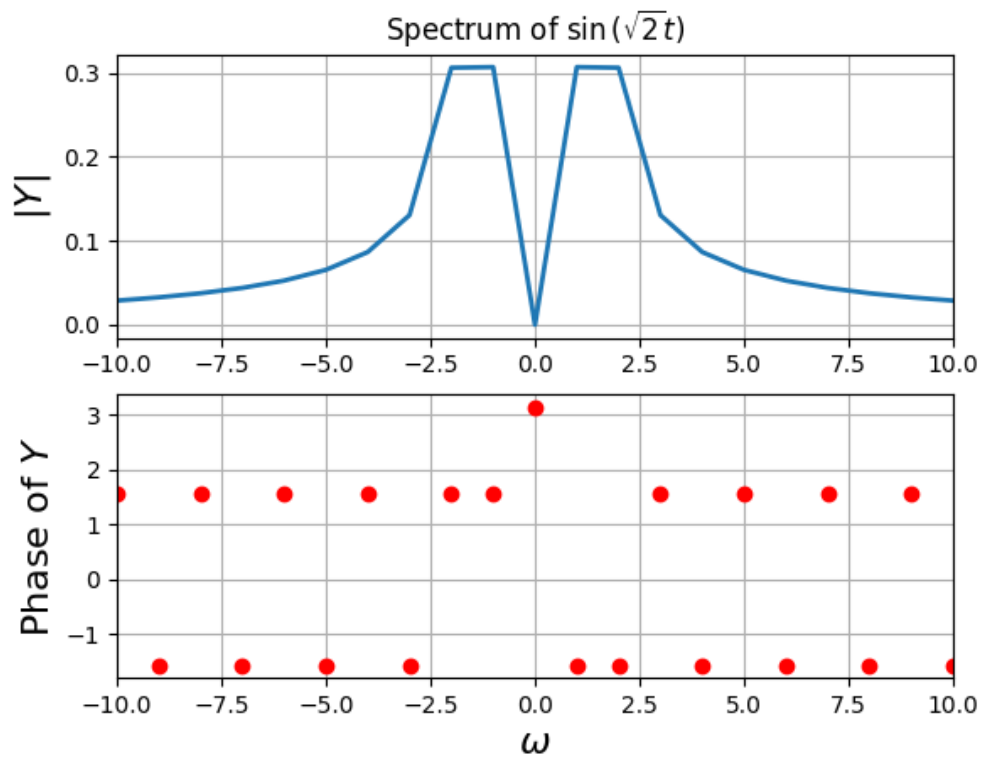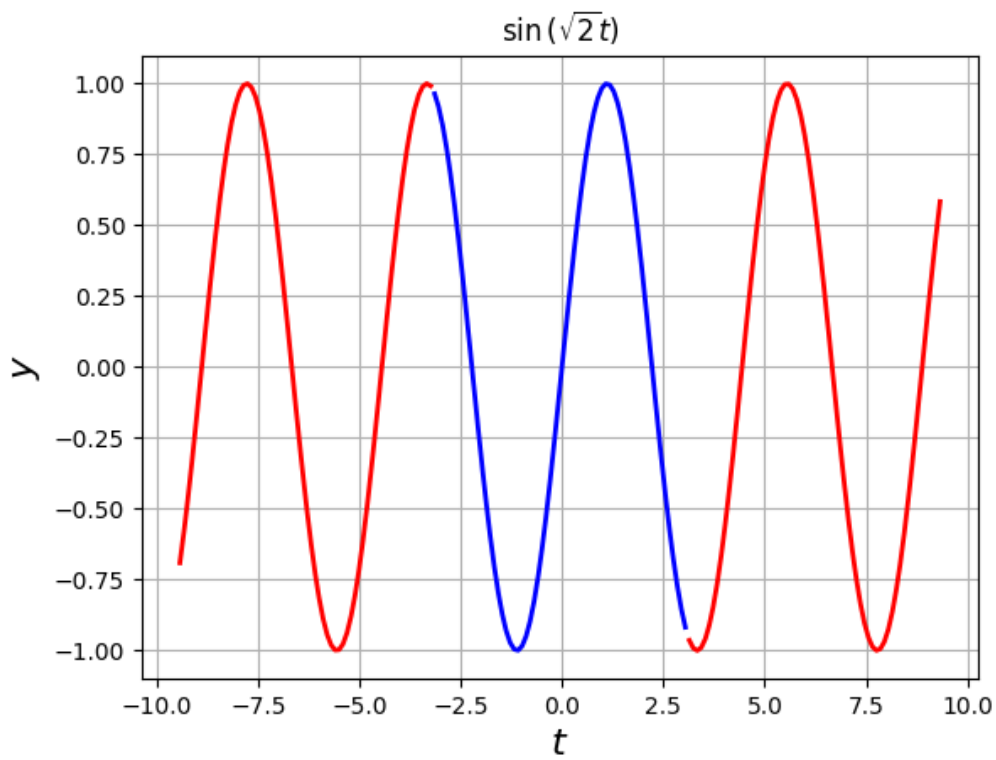# Assignment No 9

JAGAN M J EE20B047
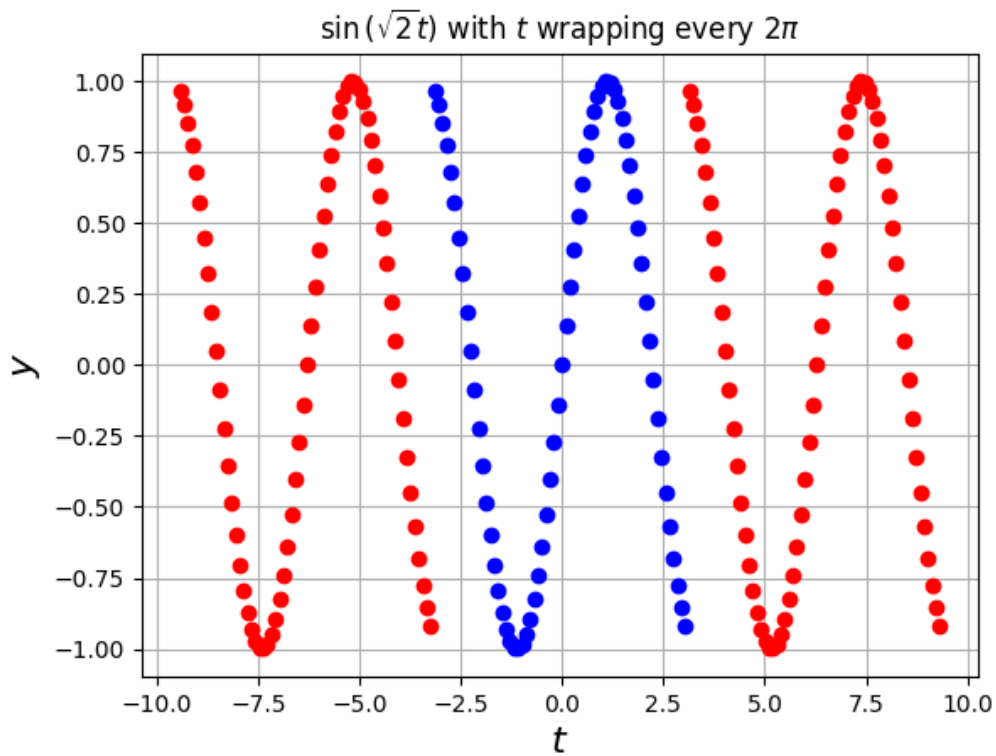
1 Worked Examples

Spectrum of $\sin(\sqrt{2}t)$ is as shown below :

The graph of function for which we want the DFT :

The graph of the function obtained by replication of the part from $[-\pi, \pi]$

Clearly this function is not $\sin(\sqrt{2}t)$ and that is why the DFT is not what we expect.These discontinuities lead to non harmonic components in FFT that decays as $\frac{1}{\omega}$.

This can be shown by plotting the spectrum of periodic ramp as shown below :
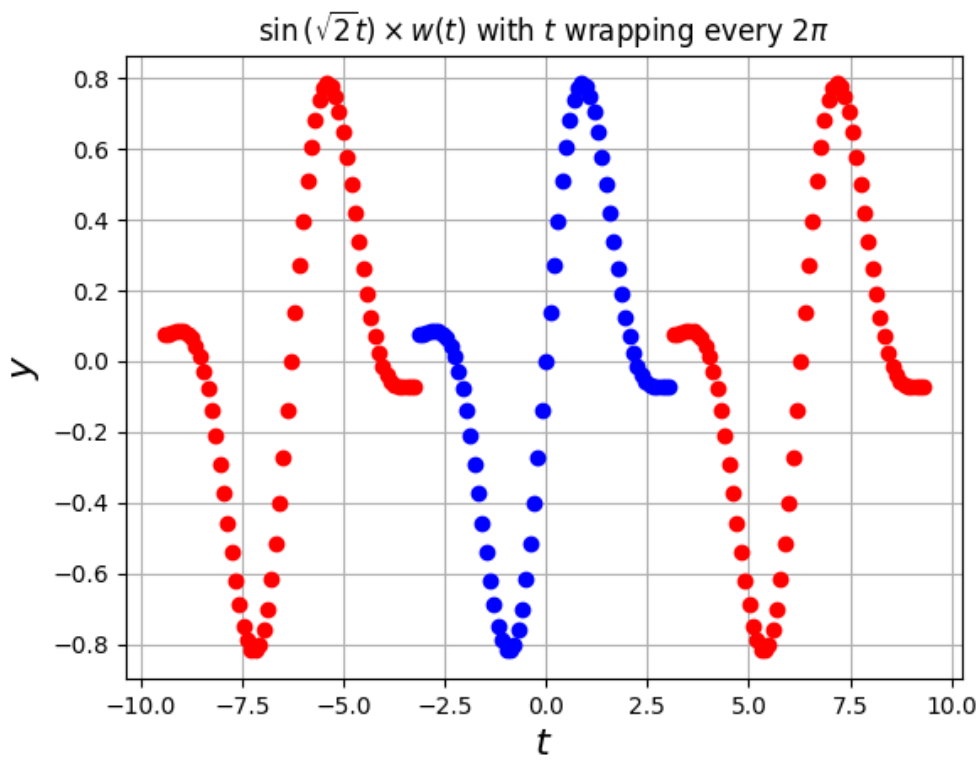
Spectrum of a digital ramp
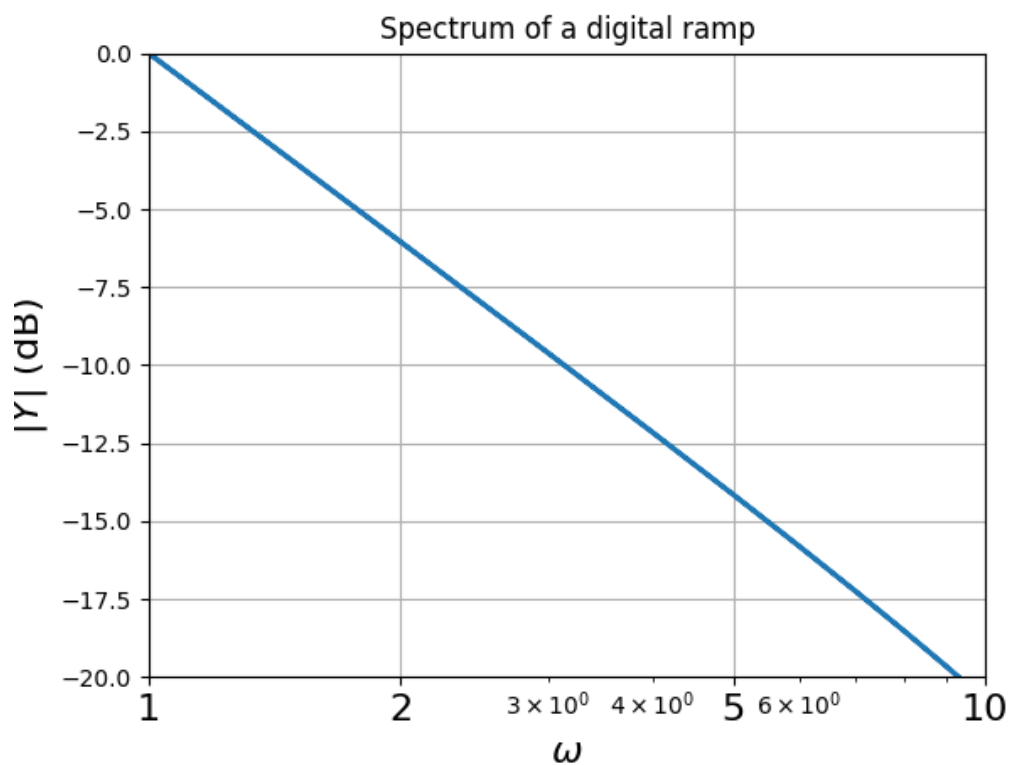
2   Hamming Window

The Hamming window removes discontinuities by attenuating the high frequency components that causes discontinuities. The Hamming window function is given by :
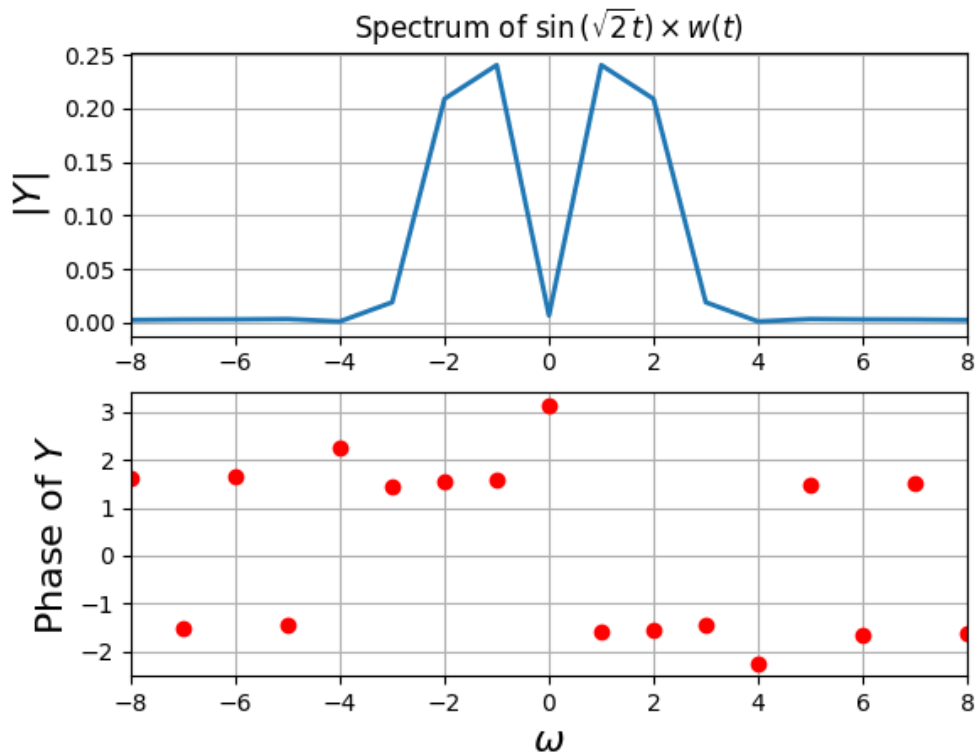
$$x[n] = 0.54 + 0.46 cos(\frac{2\pi n}{N-1})$$

This equation is multiplied by our signal and we plot the graph of the function by replication of the part from $[-\pi, \pi]$
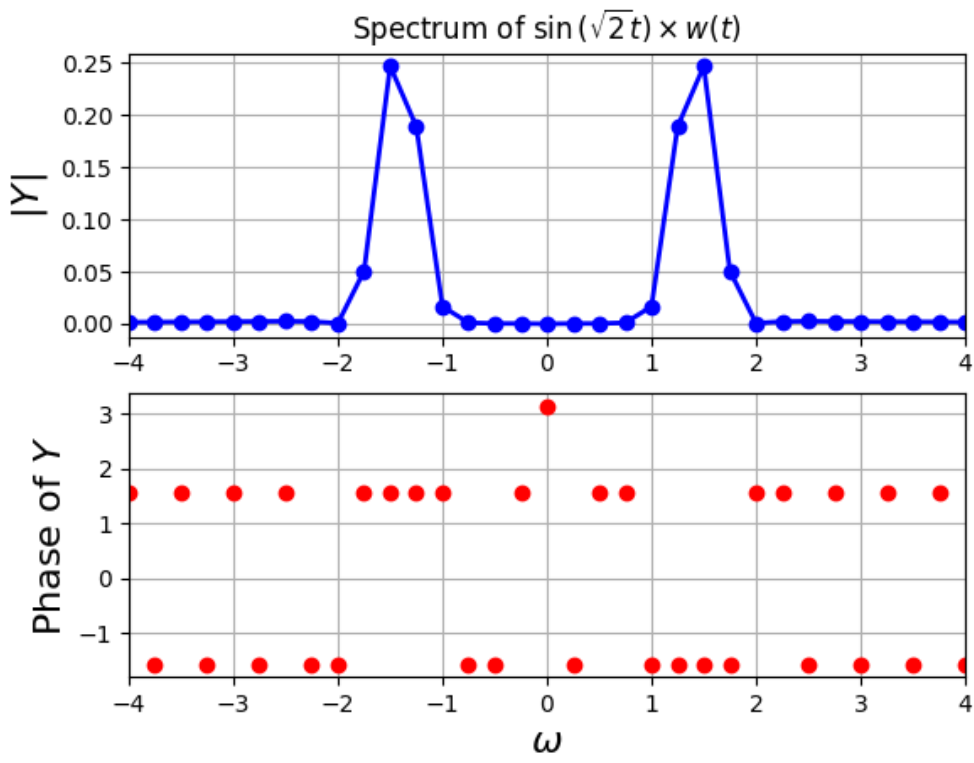
Note that the discontinuities vanish to some extent (These discontinuities vanish completely in case of raised cosine window)

We now plot the spectrum of this function and is as shown below :

The spectrum that is obtained with a time period of 8π has a slightly sharper peak and is as shown below :
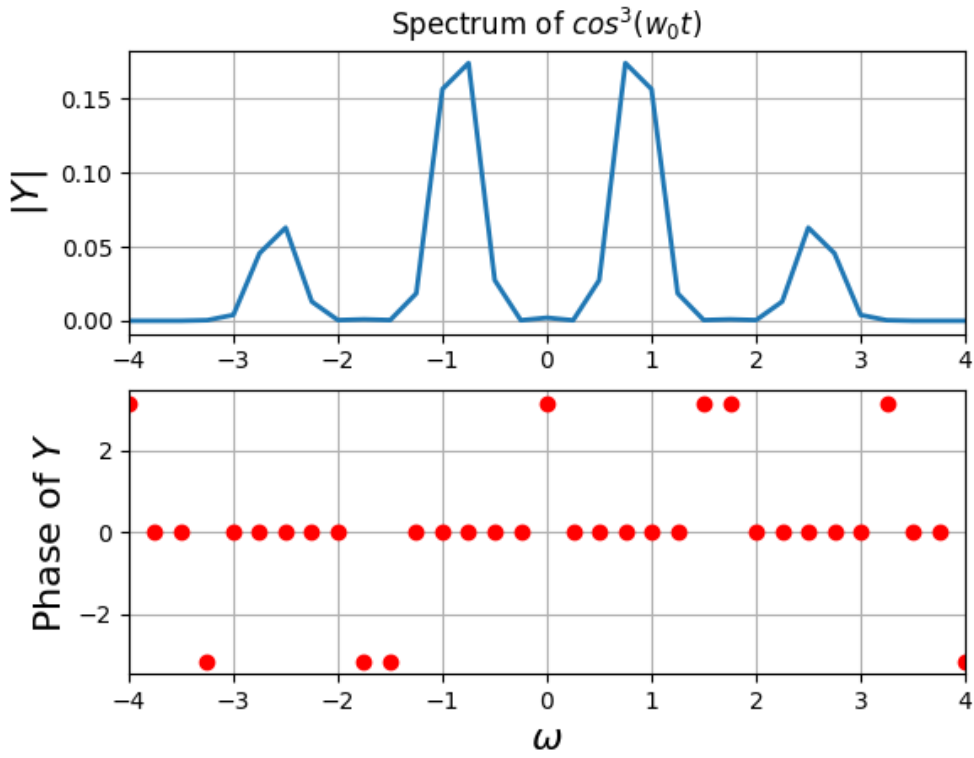
Spectrum of $\sin(\sqrt{2}t) \times w(t)$

3   DFT of $\cos^3(\omega_o t)$

Spectrum of $\cos^3(\omega_o t)$ with $\omega_o = 0.86$ without hamming window is as shown below :
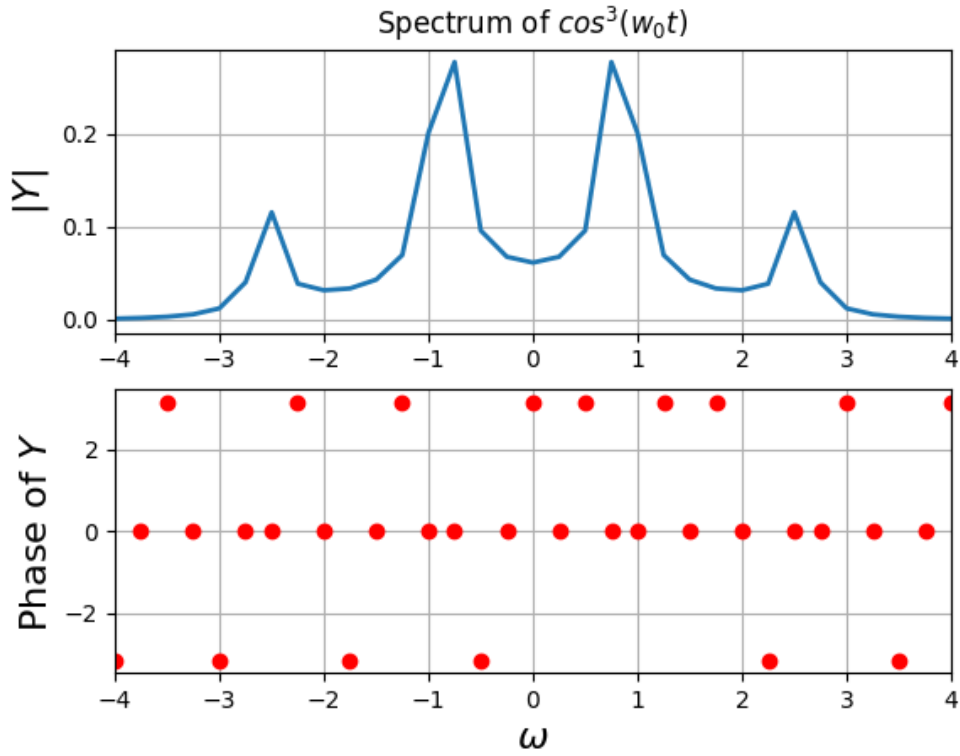
Spectrum of $\cos^3(w_0 t)$

Spectrum of $\cos^3(\omega_o t)$ with $\omega_o = 0.86$ with hamming window is as shown below :

Spectrum of $cos^3(w_0 t)$

Note that this spectrum after hamming has a narrower peak.

4 Estimation of ω_o and δ

We need to estimate the parameters for a signal $\cos(\omega t + \delta)$ for 128 samples between $[-\pi, \pi]$.
Estimation of ω_o is done by taking the weighted average of all the ω weighted with the magnitude of DFT. For estimation of δ, we find the phase of the DFT at ω_o nearest to estimated ω.

Calculated value of ω_o : 1.473027
Calculated value of δ : 0.501876

The plot of the spectrum of $\cos(\omega_o t + \delta)$ for $\omega_o = 1.5$ and $\delta = 0.5$ is as shown below :
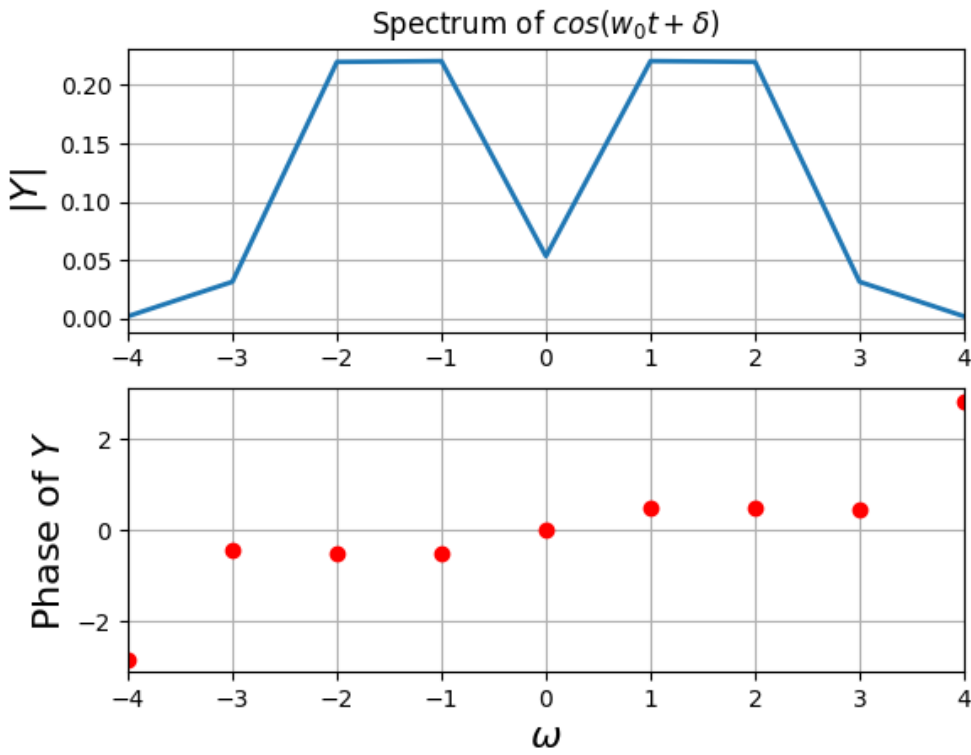
Figure 1: Spectrum of $\cos(\omega_o t + \delta)$

5   Estimation of ω_o and δ in the presence of noise

For this, we add white gaussian noise generated by $randn()$ in python to the above data. The extent of this noise is 0.1 times the amplitude. We proceed as above and calculate the values of the parameters (The estimated values are slightly different after each run due to the random error added to the data)

Calculated value of ω_o with noise : 2.014397

Calculated value of δ with noise : 0.515255

The plot of this spectrum for $\omega_o = 1.5$ and $\delta = 0.5$ is as shown below :

Spectrum of noisy function
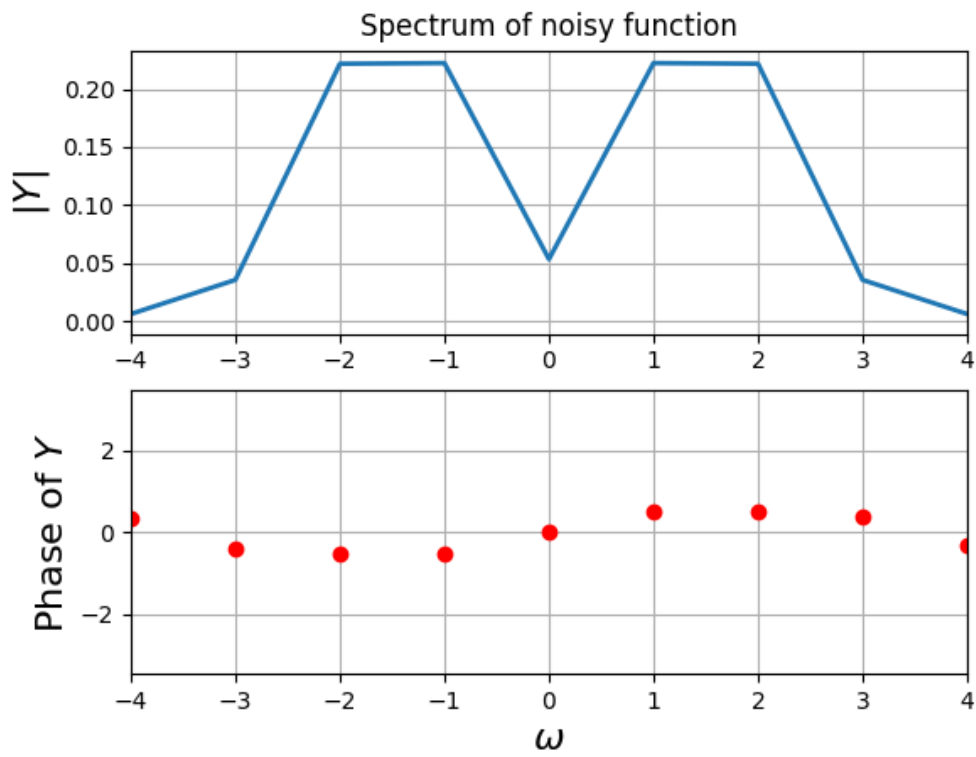
6   Analysis of Chirped Signal spectrum

We plot the DFT of the chirped signal $\cos(16(1.5 + \frac{t}{2\pi})t)$ where $-\pi \leq t \leq \pi$ in 1024 steps. Its frequency continuously changes from 16 to 32 radians per second. This also means that the period is 64 samples near $-\pi$ and is 32 samples near π

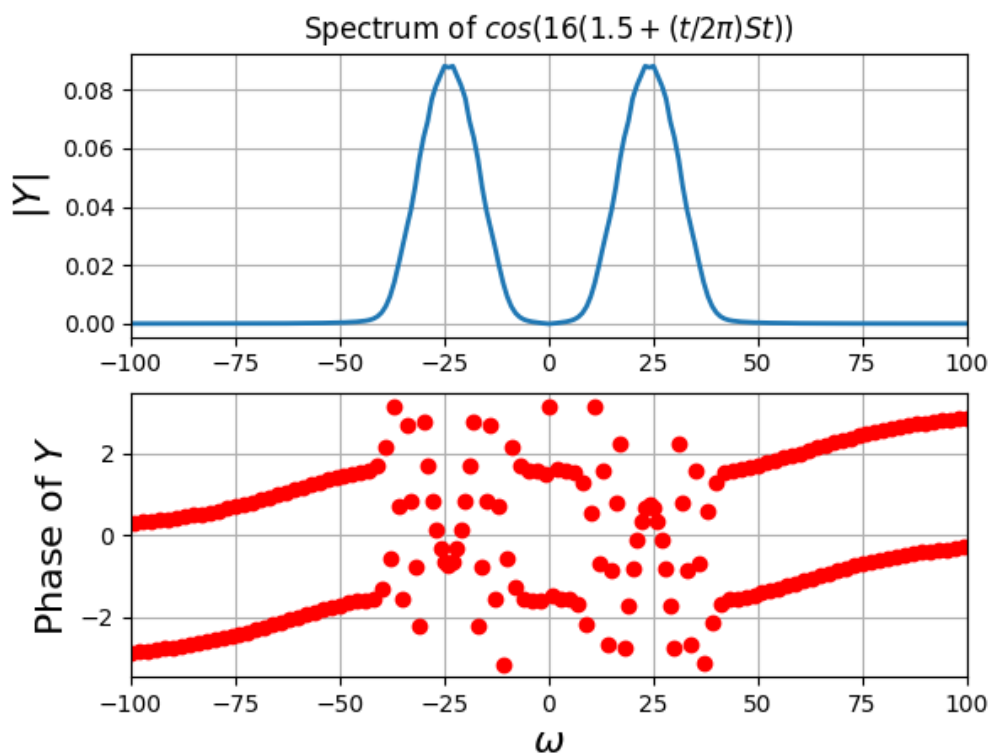The plot of the spectrum of the chirped signal is as shown below :

Figure 2: Spectrum of chirped signal with windowing

7 Surface plot of chirped signal

For the same chirped signal, break the 1024 vector into pieces that are 64 samples wide and extract the DFT of each and store as a column in a 2D array. Then we plot the surface plot to show the variation of frequency of the signal with time.
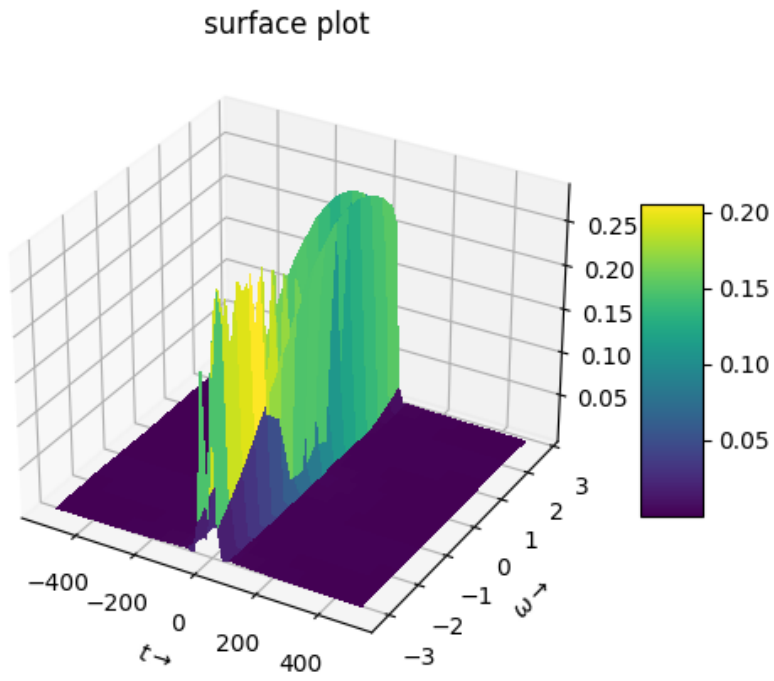
Figure 3: Surface plot of chirped signal

8 Conclusion

We obtained the DFT of non periodic functions. Also the improvement in the spectrum of DFT after the introduction of Hamming window. Estimated the value of parameters of ω_o by weighted average and δ by taking the phase of the nearest angle to the estimated ω_o. We also plot the surface plot to show the variation of frequency of the signal with time.