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MINI PROJECT REPORT

ON

A COMPARISON OF SVD AND DCT FOR IMAGE COMPRESSION

Submitted by

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PROJECT EVALUATION

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Sl.No.	Parameter	Max Marks	Marks Awarded
1	Background & Framing of the problem	4	
2	Approach and Solution	4	
3	References	4	
4	Clarity of the concepts & Creativity	4	
5	Choice of examples and understanding of the topic	4	
6	Presentation of the work	5	
	Total	25	

Name of the Course Instructor :

Signature of the Course Instructor :

A COMPARISON OF SVD AND DCT FOR IMAGE COMPRESSION

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Abstract - The Singular Value Decomposition expresses image data in terms of number of eigen vectors depending upon the dimension of an image. The psycho visual redundancies in an image are used for compression. Thus an image can be compressed without affecting the image quality. This paper present Two image compression techniques are SVD and CDT. Basic mathematics of SVD is dealt with in detail and results of applying SVD on an image are also discussed. In the conclusion part we discussed which one is the best technique for Image Compression. SVD and CDT are applied on variety of images for experimentation. We used the Matlab code for Image Compression.

Keywords – Linear Algebra, Matlab,Direct Cosine Transform, SVD , Image Compression Techniques

1. INTRODUCTION

Image compression is a type of data compression that involves encoding information in images using fewer bits than the original image representation. The main idea of image compression is reducing the redundancy of the image and the transferring data in an efficient form. The image compression takes an important place in several domains like web designing, in fact, maximally reduce an image allows us to create websites faster and saves bandwidth users, it also reduces the bandwidth of the servers and thus save time and money. When talking about compression, we generally take into account two aspects: image size in pixels and its degree of compression. The nature of the image is also playing a significant role. The main goal of such system is to reduce the storage quantity as much as possible while ensuring that the decoded image displayed in the monitor can be visually similar to original image as much as it can be.

Data for an image need more space for storage. More redundant information contains each and every format of data. Thus, the image compression has been continuing to be crucial to the development of computing. It plays a vital role of document and medical imaging. Compression of

data is very essential ingredient in any data storage. Image compression techniques can be broadly classified into two categories: - Losslessdata is very essential ingredient in any data storage. Image compression techniques can be broadly classified into two categories: - Lossless compression and lossy compression. Lossless compression techniques define entropy, which limits the reduction in the image. It tends to produce the same copy as that of the original data. Lossless compression is known as reversible as it doesn't degrade the quality of the image. Lossy compression techniques identify the minute details and variations in the system which the human eye is not fine-tuned to recognize. Amount of storage required to store the image file can be reduced by eliminating such features. Lossy compression often compromises the quality of the image. However, it can be used to achieve the storage space requirements. Lossy compression is irreversible because it degrades the data. Compression possible in lossy techniques is much higher than lossless techniques.

Here we have compared two methods of image compression, Singular Value Decomposition and Discrete Cosine Transform.

2.REVIEW OF LITERATURE

M. E. I. Tian has proposed techniques for compressing the images. They have also studied and analyzed the efficiency of SVD based image compression techniques. This paper has introduced a technique wherein efficient representation of singular vectors in each sub-block is used. This approach is called as adaptive singular value selection scheme. Three schemes are-Direct compression and decomposition scheme, adaptive singular value selection scheme, singular value subtracting one update scheme. In Direct scheme, original matrix is approximated using singular values which are fewer. As opposed to compressing the entire image in one go, this selection scheme tries to divide the original image into sub-block which are smaller in size with the goal of working with these smaller images. This approach is chosen with the aim of using the uneven complexity present in the original image to our advantage.

T. J. Peters has implemented SVD (singular value decomposition) to compress the microarray image. Huge amounts of DNA information for research purposes are stores as microarray images. These are of high-resolution images which highlights minute details of the image. Because of the high resolution, these images tend to be larger in size, which means storage on the hard disk requires lot of space. So, it is very important to reduce the size of the image without compromising the quality or compromising the amount of detail present in the image. This calls for comparatively complicated process, where in microarray images need to be clustered and classified before selecting the features. SVD can be used here to divide the image into small sub -images and on each sub-image SVD is performed. This method gives a better high peak signal to noise ratio in addition to increasing compression ratio.

Ranade suggested a variation on SVD based image compression. This approach is a slight modification to the original SVD algorithm, which gives much better compression than the standard compression using SVD method. In addition, it performs substantially better than the SVD method. Typically, for any given compression quality, this approach needs about 30% fewer singular values and vectors to be retained.

DCT

Douak et al. (2011) combined DCT transform with an adaptive block scanning to compress colour images. The algorithm started with a conversion from RGB colour space to YCbCr colour space, followed by DCT transform. Using the Bisection method, an iterative phase including thresholding, quantization was performed to compress an image. A reverse process reconstructed the original 30 images. The efficiency of the system was demonstrated through results and was compared to a block truncation-based coder.

Khali (2010) proposed a new entropy coder based on Discrete Cosine Transformation for image compression. Chen et al. (2010) similarly, used DCT with noise let information for image compression. The algorithm first transmitted DCT information sufficient to reproduce a "low-quality" version of the image at the decoder. This image was then used both at the decoder and encoder to create a mutually known list of locations of likely significant noise let coefficients. The coefficient values themselves were then transmitted to the decoder differentially, by subtracting, at the encoder, the low-quality image from the original image, obtaining the noise let values and subjecting them to quantization and entropy coding.

Fractal coding is a potential image compression scheme, which has the advantages of relatively high compression ratios and good reconstruction fidelity. However, the high computational complexity of fractal image encoding greatly restricts its application. Fu et al. (2009) proposed a DCT-based fractal image coding method, which improves the self-similarities exploiting scheme. The range and domain blocks are divided into three classes based on their DCT lower frequency coefficients, and only the domain blocks with the same class to the range block are calculated during best match exploiting process. Experimental results showed that compared with standard fractal coding scheme, the encoding time is significantly reduced and the PSNR of the reconstructed image is also improved.

3.IMPLIMENTATION

Singular Value Decomposition

Singular Value Decomposition is similar to Eigenvalue Decomposition in that the singular values of a matrix A are the square roots of the eigenvalues of the matrix A^TA. The same principle for Eigenvalue Decomposition, $A=PDP^{-1}$ is then applied to Singular Value Decomposition. The reason that we cannot use Eigenvalue Decomposition for images is because it only works for square matrices, so we use SVD because A^TA is guaranteed to be square matrix. The general formula for SVD is given as $A=USV^T$, where A is an mxn matrix of rank r, U is an mxm matrix, S is an mxn matrix, and V is an nxn orthogonal matrix. The S matrix is obtained by D, which is a diagonal matrix containing the first r singular values of A in decreasing order, and filling in zeros around D to get the correct mxn dimension. The U matrix is obtained by finding the left singular vectors of A, which are the eigenvectors of AA^T. The V matrix is obtained by finding the right singular vectors of A, which are the eigenvectors of A^TA. This can be used in image compression by throwing out values as the values are in decreasing order, the majority of the image information is contained in the first and largest values. Once the useless values are changed to zero, the matrices are multiplied together to get a new image matrix, which corresponds to a smaller file size, thus a compressed image.

Reduced SVD

After obtaining U, S and V values using above steps, we eliminate the unnecessary singular values in S matrix and obtain compressed image A with the new diagonal matrix obtained after removing some singular values.

Let A be an mxn image matrix. .

Using SVD, A can be represented as :

$$A=USV^T$$

$$A=[u_1 \ u_2 \ \ldots u_m]\begin{bmatrix} s1 & 0 \ldots & & \ldots & 0 \\ 0 & & \ldots & & \\ & & & sr & \\ 0 & & & & 0 \end{bmatrix}\begin{bmatrix} v1 \\ v2 \\ .. \\ vn \end{bmatrix}$$

Here, $s1>=s2>=\ldots s_r$.

In the above matrix S values after r terms are approximated to zero. So multiplication of the terms greater than r will be zero. If m=n , the above matrix can be represented as:

$$A= [s1u1 \ \ s2u2 \ \ \ldots \ \ s_ru_r \ \ 0 \ldots \ \ 0]\begin{bmatrix} v1 \\ v2 \\ .. \\ vn \end{bmatrix}$$

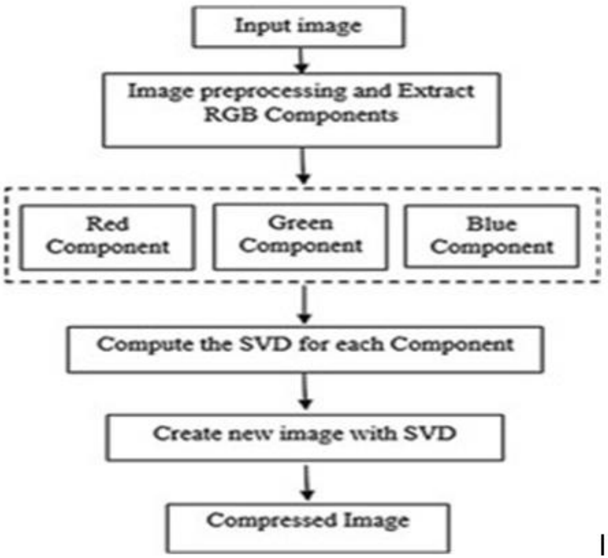
$$A=s1u1v1^T+ s2u2v2^T +\ldots\ srurvrT$$

We know that, rank of a singular matrix is equal to the number of non zero singular values. The rank of above matrix will now be obviously reduced as the number of S values is approximated to 'r' terms. Therefore, size of the matrix is reduced, which in turn reduces the memory occupied by the image.

Thus, From the above analysis, the matrix A can be approximated by adding only the first few terms (r terms) of the series. As r increases, the image quality increases, but at the same time, the amount of memory needed to store the image also increases. Optimum r value should be selected such that there is no damage to image quality and at the same time storage space occupied by image is reduced. Thus, selection of r value plays an important role in performance of Singular Value Decomposition (SVD) technique.

Original Image

ALGORITHM



<u>Compressed Images</u>

SVD=2, Size=323kb

SVD=42, size=400kb

<u>DISCRETE COSINE TRANSFORM</u>

In DCT, we express images as a series of data points as a sum of cosine functions at different frequencies. We represent image using a sinusoidal wave. It has the property that most of the significant information about the image is concentrated in just a few coefficients of the DCT.

We have used DCT transform matrix by using the function dctmtx. The M by M transform matrix T is given by:

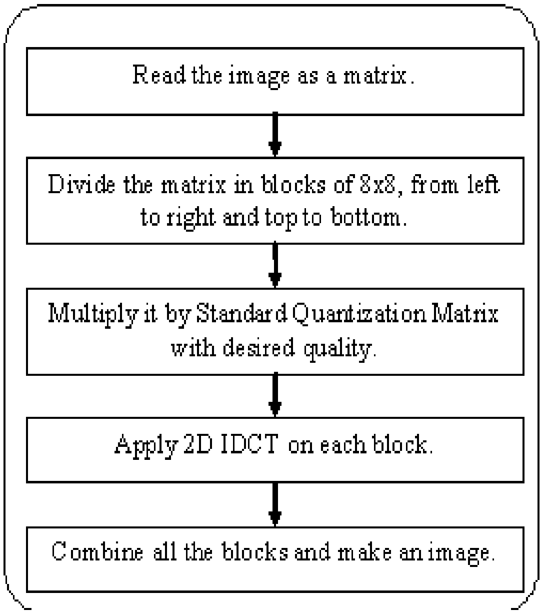
$$T_{pq} = \begin{cases} \frac{1}{\sqrt{M}} & p = 0, \quad 0 \le q \le M-1 \\ \sqrt{\frac{2}{M}} \cos \frac{\pi(2q+1)p}{2M} & 1 \le p \le M-1, \quad 0 \le q \le M-1 \end{cases}$$

For an M-by-M matrix A, T*A is an M-by-M matrix whose columns contain the one-dimensional DCT of the columns of A. The two-dimensional DCT of A can be computed as B=T*A*T'. Since T is a real orthonormal matrix, its inverse is the same as its transpose. Therefore, the inverse two-dimensional DCT of B is given by T'*B*T.

This example shows how to compress an image using the Discrete Cosine Transform (DCT).The input image is divided into 8 by 8 blocks. The example computes the two-dimensional DCT of 8-by-8 blocks in an input image, discards (sets to zero) some of the 64 DCT coefficients in each block and then reconstructs the image using the two-dimensional inverse DCT of each block.

Original Image

ALGORITHM



MATLAB CODE:

```
I=imread("Sofas.jpeg");
I=im2double(I);
T=dctmtx(8);
dct=@(block_struct)T*
block_struct.data * T';
B=blockproc(I,[8 8],dct);
mask=[ 1 1 1 1 1 1 1 1
       1 1 1 1 1 1 1 1
       1 1 1 1 0 1 1 1
       1 1 1 1 0 1 1 1
       1 1 1 1 1 1 1 1
       1 1 1 1 1 1 1 0
       1 1 1 1 1 1 1 1
       1 1 1 1 1 1 1 1];
B2=blockproc(B,[8
8],@(block_struct)mask.*block_struct
.data);
invdct=@(block_struct)T'*block_struc
t.data*T;
I2=blockproc(B2,[8 8],invdct);
imshow(I)
figure
imshow(I2)
imwrite(uint8(I),'originaldct.tif');
imwrite(uint8(I2),'comressdct.tif');
```

Compressed Images:

DCT Coefficient: 1/64

DCT Coefficient: 40/64

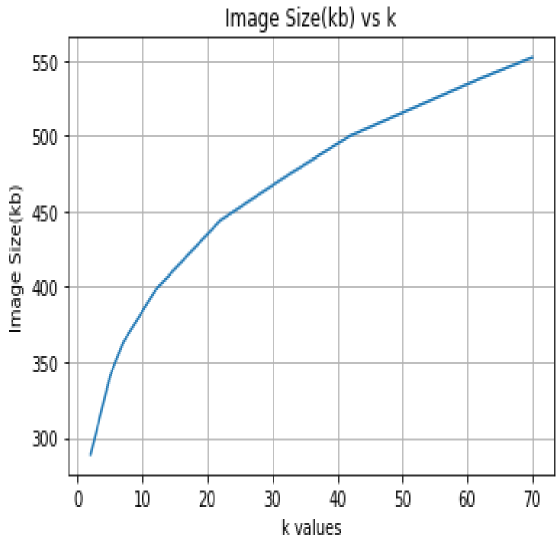
4. RESULTS AND CONCLUSION

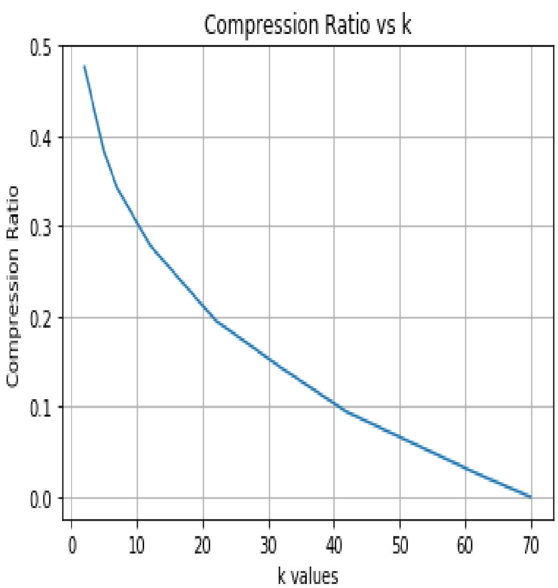
SVD Observation Table

Original Image file Size=552 Kb

Singular Values	Compared Size(kb)	Compression Ratio
2	289	0.476
5	341	0.382
7	363	0.342
12	398	0.278
22	444	0.195
32	473	0.143
42	500	0.094
52	519	0.059
62	538	0.025
70	552	0

Graphs



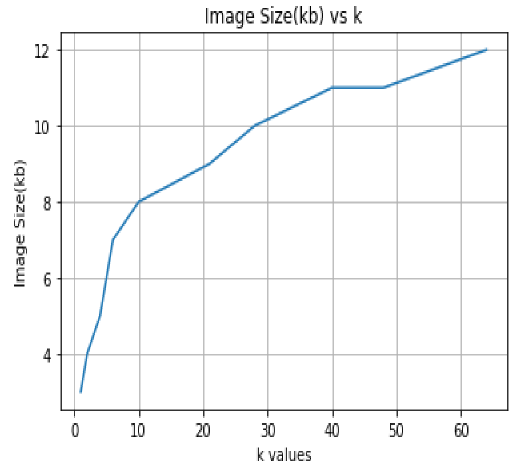


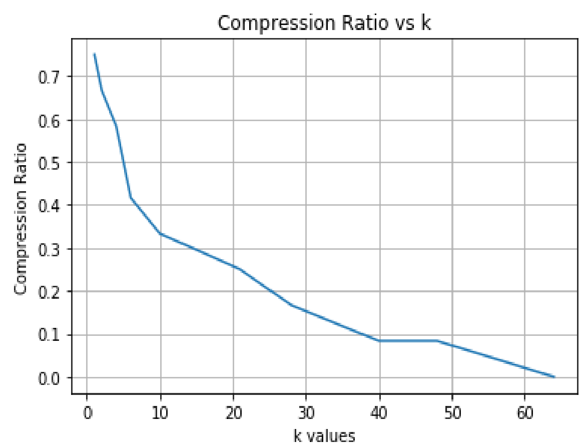
DCT Observation Table

Original Size of Image=12 Kb

Singular Values	Compared Size(kb)	Compression Ratio
1	3	0.75
2	4	0.66
4	5	0.58
6	7	0.416
10	8	0.31
21	9	0.25
28	10	0.166
40	11	0.083
64	12	0

Graphs





We can observe that in both SVD and DCT methods, with increase in singular values or DCT coefficients, the compressed image size increases leading to better image quality.

Compression ratio calculated by the formula (1-(Compressed image size/ Original image size)) decreases with increase in k values in both SVD and DCT methods.

Hence less compression ratio means greater the compressed image file size and better will be the compressed image quality.

We can observe that the compressed image with 42 singular values mostly resembles the original image in quality. And the compressed DCT image with 28 DCT coefficients mostly resembles the original image .The compressed SVD image has a compression ratio of 0.094

whereas the compressed DCT image has a compression ratio of 0.166. More the CR, lesser will be the file size and more memory can be saved.

Therefore, we finally conclude that DISCRETE COSINE TRANSFORM is a better method than SINGULAR VALUE DECOMPOSITION for image compression.

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