RAMAIAH UNIVERSITY OF APPLIED SCIENCES

Faculty of Natural Sciences

Department	**: Mathematics and Statistics**
Programme	**: B. Tech.**
Semester/Batch	**: 3rd /2023**
Course Code	**:**
Course Title	**: Engineering Mathematics – 3**

Tutorial-3

Line and Double Integral

1. Evaluate $\int_C (2 + x^2 y)\, ds$ where C is the upper half of the circle $x^2 + y^2 = 1$.
2. Evaluate
   I. $\int_C (xy^2)\, dx$
   II. $\int_C (xy^2)\, dy$
   III. $\int_C (xy^2)\, ds$ on the quarter the circle C defined by $x = 4\cos t\,, y = 4\sin t,\; 0 \le t \le \frac{\pi}{2}$.
3. Evaluate $\int_C 4x\, dx + 2y dy$ where C is given by $x = y^3 + 1$ from the point$(0, -1), to(9,2)$.
4. Evaluate $\int_C y \sin z\, ds$ where C is the helix given by, $\vec{r}(t) = \cos t\, \hat{\imath} + \sin t\, \hat{\jmath} + t\hat{k}$, $0 \le t \le 2\pi$.
5. Evaluate the integral I along the circular arc C given by $x = \cos t$, $y = \sin t$, $0 \le t \le \frac{\pi}{2}$
$$I = \int_C 2xydx + (x^2 + y^2)dy.$$
6. Evaluate $\int_C 4x^3 ds$ where C is the line segment from $(-2, -1)$ to $(1,2)$.
7. Define work done along the curve C. Find the work done in moving particle in the force field $\vec{\mathrm{F}} = 3x^2\hat{\imath} + (2xz - y)\hat{\jmath} + z\hat{k}$, over the curve defined by $x = t, y = \frac{t^2}{4}, z = \frac{3t^3}{8}$ in the interval $0 \le t \le 1$.
8. Find the total work done in moving particle in a force field $\vec{F} = 3xy\hat{\imath} - 5z\hat{\jmath} + 10x\hat{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$ and $z = t^3$ form $t = 1$ to $t = 2$.
9. If $\overrightarrow{F} = (3x^2 + 6y)\hat{\imath} - 14yz\,\hat{\jmath} + 20xz^2\hat{k}$ evaluate $\int_c \vec{F} \cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the curve c given by $x = t;\ y = t^2;\ z = t^3$.
10. If $\overrightarrow{F} = (5xy - 6x^2)\hat{\imath} + (2y - 4x)\,\hat{\jmath}$ then evaluate $\int_c \vec{F} \cdot d\vec{r}$ where c is the curve $y = x^3$ from the point (1, 1) to the point (2, 8).
11. Find the total work done by a force $\overrightarrow{F} = xy\hat{\imath} + yz\,\hat{\jmath} + zx\hat{k}$ along the curve $x = t;\ y = t^2;\ z = t^3$ from $t = -1$ to $t = 1$.
12. Evaluate the following integral over the region $R = \{(x, y): 0 \le x \le 1, 0 \le y \le \ln 2\}$
$$\iint_R ye^{xy}\, dxdy.$$
13. Evaluate the double integral
$$\iint_R (x - 3y^2)dydx \text{ where } R = \{0 \le x \le 2, 1 \le y \le 2\}$$

14. Evaluate the double integral

$\iint_D (x+2y)dA$ over the region bounded by the parabola $y = 2x^2$ and $y = 1 + x^2$

15. Evaluate following double integral

$\iint_R (e^{x+3y})dA$ over the region bounded by $y = 1$ and $y = 2, y = x, y = -x + 5$

16. By changing the order of integration evaluate

(i) $\int_{x=1}^{2}\int_{2}^{4} (xy + e^y)dydx$

(ii) $\int_{x=}^{1}\int_{x}^{1} \sin(y^2)\, dydx$

(iii)) $\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$

iv) $\int_0^a \int_0^{\sqrt{a^2-y^2}} \sqrt{a^2 - x^2 - y^2}\, dxdy$

17. Convert the following double integrals into polar coordinates by showing the region of integration and then evaluate

(i) $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$

(ii) $\int_{-a}^{a} \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\, dy\, dx$

(iii) $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}} dxdy$

ANSWERS:

1. $\frac{2(3\pi+1)}{3}$
2. (a) -64 (b) 16π (c) $\frac{256}{3}$
3. $I = 165$
4. $\sqrt{2}\,\pi$
5. $I = \frac{1}{2}$
6. $I = -15\sqrt{2}$
7. $W = \frac{141}{128}$
8. $W = 30$
9. $W = \frac{10}{7}$
10. $W = 3$
11. $W = 5$
12. $e^{ln2} - ln2 - 1$
13. $I = -12$
14. $I = \frac{32}{5}$
15. $I = \frac{e^9}{2} - \frac{e^8}{4} - \frac{e^7}{2} + \frac{e^4}{4}$
16. i) $I = \frac{9}{2} + e^4 - e^2$. ii) $I = \frac{1}{2}(1 - \cos(1))$. iii) $I = \frac{\pi a}{4}$ iv) $\frac{\pi a^3}{6}$
17. i) $I = \frac{\pi}{4}$ ii) $\frac{\pi a^3}{3}$ iii) $\frac{a^3}{3}\log(\sqrt{2}+1)$