

1. A steel wire of length 2.75 m and diameter 1mm is fixed at the top and a load of 1 kg is suspended from the lower end. The Youngs modulus of the material is $20 \times 10^{10} \text{ N/m}^2$
Calculate the elongation

- $Y = \frac{F/A}{\frac{\Delta L}{L}}$
- $F = 1 \times 9.8 \text{ N}$; $A = \pi r^2 = 3.14 \times (0.5 \times 10^{-3})^2 = 7.85 \times 10^{-7} \text{ m}^2$; $L = 2.75 \text{ m}$
- $\Delta L = \frac{F.L}{A Y} = 1.716 \times 10^{-4} \text{ m}$

2. What force is required to stretch a steel wire to double its length given the area of cross section is 1.2 cm^2 and Y for steel is $20 \times 10^{10} \text{ N/m}^2$

$$\Delta L = L ;$$
$$F = \frac{AY\Delta L}{L} = AY = 2.4 \times 10^6 \text{ N/m}^2$$



3. Your leg bones (cross sectional area 9.5 cm^2 experience a force of approximately 855 N when you walk. Find the fractional amount your leg bones are compressed by walking. Youngs modulus for bone = $1 \times 10^{10} \text{ N/m}^2$

$$\frac{\Delta L}{L} = \frac{F}{AY} = 9 \times 10^{-5}$$

4 Find the stress on a bone (1 cm in radius and 50 cm long) that supports a mass of 100 kg. Find the strain on the bone if it is compressed 0.15 mm by this weight. Find the proportionality constant C for this bone

$$\text{Stress} = F/A = 100 \times 9.8 / (3.14 \times 10^{-4}) = 3.12 \times 10^6 \text{ N/m}^2$$

$$\text{Strain} = \frac{\Delta L}{L} = 0.15 \times 10^{-3} / 50 \times 10^{-2} = 3 \times 10^{-4}$$

$$\text{Proportionality constant } C = 1.04 \times 10^{10} \text{ N/m}^2$$

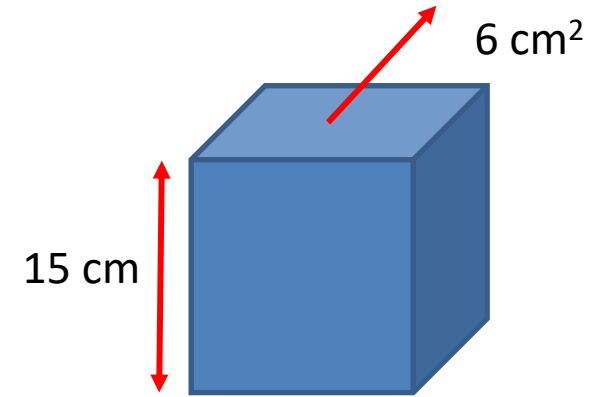


5. Calculate the shearing force required to distort a block of steel by 0.75 cms if its thickness is 15cm and surface area is 6 cm² . The shear modulus for steel is 7 x 10¹⁰ N/m²

The angle of shear = 0.75/15 = .05 radians

Shear or rigidity modulus $n = \frac{F}{A.\theta}$

$$F = n A \theta = 2.25 \times 10^6 \text{ N}$$

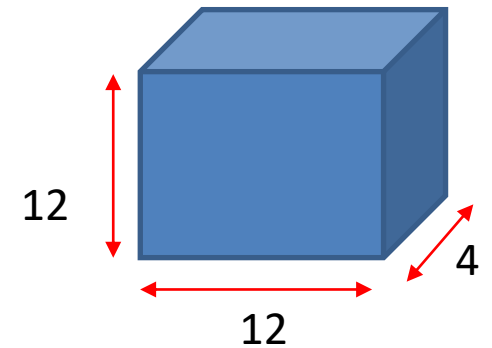


6. A rectangular solid has dimensions 12x12x4 cms. A force of 0.4 N applied tangentially to the upper surface, causes a displacement of 10 mm relative to the lower surface. Calculate shearing strain, stress and the rigidity modulus.

Tangential Stress = Tangential force/area = 0.4/ (48 x 10⁻⁴) = 83.33 N/m²

Tangential strain = 10 x 10⁻³ / 12 x 10⁻² = .0833

Rigidity Modulus = Tangential stress/ tangential strain = 999.96 N/m²



7. A copper wire has length 1.5m and diameter 0.6mm. Young's modulus and Poisson's ratio of copper are $12.5 \times 10^{10} \text{ N/m}^2$ and 0.26 respectively. The wire is fixed at one end and loaded with a weight of 2.5 kg at the other end. Calculate elongation and lateral compression.

$$\text{Elongation, } \Delta l = \frac{MgL}{Y \cdot \pi r^2} = \frac{2.5 \times 9.8 \times 1.5}{12.5 \times 10^{10} \times \pi \times (0.3 \times 10^{-3})^2} = 1.040 \times 10^{-3} \text{ m}$$

$$\frac{\Delta d}{d} = \sigma \cdot \frac{\Delta l}{l}$$

$$\Delta d = \sigma \cdot \frac{\Delta l}{l} \cdot d = 1.08 \times 10^{-7}$$

Alternate Method

$$\text{Linear strain } \alpha = \frac{1}{Y} = 8 \times 10^{-12}$$

$$\text{Stress} = Mg/\pi r^2 = 8.7 \times 10^7 \text{ N/m}^2$$

$$\text{Elongation} = \alpha \cdot T \cdot L = 8 \times 10^{-12} \times 8.7 \times 10^7 \times 1.5 = 1.044 \times 10^{-3} \text{ m}$$

$$\text{Lateral strain } \beta = \alpha \times \sigma = 8 \times 10^{-12} \times .26 = 2.08 \times 10^{-12}$$

$$\text{Lateral Compression} = \beta \cdot T \cdot d = 1.085 \times 10^{-7}$$



8. Two masses are suspended on a copper and on an iron wire (see Figure). What is the stress in each wire? What is the elongation for each? What is elastic Potential energy in each wire

Stress in copper wire = $9 \times 9.8 / 3.14 \times (0.25 \times 10^{-3})^2 = 4.49 \times 10^8$ N/m²

Elongation in copper wire = $4.49 \times 10^8 \times 1 / (12 \times 10^{10}) = 3.74 \times 10^{-3}$

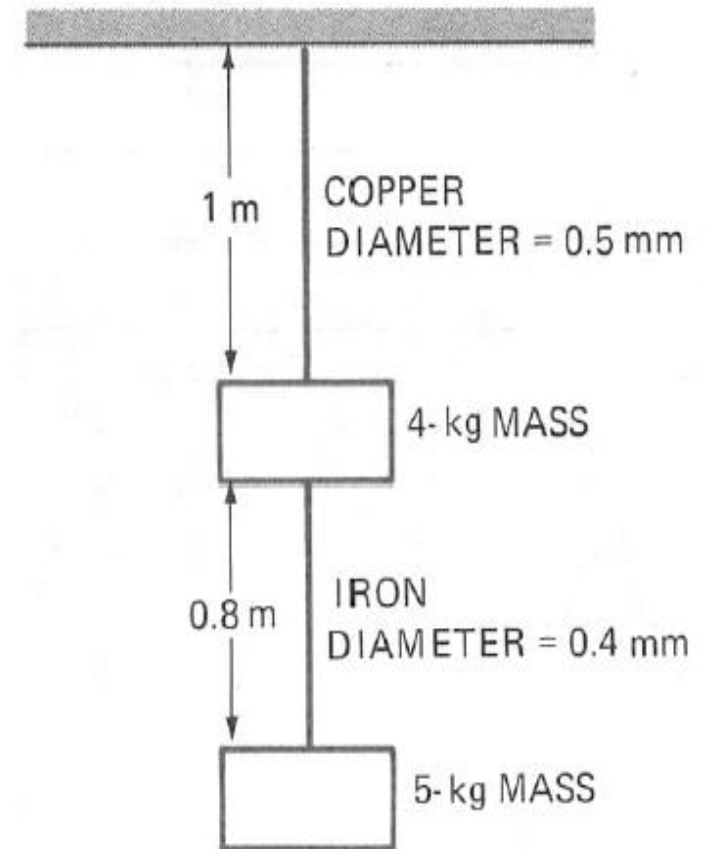
Elongation in copper = 3.745 mm

Elastic potential energy = $\frac{1}{2} K x^2 = \frac{1}{2} F x$ where x is elongation
= 0.1649 J

Stress in iron wire = $5 \times 9.8 / 3.14 \times (0.2 \times 10^{-3})^2 = 3.90 \times 10^8$ N/m²

Elongation in iron wire = $3.90 \times 10^8 \times 0.8 / 20 \times 10^{10} = 1.560 \times 10^{-3}$ m

Elastic Potential energy in iron wire = .03822 J



9. Calculate Poisson's ratio for silver, given that Young's modulus is $7.25 \times 10^{10} \text{ N/m}^2$ and Bulk Modulus is $11 \times 10^{10} \text{ N/m}^2$

$$3K(1-2\sigma) = Y$$

$$\sigma = \frac{1}{2} \left(1 - \frac{Y}{3K} \right) = 0.5 \times \left(1 - \frac{7.25 \times 10^{10}}{3 \times 11 \times 10^{10}} \right) = 0.390$$

10. A material has Poisson's ratio 0.2. If a uniform rod of it suffers a longitudinal strain 4×10^{-3} , calculate the percentage change in volume.

$$\text{Volume of a rod } V = \pi r^2 l$$

$$\text{Change in volume, } dV = \pi r^2 dl + \pi l 2r dr$$

$$\frac{dV}{V} = \frac{\pi r^2 dl + \pi l 2r dr}{\pi r^2 l}$$

$$\frac{dV}{V} = \frac{dl}{l} + 2 \frac{dr}{r} \quad (\text{dr is negative because cross section will decrease})$$

$$\frac{dl}{l} \text{ is the linear strain } \alpha \text{ and } \frac{dr}{r} \text{ is lateral strain } \beta = \sigma \cdot \alpha$$

$$\text{Percentage change in volume} = \frac{dV}{V} \times 100 = \alpha (1 - 2\sigma) \times 100 = .24 \%$$



11. What couple must be applied to a wire of length 1m and diameter 1mm in order to twist one end through 90° when the other end is fixed? The rigidity modulus is $2.8 \times 10^{10} \text{ N/m}^2$

$$C = \frac{\pi n r^4 \theta}{2l}$$

$$\theta = 90^\circ = \frac{\pi}{2} \text{ radians}$$

$$\text{Couple, } C = \frac{\pi \times 2.8 \times 10^{10} \times (.5 \times 10^{-3})^4 \times \pi}{2 \times 1 \times 2} = 4.31 \times 10^{-3} \text{ N-m}$$

12. An iron wire of length of 1m and radius 0.5 mm elongates by 0.32 mm when stretched by a force 49 N and twists through 0.4 radian when a couple of $3 \times 10^{-3} \text{ N-m}$ is applied to its ends. Calculate the elastic constants of iron.

$$Y = \frac{FL}{A \Delta l} = 1.95 \times 10^{11} \text{ N/m}^2 ; \quad n = \frac{2LC}{\pi r^4 \theta} = 7.6 \times 10^{10} \text{ N/m}^2$$

$$\sigma = \frac{Y}{2n} - 1 = 0.282 ; \quad K = \frac{Y}{3(1-2\sigma)} = 1.49 \times 10^{11} \text{ Nm}^{-2}$$

