Basic Laws of Electric Circuits

Mesh Analysis

Mesh analysis

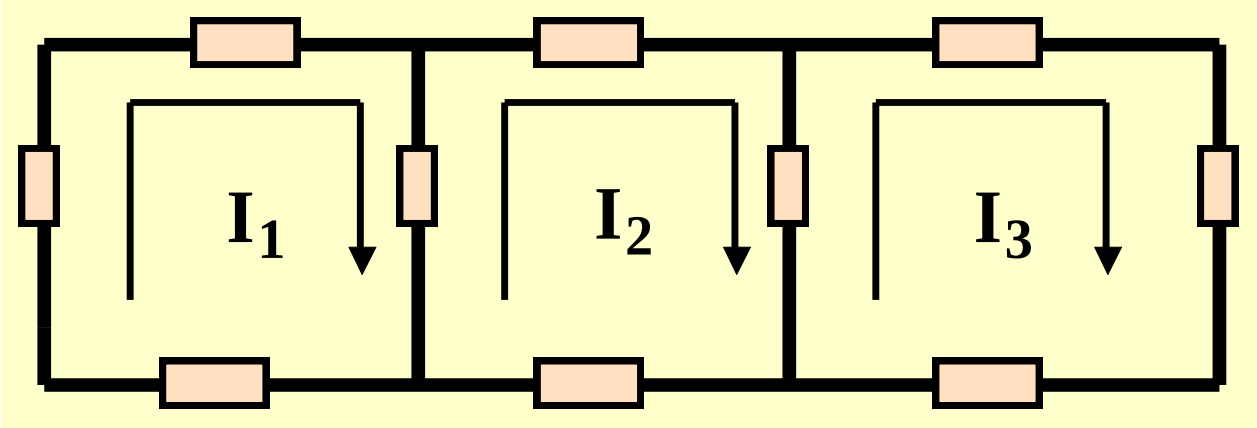
Analysis using KVL to solve for the currents around each closed loop of the network.

Mesh analysis procedure:

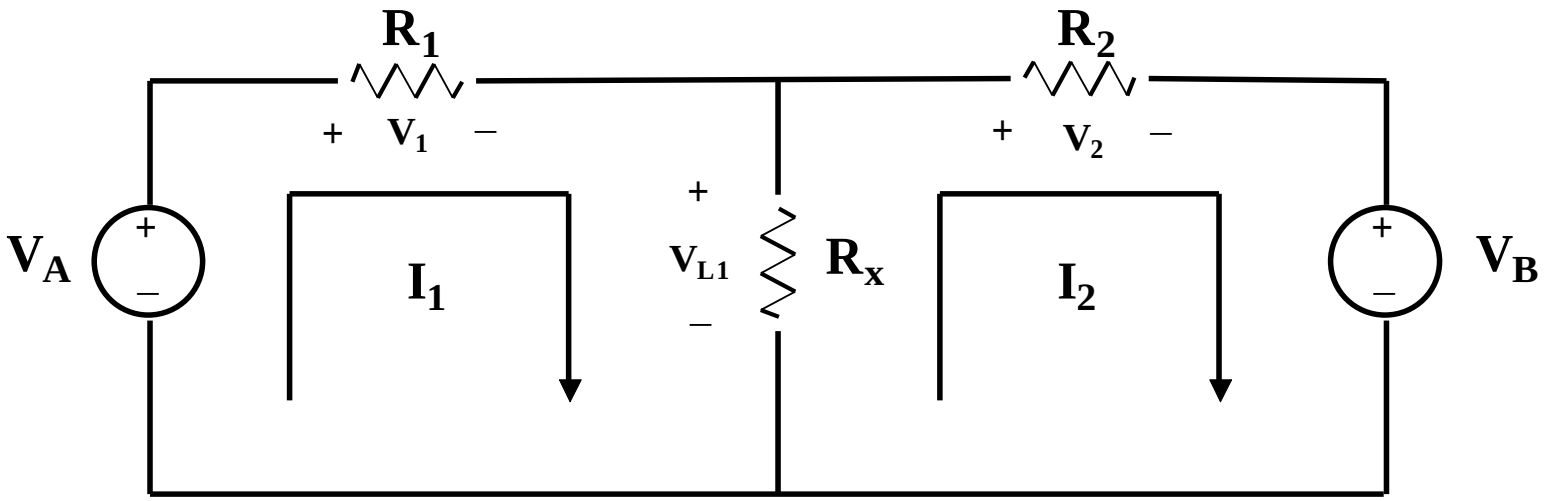
1. Assign currents to each closed loop of the network.
2. Apply KVL around each closed loop of the network.
3. Solve the resulting simultaneous linear equation for the loop currents.

Mesh Analysis: Basic Concepts:

☑ In formulating mesh analysis we assign a mesh current to each mesh.



Mesh Analysis: Basic Concepts:



A circuit for illustrating mesh analysis.

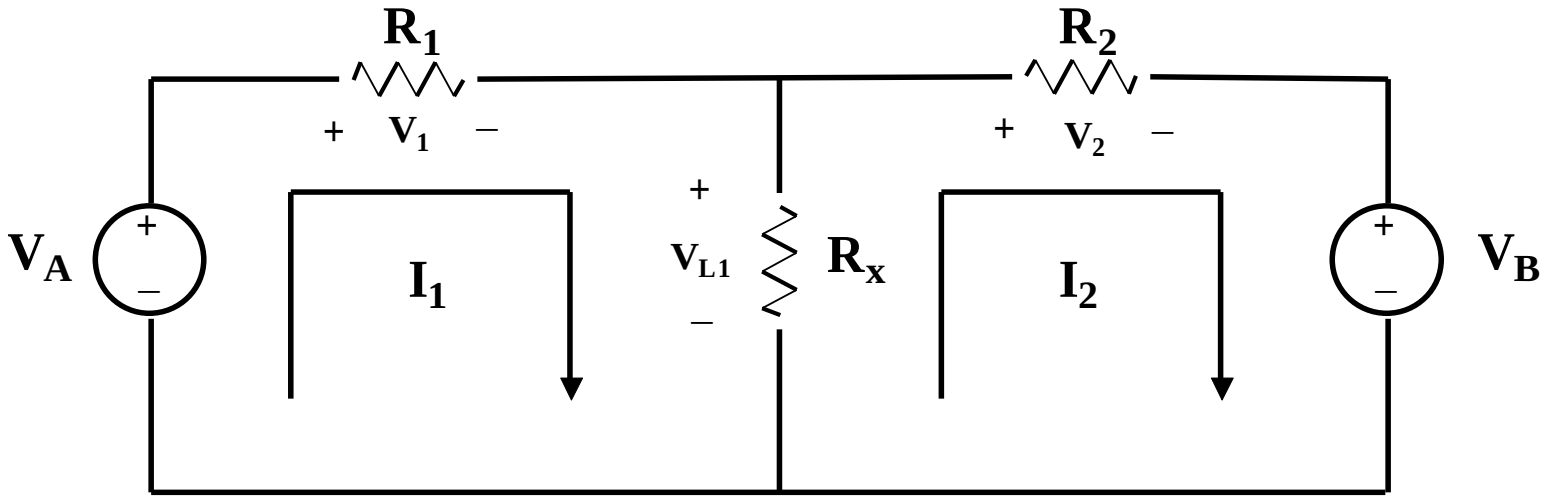
Around mesh 1:

$$V_1 + V_{L1} = V_A$$

$$\textit{where} \quad V_1 = I_1 R_1; \quad V_{L1} = (I_1 - I_2) R_X$$

$$\textit{so,} \quad (R_1 + R_X) I_1 - R_X I_2 = V_A \qquad \text{Eq 1}$$

Mesh Analysis: Basic Concepts:



Around mesh 2 we have

$$V_{L1} - V_2 = V_B \qquad \text{Eq 2}$$

with; $V_{L1} = -(I_2 - I_1)R_X$; $\quad V_2 = I_2 R_2$ Eq 3

Substituting Eq 3 in Eq 2 gives,

$$R_X I_1 - (R_X + R_2) I_2 = V_B$$

or $$-R_X I_1 + (R_X + R_2) I_2 = -V_B \qquad \text{Eq 4}$$

Mesh Analysis: Basic Concepts:

We are left with 2 equations: From (1) and (4) we have,

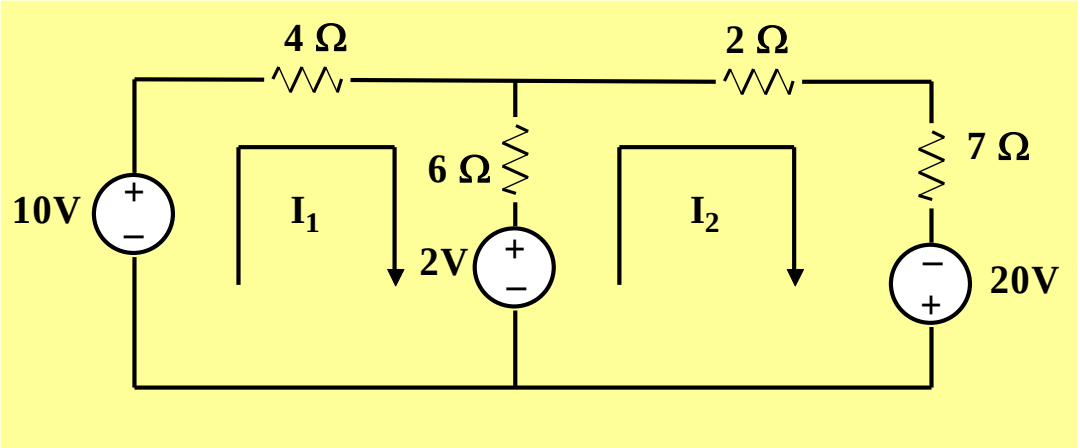
$$(\boldsymbol{R_1 + R_X})\boldsymbol{I_1} - \boldsymbol{R_X I_2} = \boldsymbol{V_A} \qquad \text{Eq 1}$$

$$-\boldsymbol{R_X I_1} + (\boldsymbol{R_X + R_2})\boldsymbol{I_2} = -\boldsymbol{V_B} \qquad \text{Eq 4}$$

We can easily solve these equations for I_1 and I_2.

Mesh Analysis: Example 1

Write the mesh equations and solve for the currents I_1 and I_2.



Circuit for Example 1

Mesh 1 $4I_1 + 6(I_1 - I_2) = 10 - 2$ Eq (1)

Mesh 2 $6(I_2 - I_1) + 2I_2 + 7I_2 = 2 + 20$ Eq (2)

Mesh Analysis: Example 1, continued.

Simplifying Eq (1) and (2) gives,

$$10I_1 - 6I_2 = 8 \qquad \text{Eq (3)}$$

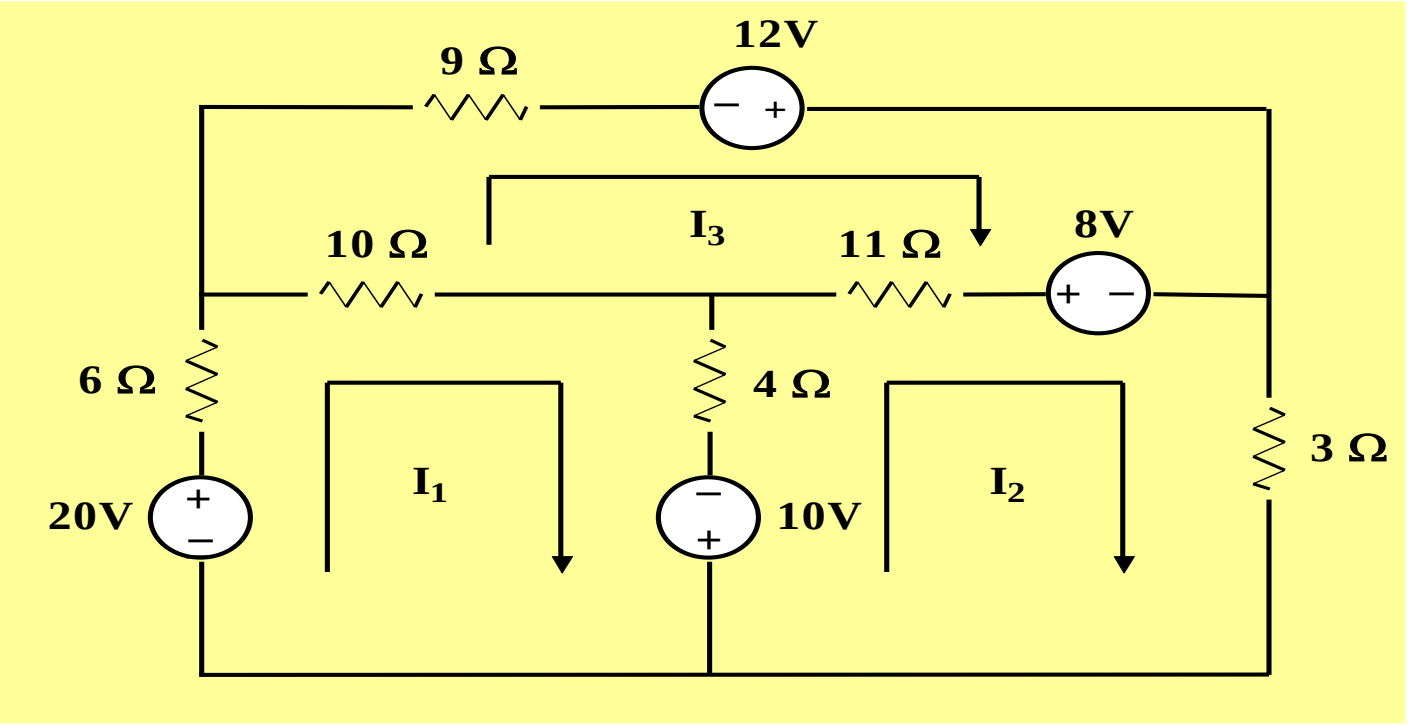
$$-6I_1 + 15I_2 = 22 \qquad \text{Eq (4)}$$

$I_1 = 2.2105$ A

$I_2 = 2.3509$ A

Mesh Analysis: Example 2

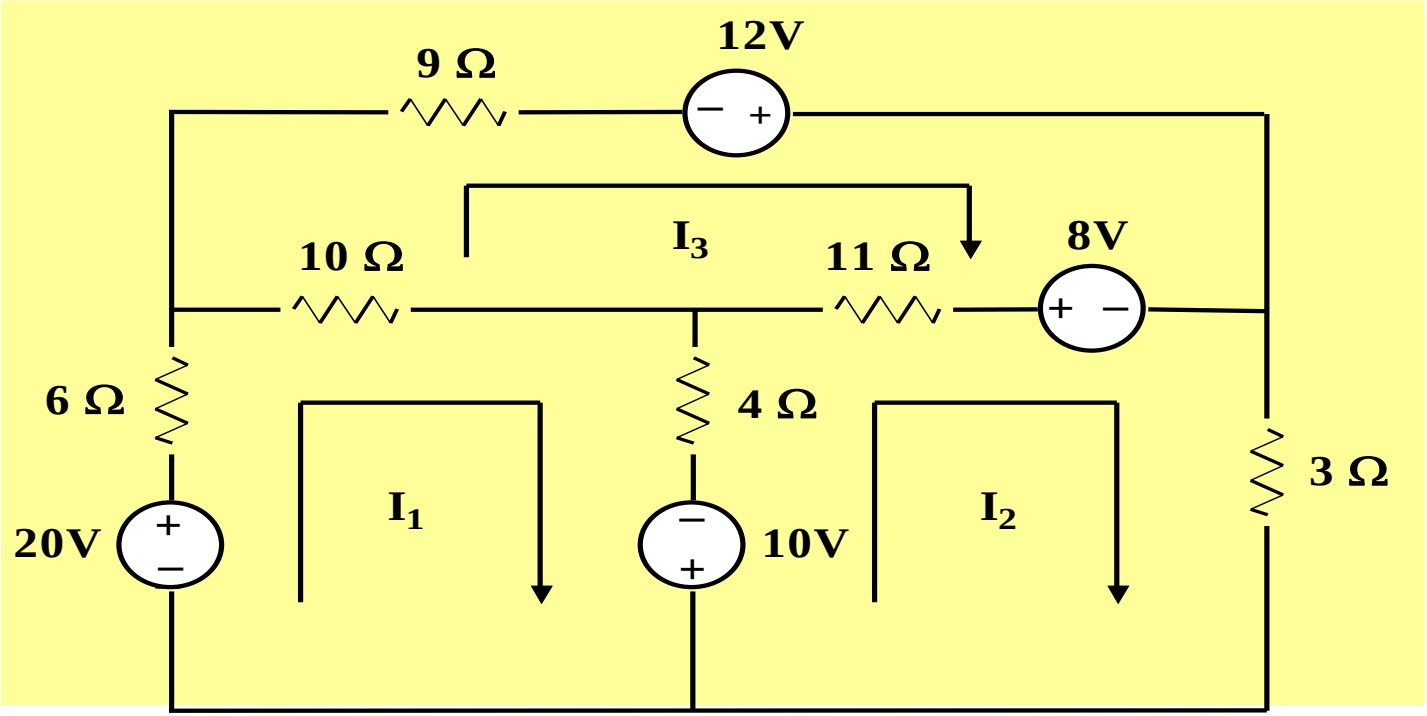
Solve for the mesh currents in the circuit below.



Circuit for Example 2

Mesh Analysis: Example 2

Write KVL for each mesh.



Mesh 1: $6I_1 + 10(I_1 - I_3) + 4(I_1 - I_2) = 20 + 10$ Eq (1)

Mesh 2: $4(I_2 - I_1) + 11(I_2 - I_3) + 3I_2 = -10 - 8$ Eq (2)

Mesh 3: $9I_3 + 11(I_3 - I_2) + 10(I_3 - I_1) = 12 + 8$ Eq (3)

Mesh Analysis: Example 2

Clearing Equations (1), (2) and (3) gives,

Standard Equation form

$$20I_1 - 4I_2 - 10I_3 = 30$$

$$-4I_1 + 18I_2 - 11I_3 = -18$$

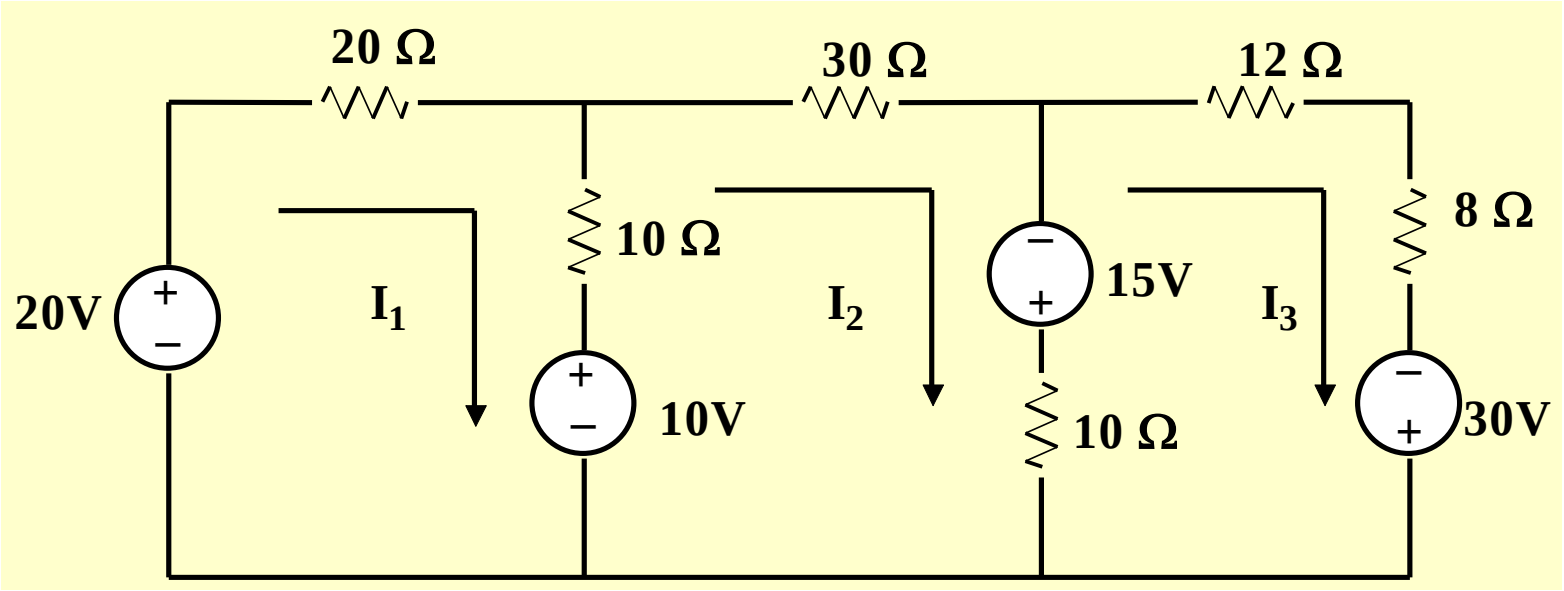
$$-10I_1 - 11I_2 + 30I_3 = 20$$

In matrix form:

$$\begin{bmatrix} 20 & -4 & -10 \\ -4 & 18 & -11 \\ -10 & -11 & 30 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 30 \\ -18 \\ 20 \end{bmatrix}$$

Mesh Analysis: Example 3

Use the direct method to write the mesh equations for the following.



Circuit diagram for Example 3

Mesh Analysis: Example 3

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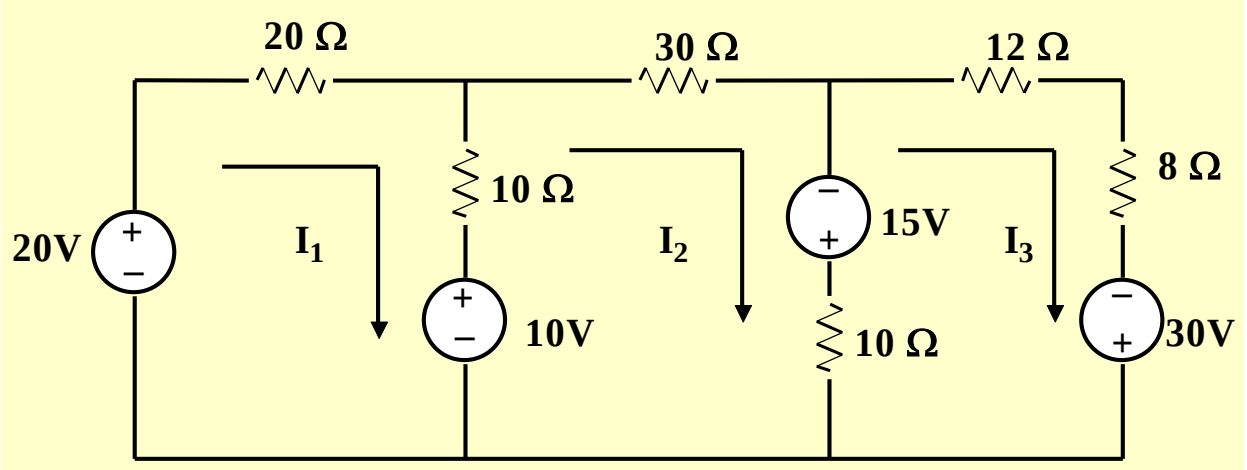


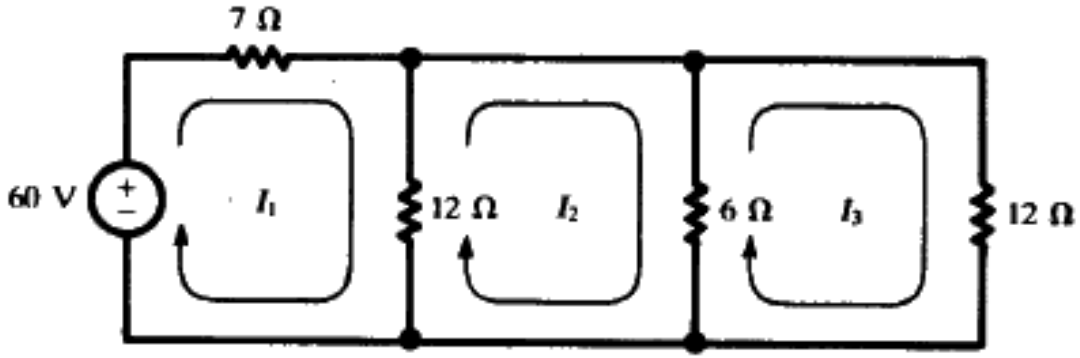
Circuit diagram for Example 3

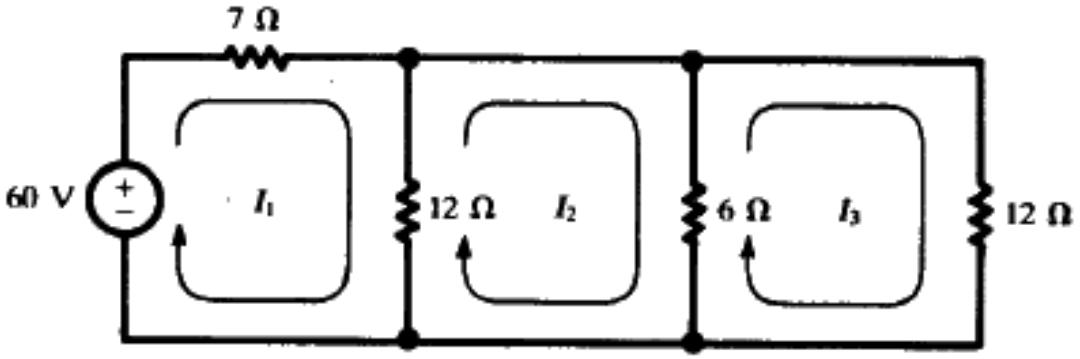
$$\begin{bmatrix} 30 & -10 & 0 \\ -10 & 50 & -10 \\ 0 & -10 & 30 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 25 \\ 15 \end{bmatrix}$$

Eq (1)

Problem

Solve I_1 by using mesh current method





$$60 = 7I_1 + 12(I_1 - I_2)$$
$$0 = 12(I_2 - I_1) + 6(I_2 - I_3)$$
$$0 = 6(I_3 - I_2) + 12I_3$$

$$\begin{bmatrix} 19 & -12 & 0 \\ -12 & 18 & -6 \\ 0 & -6 & 18 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ 0 \end{bmatrix}$$

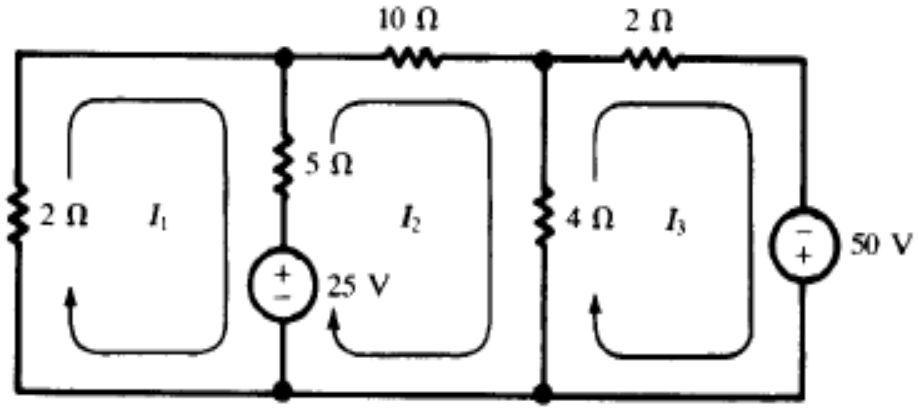
Using Cramer's rule to find I_1,

$$I_1 = \begin{vmatrix} 60 & -12 & 0 \\ 0 & 18 & -6 \\ 0 & -6 & 18 \end{vmatrix} \div \begin{vmatrix} 19 & -12 & 0 \\ -12 & 18 & -6 \\ 0 & -6 & 18 \end{vmatrix} = 17280 \div 2880 = 6\,\text{A}$$

Similarly I_2 = 4.5 A and I_3 = 1.5 A

Problem

Write the mesh current matrix equation for the network of Fig by inspection, and solve for Currents.



Ans: I_1 = -1.31 A, I_2 = 3.17 A and I_3 = 10.45 A