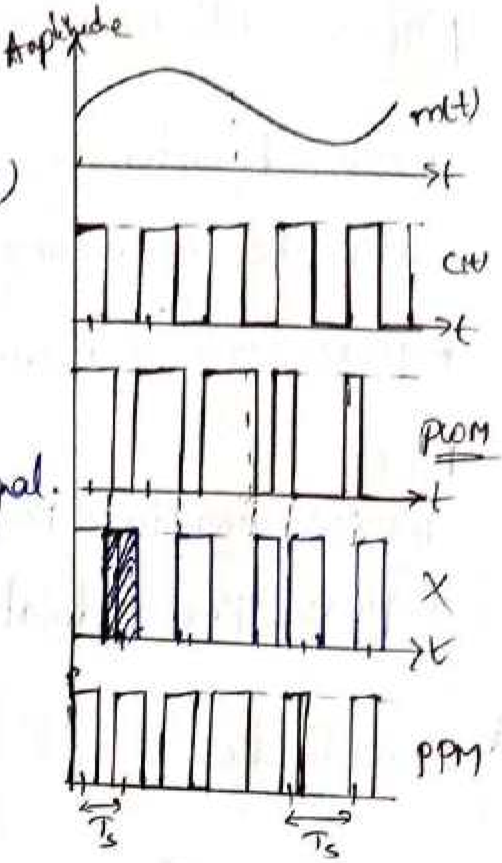
PAM:- The Amplitude of the pulse is varied inaccordance with the message signal.



In PAM, only amplitude is changing.

PWM:- The width of the pulse is varied inaccordance with the message signal.

* Here T_s is same, only 'T' (pulse ON Time) is changing.



PPM:- the ~~pulse~~ position of the pulse is varied inaccordance with the message signal.

* Here T_s is changing, and T is same.

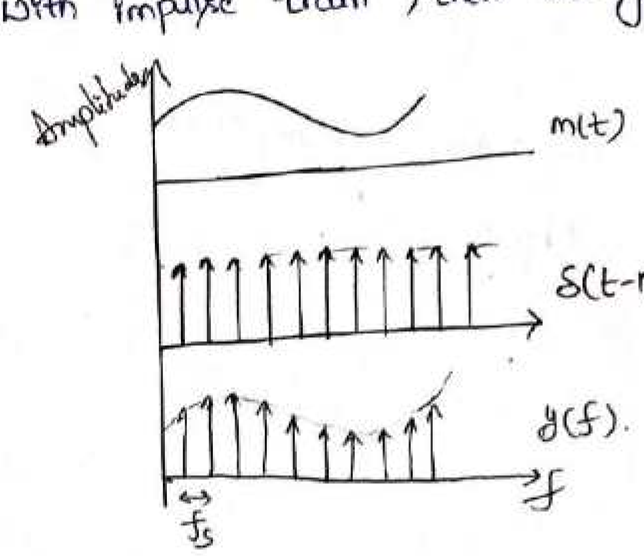
* To recover the original signal without any disturbances, the sampling frequency (f_s) should be greater than 2W.

$$f_s \geq 2W$$

$$\frac{1}{f_s} \leq \frac{1}{2W}$$

$$\therefore T_s \leq \frac{1}{2W}$$

Instantaneous sampling:- the message signal m(t) is multiplied with impulse train, then we get instantaneous sampling

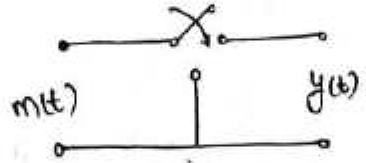


$$y(t) = m(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$$

$$y(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t-nT_s)$$

$$y(f) = f_s \cdot \sum_{n=-\infty}^{\infty} m(f-nf_s)$$

Circuit



Limitations
* If the width is extending to zero, then noise interference is very high
* It cannot travel long distances.

Natural Sampling: Generation of PAM using Natural sampling.

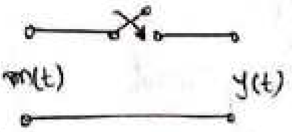


fig: circuit for generation of natural sampling.

$y(t) = m(t)$; $p(t) = A$
$y(t) = 0$; $p(t) = 0$

* T and T_s are changed according to the switch.
* Fourier series representation of pulse is

$$p(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi nt/T_s} \longrightarrow (1)$$

$$p(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi nt \cdot f_s}$$

$$C_n(t) = \frac{AT}{T_s} \operatorname{sinc}(nf_s t) \longrightarrow (2)$$

eq (2) in eq (1)

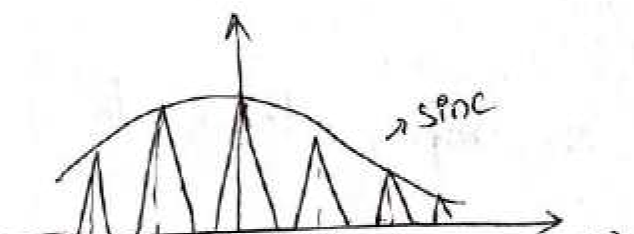
$$\therefore p(t) = \sum_{n=-\infty}^{\infty} \frac{AT}{T_s} \operatorname{sinc}(nf_s t) \cdot e^{j2\pi nf_s t}.$$

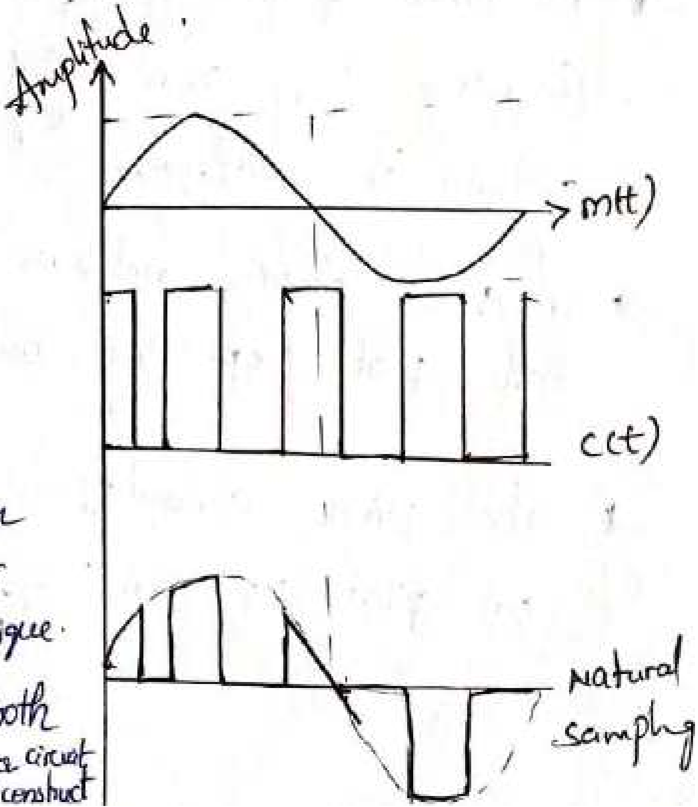
$$y(t) = m(t) \cdot p(t)$$

$$= \sum_{n=-\infty}^{\infty} \frac{AT}{T_s} \operatorname{sinc}(nf_s t) e^{j2\pi nf_s t} \cdot m(t)$$

$$e^{j2\pi nf_s t} \cdot m(t) = M(f - nf_s)$$

$$\therefore y(t) = \frac{AT}{T_s} \sum_{n=-\infty}^{\infty} \operatorname{sinc}(nf_s t) \cdot M(f - nf_s)$$

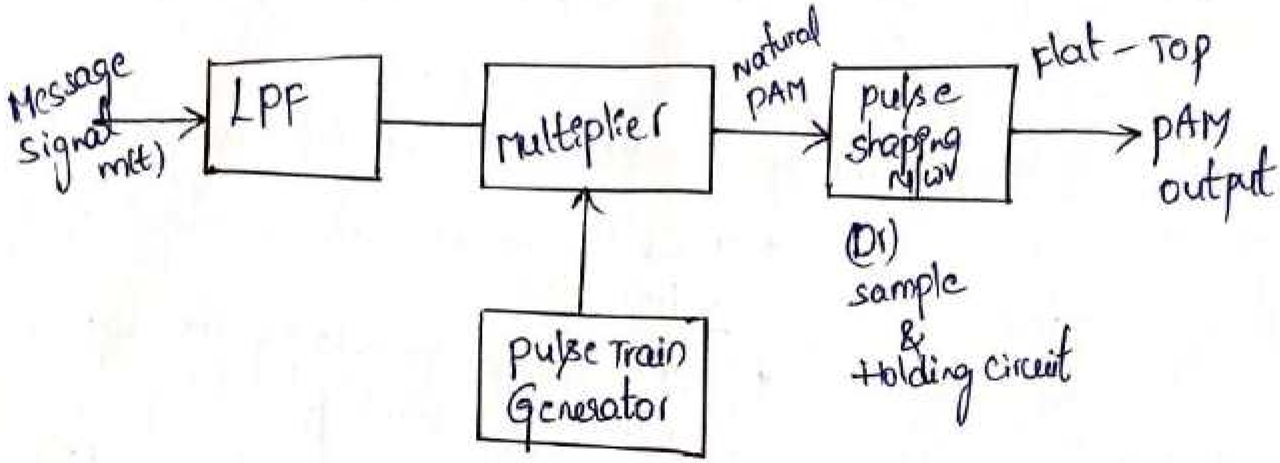




Advantages of PAM

* It is the base for all digital modulation techniques and it is simple process for both modulation and demodulation technique.
* No complex circuitary is required for both transmission & Reception. Transmitter & Receiver circuit is simple & easy to construct

Generation of PAM:-



* It consists of a LPF, multiplier and pulse Train generator, pulse shaping network.
* The LPF removes all the frequency components which are higher higher than frequency f_m. This is known as bandlimiting. The band limiting is necessary to avoid the aliasing effect in the sampling process.
* The pulse train generator generates a pulse train at a frequency f_s, such that $f_s \geq 2f_m$. Thus the nyquist criteria is satisfied.
* The pulse shaping network does the shaping network to give Flat-Top PAM output.
* The PAM generates either using natural sampling (or) Flat Top sampling.

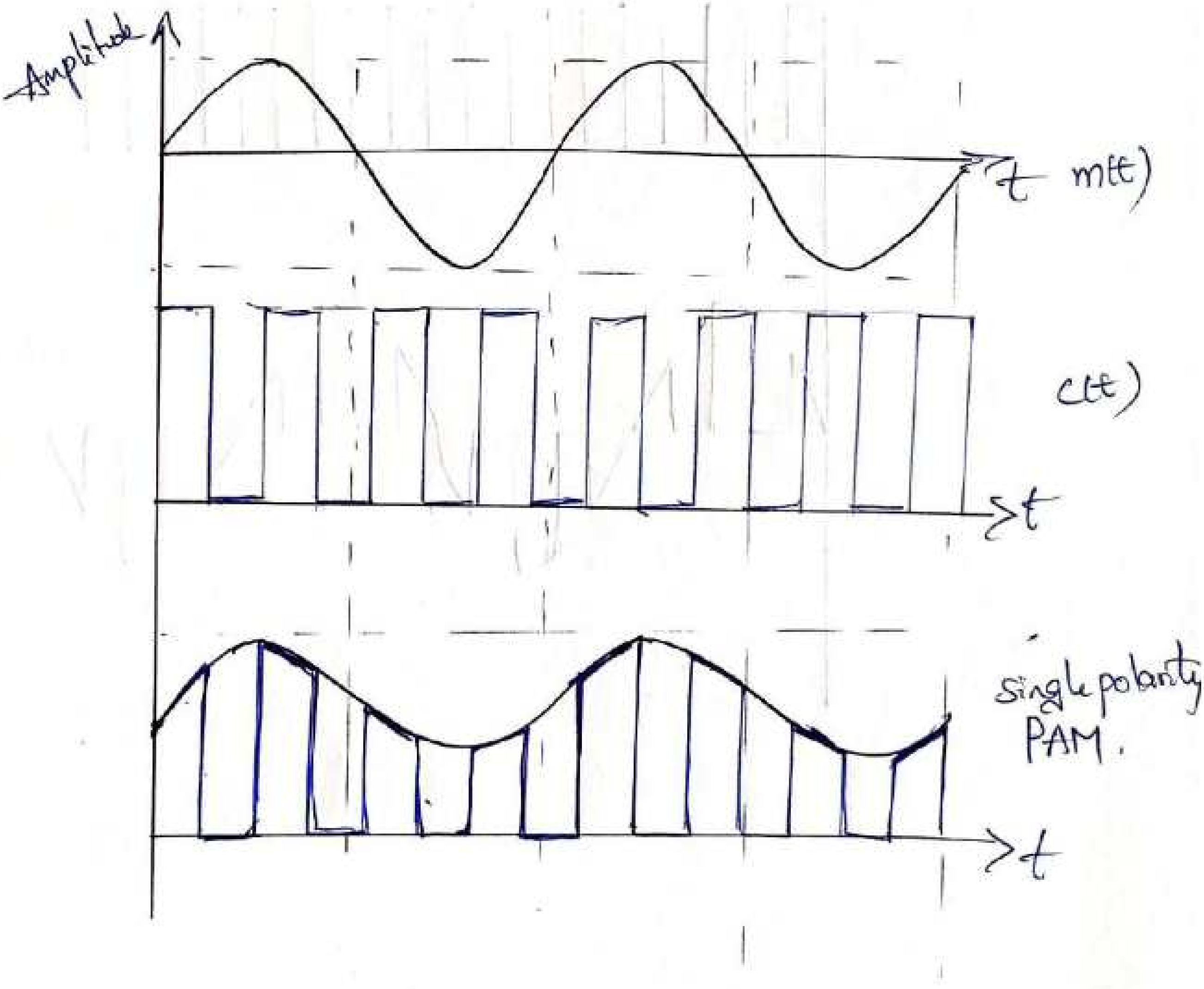
Classifications of PAM

* The PAM signal can be classified according to polarity

    1. Single polarity PAM
    2. Double polarity PAM

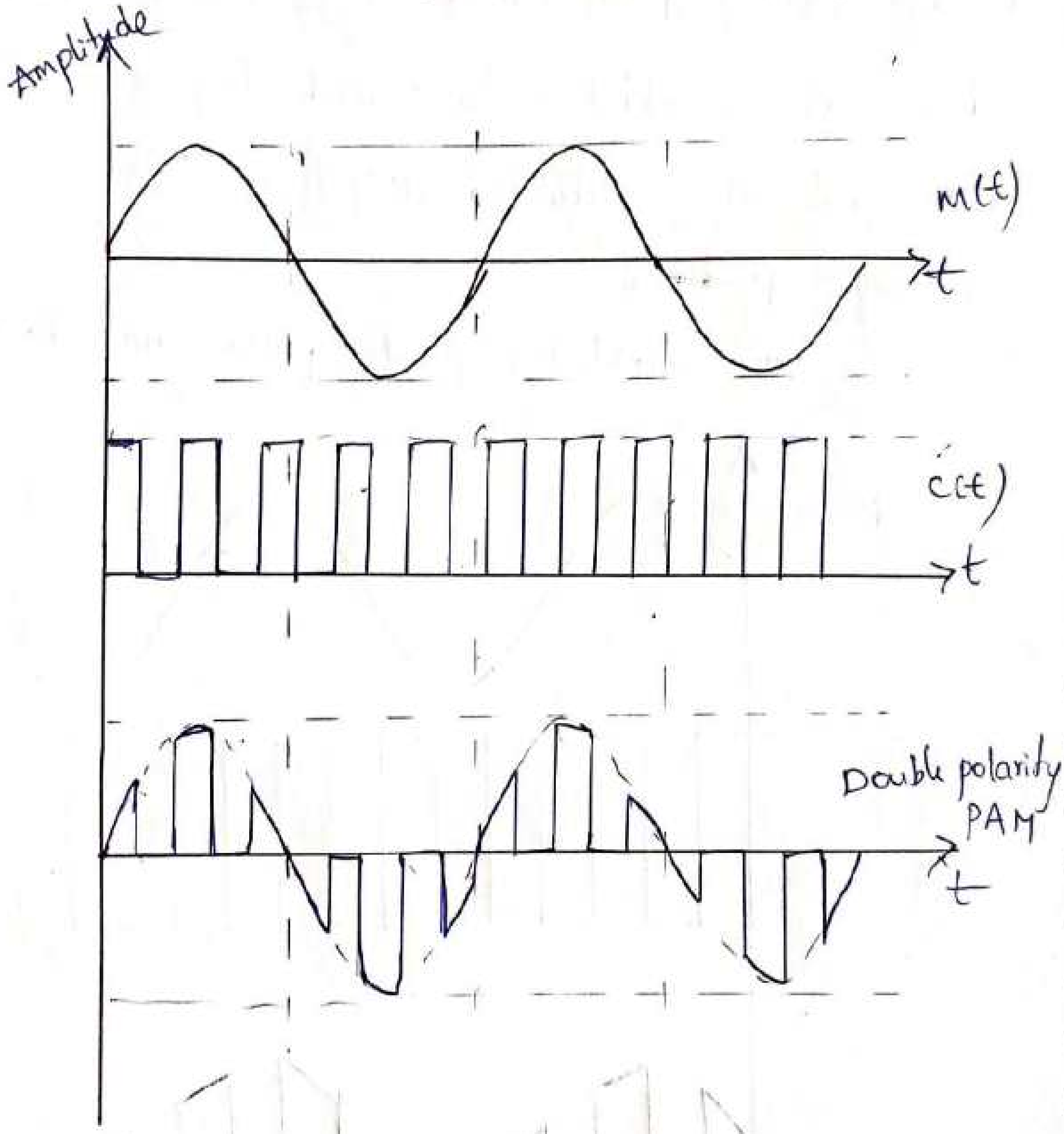
1. <u>Single polarity PAM</u>:- In a single polarity PAM, a fixed d.c level is added to the modulating signal m(t), such that the modulated output i.e PAM signal is always positive.

The amplitudes of the pulses are always positive.



2. Double polarity PAM

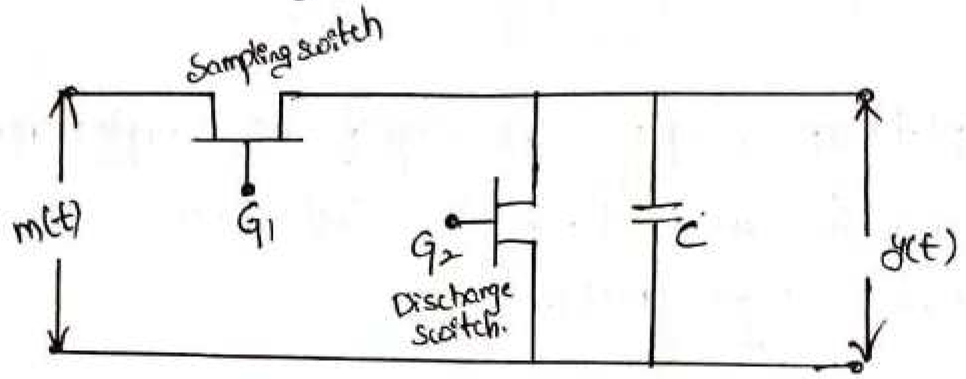
* In double polarity PAM signal, signal has positive as well as negative polarity.
* Amplitude of the pulses may be positive or negative depending on the sample value.



Generation of PAM : Flat Top PAM

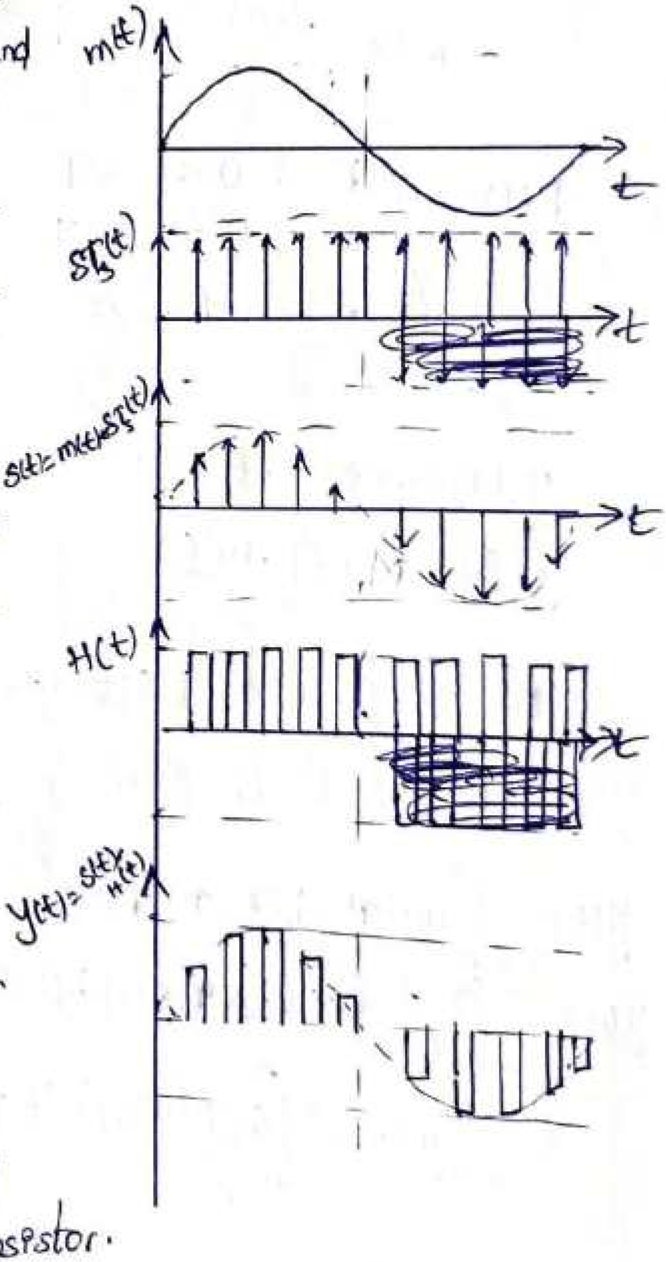
* It uses sample and hold circuit. It is practically possible like natural sampling but flat Top sampling is easier compared to natural sampling.

*



* The sample and Hold circuit (S&H) consists of two FET switches and a capacitor.
* The sampling switch is closed for a short duration by a short pulse applied to the gate G_1 of the transistor.

During this period, the capacitor 'C' is quickly charged upto a voltage equal to the instantaneous sample value of the incoming signal m(t).

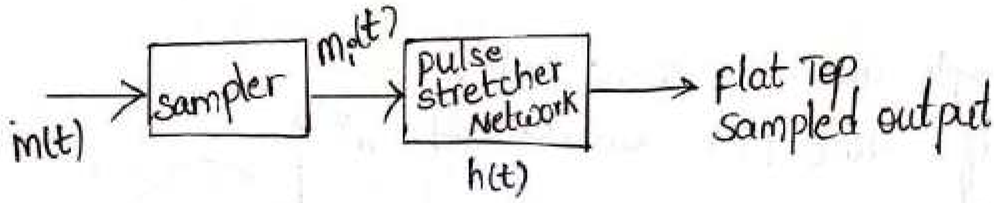


* Now the sampling switch is opened and the capacitor 'c' holds the charge.
* the discharge switch is then closed by a pulse applied to gate 'G_2' of the other transistor.

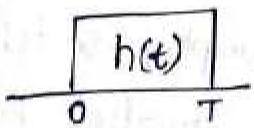
* Due to this, the capacitor 'C' is discharged to zero volts.
* the discharge switch is then opened, and thus the capacitor has no voltage. Hence the output contains flat-Top samples.

Mathematical Analysis of Flat-Top Sampling

* In a Flat-Top samples, the top of the samples remain constant and it's equal to the instantaneous value of the base band signal m(t).



$$h(t) = \begin{cases} 1 & : 0 < t < T \\ 1/2 & : t = 0 ; t = T \\ 0 & : \text{otherwise} \end{cases}$$



$$y(t) = m_i(t) * h(t) \longrightarrow (1)$$

$m_i(t) \rightarrow$ o/p of instantaneous Sampling

$$y(f) = M_i(f) \cdot H(f) \longrightarrow (2)$$

$$m_i(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\,\delta(t - nT_s) \longrightarrow (3)$$

$$M_i(f) = f_s \sum_{n=-\infty}^{\infty} M(f - nf_s) \longrightarrow (4)$$

$$y(t) = \int_{-\infty}^{\infty} m_i(\tau)\, h(t-\tau)\, d\tau \longrightarrow (5)$$

eq(3) in eq(5)

$$y(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} m(nT_s)\,\delta(\tau - nT_s)\, h(t-\tau)\, d\tau$$

$$= \sum_{n=-\infty}^{\infty} m(nT_s) \int_{-\infty}^{\infty} \delta(\tau - nT_s)\, h(t-\tau)\, d\tau$$

* According to the shifting property of delta function

$$\int_{-\infty}^{\infty} \delta(\tau - nT_s) = 1 \text{ at } \tau = nT_s$$
$$= 0 \text{ ; otherwise}$$

$$\boxed{\begin{aligned} \int_{-\infty}^{\infty} \delta(\tau)\, h(t-\tau)\, d\tau &= \delta(t) * h(t) \\ &= h(t) \\ \int_{-\infty}^{\infty} \delta(\tau - nT_s)\, h(t-\tau) &= \delta(t - nT_s) * h(t) \\ &= h(t - nT_s) \end{aligned}}$$

$$y(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\, h(t - nT_s) \longrightarrow (6)$$

Here

$$m_i(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\, \delta(t - nT_s)$$

$$\Big\downarrow F.T$$

$$M_i(f) = f_s \sum_{n=-\infty}^{\infty} M(f - nf_s)$$

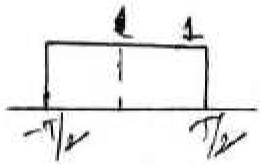
From eq (2)

$$Y(f) = M_i(f) \cdot H(f) \longrightarrow (7)$$

eq (4) is substituted in eq (7)

$$Y(f) = f_s \sum_{n=-\infty}^{\infty} M(f - nf_s) \cdot H(f)$$

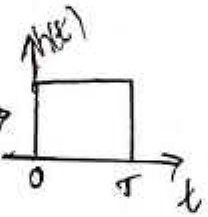
$$Y(f) = f_s\, M(f) \cdot H(f)$$



$$\xRightarrow{F.T} \quad T\,\mathrm{sinc}(fT)\, e^{-j2\pi f \cdot \frac{T}{2}}$$

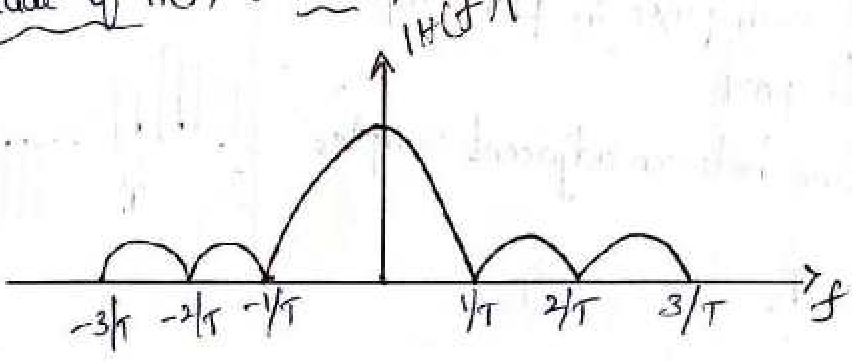
$\Downarrow$

shifted to $\Rightarrow$

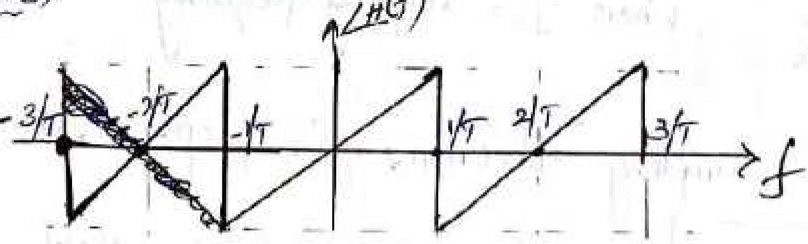


$$\therefore\ H(f) = T\,\mathrm{sinc}(fT)\, e^{-j2\pi f T}$$

Amplitude of H(f) : $|H(f)|$



Phase of H(f)



* Therefore magnitude and phase of H(f) are not constant. Due to magnitude and phase of H(f), distortion present in output of reconstruction, hence it is called <u>Aperture effect</u>

Drawbacks of PAM

(i) Aperture effect :- Due to transfer function of pulse stretcher network, it causes amplitude and phase variations. It can be overcome by using equalizer.

(ii) In PAM, information present in amplitude, so that which can be easily affected by noise.

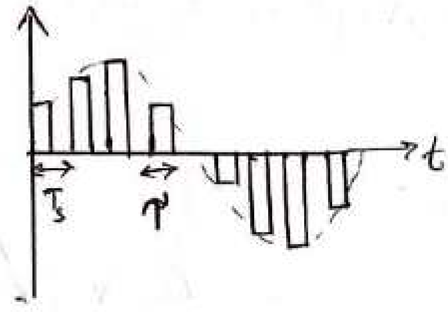
Therefore more interference and crosstalk will present at the output.

(iii) Variable power for transmission due to changing amplitude of message signal.

Transmission Bandwidth of PAM Signal

* Let τ = Width of each pulse in Flat-Top sampled PAM

T_s = Duration between adjacent samples



Here $\tau << T_s \longrightarrow (1)$

Here $T_s = \frac{1}{f_s}$, where f_s = sampling frequency

* According to sampling theorem, for proper recovering of signal $\boxed{f_s \geq 2W}$, where W = message B.W.

So, $\frac{1}{T_s} \geq 2W$ (or) $\boxed{T_s \leq \frac{1}{2W}} \longrightarrow (2)$

eq (2) substitute in eq (1)

$\therefore \tau << \frac{1}{2W}$ (or) $W << \frac{1}{2\tau}$

* To transmitt and receive the PAM signal without signal distortion, the transmission bandwidth B_T need to satisfy the following equation

$$\boxed{B_T \geq \frac{1}{2\tau} >> W}$$

* thus the transmission B.W of PAM signal is very large compared to highest frequency in the signal m(t).

* If B_T decreases, then τ increases, therefore it increases the aputure effect. To avoid this one, B_T should be very large

Problem

For a PAM transmission of voice signal having maximum frequency equal to $f_m = 3$ KHz. Calculate the transmission bandwidth. sampling frequency $f_s = 8$ KHz, pulse duration $\tau = 0.1T_s$

Sol

$$T_s = \frac{1}{f_s} = \frac{1}{8 \times 10^3} \text{ sec}$$

$$T_s = 0.125 \times 10^{-3} = 125 \mu sec.$$

$$\tau = 0.1 T_s$$

$$\tau = 0.1 \times 125 = 12.5 \mu sec.$$

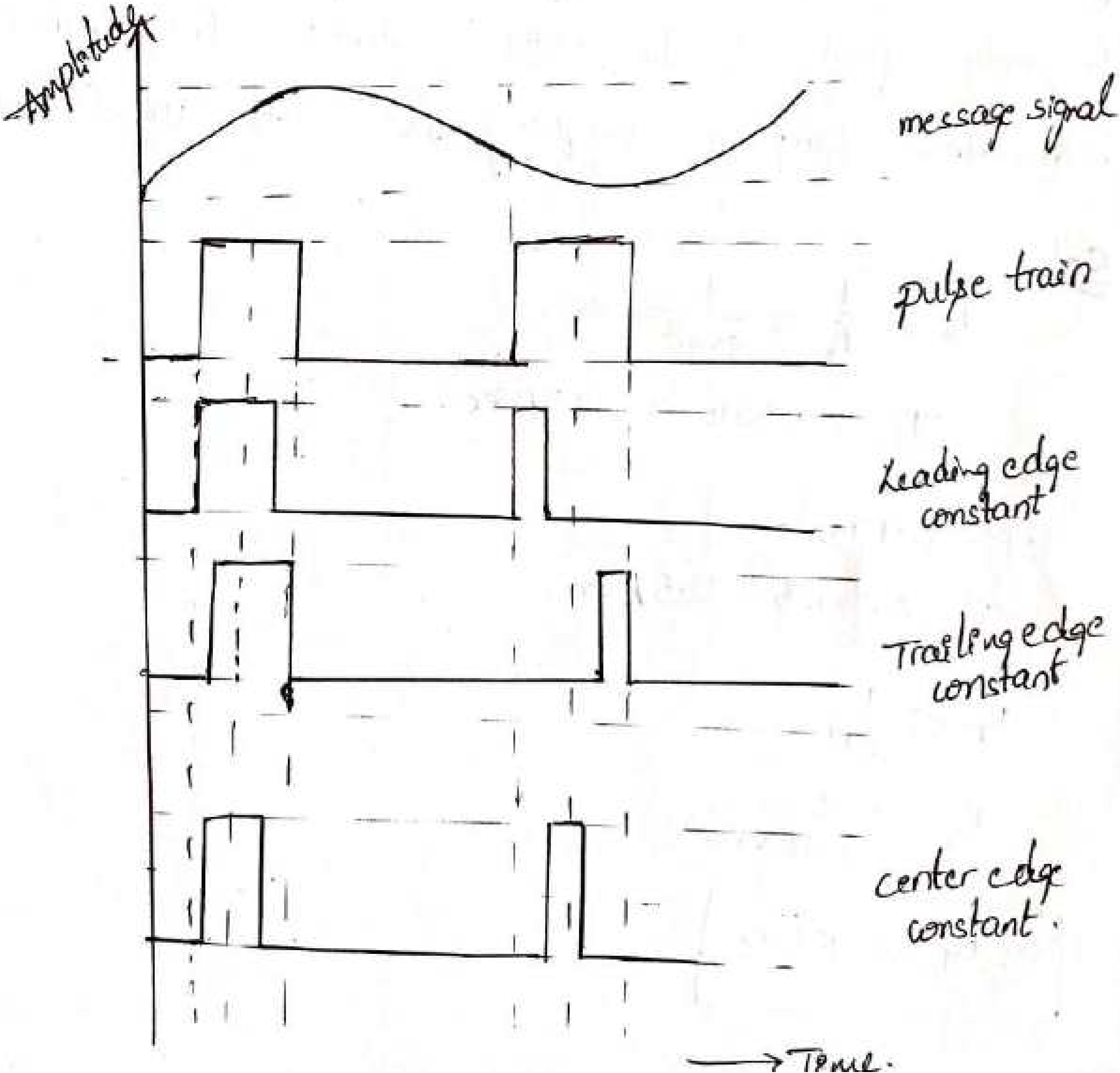
$$B_T \geq \frac{1}{2\tau}$$

$$B_T \geq \frac{1}{2 \times 12.5 \times 10^{-6}}.$$

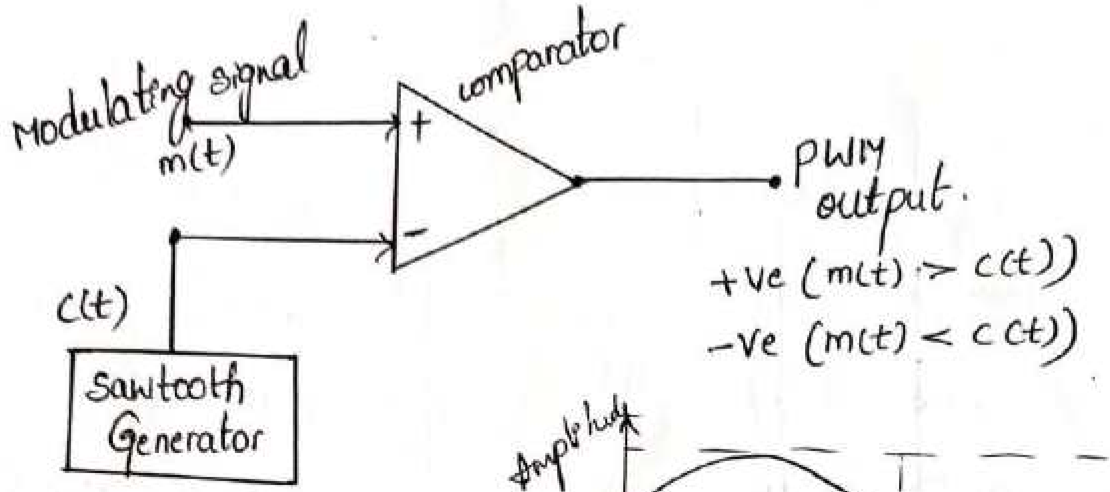
$$\boxed{\therefore B_T \geq 40 KHz}$$

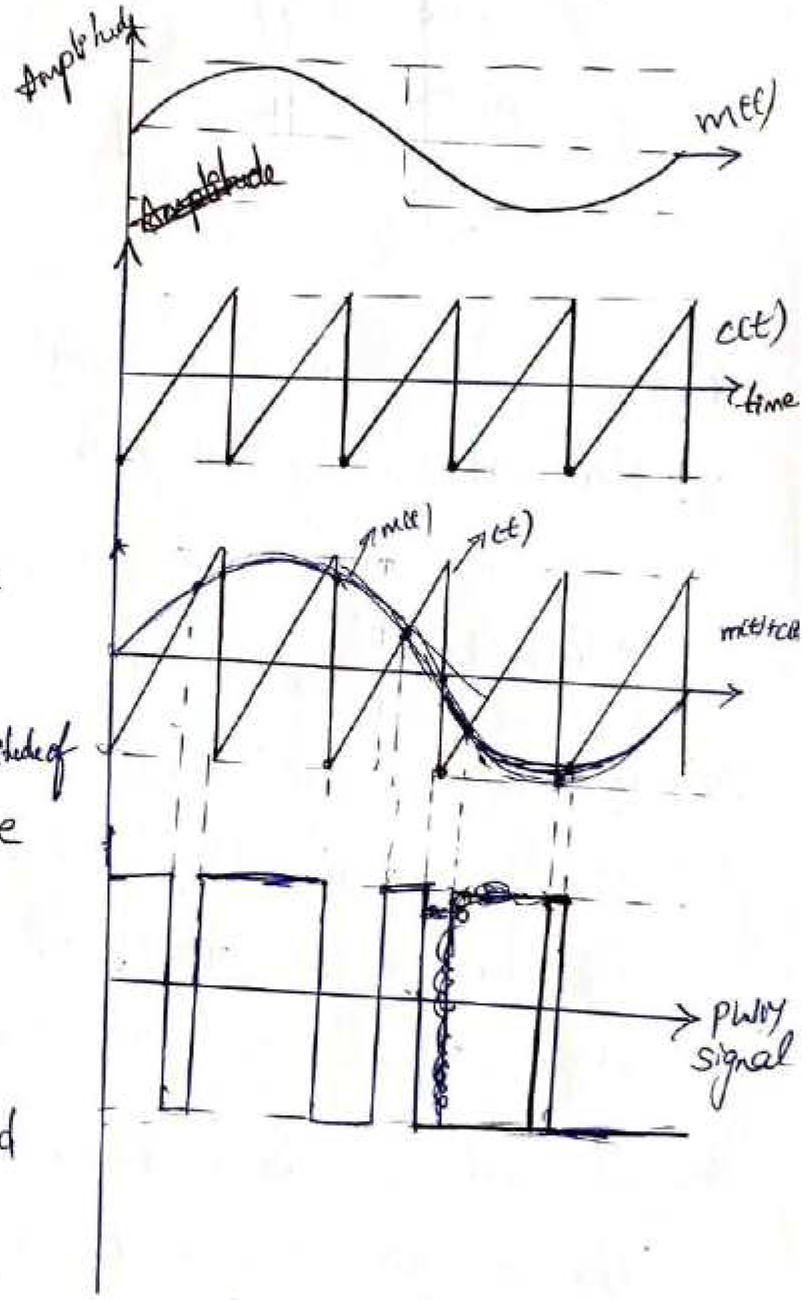
Pulse Width Modulation (PWM)

* PWM is also called as pulse Duration modulation (PDM) and pulse length modulation (PLM)
* The width of the pulse is varied inaccordance with the instantaneous value of the message signal.
* The modulating signal may vary with time of occurance of the leading edge, the trailing edge (or) both edge of the pulse.
* Three variations of PWM are possible.
    1. Leading edge of pulse kept constant.
    2. Trailing edge of pulse kept constant
    3. center edge of pulse kept constant.



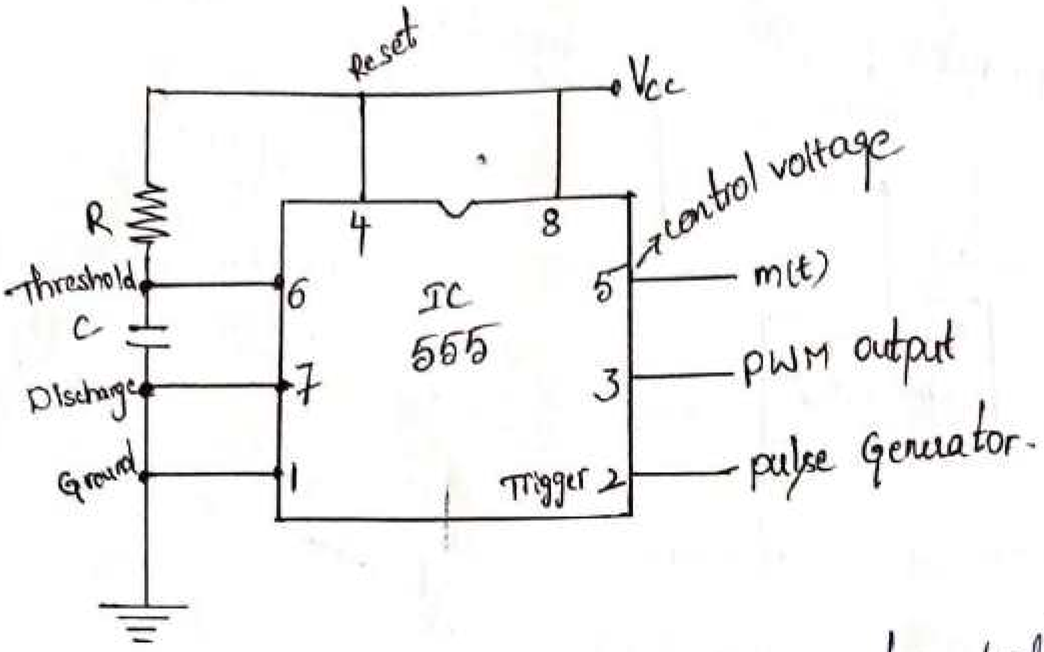
Generation of PWM





* The message signal m(t) and sawtooth generator c(t) is given to comparator.
* If amplitude of m(t) is greater than the c(t) amplitude, then it will produce positive output.
* If m(t) amplitude is less than amplitude of c(t), then it will produce negative output.
* Therefore finally it generates the pulse width modulated signal.
* IC 710 can be used as a comparator.

PWM Generation by using 555 timer IC (Practical PWM Generation)

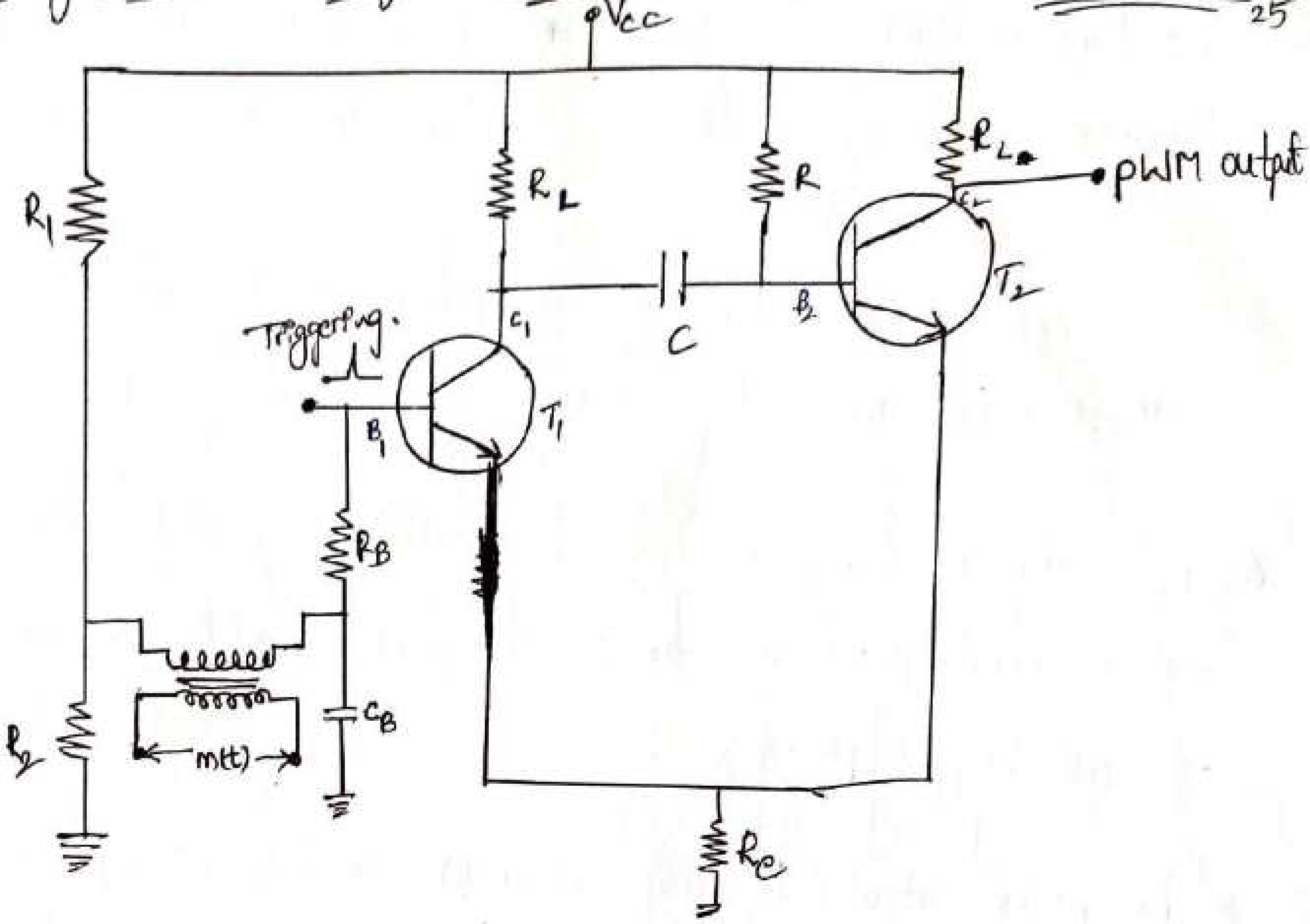


* when the modulating signal m(t) is a positive half cycle, then the control voltage of 555 timer is increases and the time duration of ON period is charging upto the threshold level of the input signal.

* If the message signal m(t) is a negative half cycle, then the control voltage of 555 timer will reduce and time duration of OFF period is ~~[illegible]~~ changes. Hence the output signal is PWM signal.

* the external side capacitor and resistor produce Synchronization between one element to other element in the entire parts of the IC.

PWM generation using Transistor (or) Emitter coupled monostable multivibrator



* The monostable multivibrator operates when positive pulse is given. It has two inputs, one is m(t) and other input is trigger input.
* Input is applied at the base of transistor T_1, and the output will present at the collector of transistor T_2.
* When positive trigger pulse, the transistor T_1 is 'ON', when T_1 is 'ON', the collector current (I_{C_1}) begins to increase, as a result the voltage at B_2 falls, that makes the transistor T_2 is OFF.

T_2 is OFF means, it is open circuit, the capacitor 'C' begins to charge upto the collector supply voltage through 'R'. Therefore the output voltage increases

* After a time determined by the supply voltage and the RC time constant of the charging network, the base of T_2 becomes sufficiently positive to switch T_2 is ON.

(or)

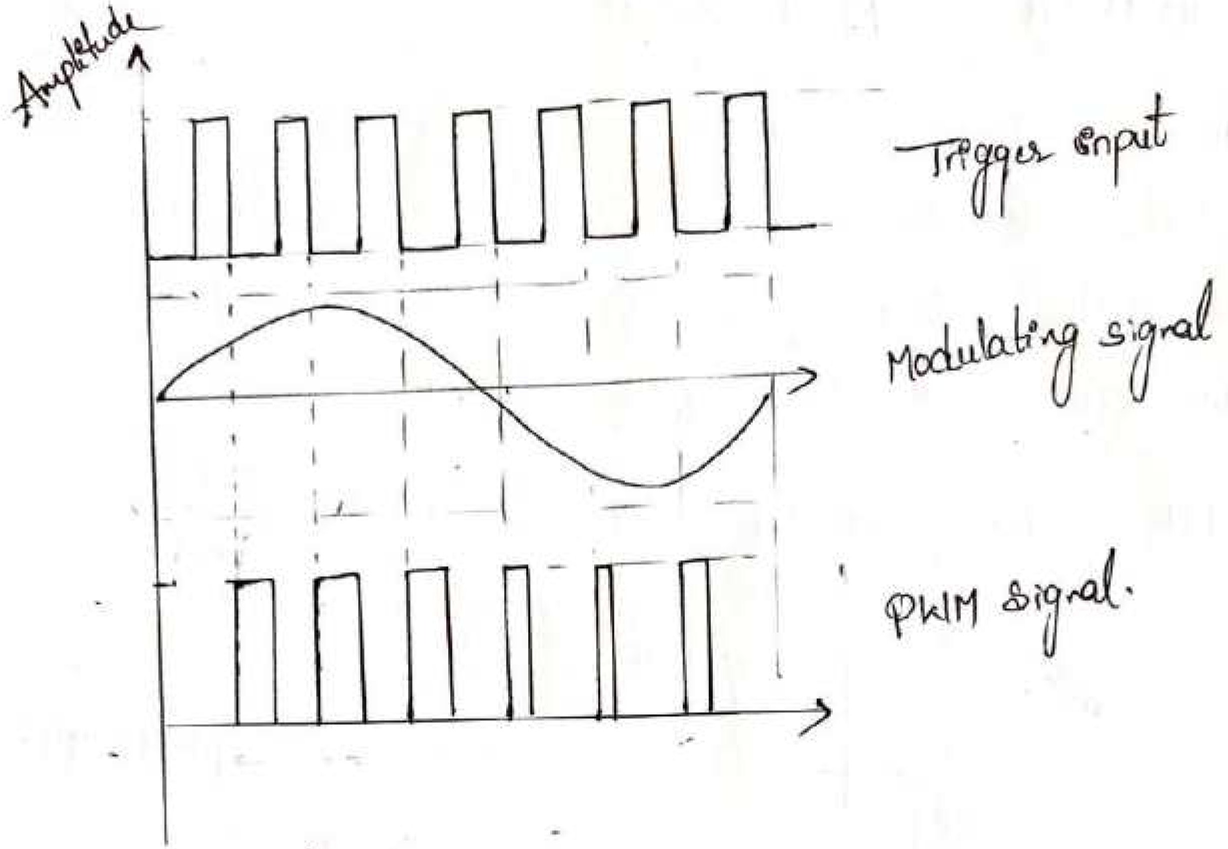
once the capacitor voltage reaches to a minimum input voltage for T_2 to turn ON.

* T_2 is ON, the Transistor T_1 is simultaneously switched OFF by regenerative action and it stays OFF until the arrival of next trigger pulse.

* The PWM output depends on m(t) (signal voltage) : If signal voltage is maximum, the voltage ~~that~~ capacitor should charge to turn ON T_2 is also maximum, therefore at maximum signal voltage, capacitor has to charge to maximum voltage requiring maximum time to charge. This gives us maximum pulse width at maximum input signal voltage.

* At minimum signal voltage, the capacitor has to charge for a minimum voltage and we get minimum pulse width at the output.

* therefore the pulse width is controlled by the input signal voltage and we get pulse width modulated waveform



Advantages of PWM

* Noise interference is less due to amplitude has been made constant.
* signal can be separated ~~very~~ easily at demodulation and noise can also be separated easily.
* Synchronization between Transmitter and receiver is not required.

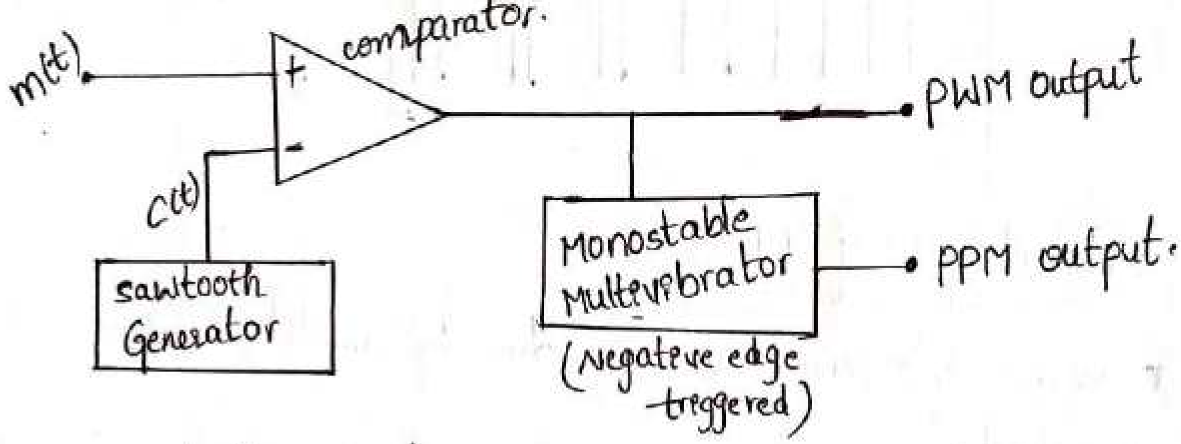
Disadvantages of PWM

* Power will be variable because of varying in width of pulse. Transmitter can handle the more power for maximum width of the pulse
* B.W should be large to use in communication, should be huge even when compared to the pulse amplitude modulation.

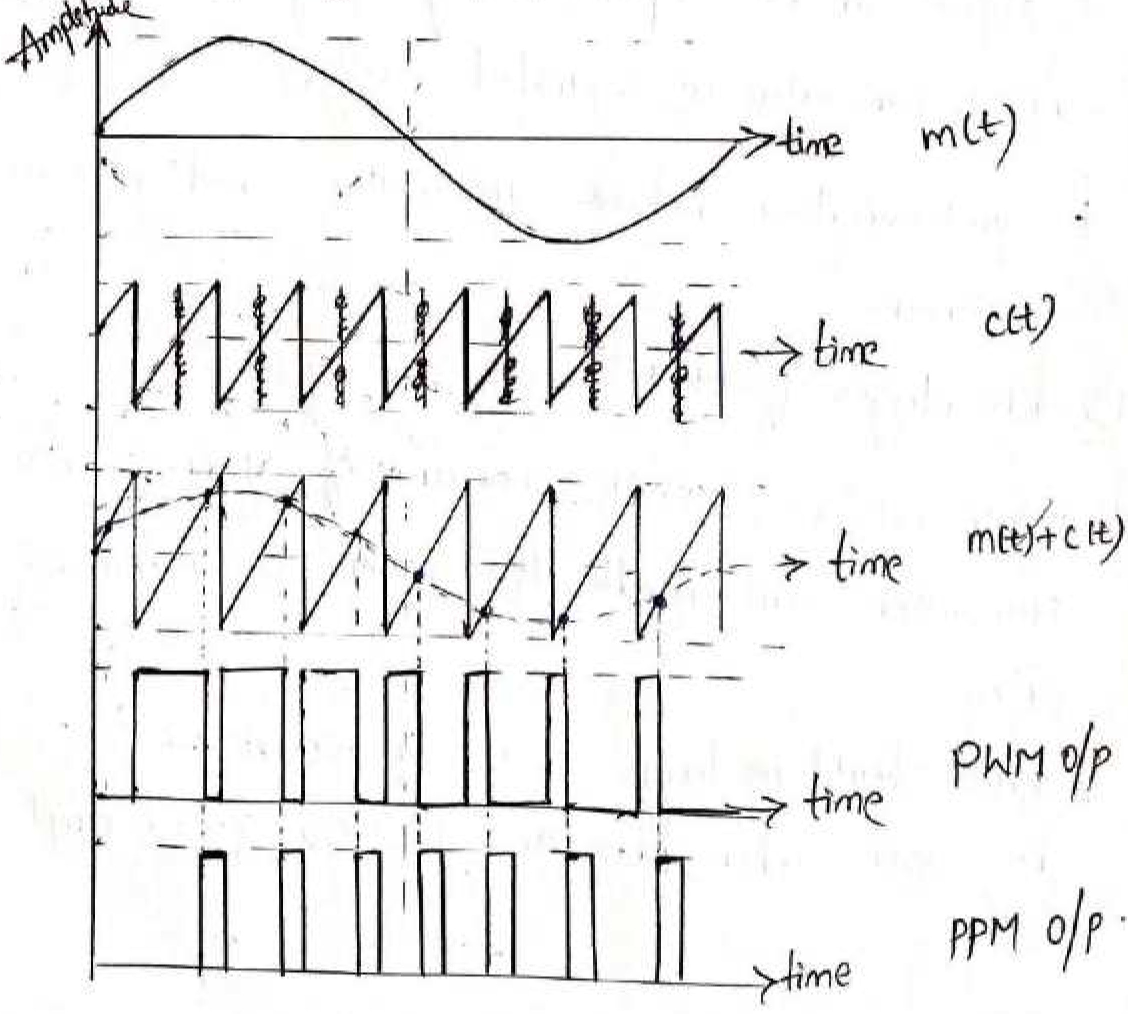
Generation of PPM signal

* In PPM, the width and amplitude of the pulses are kept constant, while the position of each pulse, with reference to the position of a reference pulse varies according to the instantaneous value of the message signal.

* PPM is done in accordance with the PWM signal.



Fig(1) circuit diagram of PPM

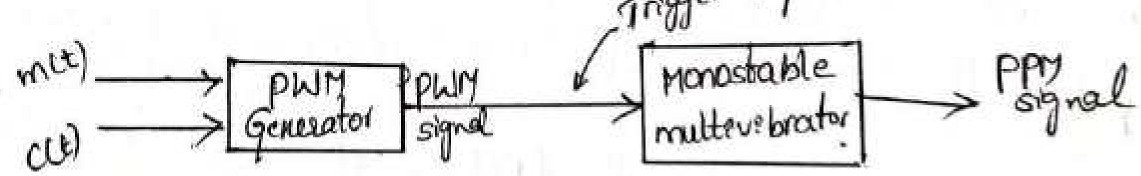


* comparator has two inputs one is m(t) and other is c(t). comparator compares the two inputs.
* If amplitude of m(t) > Amplitude of C(t), then output will generate positive output. If the amplitude of m(t) is less than amplitude of c(t), then the output will generate negative output.
* therefore the pulse width modulated (PWM) signal is generated
* this PWM signal is applied to the input of monostable multivibrator. It is used as the trigger input to a monostable multivibrator.

  Its output remains zero until it is triggered on the trailing edge of PWM

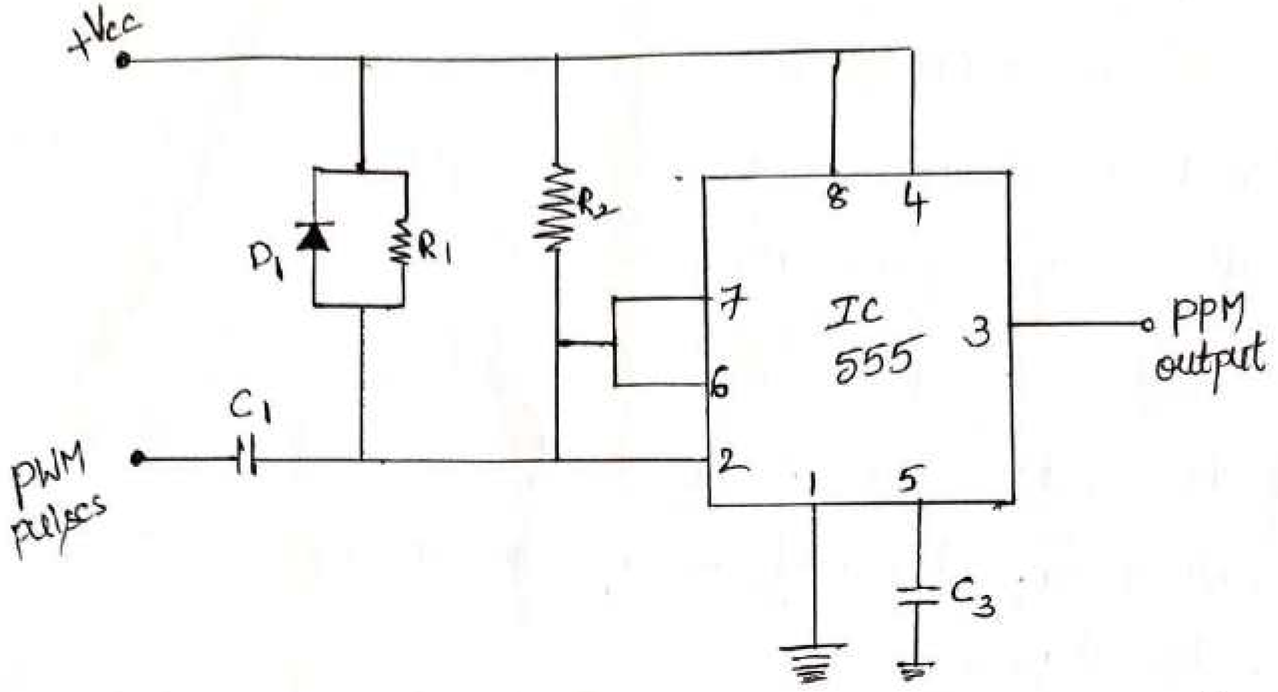
* Hence, the position of these pulses is proportional to the width of the trailing edge of PWM signal.
* the transmitter has to send synchronizing pulses to keep the transmitter and receiver is synchronism.

  These sync pulses help to maintain the position of the pulses.



Block diagram of PPM Generator

Generation of PPM by using 555 timer IC (Practical PPM generation)



* It consists of differentiator and a monostable multivibrator. the input of to the differentiator is a PWM waveform. the differentiator generates positive and negative spikes corresponding to leading and trailing edges of the PWM waveform.
* Diode 'D1' is used to bypass the positive spikes. the negative spikes are used to trigger the monostable multivibrator.
* the monostable multivibrator then generates the pulses of same width and amplitude with reference to trigger pulse to give pulse position modulated waveform.
* Trailing edge of PWM pulses will generate the pulse position modulated (PPM) signal.

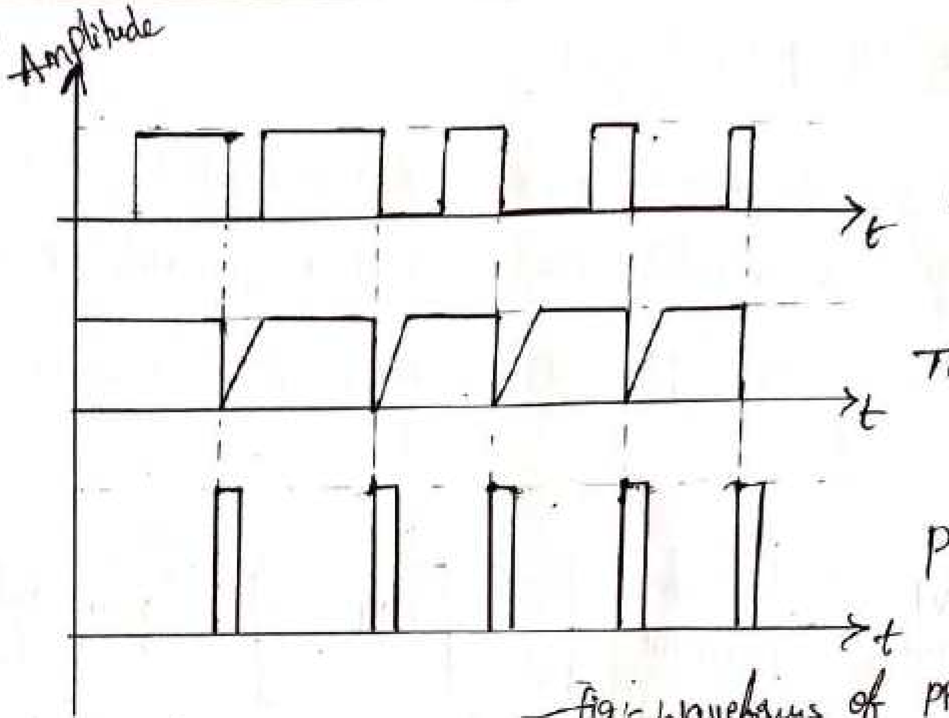


fig:- Waveforms of PPM

Advantages of PPM

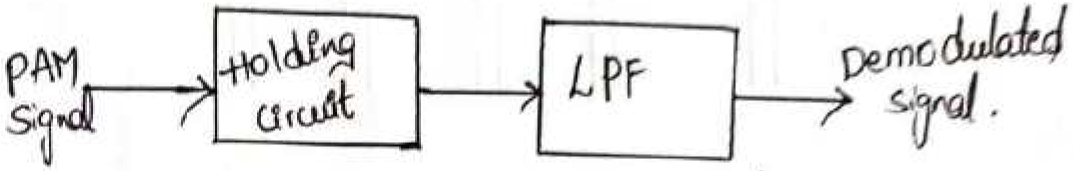
* Noise interference is less (or) minimum due to constant amplitude.
* It is easy to separate out signal from noisy signal.
* Instantaneous power of PPM modulated signal remains constant due to constant pulse widths and pulse amplitudes.
* It requires less power compare to PAM due to short duration pulses.

Disadvantages of PPM

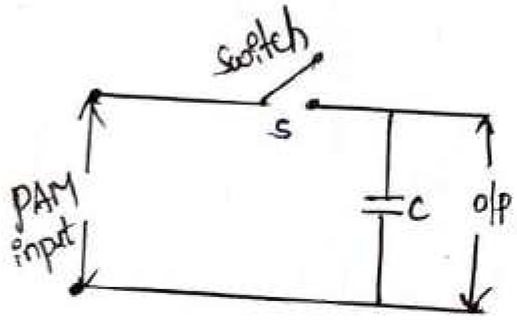
* The synchronization between transmitter and receiver is required, which is not possible for every time and we need dedicated channel for it.
* Large BW is required for transmission same as pulse amplitude modulation.

Demodulation of PAM signal

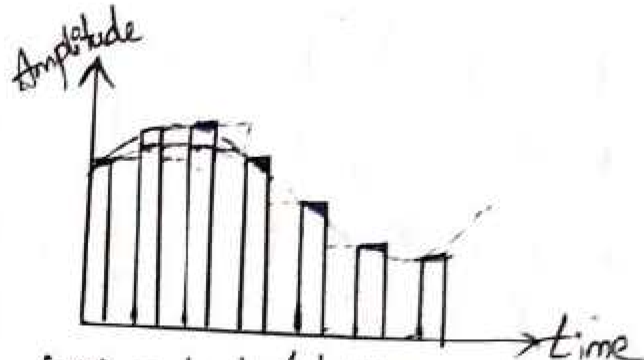
* Demodulation is the reverse process of modulation in which the modulating signal is received back from a modulated signal.
* For PAM signals, the demodulation is done using a holding circuit.



fig(1) Block diagram of detection of PAM

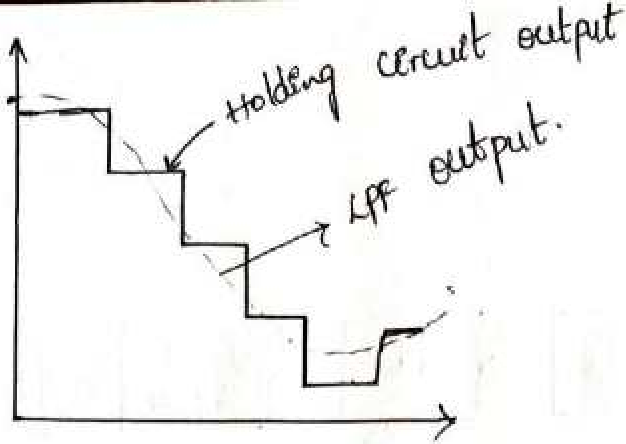


fig(2): Zero order holding circuit

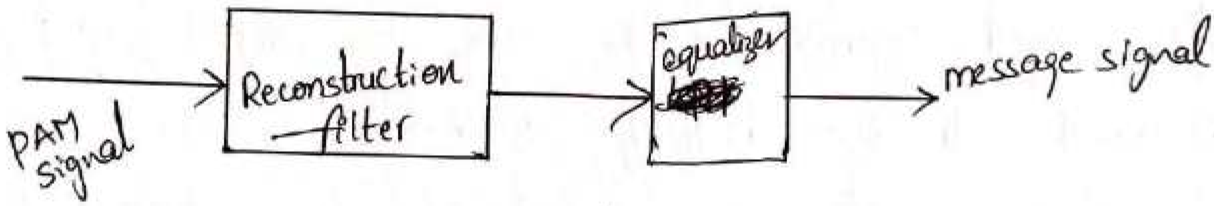


fig(3) output of holding circuit.

* In this method, the received PAM signal is allowed to pass through a holding circuit and a lowpass filter.
* The switch 'S' is closed after the arrival of the pulse and it is opened at the end of the pulse.
* In this way, the capacitor 'C' is charged to the pulse amplitude value and it holds this value during the interval between the two pulses. Hence the samples values are held.
* After this holding circuit output, the output is smoothened by using LPF i.e to reconstruct the original signal m(t).

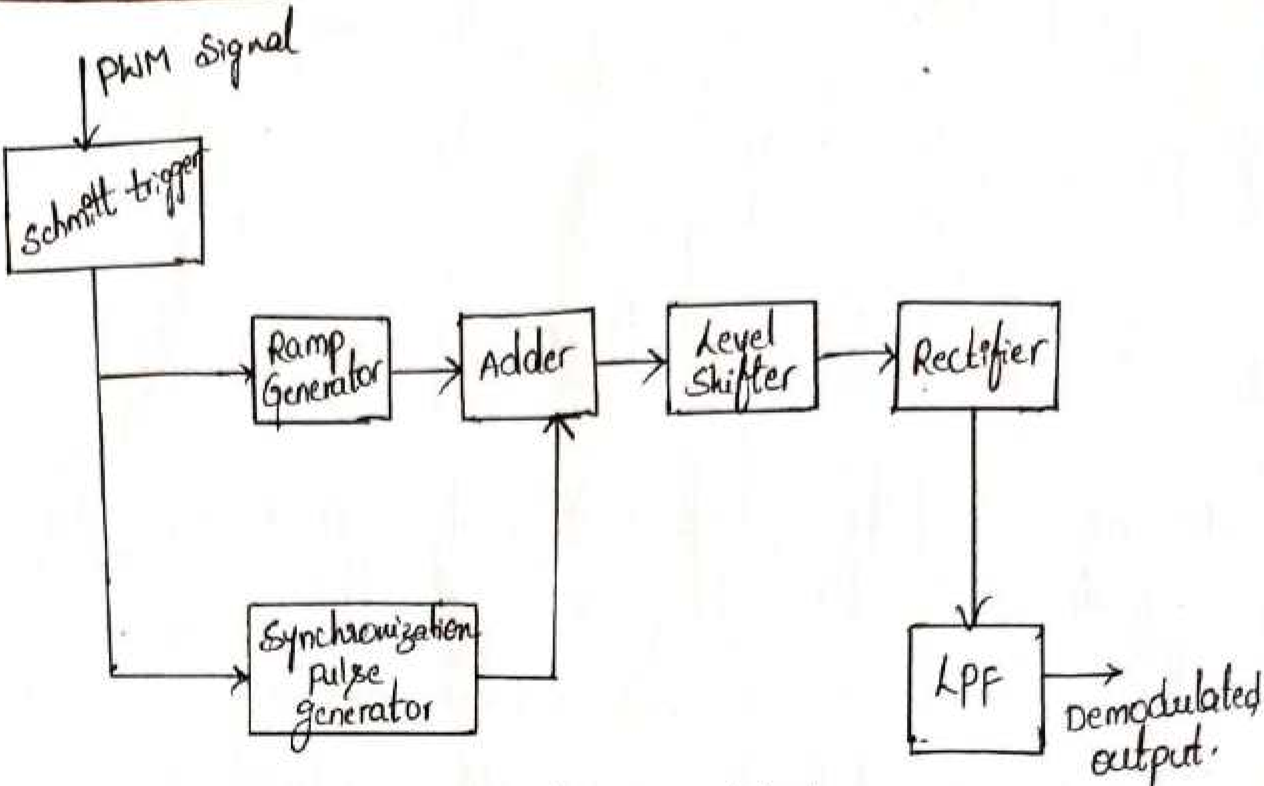


* Incase of Flat-Top PAM to reduce aperture effect, an equalizer is used.
* the Receiver consists of low pass reconstruction filter with cut-off frequency slightly higher than the maximum frequency in message signal.
* The equalizer compensates the aperture effect, it means that decreasing the losses of the reconstruction filter within ~~the~~ Bandwidth.



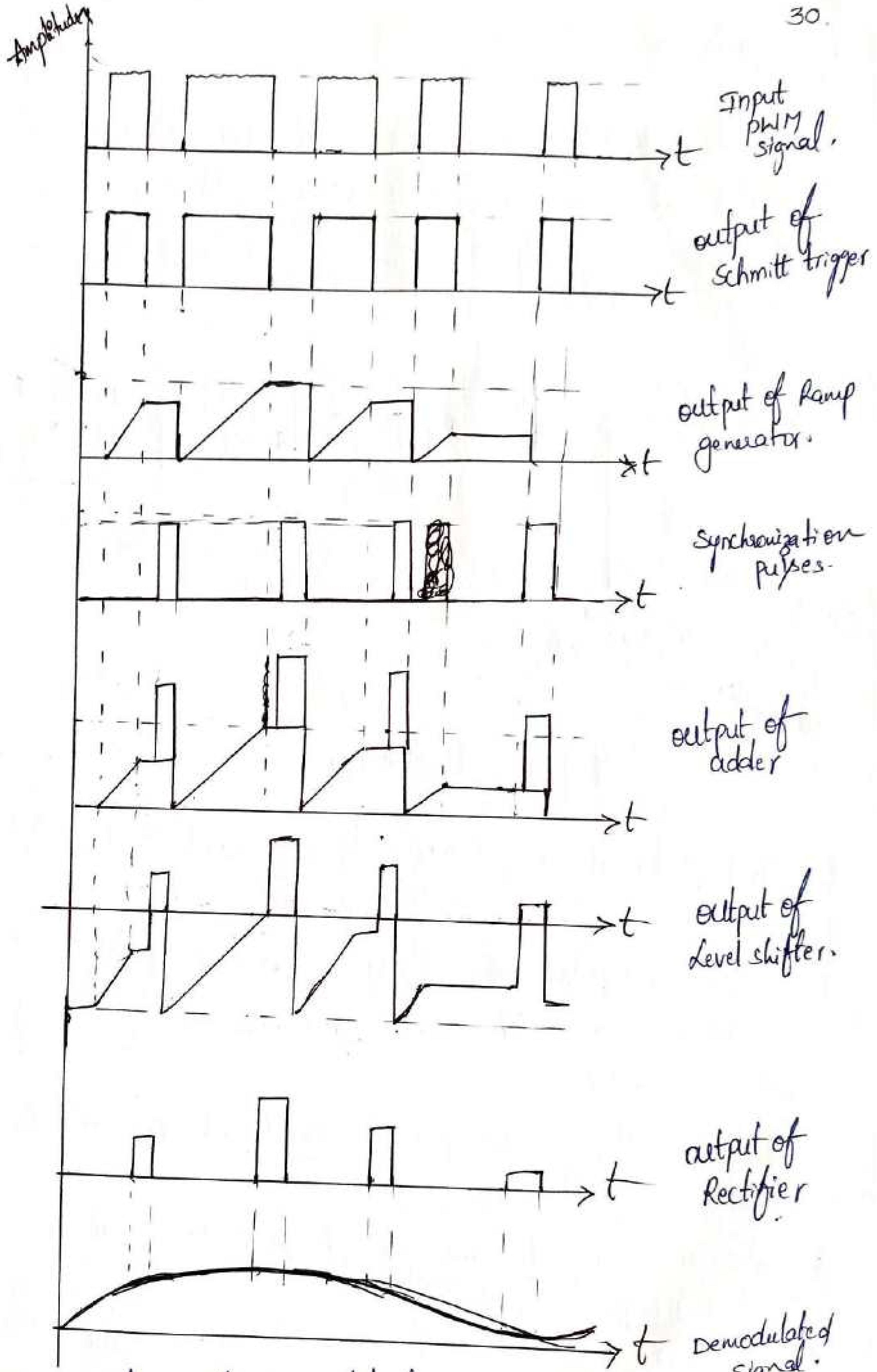
Demodulation of PWM signal

* The received PWM signal is applied to the schmitt trigger
* schmitt trigger circuit removes noise from PWM wave. therefore the regenerated PWM is applied to the ramp generator and the synchronization pulse generator.
* The ramp generator produces ramp for the duration of pulses such that height of the ramps are proportional to the width of PWM pulses.



fig(1): Block diagram of PWM detector

* the maximum ramp voltage is retained till the next pulse.
* synchronization pulse generator produces reference pulses with constant amplitude and pulse width. These pulses are delayed by specific amount of delay as shown in fig(2)
* The delayed reference pulses and the output of ramp generator is added with the help of adder.
* the output of adder is given to the level shifter. Level shifter offset the DC level
* the output of level shifter is given to the ~~dep~~ rectifier, therefore the negative part of the waveform is clipped by the rectifier.
* Finally the output of the rectifier is passed through LPF to recover the modulating signal.



Fig(2) Waveforms of PWM detector.

Demodulation of PPM signal

* For the demodulation of PPM signal, PPM pulses are first converted into corresponding PWM pulses with the help of SR flipflop as shown in fig.

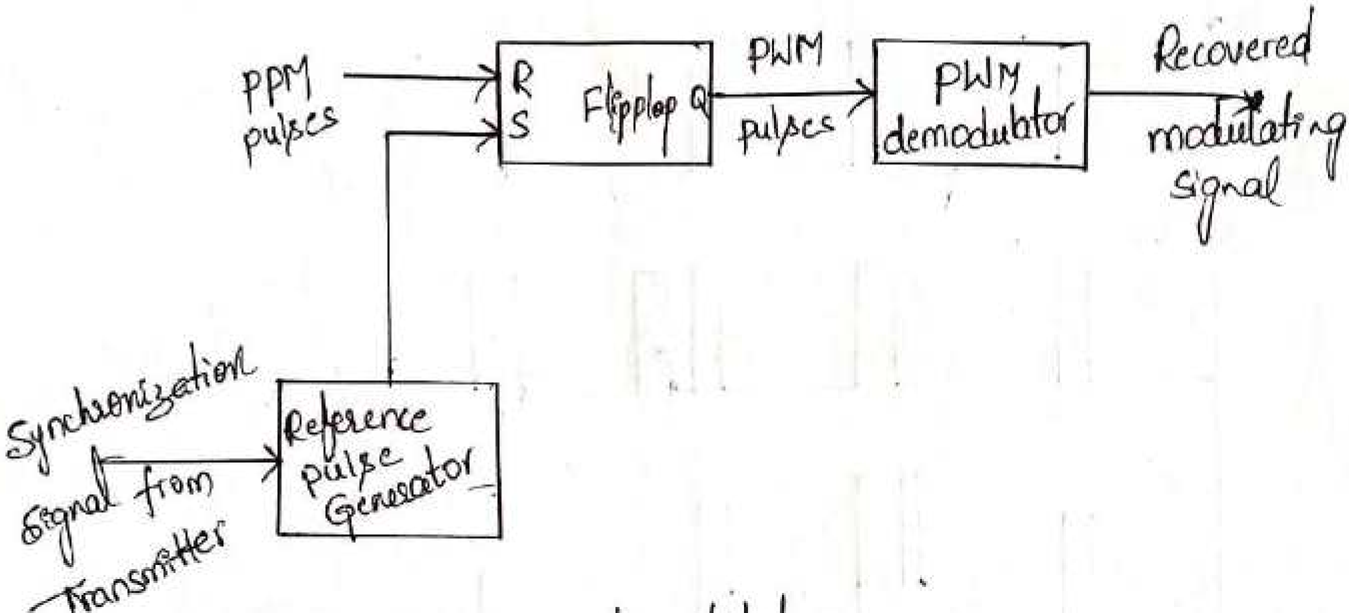
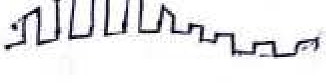
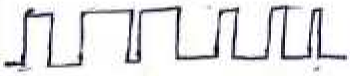


fig: PPM demodulator

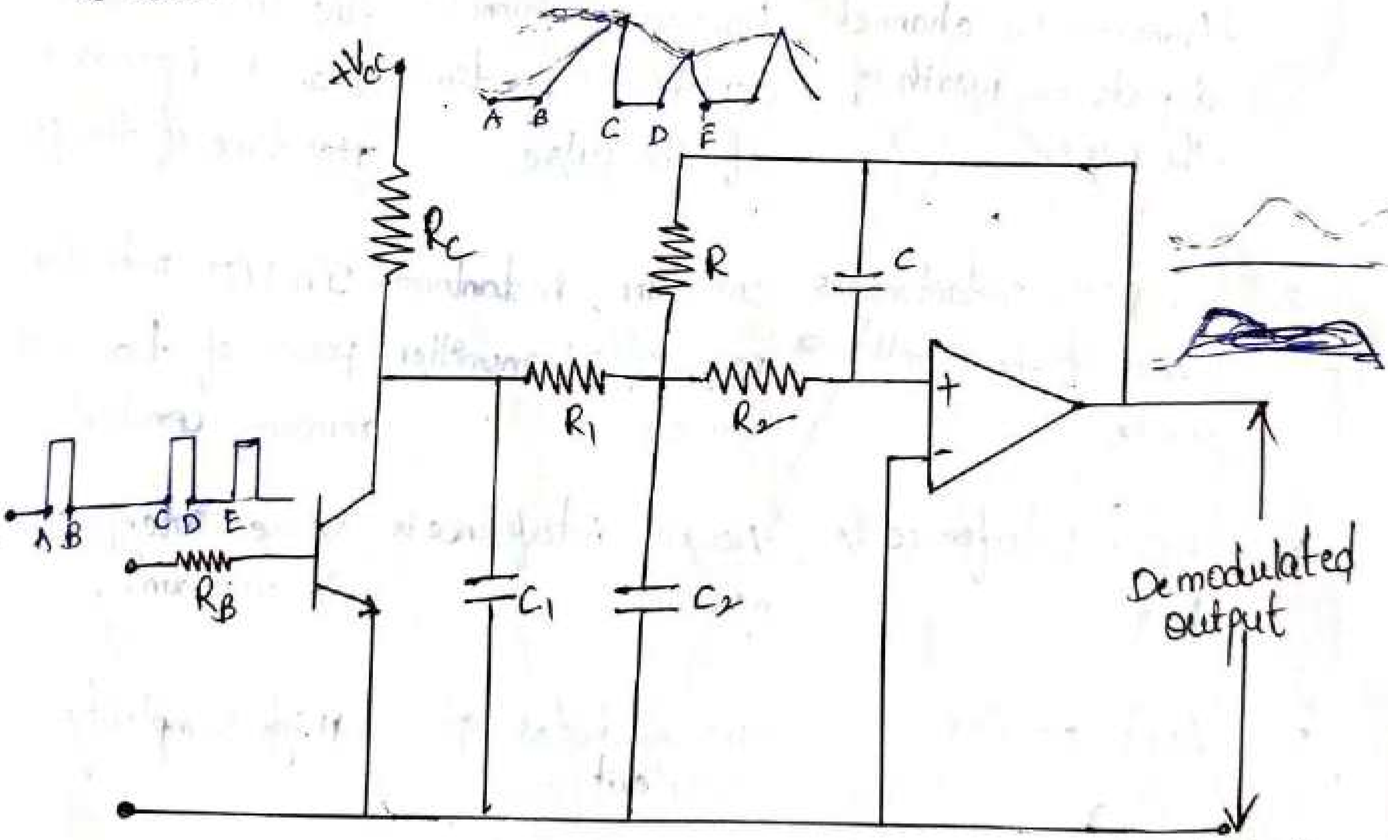
* Flip flop circuit is set (or) Turned ON when the reference pulse arrives.
* This reference pulse is generated by reference pulse generator of the receiver with the synchronization signal from the transmitter.
* The flip flop circuit is reset (or) turned OFF at the leading edge of the PPM pulse.
* This process repeats and we get PWM pulses at the output of flip flop.
* The PWM pulses are then demodulated by using PWM demodulator to get original modulating signal.

Comparision of PAM, PWM and PPM signals

S.No.	PAM	PWM (or) PDM	PPM
1.	Amplitude of carrier pulse is proportional to the amplitude of modulating signal. 	width of the carrier pulse is proportional to amplitude of modulating signal. 	The relative position of the pulse is proportional to the amplitude of modulating signal. 
2.	The Bandwidth of the transmission channel depends on width of the pulse.	The Bandwidth of the transmission channel depends on rise time of the pulse	The Bandwidth of the transmission channel depends on rise time of the pulse
3.	In PAM, instantaneous power of transmitter varies	In PWM, instantaneous power of transmitter varies	In PPM, instantaneous power of transmitter remains constant.
4.	Noise Interference is high	Noise interference is minimum	Noise interference is minimum.
5.	Least complex	Intermediate complexity	High complexity.
6.	Least SNR	Intermediate SNR	Maximum SNR.
7.	synchronization is not required with transmitter	synchronization is not required with transmitter.	synchronization is required with transmitter. Receiver fails incase synchronization is not present.

Demodulation of PPM Signal using Transistor.

* In this circuit, during the gap from "A" to "B" between the pulses, duration from A to B, its amplitude is high, therefore transistor is 'ON', then it is short circuited, entire Vcc is connected to ground, therefore the output is '0'.
* similarly during the pulse duration from B to C, its amplitude is zero, the transistor is cut-off and the capacitor gets charged through R-C combination.
* Hence this waveform at the collector is approximately a sawtooth waveform.



* From that waveform, envelope contains information related to message signal.
* Now when this is passed through a second order op-Amp LPF, therefore the desired demodulated output is obtained.