Unit-IV (Part-II) Number System

→ The Number System contains numbers formed with 0 to 9 and signed & Unsigned numbers.

→ The following are the properties of Number System

(i) Closure Property
(ii) Associative Property
(iii) Identity Property
(iv) Inverse Property
(v) Commutative Property
(vi) Distributive Property

1. Closure Property :-

$a \in I, b \in I$

i) $a + b \in I$

ii) $a * b \in I$

2. Associative Property :-

$a + (b + c) = (a + b) + c$

$a * (b * c) = (a * b) * c$

3. Identity Property :-

$e = 0$ w.r.t '+' operator

$e = 1$ w.r.t. '*' operator

Ex. (i) $a + 0 = a$

(ii) $a * 1 = a$

4. Inverse Property :- i) $a + (-a) = 0$

ii) $a * (a^{-1}) = 1$

5. Commutative Property :- i) $a + b = b + a$

ii) $a * b = b * a$

6. Distributive Property :-

$a * (b + c) = (a * b) + (a * c)$

$a + (b * c) = (a + b) * (a + c)$

* Division Theorem [Euclidean Algorithm] :-

→ Let a, b, q and r are 4 integers then a is divisible by b with quotient 'q' and remainder 'r' is defined as $a = b * q + r$

$$\boxed{a = b * q + r}$$

→ It is Symbollically denoted as a/b or b/a.

Ex:- Find the remainder if 99 is divisible by 10.

$10\overline{)99}(9$
$\underline{90}$
9

$a = b * q + r$

$99 = 10 * 9 + 9$

where remainder = 9 and quotient = 9

* Greatest Common Divisor (G.C.D):-

→ GCD is a Combination of Several division theorems ie.

$a = b * q_1 + r_1$

$b = r_1 * q_2 + r_2$

$r_1 = r_2 * q_3 + r_3$

⋮

$r_{n-2} = r_{n-1} * q_n + r_n$

where r_1 is the remainder when 'a' is divisible by 'b' with quotient 'q_1'.

→ r_2 is the remainder when 'b' is divisible by 'r_1' with quotient 'q_2'

⋮

→ r_n is the remainder when 'r_{n-2}' is divisible by "r_{n-1}" with quotient 'q_n'.

* Integral Linear Combination :-

→ The following are the steps involved in Integral Linear Combination (ILC)

Step ①: Find all the remainder expressions except the remainder zero value.

Step ②: Consider the bottom remainder expression and substitute remainder expressions from bottom to top to get the values of a & b.

Ex:- Find the values of u & v from the eqn $19u + 17v$

Sol:-

$a = 19$

$b = 17$

17 | 19 | 1
17
2 | 17 | 8
16
1 | 2 | 2
2
0

GCD value $\boxed{d = 1}$

Euclidean Algorithm Process :

$a = b \times q + r$

$19 = 17 \times 1 + 2$

$17 = 8 \times 2 + 1$

$\boxed{2 = 1 \times 2 + 0}$ → stop

ILC

$19 = 17 \times 1 + 2 \Rightarrow \boxed{2 = 19 - 17}$

$17 = 2 \times 8 + 1$

Consider $17 = 2 \times 8 + 1$

$1 = 17 - (2 \times 8)$

Sub '2' value

$1 = 17 - (19 - 17 \times 8)$

$1 = 17 - 19 \times 8 + 17 \times 8$

$= 17 + 17 \times 8 - 19 \times 8$

$= 17 \times 9 + 19 \times (-8)$

$1 = 19 \times (-8) + 17 \times 9$

Given, $19u + 17v$

$u = -8$ and $v = 9$

$\Rightarrow \boxed{d = 1 \quad u = -8 \quad v = 9}$

Ex:② Find the values of u & v from the Eqn $1080u + 615v$

Sol: Given $a = 1080$

$b = 615$

$$
\begin{array}{l}
615 \,|\, 1080 \,|\, 1 \\
\quad\;\; 615 \\
\quad\;\; 465 \,|\, 615 \,|\, 1 \\
\qquad\quad\;\; 465 \\
\qquad\quad\;\; 150 \,|\, 465 \,|\, 3 \\
\qquad\qquad\quad\;\; 450 \\
\qquad\qquad\quad\;\; 15 \,|\, 150 \,|\, 10 \\
\qquad\qquad\qquad\;\; 150 \\
\qquad\qquad\qquad\;\; 0
\end{array}
$$

GCD $= d = 15 \Rightarrow \boxed{d = 15}$

Euclidean Algorithm Process

$a = b \times q + r$

$1080 = 615 \times 1 + 465$

$615 = 465 \times 1 + 150$

$465 = 150 \times 3 + 15$

$150 = 15 * 10 + 0$ (stop)

ELC

$615 = 465 * 1 + 150 \Rightarrow \boxed{150 = 615 - 465}$

Consider, ~~1080~~ $465 = 150 * 3 + 15$

$15 = 465 - (150 * 3)$

$15 = 465 - (615 * 3 - 465 * 3)$

$= 465 - 615 * 3 + 465 * 3$

$= \underbrace{465 + 465 * 3}_{} - 615 * 3$

$15 = 465 * 4 - 615 * 3$

$\because 1080 = 615 * 1 + 465 \Rightarrow \boxed{465 = 1080 - 615}$

Sub '465' value.

$15 = (1080 - 615) * 4 - 615 * 3$

$= 1080 * 4 - 615 * 4 - 615 * 3$

$= 1080 * 4 - 615 * 7$

$15 = 1080 * 4 + 615 * (-7)$

$d = 1080u + 615v$

$$\boxed{\begin{array}{l} d = 15 \\ u = 4 \\ v = -7 \end{array}}$$

Ex: 3 Solve the eqn $19u + 29v$

Sol:

$a = 29$

$b = 19$

$19 \,|\, 29 \,|\, 1$
$\quad 19$
$10 \,|\, 19 \,|\, 1$
$\quad 10$
$9 \,|\, 10 \,|\, 1$
$\quad 9$
$1 \,|\, 9 \,|\, 9$
$\quad 9$
$\quad 0$

$\boxed{d = 1}$

Euclidean Algorithm Process

$$a = b * q + r$$

$$29 = 19 * 1 + 10$$

$$19 = 10 * 1 + 9$$

$$10 = 9 * 1 + 1$$

$$9 = 1 * 9 + 0 \rightarrow \text{stop}$$

ELC

$$29 = 19 * 1 + 10 \Rightarrow \boxed{10 = 29 - 19}$$

$$19 = 10 * 1 + 9 \Rightarrow \boxed{9 = 19 - 10}$$

$$10 = 9 * 1 + 1$$

Consider, $10 = 9 * 1 + 1$

$$1 = 10 - (9 * 1)$$

Sub '9' value

$$1 = 10 - (19 - 10 * 1)$$

$$= 10 - (19 - 10)$$

$$= 10 - 19 + 10$$

$$1 = 10 * 2 - 19.$$

Sub '10' value

$$1 = (29 - 19) * 2 - 19$$

$$= 29 * 2 - 19 * 2 - 19$$

$$= 29 * 2 - 19 * 3 = -19 * 3 + 29 * 2$$

$$1 = 19 * (-3) + 29 * 2$$

$$d = 19u + 29v$$

$$\boxed{\begin{array}{l} d = 1, \\ u = -3 \\ v = 2 \end{array}}$$

* Primality Testing :-

→ The Primality testing is used to verify whether the given number is prime or not.

→ The following are the steps in Primality testing.

(i) →: Find all the prime numbers, less than or equal to square root of given number. check whether the given no. is divisible by above Prime numbers or not.

(ii) → If the given no. is divisible by any of the prime number is not a prime otherwise it is a Prime.

→ Example :- check 133, 143, 153 are Primes or not by using Primality Testing.

Sol:- (i) 133

prime numbers $\leq \sqrt{133}$

≤ 12

2, 3, 5, 7, 11

Prime no. 2	Prime no. 3	Prime no. 5	P. no. 7
2)133(66 12 13 12 (1)	3)133(44 12 13 12 (1)	5)133(26 10 33 30 (3)	7)133(19 7 63 63 (0)
133 is not divisible by 2	133 is not divisible by 3	not divisible	133 is divisible by '7'

∴ 133 is not a Prime.

(ii) 143

Prime numbers $\leq \sqrt{143}$

≤ 12

2, 3, 5, 7, 11

Prime no. 2	Prime no. 3	Prime no. 5	Prime no. 7
2)143(71 14 3 2 1 not divisible	3)143(47 12 23 21 2 143 is not divisible by 3	5)143(28 10 43 40 3 not div.	7)143(2 14 3 not divisible

11)143(13
11
33
33
0

143 is divisible by '11'

$\therefore$ 143 is not a Prime.

(iii) 153

Prime numbers $\leq \sqrt{153}$

≤ 13

2, 3, 5, 7, 11

Prime no. 2

2)153(76
14
13
12
1

3 is not div. by 2.

3)153(51
15
3
3
0

'153' is divisible by "3"

$\therefore$ 153 is not a Prime.

Ex: ② Check whether 101, 251 are primes or not by using Primality testing.

* Factorization:-

→ Let 'n' be a positive integer and $P_1, P_2, P_3 \ldots P_n$ are 'n' prime numbers. $e_1, e_2, \ldots e_n$ are 'n' integers then the factorization of 'n' is defined as

$$\boxed{n = P_1^{e_1} * P_2^{e_2} * P_3^{e_3} * \cdots * P_n^{e_n}}$$

Ex: Find the Prime factors of 81.

Sol:

$$\begin{array}{r|l} 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$81 = 3^4$

∴ if $e = 1$ (uniqueness)
if $e > 1$ (Existence)

here $e = 4$ ~~or~~ $e > 1 \Rightarrow$ Existence factorization

Ex: Find the Prime factors of 50

$$\begin{array}{r|l} 2 & 50 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

$50 = 2^1 * 5^2 \Rightarrow 2^1$ (Uniqueness)
5^2 (Existence)

Ex: Find the prime factors of 10 $\Rightarrow \begin{array}{r|l} 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \Rightarrow 2^1 * 5^1$

It is Uniqueness factorization.

Ex: Find the LCM of 10, 25

$$\begin{array}{r|l} 2 & 10, 25 \\ \hline 5 & 5, 25 \\ \hline & 1, 5 \end{array}$$

$LCM = 2 \times 5 \times 1 \times 5$
$= 50$
ie $2^1 * 5^2$

Ex: LCM of 225, 25, 625

$$\begin{array}{r|l} 5 & 225, 25, 625 \\ \hline 5 & 45, 5, 125 \\ \hline 3 & 9, 1, 25 \\ \hline 3 & 3, 1, 25 \\ \hline 5 & 1, 1, 25 \\ \hline 5 & 1, 1, 5 \end{array}$$

LCM
$= 3^2 * 5^4$
$= 5625$

* **Fundamental Theorem of Arithmetic :-**

→ According to Fundamental Theorem of Arithmetic, any positive integer greater than 1, can be written uniquely in the following Prime factorization form

$$n = P_1^{e_1} * P_2^{e_2} * P_3^{e_3} * \cdots\cdots * P_k^{e_k}$$

where $P_1, P_2, \ldots P_k$ are 'k' prime numbers
$e_1, e_2, \ldots e_k$ are 'k' +ve integers.

→ Let us Consider $a = P_1^{a_1} * P_2^{a_2} * P_3^{a_3} * \cdots * P_k^{a_k}$

and $b = P_1^{b_1} * P_2^{b_2} * P_3^{b_3} * \cdots\cdots * P_k^{b_k}$ Then

The GCD of (a, b) is $P_1^{\min(a_1,b_1)} * P_2^{\min(a_2,b_2)} * \cdots$

$$\boxed{GCD_{(a,b)} = P_1^{\min(a_1,b_1)} * P_2^{\min(a_2,b_2)} * \cdots\cdots * P_k^{\min(a_k,b_k)}}$$

→ LCM can be calculated as

$$\boxed{LCM_{(a,b)} = P_1^{\max(a_1,b_1)} * P_2^{\max(a_2,b_2)} * \cdots P_k^{\max(a_k,b_k)}}$$

Ex: ① Find GCD and LCM of (10, 50) by using fundamental theorem of Arithmetic

Sol:- $a = 10, b = 50$

$$\begin{array}{r|l} 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad a = 2^1 * 5^1$$

$$\begin{array}{r|l} 2 & 50 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \Rightarrow b = 2^1 * 5^2$$

$$GCD(10, 50) = 2^{\min(1,1)} * 5^{\min(1,2)}$$
$$= 2^1 * 5^1 = 10.$$

$$LCM(10, 50) = 2^{\max(1,1)} * 5^{\max(1,2)}$$
$$= 2^1 * 5^2 = 2 * 25 = 50.$$

** (Fermatt's Theorem :)

* **Relative Prime** :-

→ Let a and b are two +ve integers and a, b are said to be relatively prime to each other if GCD of (a, b) is '1'.

* **Fermatt's Theorem** :-

→ Let 'p' be a prime no. and 'a' is any +ve integer is not a factor of 'p' and GCD of GCD(a, b) = 1 then the Fermatt's 1st eqn is

(i) $a^{p-1} \cong 1 \pmod{p}$

$\Rightarrow a^{p-1} \bmod p = 1$

2nd eqn is (ii) $a^p \cong a \pmod{p}$

$\Rightarrow a^p \bmod p = a$

Ex: Find the value of $5^{18} \bmod 19$

Note: To apply the Fermatt's Theorem, the GCD (a, b) must be equal to 1.

Sol: $5^{18} \bmod 19$, $a = 5$; $b = 19$

$\gcd(5, 19) = 1$ $\boxed{\gcd(a,b) \text{ must be } 1}$

$a^{P-1} \bmod P = 1$

$5^{18} \bmod 19 = 1$

$\therefore 5^{18} \bmod 19 = 5^{19-1} \bmod 19$

$= 1$

(2) $5^{19} \bmod 19$. find the value.

$\gcd(5, 19) = 1$

$\boxed{a^P \bmod P = a}$

$5^{19} \bmod 19 = 5$.

(3) Find the value of $5^{20} \bmod 19$

$\gcd(5, 19) = 1$

$5^{20} \bmod 19 = (5^{19} \cdot 5^1) \bmod 19$

$= (5^{19} \bmod 19)(5^1 \bmod 19)$

$= 5 * 5$

$= 25$

(4) Find the value of $9^{794} \bmod 73$

$\gcd(a, P) = 1$ $\Rightarrow$ GCD(.

$73)\ 793\ (10$
$\ \ \ \ \ 730$
$\ \ \ \ \ \ 63$

$793 = 73 * 10 + 63$

$9^{(73*10+63)+1} \bmod 73$

$= ((9^{73*10+63}) \cdot 9^1) \bmod 73$

$= (9 \bmod 73) \cdot (9^{73*10+63}) \bmod$

$a = 5 ; b = 19$

$\boxed{GCD(a,b) \text{ must be } 1}$

$5^{18} \bmod 19$

$GCD(5, 19) = 1$

$a^{P-1} \bmod P = 1$

$5^{18} \bmod 19 = 1$

$\therefore 5^{18} \bmod 19 = 5^{19-1} \bmod 19$

$= 1$

(2) $5^{19} \bmod 19$. find the value.

$gcd(5, 19) = 1$

$\boxed{a^P \bmod P = a}$

$5^{19} \bmod 19 = 5$.

(3) find the value of $5^{20} \bmod 19$

$gcd(5, 19) = 1$

$5^{20} \bmod 19 = (5^{19} . 5^{1}) \bmod 19$

$= (5^{19} \bmod 19)(5^{1} \bmod 19)$

$= 5 * 5$

$= 25$

(4) find the value of $9^{794} \bmod 73$

$GCD(a, P) = 1 \Rightarrow GCD($.

$73) \; 793 \; (10$

$\underline{730}$

63

$793 = 73 * 10 + 63$

$9^{(73*10+63)+1} \bmod 73$

$= ((9^{73*10+63}) . 9^{1}) \bmod 73$

$= (9 \bmod 73) \cdot (9^{73*10+63}) \bmod 73$

c) $9^{73*10+63} \bmod 73 = (9^{73})^{10} \bmod 73 \cdot 9^{63} \bmod 73$

$= (9)^{10} \cdot 9^{63} \bmod 73$

$9^{73} \bmod 73 \Rightarrow 9 \bmod 73.$

$9^{794} \bmod 73 = 9^{1} \cdot 9^{793} \bmod 73$

$= (9 \bmod 73) \cdot 9^{793} \bmod 73$

$= (9 \times 9) \bmod 73 = 81 \bmod 73 = 8.$

5) Find the value of x if $x^{86} \cong 6 \pmod{29}$

c) $9^{73 \times 10 + 63} \bmod 73 = (9^{73})^{10} \bmod 73 \cdot 9^{63} \bmod 73$

$= (9)^{10} \cdot 9^{63} \bmod 73$

$9^{73} \bmod 73 \Rightarrow 9 \bmod 73$

$9^{794} \bmod 73 = 9^{1} \cdot 9^{793} \bmod 73$

$= (9 \bmod 73) \cdot 9^{793} \bmod 73$

$= (9 \times 9) \bmod 73 = 81 \bmod 73 = 8$

5) Find the value of x if $x^{86} \cong 6 \pmod{29}$

* Chinese Remainder Theorem :-

→ Chinese Remainder theorem is used to solve a set of congruent equations with unique variable but different moduli which are relatively prime to each other as shown below.

$$x \cong a_1 (\mathrm{mod}\ m_1)$$
$$x \cong a_2 (\mathrm{mod}\ m_2)$$
$$x \cong a_3 (\mathrm{mod}\ m_3)$$
$$\vdots$$
$$x \cong a_K (\mathrm{mod}\ m_K)$$

→ The chinese remainder theorem states that the above eqns have a unique soln, if the moduli are relatively prime to each other.

→ The following are the steps involved in Chinese Remainder Theorem.

Step ①:- Find the value of "M" by using

$$M = m_1 \times m_2 \times m_3 \cdots \times m_K$$

Step ②: Find the value of $M_1 = M/m_1$, $M_2 = M/m_2$, $\cdots M_K = M/m_K$

Step ③: Find the multiplicative inverse of $M_1, M_2, \cdots M_K$ as $M_1^{-1}, M_2^{-1}, \cdots M_K^{-1}$ by using moduli $(m_1, m_2, \cdots m_K)$

Step ④ :- Find the value of x by using the formula

$$x = \left[(a_1 \times M_1 \times M_1^{-1}) + (a_2 \times M_2 \times M_2^{-1}) + \dots + (a_k \times M_k \times M_k^{-1})\right] \bmod m$$

$$x = \left[(a_1 \times M_1 \times M_1^{-1}) + (a_2 \times M_2 \times M_2^{-1}) + \dots + (a_k \times M_k \times M_k^{-1})\right] \bmod m$$

Example :- Find the value of x by using Chinese Rem. Theorem

$$x \equiv 2 \pmod 3$$
$$x \equiv 3 \pmod 5$$
$$x \equiv 2 \pmod 7$$

Sol:- From Given, $a_1 = 2$, $M_1 = 3$
$a_2 = 3$; $M_2 = 5$
$a_3 = 2$; $M_3 = 7$

$\gcd(M_1; M_2) = \gcd(3, 5) = 1$
$\gcd(M_2, M_3) = \gcd(5, 7) = 1$
$\gcd(M_1, M_3) = \gcd(3, 7) = 1$

M_1, M_2, M_3 are Relatively Prime to each other.
(So, we can apply chinese remainder theorem)

By using CRT,

(i) $M = m_1 \times m_2 \times m_3 \times \dots m_k$
$= 3 \times 5 \times 7 = 105$

(ii) $M_1 = M/m_1 = 105/3 = 35$
$M_2 = M/m_2 = 105/5 = 21$
$M_3 = M/m_3 = 105/7 = 15$

(iii) Multiplicative Inverse values

$$\boxed{M_1 \times M_1^{-1} \cong 1 \pmod{m_1}}$$

(verify, if $M_1^{-1} = 1$

$M_1^{-1} = 2$ $\quad$ $35 \times 1 \cong 1 \pmod 3$ - false

$M_2^{-1} = 1$ $\quad$ $35 \times 2 \cong 1 \pmod 3$

$M_3^{-1} = 1$ $\quad$ $70 \cong 1 \pmod 3$ - True

3) 70 (23
  69
  rem = 1

$\therefore M_1^{-1} = 2$)

(iv) $x = [(a_1 \times M_1 \times M_1^{-1}) + (a_2 \times M_2 \times M_2^{-1}) + (a_3 \times M_3 \times M_3^{-1})] \bmod m$

$x = [(2 \times 35 \times 2) + (3 \times 21 \times 1) + (2 \times 15 \times 1)] \bmod 105$

$= (140 + 63 + 30) \bmod 105$

$= (233) \bmod 5 = 23$

* Fermatt's Little Theorem:-

→ Let 'p' is a non-prime number, 'a' is any +ve integer then Fermatt's Little theorem defined as

$a^{P_1 - 1} * a^{P_2 - 1} * a^{P_3 - 1} \cdots\cdots * a^{P_n - 1} \cong 1 \bmod (P_1 * P_2 * \cdots P_n)$

→ $\boxed{a^{P_1 - 1} * a^{P_2 - 1} * \cdots a^{P_n - 1} \cong 1 \bmod (P_1 * P_2 * \cdots P_n)}$

where $P_1, P_2, \ldots P_n$ are prime factors of 'P'

Ex: ① Solve the eqn $17^{80} \bmod 91$

Sol:- $a = 17$, $P = 91$

7 | 91
  13

$91 = 7 \times 13$

$\therefore \boxed{a^{P_1 - 1} * a^{P_2 - 1} \cong 1 \bmod (P_1 * P_2)}$

here $\boxed{P_1 = 7, \quad P_2 = 13}$

($\because$ here P = 91 is not a prime no. then divide 'P' into multiplication of Prime factors)

$a^{7-1} \times a^{13-1} \cong 1 \bmod (7 \times 13)$

$a^6 \times a^{12} \cong 1 \bmod (7 \times 13)$ $(a = 17)$

$17^6 \times 17^{12} \cong 1 \bmod (7 \times 13)$

$\Rightarrow 17^{18} \cong 1 (\bmod 91)$

Multiply with 17^{62} on Both sides $\left(\because 17^{18} \cdot 17^{62} = 17^{18+62} = 17^{80} \right)$

$17^{80} \cong 17^{62} \cdot 1 (\bmod 91)$

$\cong (17^{18 \times 3 + 8}) \cdot \bmod 91$

$\cong (17^{18})^3 \cdot 17^8 \bmod 91$

$\cong (17^8 \bmod 91)^3 \cdot 17^8 \bmod 91.$

$= (1) \cdot 17^8 \bmod 91.$

(2) Find the value of $2^{243} \bmod 3$ (Ans. 1)

(3) Find the value of $3^{343} \bmod 5$ (Ans. 2)

(4) Find the value of $7^{22} \bmod 21$ (Ans. 0)

Find the value of $5^{240} \bmod 7$.

* Euler's Totient function :-

→ The Euler's function is used to find out the no. of integers that are both smaller than 'n' and relatively prime to 'n' where 'n' is a given number.

→ The Euler's totient function is denoted with the symbol " $\phi(\,)$ "

Ex: $\phi(1) = 0$ | $\phi(3) = 2$
$\phi(2) = 1$

→ If 'n' is a prime no, then $\boxed{\phi(n) = n-1}$

$\therefore \phi(7) = 6$, $\phi(13) = 12$, $\phi(10) = \phi(2\times5)$

$\Rightarrow \phi(10) = \phi(2\times5) = \phi(2)\cdot\phi(5) = 1.(4) = 4$.

(∵ '10' is not a prime no.)

Ex: Find the value of $\phi(20)$

Sol:- $\phi(20) = \phi(2\times2\times5)$
$= \phi(2)\cdot\phi(2)\cdot\phi(5)$
$= 1\times1\times4$
$= 4.$

(or) $\phi(20) = \phi(2\times2\times5)$
$= \phi(4)\times\phi(5)$
$= 2\times4$
$= 8.$

* Eulers Theorem :-

→ Eulers Theorem is used to find the co-factors of given number where the given no. is not a prime no.

→ The Euler's Theorem states that

$$\boxed{\phi(n) = \left[(P_1-1) * P_1^{e_1-1}\right] * \left[(P_2-1) * P_2^{e_2-1}\right] * \cdots}$$

where, P_1, P_2, P_3 are Prime factors of 'n' &
e_1, e_2, e_3 are Positive integers.

Ex: Find value of $\phi(20)$ using Euler's formula.

Sol:- $\phi(20)$.

$P_1 = 2, P_2 = 5$

$e_1 = 2, e_2 = 1$

$$\phi(20) = [(2-1) \times 2^{2-1}] \times [(5-1) \times 5^0]]$$

$$= ((1 \times 2) \times 4) = 8.$$

$$\begin{array}{r|l} 2 & 20 \\ \hline 2 & 10 \\ \hline & 5 \end{array}$$

$= 2, 5$

$10 = 2^2 \times 5^1$

Ex: Find the value of $\phi(2008)$

$P_1 = 2, P_2 = 251$

$2008 = 2^3 \times (251)^1$

$e_1 = 3, e_2 = 1$

$\phi(2008) =$

$$\begin{array}{r|l} 2 & 2008 \\ \hline 2 & 1004 \\ \hline 2 & 502 \\ \hline & 251 \end{array}$$

$= 2, 251$

Ex: Find the Value of $\phi(1260)$

$1260 = 2^2 \times 3^2 \times 5^1 \times 7^1$

$P_1 = 2, P_2 = 3, P_3 = 5, P_4 = 7$

$e_1 = 2, e_2 = 2, e_3 = 1, e_4 = 1.$

$$\phi(1260) = ((1 \times 2^1) \times (2 \times 3^1) \times (4 \times 5^0) \times (6 \times 7^0))$$

$$= 2 \times 6 \times 4 \times 6$$

$=$

$$\begin{array}{r|l} 2 & 1260 \\ \hline 2 & 630 \\ \hline 3 & 315 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$