# Basic Laws of Electric Circuits

# Nodal Analysis

**Nodal analysis**

Analysis using KCL to solve for voltages at each common node of the network

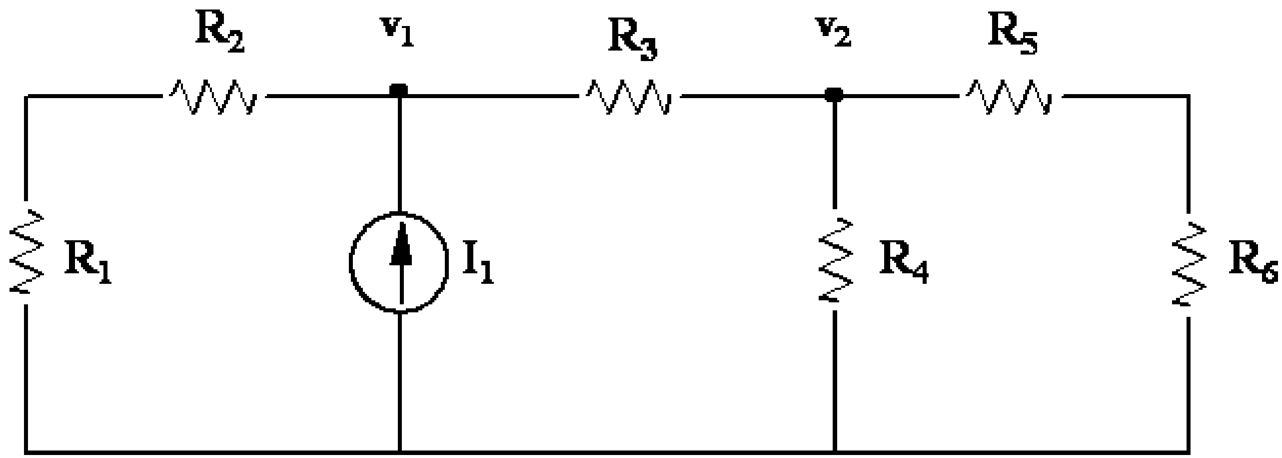
Nodal analysis procedure:

1. Determine the number of common nodes and reference node within the network.
2. Apply KCL at each of the common nodes in the network
3. Solve the resulting simultaneous linear equation for the nodal voltages.

# Circuit Analysis

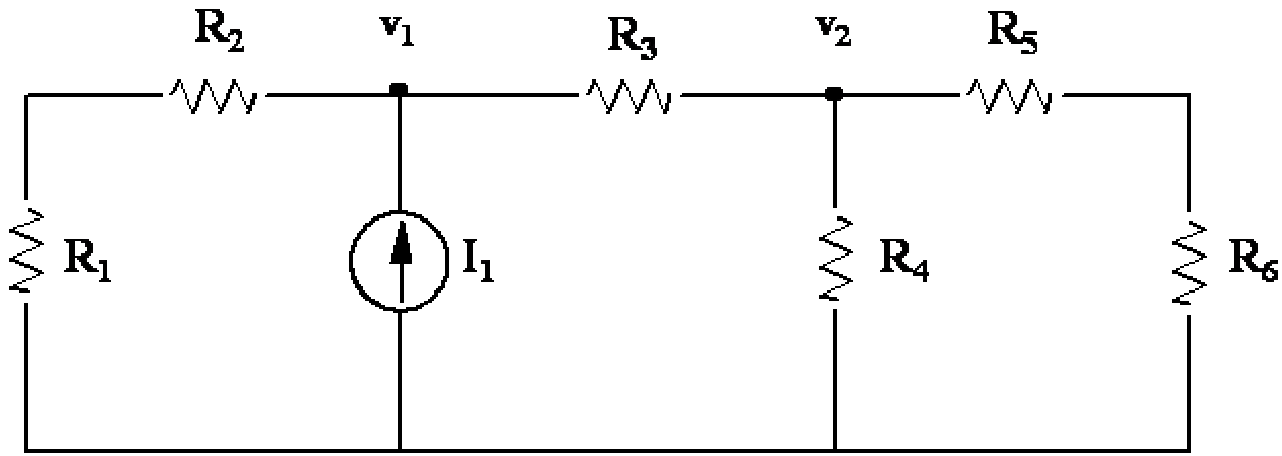
## Nodal Analysis:

**Given the following circuit. Set-up the equations to solve for $V_1$ and $V_2$.**



# Circuit Analysis

## Nodal Analysis: Nodal equations.




$$\frac{V_1}{R_1 + R_2} + \frac{V_1 - V_2}{R_3} = I_1 \qquad \textbf{Eq 1}$$

$$\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_4} + \frac{V_2}{R_5 + R_6} = 0 \qquad \textbf{Eq 2}$$

# Circuit Analysis

## Nodal Analysis: Set up for solution.

$$\frac{V_1}{R_1 + R_2} + \frac{V_1 - V_2}{R_3} = I_1 \qquad \textbf{Eq 1}$$

$$\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_4} + \frac{V_2}{R_5 + R_6} = 0 \qquad \textbf{Eq 2}$$

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$$\left(\frac{1}{R_1 + R_2} + \frac{1}{R_3}\right)V_1 - \left(\frac{1}{R_3}\right)V_2 = I_1 \qquad \textbf{Eq 3}$$

$$-\left(\frac{1}{R_3}\right)V_1 + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5 + R_6}\right)V_2 = 0 \qquad \textbf{Eq 4}$$

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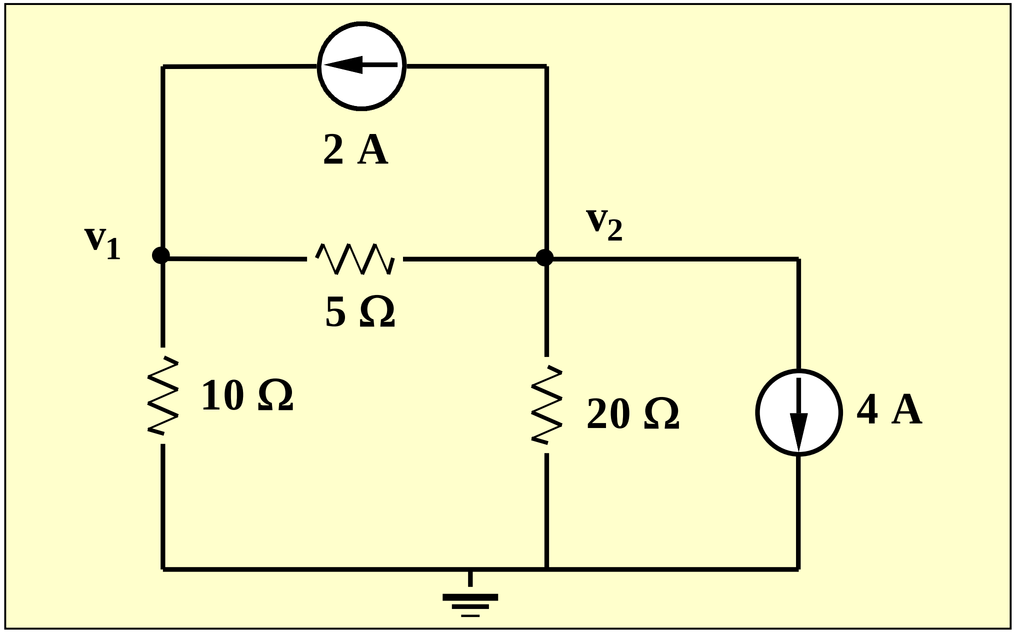
# Circuit Analysis

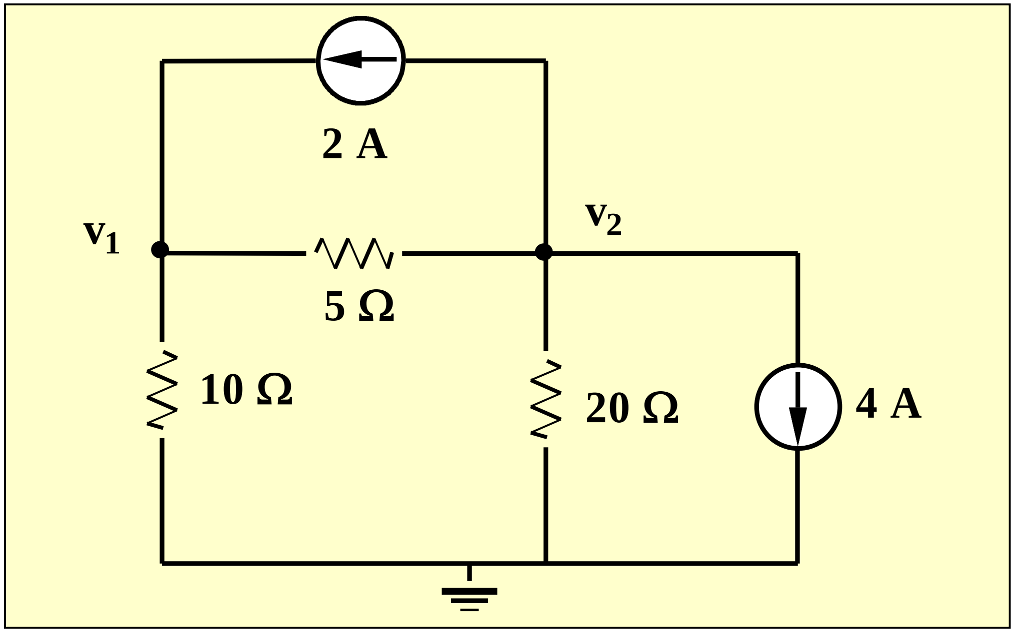
## Nodal Analysis:




For the given circuit
Find $V_1$ and $V_2$.

# Circuit Analysis

## Nodal Analysis:




**At $v_1$:**

$$\frac{V_1}{10} + \frac{V_1 - V_2}{5} = 2$$

**Eq 1**

**At $v_2$:**

$$\frac{V_2 - V_1}{5} + \frac{V_2}{20} = -6$$

**Eq 2**

# Circuit Analysis

**Nodal Analysis: Clearing Equations;**

**From Eq 1:**

$$V_1 + 2V_1 - 2V_2 = 20$$

or

$$3V_1 - 2V_2 = 20 \qquad \text{Eq 3}$$

**From Eq 2:**

$$4V_2 - 4V_1 + V_2 = -120$$

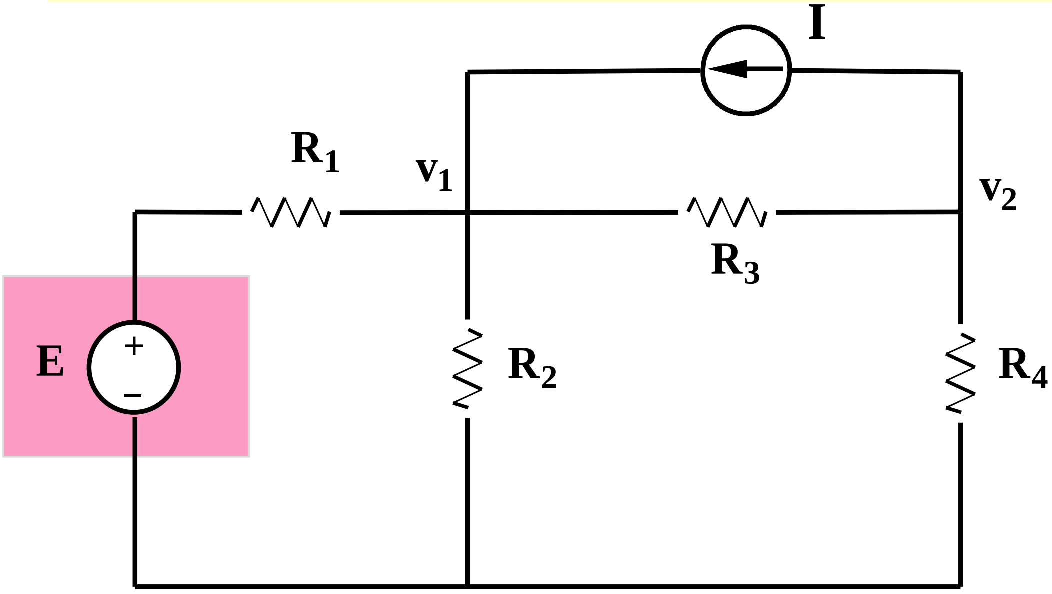
or

$$-4V_1 + 5V_2 = -120 \qquad \text{Eq 4}$$

**Solution: $V_1 = -20$ V, $V_2 = -40$ V**

# Circuit Analysis

## Nodal Analysis: With voltage source.




**At $V_1$:**

$$\frac{V_1 - E}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_2}{R_3} = I$$

**Eq 1**

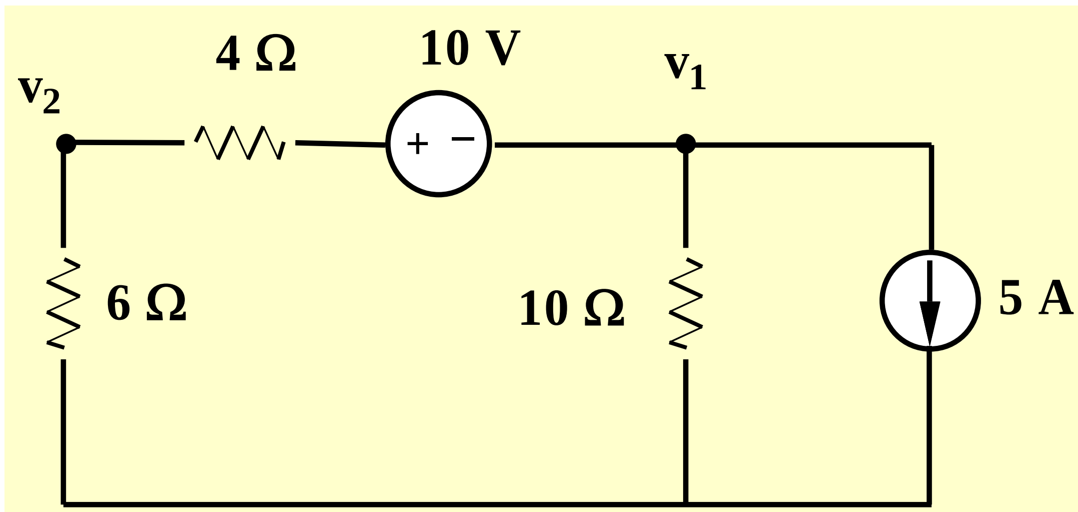
**At $V_2$:**

$$\frac{V_2}{R_4} + \frac{V_2 - V_1}{R_3} = -I$$

**Eq 2**

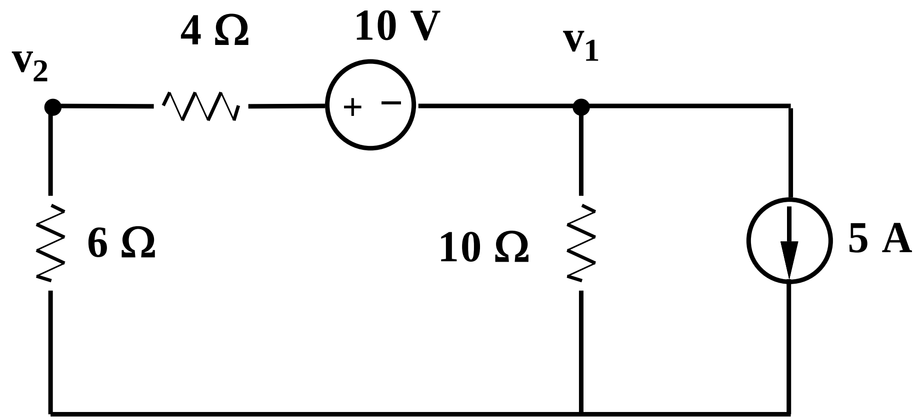
# Circuit Analysis

**Nodal Analysis:** **Numerical example with voltage source.**



# Circuit Analysis

## Nodal Analysis: Continued…




**At $v_1$:**

$$\frac{V_1}{10}+\frac{V_1+10-V_2}{4}=-5 \qquad \text{Eq 1}$$

**At $v_2$:**

$$\frac{V_2}{6}+\frac{V_2-10-V_1}{4}=0 \qquad \text{Eq 2}$$

# Circuit Analysis

## Nodal Analysis: Continued

**Clearing Eq 6.15**

$$4V_1 + 10V_1 + 100 - 10V_2 = -200$$

or

$$14V_1 - 10V_2 = -300 \qquad \text{Eq 3}$$

**Clearing Eq 6.16**

$$4V_2 + 6V_2 - 60 - 6V_1 = 0$$

**or**

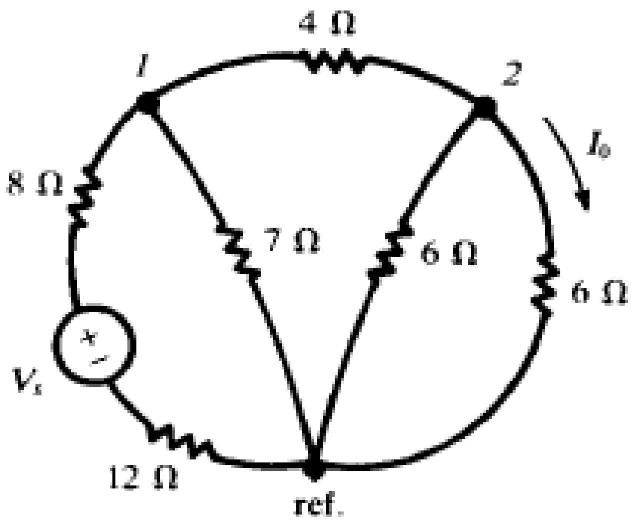
$$-6V_1 + 10V_2 = 60 \qquad \text{Eq 4}$$

$$V_1 = -30\text{ V}, \ V_2 = -12\text{ V}$$

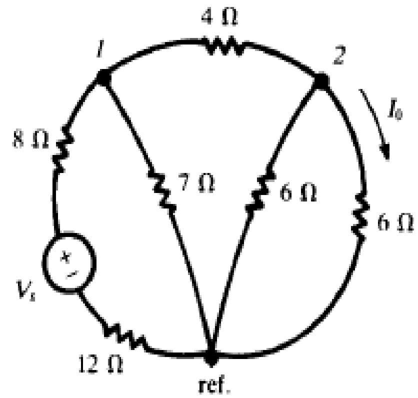
# Circuit Analysis

Problem

For the network shown in Fig, find Vs which makes Io = 7.5 mA.



## Solution:




$$\begin{bmatrix} \frac{1}{20}+\frac{1}{7}+\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4}+\frac{1}{6}+\frac{1}{6} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_s/20 \\ 0 \end{bmatrix}$$

Solving for $V_2$,

$$V_2 = \frac{\begin{vmatrix} 0.443 & V_s/20 \\ -0.250 & 0 \end{vmatrix}}{\begin{vmatrix} 0.443 & -0.250 \\ -0.250 & 0.583 \end{vmatrix}} = 0.0638 V_s$$

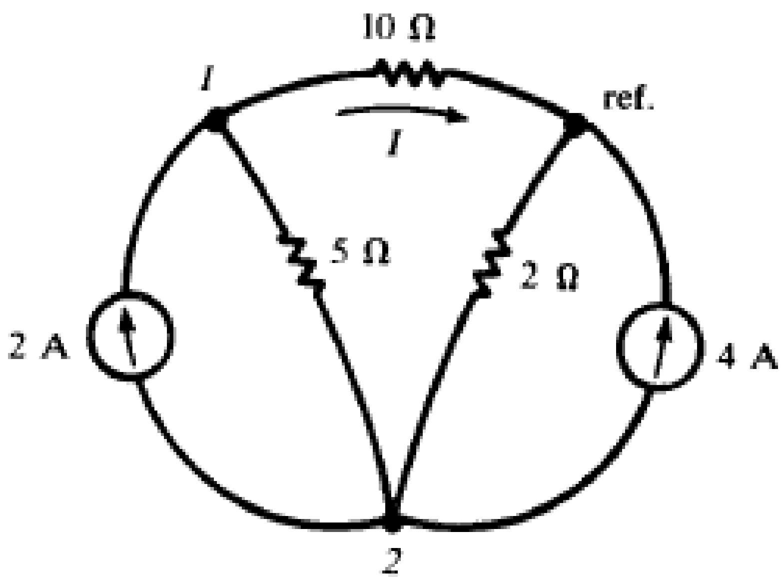
Then $$7.5 \times 10^{-3} = I_0 = \frac{V_2}{6} = \frac{0.0638\, V_s}{6}$$

from which $V_s = 0.705$ V.

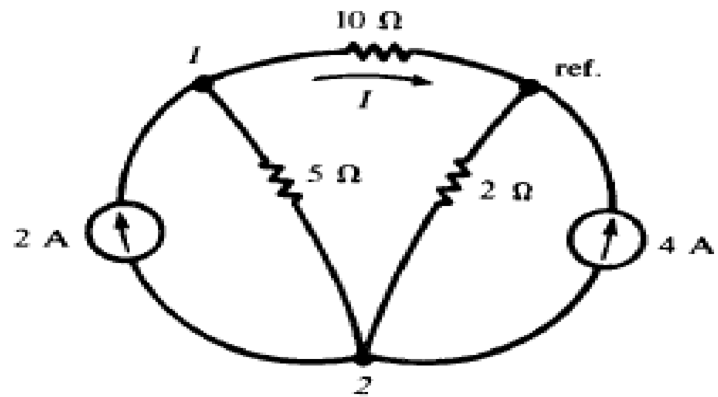
# Circuit Analysis

Problem

In the network shown, find the current in the 10 Ω resistor.



Solution:




The nodal equations in matrix form are written by inspection.

$$\begin{bmatrix} \frac{1}{5}+\frac{1}{10} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{5}+\frac{1}{2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

$$V_1 = \frac{\begin{vmatrix} 2 & -0.20 \\ -6 & 0.70 \end{vmatrix}}{\begin{vmatrix} 0.30 & -0.20 \\ -0.20 & 0.70 \end{vmatrix}} = 1.18\,\text{V}$$

Then, $I = V_1/10 = 0.118$ A.

## Problem

Solve Problem by the node voltage method