Tutorial -1

1. Let p and q be the propositions

   p : It is below freezing.
   q : It is snowing.

   Write these propositions using p and q and logical connectives (including negations).

   a) It is below freezing and snowing.
   b) It is below freezing but not snowing.
   c) It is not below freezing and it is not snowing.
   d) It is either snowing or below freezing (or both).
   e) If it is below freezing, it is also snowing.
   f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
   g) That it is below freezing is necessary and sufficient for it to be snowing.

2. Write each of these statements in the form "if p, then q" in English. [*Hint:* Refer to the list of common ways to express conditional statements provided in this section.]

   a) I will remember to send you the address only if you send me an e-mail message.
   b) To be a citizen of this country, it is sufficient that you were born in the United States.
   c) If you keep your textbook, it will be a useful reference in your future courses.
   d) The Red Wings will win the Stanley Cup if their goalie plays well.
   e) That you get the job implies that you had the best credentials.
   f) The beach erodes whenever there is a storm.
   g) It is necessary to have a valid password to log on to the server.
   h) You will reach the summit unless you begin your climb too late.

3. Construct a truth table for each of these compound propositions.

a) $p \wedge \neg p$

b) $p \vee \neg p$

c) $(p \vee \neg q) \rightarrow q$

d) $(p \vee q) \rightarrow (p \wedge q)$

e) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

f) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

4. State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows today, I will ski tomorrow.

b) I come to class whenever there is going to be a quiz.

c) A positive integer is a prime only if it has no divisors other than 1 and itself.

5. Express these system specifications using the propositions p "The user enters a valid password," q "Access is granted," and r "The user has paid the subscription fee" and logical connectives (including negations).

a) "The user has paid the subscription fee, but does not enter a valid password."

b) "Access is granted whenever the user has paid the subscription fee and enters a valid password."

c) "Access is denied if the user has not paid the subscription fee."

d) "If the user has not entered a valid password but has paid the subscription fee, then access is granted."

6. Use De Morgan's laws to find the negation of each of the following statements.

   a) Kwame will take a job in industry or go to graduate school.
   b) Yoshiko knows Java and calculus.
   c) James is young and strong.
   d) Rita will move to Oregon or Washington.

7. Show that each of these conditional statements is a tautology by using truth tables.

   a) $(p \wedge q) \rightarrow p$
   b) $p \rightarrow (p \vee q)$
   c) $\neg p \rightarrow (p \rightarrow q)$
   d) $(p \wedge q) \rightarrow (p \rightarrow q)$
   e) $\neg(p \rightarrow q) \rightarrow p$
   f) $\neg(p \rightarrow q) \rightarrow \neg q$

8. Use truth tables to verify the absorption laws.

   a) $p \vee (p \wedge q) \equiv p$
   b) $p \wedge (p \vee q) \equiv p$

9. Determine whether $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ is a tautology.

10. Determine whether $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology.

11. Show that $(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are logically equivalent.

12. Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent.

13. Show that $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are logically equivalent.

14. Let $P(x)$ be the statement "the word x contains the letter a." What are these truth values?

a) $P(\text{orange})$
b) $P(\text{lemon})$
c) $P(\text{true})$
d) $P(\text{false})$

15. Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

a) $\exists x P(x)$
b) $\forall x P(x)$
c) $\exists x \neg P(x)$
d) $\forall x \neg P(x)$

16. Determine the truth value of each of these statements if the domain consists of all integers.

a) $\forall n(n + 1 > n)$
b) $\exists n(2n = 3n)$
c) $\exists n(n = -n)$
d) $\forall n(3n \leq 4n)$

17. Determine the truth value of each of these statements if the domain consists of all real numbers.

a) $\exists x(x^3 = -1)$
b) $\exists x(x^4 < x^2)$
c) $\forall x((-x)^2 = x^2)$
d) $\forall x(2x > x)$

18. Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

a) $\forall x(C(x) \rightarrow F(x))$
b) $\forall x(C(x) \wedge F(x))$
c) $\exists x(C(x) \rightarrow F(x))$
d) $\exists x(C(x) \wedge F(x))$

19. Translate each of these statements into logical expressions in three different ways by varying the domain and by using predicates with one and with two variables.

a) Someone in your school has visited Uzbekistan.

b) Everyone in your class has studied calculus and C++.

c) No one in your school owns both a bicycle and a motorcycle.

d) There is a person in your school who is not happy.

e) Everyone in your school was born in the twentieth century.

20. For each of these statements find a domain for which the statement is true and a domain for which the statement is false.

a) Everyone is studying discrete mathematics.

b) Everyone is older than 21 years.

c) Every two people have the same mother.

d) No two different people have the same grandmother.

21. Express the negation of these propositions using quantifiers, and then express the negation in English.

a) Some drivers do not obey the speed limit.

b) All Swedish movies are serious.

c) No one can keep a secret.

d) There is someone in this class who does not have a good attitude.

22. Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

a) $\forall x(x^2 \geq x)$

b) $\forall x(x > 0 \lor x < 0)$

c) $\forall x(x = 1)$

23. Translate these statements into English, where the domain for each variable consists of all real numbers.

a) $\forall x \exists y(x < y)$

b) $\forall x \forall y(((x \geq 0) \land (y \geq 0)) \rightarrow (xy \geq 0))$

c) $\forall x \forall y \exists z(xy = z)$

24. Translate these statements into English, where the domain for each variable consists of all real numbers.

a) $\exists x \forall y(xy = y)$

b) $\forall x \forall y(((x \geq 0) \land (y < 0)) \rightarrow (x - y > 0))$

c) $\forall x \forall y \exists z(x = y + z)$

25. Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

a) $\forall x \exists y(x^2 = y)$

b) $\forall x \exists y(x = y^2)$

c) $\exists x \forall y(xy = 0)$

d) $\exists x \exists y(x + y \neq y + x)$

e) $\forall x(x \neq 0 \rightarrow \exists y(xy = 1))$

f) $\exists x \forall y(y \neq 0 \rightarrow xy = 1)$

g) $\forall x \exists y(x + y = 1)$

h) $\exists x \exists y(x + 2y = 2 \land 2x + 4y = 5)$

i) $\forall x \exists y(x + y = 2 \land 2x - y = 1)$

j) $\forall x \forall y \exists z(z = (x + y)/2)$

26. Let $S(x)$ be the predicate "x is a student," $F(x)$ the predicate "x is a faculty member," and $A(x, y)$ the predicate "x has asked y a question," where the domain consists of all people associated with your school. Use quantifiers to express each of these statements.

a) Lois has asked Professor Michaels a question.

b) Every student has asked Professor Gross a question.

c) Every faculty member has either asked Professor Miller a question or been asked a question by Professor Miller.

d) Some student has not asked any faculty member a question.

e) There is a faculty member who has never been asked a question by a student.

f) Some student has asked every faculty member a question.

g) There is a faculty member who has asked every other faculty member a question.

h) Some student has never been asked a question by a faculty member.

27. Express each of these statements using quantifiers. Then form the negation of the statement, so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that.")

a) Some old dogs can learn new tricks.

b) No rabbit knows calculus.

c) Every bird can fly.

d) There is no dog that can talk.

e) There is no one in this class who knows French and Russian.

28. Use rules of inference to show that the hypotheses "Randy works hard," "If Randy works hard, then he is a dull boy," and "If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job."

29. Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained."

30. Determine whether these are valid arguments.

a) If x is a positive real number, then x^2 is a positive real number. Therefore, if a^2 is positive, where a is a real number, then a is a positive real number.

b) If $x^2 \neq 0$, where x is a real number, then $x \neq 0$. Let a be a real number with $a^2 \neq 0$; then $a \neq 0$.

31. Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(\neg Q(x) \vee S(x))$, $\forall x(R(x) \rightarrow \neg S(x))$, and $\exists x \neg P(x)$ are true, then $\exists x \neg R(x)$ is true.

32. Determine whether this argument, taken from Kalish and Montague [KaMo64], is valid.

If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent; if he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exist.