Assignment

CSA0669 - Design and Analysis of Algorithm

By
R. Jagan
192321102
B Tech IT

1. Calculate the number of ways to achieve a sum of 15 when rolling four six sided dice. provide a detailed step by step solution.

$\{1, 2, 3, 4, 5, 6\}$

$m = 24 \rightarrow 4 \times 6$

$K = 4 \rightarrow$ No. of dice

$x = 15 \rightarrow$ sum

$j = 6 \rightarrow$ four dice

$x_1 + x_2 + x_3 + x_4 = 15$

$y_i = x_i - 1$

$y_i + 1 = x_i$

$(y_1 + 1) + (y_2 + 1) + (y_3 + 1) + (y_4 + 1) = 15$

$y_1 + y_2 + y_3 + y_4 + 4 = 15$

$y_1 + y_2 + y_3 + y_4 = 15 - 4$

$y_1 + y_2 + y_3 + y_4 = 11$

$K = 4, \; n = 11$

$$\binom{n+K-1}{K-1} = \binom{11+4-1}{4-1} = \binom{14}{3} = \frac{14 \times 13 \times \cancel{12}^{6}}{1 \cancel{\times 2} \times 3}$$

$= 364$

$y_i > 5$

$y_1 + y_2 + y_3 + y_4 = 5$

$$\frac{(5+4-1)!}{4-1!} = \frac{8!}{3!} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

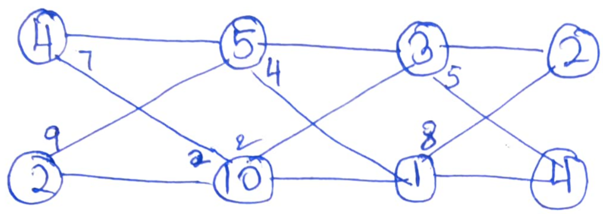
$4 \times 56 = 224$

No. of valid solution

$364 - 224 = 140$

There are 140 possible solution to get a sum of 15 when 4 six sided dices are rolled.

2. Two assembly lines have station times as follows: Line : 1 [4 5 3 2] Line 2 : [2 10 1 4] Transfer times between lines are From Line 1 to Line 2 : [7 4 5], From Line 2 to Line 1 : [9 2 8]. Calculate the minimum time to assemble a product.



	1	2	3	4
$f_1[j]$	4	9	12	14
$f_2[j]$	2	12	11	15
$l_1[j]$	1	1	1	1
$l_2[j]$	2	2	2	2

Min (14, 15) = 14

∴ path is

(4) → (5) → (3) → (2) = 14

Line 1 1 1 1

3. Given Key {10, 20, 30, 40} \ {10, 20, 30, 40} with access probabilities {0.1, 0.2, 0.4, 0.3} \ {0.1, 0.2, 0.4, 0.3} respectively Construct the optimal binary search tree. Calculate the total cost of the tree.

Serial	1	2	3	4
Nodes	10	20	30	40
value	0.1	0.2	0.4	0.3

i \ j	0	1	2	3	4
0	0	0.1	[2] 0.4	[3] 1.1	[3] 1.7
1		0	0.2	[3] 0.8	[3] 1.4
2			0	0.4	[3] 1.0
3				0	0.3
4					0

j - i = 0
0 - 0 = 0
1 - 1 = 0
2 - 2 = 0
3 - 3 = 0
4 - 4 = 0

j - i = 1
1 - 0 = 1
2 - 1 = 1
3 - 2 = 1
4 - 3 = 1

j - i = 2
2 - 0 = 2 (0, 2)
3 - 1 = 2 (1, 3)
4 - 2 = 2 (2, 4)

$$\text{cost}(0,2)^{K=1,2} = \text{Min}\begin{cases}\text{cost}(0,0) + \text{cost}(1,2)\\ \text{cost}(0,1) + \text{cost}(2,2)\end{cases} + 0.3$$

$$= \text{Min}\begin{Bmatrix}0.2\\0.1\end{Bmatrix} + 0.3 = \min\{0.5\}$$

0.5 → 2

$$\text{cost}(1,3) = \underset{\min}{K = 2,3}\begin{cases}\text{cost}(1,1) + \text{cos}(2,3)\\ \text{cost}(1,2) + \text{cost}(3,3)\end{cases} + 0.6 = \min\begin{Bmatrix}1.0\\0.8\end{Bmatrix}$$

$$\text{cost}(2,4) = \text{Min}\begin{cases}\text{cost}(2,2) + \text{cost}(3,4)\\ \text{cost}(2,3) + \text{cost}(4,4)\end{cases} + 0.7 \quad \min\begin{Bmatrix}1.0\\1.1\end{Bmatrix}$$

R = 3, 4

$j - i = 3$

$3, 0 = 3 \Rightarrow cost(0,3) \quad k = 1,2,3$

$4.1 = 3$

$$= \min \begin{Bmatrix} cost(0,0) + cost(1,3) \\ cost(0,1) + cost(2,3) \\ cost(0,2) + cost(3,3) \end{Bmatrix} + 0.7$$

$$= \begin{Bmatrix} 0.8 \\ 0.5 \\ 0.4 \end{Bmatrix} + 0.7$$

$$\min \begin{Bmatrix} 1.5 \\ 1.2 \\ 1.1 \end{Bmatrix}$$

$$Cost(1,4) = K = 2,3,4 \quad \min \begin{Bmatrix} cost(1,1) + cost(2,4) \\ cost(1,2) + cost(3,4) \\ cost(1,3) + cost(4,4) \end{Bmatrix} + 0.9$$

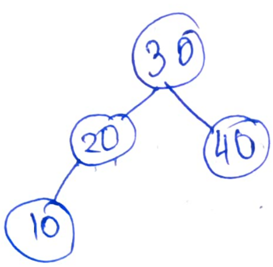
$$= \min \begin{Bmatrix} 1.0 \\ 0.5 \\ 0.8 \end{Bmatrix} + 0.9 = \min \begin{Bmatrix} 1.9 \\ \boxed{1.4} \\ 1.7 \end{Bmatrix}$$

$j - i = 4$

$4 - 0 = 4 \quad (0,4)$

$$Cost(0,4) : Key = 1,2,3 \quad \min \begin{Bmatrix} cost(0,0) + cost(1,4) \\ cost(0,1) + cost(2,4) \\ cost(0,2) + cost(3,4) \\ cost(0,3) + cost(4,4) \end{Bmatrix} + 1.0$$

$$= \min \begin{Bmatrix} 1.4 \\ 0.7 \\ 1.1 \end{Bmatrix} + 1.0 = \min \begin{Bmatrix} 2.4 \\ 2.1 \\ 1.7 \\ 2.1 \end{Bmatrix}$$



$$\text{Total Cost} = (0.1 \times 3) + (0.2 \times 2) + (0.3 \times 2) + 0.4$$

$$= 0.3 + 0.4 + 0.6 + 0.4$$

$$= 1.7$$

4. Solve the TSP for the following 5 city distance matrix using dynamic programming.

A [0, 29, 20, 21, 17]

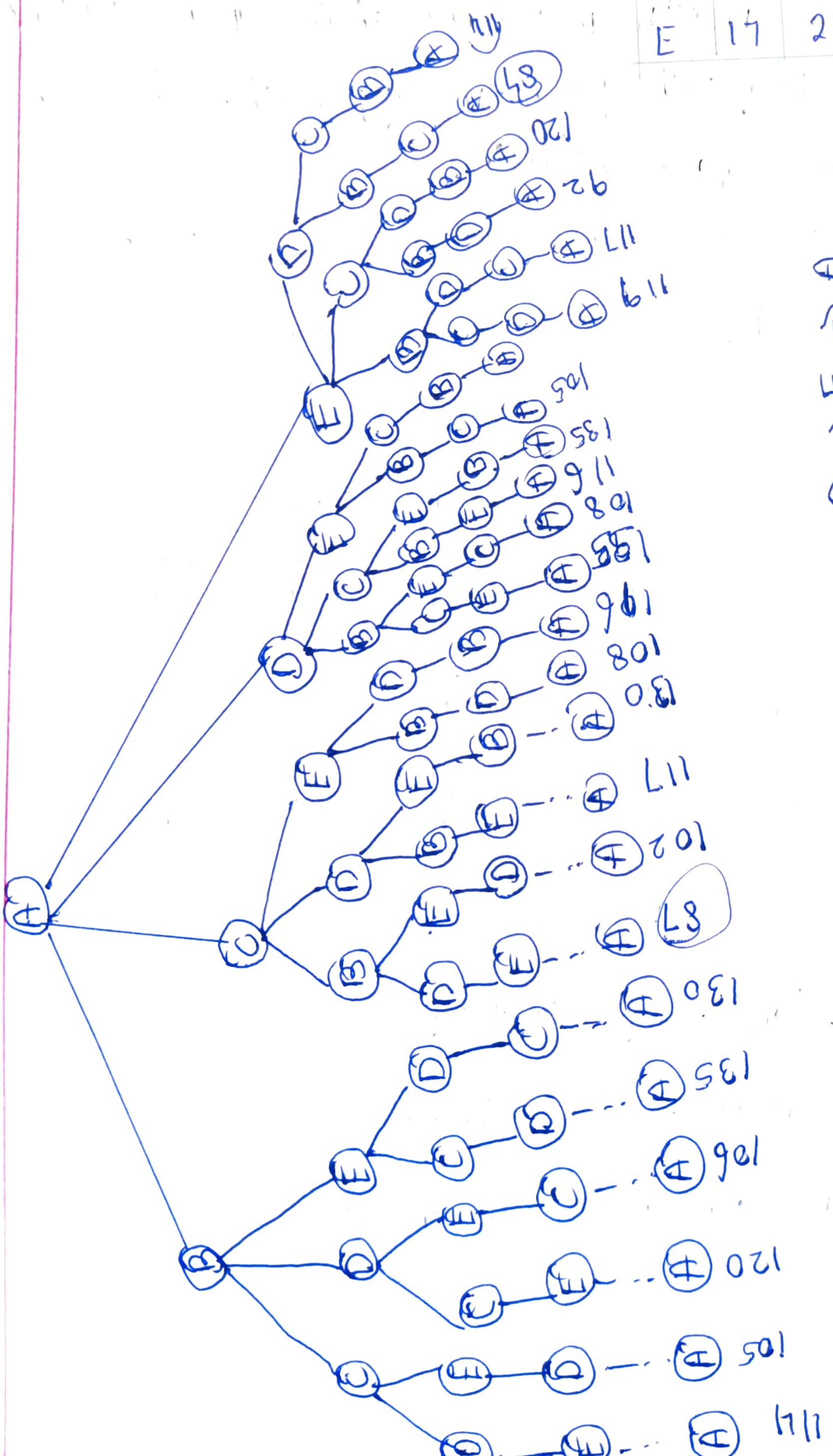
B [29, 0, 15, 17, 28]

C [20, 15, 0, 35, 22]

D [21, 17, 35, 0, 18]

E [17, 28, 22, 18, 0]

	A	B	C	D	E
A	0	29	20	21	17
B	29	0	15	17	28
C	20	15	0	35	22
D	21	17	35	0	18
E	17	28	22	18	0



Shortest path ⇒ A → C → B → D → E → A

A → E → D → B → C → A

Cost : 87.

5. You have a Knapsack with capacity of 50 units. There are 4 items with the following weights and values:

Item 1 : weight = 10 value = 60
Item 2 : weight = 20 value = 100
Item 3 : weight = 30 value = 120
Item 4 : weight : 40 value = 200

Determine the maximum value that can be obtained using tho 0/1 Knapsack problem approach. show the steps and the final solution.

$W = 50$, $n = 4$, $w = [10, 20, 30, 40]$. $V = [60, 100, 120, 200$

	0	10	20	30	40	50
0	0	0	0	0	0	0
1	6	60	60	60	60	60
2	0	60	100	160	160	160
3	0	60	100	160	180	220
4	0	60	100	160	200	260

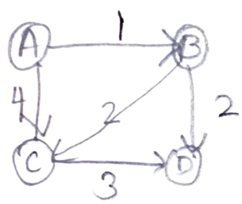
total weight = 50 (10 + 40)

Maxi value = 260 (60 + 200)

Items included = 1 and 4

6. Given the following directed graph with vertices A, B, C, D, A, B, C, DA, B, C, D and edges with weights.

A -> B A \ right arrow BA -> B with weight 1
A -> CA \ right arrow CA -> C with weight 4
B -> CB. \ right arrow CB -> C with weight 2
B -> CB \ right arrow CB -> C with weight 2
B -> DB \ right arrow DB -> D with weight 6
C -> DC \ right arrow DC -> D with weight 3
use the Bellman Ford algorithm to find the shortest path from vertex AAA to all other vertices. show the steps and final distances.



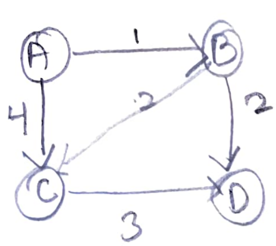
A - B = 1
B - C = 2
B - D = 2
A - C = 4
C - D = 3

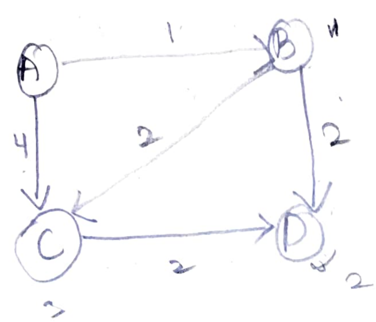
i)

V	A	B	C	D
d	0	∞	∞	∞
P	-	-	-	-

ii)

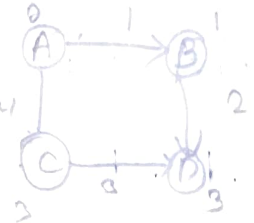
V	A	B	C	D
d	0	1	3	3
P	-	A	B	B





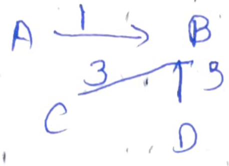
iii)

V	A	B	C	D
d	0	1	3	3
p	-	A	B	B



Shortest path / cost = 7

Shortest path =



7. Determine the probability of rolling five dice such that the sum is exactly 20. Include a combinatorial approach to arrive at the solution.

$6^5 = 7776$

$x_1 + x_2 + x_3 + x_4 + x_5 = 20$

$(y_1+1)+(y_2+1)+(y_3+1)+(y_4+1)+(y_5+1) = 20$

$y_1 + y_2 + y_3 + y_4 + y_5 = 15$

where $0 \le y_i \le 5$

$$\binom{15+5-1}{5-1} = \binom{19}{4}$$

$$\binom{19}{4} = \frac{19 \times 18 \times 17 \times 16}{4 \times 3 \times 2 \times 1} = 3876$$

If $y_1 \ge 6$, let $u_1 = y_1 - 6$

$y_1 + y_2 + y_3 + y_4 + y_5 = 9$

$$\binom{9+5-1}{5-1} = \binom{13}{4}$$

$\binom{13}{4} = \frac{13 \times 12 \times 11 \times 10}{1 \times 2 \times 3 \times 4} = 715$

There are 5 such variables.

$5 \times 715 = 3575$

If two variables $y_i, y_j \geq 6$, let $y_1' = y_1 - 6$ and $y_2' = y_2 - 6$

$y_1 + y_2 + y_3 + y_4 + y_5 = 3$

$\binom{3+5-1}{5-1} = \binom{7}{4}$

$\binom{7}{4} = \frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2 \times 1} = 35$

$\binom{5}{2} \times 35 = 10 \times 35 = 350$

using the Inclusion - Exclusion principle

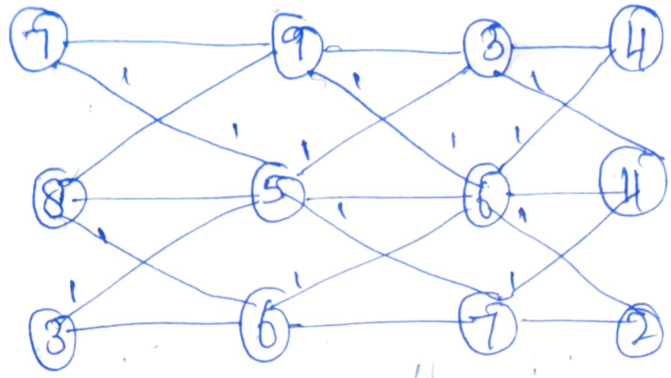
$3876 - 3575 + 350 = 651$

$\frac{651}{7776} = \frac{651}{7776} = 0.0837$.

8. For three assembly lines with station times:

Line 1: [7, 9, 3, 4] Line 2 [8, 5, 6, 4] Line 3: [3, 6, 7, 2]

and transfer times between lines given, determine the optimal scheduling and the total minimum assembly time.



	1	2	3	4
$f_1[j]$	7	16	13	17
$f_2[j]$	8	9	15	19
$f_3[j]$	3	9	16	18

$$\text{Min}(f_1[j], f_2[j], f_3[j]) = 17$$

	0	1	2	3	4
l_1	1	1	2	1	1
l_2	2	3	2	2	1
l_3	3	3	3	3	1

9. Consider Keys {15, 25, 35, 45, 55} | {15, 25, 35, 45, 55} {15, 25, 35, 45, 55} with access probabilities {0.05, 0.15, 0.4, 0.25, 0.15} | {0.05, 0.15, 0.4, 0.25, 0.15} {0.05, 0.15, 0.4, 0.25, 0.15}. Determine the structure of the OBST and compute the expected cost.

	1	2	3	4	5
Keys	15	25	35	45	55
Frequency	0.05	0.15	0.4	0.25	0.15

	0	1	2	3	4	5
0	0	0.05	$0.25^{(1)}$	$0.85^{(3)}$	$1.65^{(3)}$	$1.8^{(3)}$
1		0	0.15	$0.7^{(3)}$	$1.2^{(3)}$	$1.65^{[3]}$
2			0	0.4	$0.9^{(3)}$	$1.7^{[5]}$
3				0	0.25	$0.55^{[4]}$
4					0	0.15
5						0

j - i = 0
0 - 0 = 0
1 - 1 = 0
2 - 2 = 0
3 - 3 = 0
4 - 4 = 0
5 - 5 = 0

j - i = 1
1 - 0 = 1
2 - 1 = 1
3 - 2 = 1
4 - 3 = 1
5 - 4 = 1

j - i = 2
2 - 0 = 2
3 - 1 = 2
4 - 2 = 2
5 - 3 = 2

j - i
3 - 0 = 3
4 - 1 = 3
5 - 2 = 3

Cost (0, 3)
R = 1, 2, 3

$$= \min \begin{cases} \text{Cost}(0,0) + \text{cost}(1,3) \\ \text{cost}(0,1) + \text{Cost}(2,3) \\ \text{Cost}(0,2) + \text{Cost}(3,3) \end{cases} + 0.6$$

$$= \min \begin{Bmatrix} 0.7 \\ 0.45 \\ 0.25 \end{Bmatrix} + 0.6 = \min \begin{Bmatrix} 1.3 \\ 1.05 \\ 0.85 \end{Bmatrix}_{R=3}$$

$$\text{Cost}(1,4) = \min \begin{Bmatrix} \text{Cost}(1,1) + \text{cost}(2,4) \\ \text{Cost}(1,2) + \text{cost}(3,4) \\ \text{cost}(1,3) + \text{Cost}(4,4) \end{Bmatrix} + 0.8$$

$k = 2, 3, 4$

$$= \min \begin{Bmatrix} 0.9 \\ 0.4 \\ 0.7 \end{Bmatrix} + 0.8$$

$$= \min \begin{Bmatrix} 1.7 \\ 1.2 \\ 1.5 \end{Bmatrix} k = 3$$

$$\text{Cost}(2,5) = \min \begin{Bmatrix} \text{Cost}(2,2) + \text{Cost}(3,5) \\ \text{Cost}(2,3) + \text{Cost}(4,5) \\ \text{Cost}(2,4) + \text{Cost}(5,5) \end{Bmatrix} + 0.8$$

$k = 3, 4, 5$

$$= \min \begin{Bmatrix} 0.55 \\ 0.55 \\ 0.9 \end{Bmatrix} + 0.8 = \min \begin{Bmatrix} 1.35 \\ 1.35 \\ 1.7 \end{Bmatrix} k = 5$$

$j - i = 4$

$4 - 0 = 4$

$5 - 1 = 4$

$$\text{Cost}(0,4) = \underset{k = 1,2,3,4}{\min} \begin{Bmatrix} \text{Cost}(0,0) + \text{Cost}(1,4) \\ \text{Cost}(0,1) + \text{Cost}(2,4) \\ \text{Cost}(0,2) + \text{Cost}(3,4) \\ \text{Cost}(0,3) + \text{Cost}(4,4) \end{Bmatrix} + 0.85$$

$$= \min \begin{Bmatrix} 1.2 \\ 0.95 \\ 0.8 \\ 0.85 \end{Bmatrix} + 0.85 = \min \begin{Bmatrix} 2.05 \\ 1.8 \\ 1.65 \\ 1.7 \end{Bmatrix} \cdot k = 3$$

$$\text{Cost}(1,5) = \min \begin{Bmatrix} \text{Cost}(1,1) + \text{Cost}(2,5) \\ \text{Cost}(1,2) + \text{Cost}(3,5) \\ \text{Cost}(1,3) + \text{Cost}(4,5) \\ \text{Cost}(1,4) + \text{Cost}(5,5) \end{Bmatrix} + 0.95$$

$k = 2, 3, 4, 5$

$$= \min \begin{Bmatrix} 1.7 \\ 0.7 \\ 0.85 \\ 1.2 \end{Bmatrix} + 0.95$$

$$= \min \begin{Bmatrix} 2.65 \\ 1.65 \\ 1.8 \\ 2.15 \end{Bmatrix} \rightarrow k = 3$$

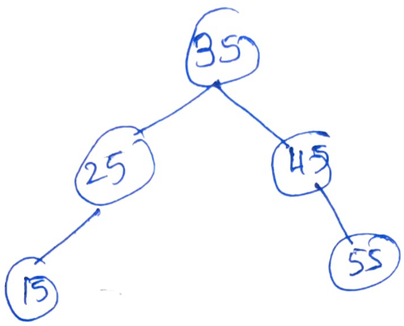
$j - i = 5$

$5 - 0 = 5$

$$cost(0,5) = \min \begin{Bmatrix} cost(0,0) + cost(1,5) \\ cost(0,1) + cost(2,5) \\ cost(0,2) + cost(3,5) \\ cost(0,3) + cost(4,5) \\ cost(0,4) + cost(5,5) \end{Bmatrix} + 1$$

$K = 1,2,3,4,5$

$$= \min \begin{Bmatrix} 1.65 \\ 1.75 \\ 0.8 \\ 1.0 \\ 1.65 \end{Bmatrix} + 1 = \min \begin{Bmatrix} 2.65 \\ 2.75 \\ \boxed{1.8} \\ 2 \\ 2.65 \end{Bmatrix} \rightarrow K = 3$$



$= (0.4) + (0.25 \times 2) + (0.15 \times 2) + (0.15 \times 3) + (0.05 \times 3)$

$= 0.4 + 0.5 + 0.3 + 0.45 + 0.15$

Min cost $= 1.8$

10. Given a distance matrix for 6 cities, Finding the shortest path using the nearest neighbor heuristic

A [0, 10, 8, 9, 7, 5]

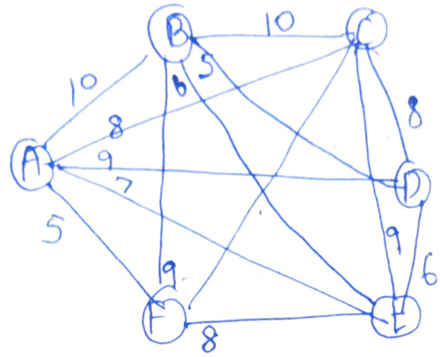
B [10, 0, 10, 5, 6, 9]

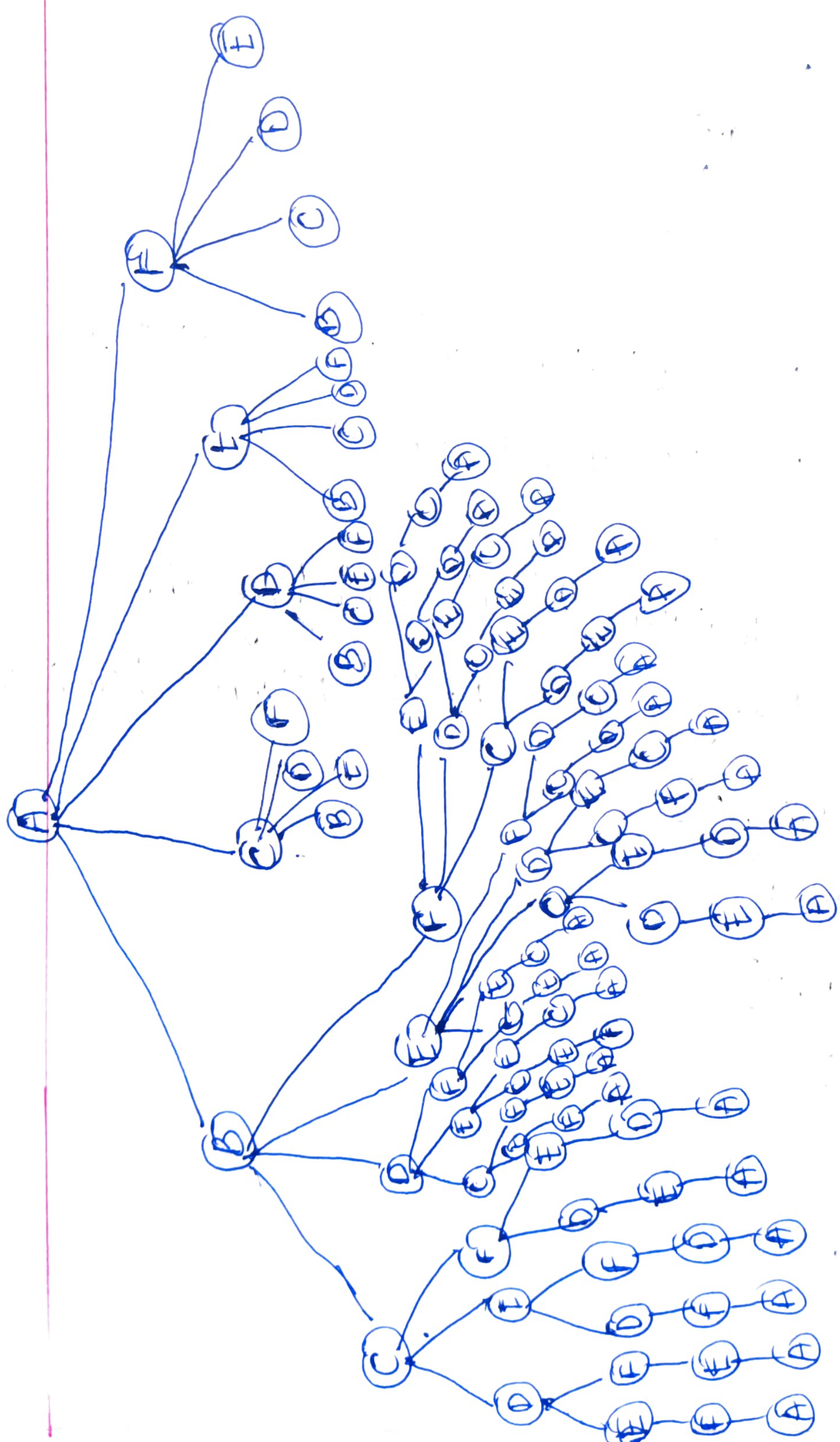
C [8, 10, 0, 8, 9, 7]

D [9, 5, 8, 0, 6, 5]

E [7, 6, 9, 6, 0, 8]

F [5, 9, 7, 5, 8, 0]





A — F — E — D — C — B — A = 47

11. Solve the Fractional Knapsack problem for a Knapsack with a capacity of 60 units and the following items.

Item 1 : weight = 20 value = 100
Item 2 : weight = 30 value = 120
Item 3 : weight = 10 value = 60

Calculate the maximum value that can be achieved and describe the fractions of items taken.

Item	weight	value (p)
1	20	100
2	30	120
3	10	60

$W = 60$ units

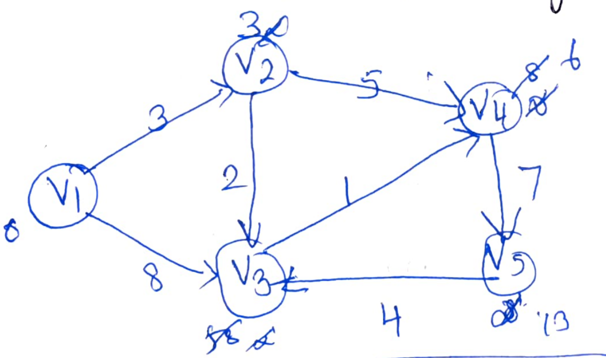
	0	10	20	30	40	50	60
0	0	0	0	0	0	0	0
1	0	0	100	100	100	100	100
2	0	0	100	120	120	220	220
3	0	10	100	120	180	220	280

Max value : 280.

12. Consider a directed grap with 5 vertices $V_1, V_2, V_3, V_4, V_5 V_1$, $V_2, V_3, V_4, V_5 V_1, V_2 V_3, V_4, V_5$ and the following edges with weights.

$V_1 \to V_2 V_1$ \ right arrow $V_2 V_1 \to V_2$ weight 3
$V_1 \to V_3 V_1$ right arrow $V_3 V_1 \to V_3$ weight 8
$V_2 \to V_3 V_2$ right arrow $V_3 V_2 \to V_3$ weight 2
$V_2 \to V_4 V_2$ right arrow $V_4 V_2 \to V_4$ weight 5
$V_3 \to V_4 V_3$ right arrow $V_4 V_3 \to V_4$ weight 1
$V_4 \to V_5 V_4$ right arrow $V_5 V_4 \to V_5$ weight 7
$V_5 \to V_3 V_5$ right arrow $V_3 V_5 \to V_3$ weight 4

Apply the Bellman. Ford algorithm starting from vertex $V_1 V_1 V_1$ to determine the shortest paths to all other vertices and check for negative weight cycles.



$V_1 \to V_2$ 3
$V_1 \to V_3$ 8
$V_2 \to V_3$ 2
$V_2 \to V_4$ – 5
$V_3 \to V_4$ – 1
$V_4 \to V_5$ 7
$V_5 \to V_3$ 4

V	V_1	V_2	V_3	V_4	V_5
d	0	α	α	α	α
P	–	–	–	–	–

①

V	V_1	V_2	V_3	V_4	V_5
d	0	3	8	α	α
P	–	V_1	V_1	V_2	V_4

(2)

V	V_1	V_2	V_3	V_4	V_5
d	0	3	5	8	∞
p	–	V_1	V_2	V_2	V_4

(3)

V	V_1	V_2	V_3	V_4	V_5
d	0	3	5	6	15
p	–	V_1	V_2	V_3	V_4

(4)

V	V_1	V_2	V_3	V_4	V_5
d	0	3	5	6	13
p	-	V_1	V_2	V_3	V_4

O/p → $V_1 \to V_2 \to V_3 \to V_4 \to V_5$

path	Shortest distance	Shortest path
$V_1 - V_2$	3	$V_1 - V_2$
$V_1 - V_3$	5	$V_1 - V_2 - V_3$
$V_1 - V_4$	6	$V_1 - V_2 - V_3 - V_4$
$V_1 - V_5$	13	$V_1 - V_2 - V_3 - V_4 - V_5$

13. Given two eight-sided dice, Compute the number of ways to achieve a sum of 10. then extend this to three dice and find the new number of ways to get the same sum.

$x + y + z = 10$

$x = 1$ $y + z = 9$

(1,1,8) (1, 2,7), (1,3,6), (1,4,5), (1,5,4), (1,6,3) (1,7,2) (1,8,1)

$x = 2$ $y + z = 8$

(2,1,7), (2,2,6), (2,3,5), (2,4,4), (2,5,3), (2,6,2), (2,7,1)

$x = 3$ $y + z = 7$

(3,1,6), (3,2,5), (3,3,4), (3,4,3), (3,4,3), (3,5,2) (3,6,1)

$x = 4$ $y + z = 6$

(4,1,5) (4,2,4), (4,3,3), (4,4,2), (4,5,1)

$x = 5$ $y + z = 5$

(5,1,4). (5,2,3), (5,3,2), (5,4,1)

$x = 6$ $y + z = 4$

(6,1,3), (6,2,2), (6,3,1)

$x = 7$ $y + z = 3$

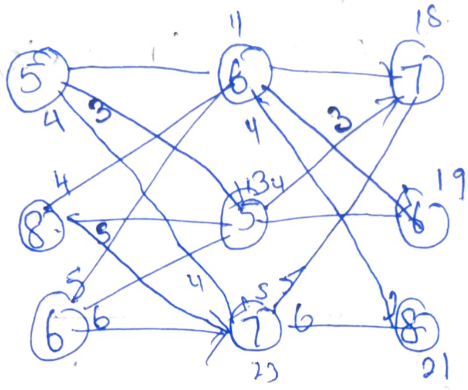
(7,1,2) (7,2,1)

$x = 8$ $y + z = 2$ (8,1,1)

Sum = $8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36$

the no. of ways to a sum of 10 = 36

14. Given station times for Line 1: [5, 6, 7], Line 2 [8, 5, 6] and Line 3 [6, 7, 8], and transfer times between lines: [3, 4], [4, 5] and [5, 6] calculate the minimum time required to complete the product assembly.



$F_1[j]$	5	11	18
$F_2[j]$	8	13	19
$F_3[j]$	6	13	21
	1	2	3

$l_1[j]$	1	1	1
$l_2[j]$	2	2	2
$l_3[j]$	3	3	3

$$F_1[j] = \min \{ (f_1(j-1) + a_1 j),\ f_2(j-1) + (t_2, j-1) + a_1 j),\ f_3(j-1) + (t_3, j-1) + (a_1 j) \}$$

$$= \min \{11, 18, 17\}$$

$$= 11$$

15. Given Keys [5, ~~15~~ 15, 25, 35, 45, 55] 1 {5, 15, 25, 35, 45, 55] {5, 15, 25, 45, 55) ~~with~~ with access probabilities {0.1, 0.05, 0.2, 0.25, 0.3, 0.1} {0.1, 0.05, 0.2, 0.25, 0.3, 0.1} {0.1, 0.05, 0.2, 0.25, 0.3, 0.1}. use dynamic programming to find the OBST. Show the steps of your Calculation and resulting cost.

{5, 15, 25, 35, 45, 55}
{0.1, 0.05, 0.2, 0.25, 0.3, 0.1}

j - i = 1
1 - 0 = 1
2 - 1 = 1
3 - 2 = 1
4 - 3 = 1
5 - 4 = 1
6 - 5 = 1

	0	1	2	3	4	5	6
0	0	0.1	0.2 (1)	(2) 0.55	(3) 1.05	(4) 1.7	4 2.05
1		0	0.05	(3) 0.3	(3,4) 0.8	(4) (1,4)	(4) 1.7
2			0	0.2	(4) 0.65	(4) 1.25	(4) 1.55
3				0	0.25	(5) 0.8	(5) 1
4					0	0.3	5 0.5
5						0	0.1
6							0

j - i = 2
2 - 0 = 2 (0,2) (1,2)
3 - 1 = 2 (1,3) (2,3)
4 - 2 = 2 (2,4) (3,4)
5 - 3 = 2 (3,5) (4,5)
6 - 4 = 2 (4,6) (5,6)

5: 1×0.1 = 0.1; 15: 2×0.05 = 0.1; = 0.15

15: 1×0.05 = 0.05; 5: 2×0.1 = 0.2; = 0.25

15: 1×0.05 = 0.05; 25: 2×0.2 = 0.4; = 0.45

25: 1×0.2 = 0.2; 15: 2×0.05 = 0.1; = 0.3

25: 1×0.2 = 0.2; 35: 2×0.25 = 0.5; = 0.7

35: 1×0.25 = 0.25; 25: 2×0.2 = 0.4; = 0.65

35: 1×0.25 = 0.25; 45: 2×0.3 = 0.6; = 0.85

45: 1×0.3 = 0.3; 35: 2×0.25 = 0.5; = 0.8

45: 1×0.3 = 0.3; 55: 2×0.1 = 0.2; = 0.5

55: 1×0.1 = 0.1; 45: 2×0.3 = 0.6; = 0.7

$$Cost(i,j) = \min\{cost(i,k-1) + cost(k,j)\} + w_i$$

$$Cost(0,3) = \min \begin{Bmatrix} Cost(0,0) + Cost(1,3) \\ Cost(0,1) + Cost(2,3) \\ Cost(0,2) + Cost(3,3) \end{Bmatrix} + 0.35$$

$k = 1,2,3$

$$= \min \begin{Bmatrix} 0.65 \\ 1.1 \\ 0.55 \end{Bmatrix} \rightarrow key = 3$$

$$Cost(1,4) = \min \begin{Bmatrix} Cost(1,1) + Cost(2,4) \\ Cost(1,2) + Cost(3,4) \\ Cost(1,3) + Cost(4,4) \end{Bmatrix} + 0.5$$

$k = 2,3,4$

$$= \min \begin{Bmatrix} 1.15 \\ 0.8 \\ 0.8 \end{Bmatrix} = 0.8$$

$$Cost(2,5) = \min \begin{Bmatrix} Cost(2,2) + Cost(3,5) \\ Cost(2,3) + Cost(4,5) \\ Cost(2,4) + Cost(5,5) \end{Bmatrix} + 0.75$$

$k = 3,4,5$

$$= \min \begin{Bmatrix} 1.55 \\ 1.25 \\ 1.4 \end{Bmatrix} \rightarrow = 1.25$$

$$Cost(3,6) = \min \begin{Bmatrix} Cost(3,3) + Cost(4,6) \\ Cost(3,4) + Cost(5,6) \\ Cost(3,5) + Cost(6,6) \end{Bmatrix} + 0.65$$

$k = 4,5,6$

$$= \min \begin{Bmatrix} 1.15 \\ 1 \\ 1.45 \end{Bmatrix} \rightarrow = 1$$

$j - i = 4$

$4 - 0 = 4 \quad (0,4)(1,4)$

$5 - 1 = 4 \quad (1,5)(2,5)$

$6 - 2 = 4 \quad (2,6)(3,6)$

$$Cost(0,4) = \min \begin{Bmatrix} Cost(0,0) + Cost(1,4) \\ Cost(0,1) + Cost(2,4) \\ Cost(0,2) + Cost(3,4) \\ Cost(0,3) + Cost(4,4) \end{Bmatrix} + 0.6$$

$k = 1,2,3,4$

$$= \min \begin{Bmatrix} 1.4 \\ 1.35 \\ 1.05 \\ 1.15 \end{Bmatrix} \rightarrow = 1.05$$

$$\text{Cost}(1,5) = \min \begin{cases} \text{Cost}(1,1) + \text{Cost}(2,5) \\ \text{Cost}(1,2) + \text{Cost}(3,5) \\ \text{Cost}(1,3) + \text{Cost}(4,5) \\ \text{Cost}(1,4) + \text{Cost}(5,5) \end{cases} + 0.8$$

2.3.4.5

$$= \min \begin{cases} 0 + 1.25 \\ 0.05 + 0.8 \\ 0.3 + 0.3 \\ 0.5 + 0 \end{cases} + 0.8 = \min \begin{cases} 2.05 \\ 1.65 \\ 1.4 \\ 1.6 \end{cases} = 1.4$$

$$\text{Cost}(2,6) = \min \begin{cases} \text{Cost}(2,2) + \text{Cost}(3,6) \\ \text{Cost}(2,3) + \text{Cost}(4,6) \\ \text{Cost}(2,4) + \text{Cost}(5,6) \\ \text{Cost}(2,5) + \text{Cost}(6,6) \end{cases} + 0.85$$

$k = 3,4,5,6$

$$= \min \begin{cases} 1.85 \\ 1.55 \\ 1.6 \\ 2.1 \end{cases} = 1.55$$

$j - i = 5$

$5 - 0 = 5$ (0,5), (1,5)

$6 - 1 = 5$ (1,6) (2,6)

$$\text{Cost}(0,5) = \min \begin{cases} \text{Cost}(0,0) + \text{Cost}(1,5) \\ \text{Cost}(0,1) + \text{Cost}(2,5) \\ \text{Cost}(0,2) + \text{Cost}(3,5) \\ \text{Cost}(0,3) + \text{Cost}(4,5) \\ \text{Cost}(0,4) + \text{Cost}(5,5) \end{cases} + 0.9$$

$k = 1,2,3,4,5$

$$= \min \begin{cases} 2.3 \\ 2.25 \\ 1.9 \\ 1.75 \\ 1.95 \end{cases} = 1.75$$

$$\text{Cost}(1,6) = \min \begin{cases} \text{Cost}(1,1) + \text{Cost}(2,6) \\ \text{Cost}(1,2) + \text{Cost}(3,6) \\ \text{Cost}(1,3) + \text{Cost}(4,6) \\ \text{Cost}(1,4) + \text{Cost}(5,6) \\ \text{Cost}(1,5) + \text{Cost}(6,6) \end{cases} + 0.9$$

$k = 2,3,4,5,6$

$$= \min \begin{cases} 2.45 \\ 2 \\ 1.7 \\ .8 \\ 2.3 \end{cases} = 1.7$$

$$\text{Cost}(0,6) = \min \begin{cases} \text{Cost}(0,0) + \text{Cost}(1,6) \\ \text{Cost}(0,1) + \text{Cost}(2,6) \\ \text{Cost}(0,2) + \text{Cost}(3,6) \\ \text{Cost}(0,3) + \text{Cost}(4,6) \\ \text{Cost}(0,4) + \text{Cost}(5,6) \\ \text{Cost}(0,5) + \text{Cost}(6,6) \end{cases} + 1$$

k 1,2,3,4,5,6

$$= \min \begin{cases} 2.7 \\ 2.65 \\ 2.2 \\ 2.05 \\ 2.15 \\ 2.75 \end{cases} = 2.05$$

16. Extend the following distance matrix to 7 cities and solve TSP

A [0, 12, 10, 19, 8, 16]

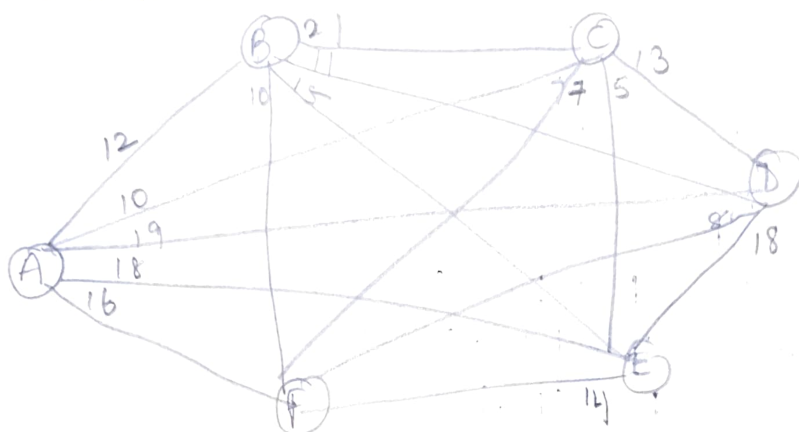
B [12, 0, 21, 11, 15, 10]

C [10, 21, 0, 13, 5, 7]

D [19, 11, 13, 0, 18, 8]

E [8, 15, 5, 18, 0, 14]

F [16, 10, 7, 8, 14, 0]



O/P

A – B – C – D – E – F = 83

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17. Given a Knapsack capacity of 70 units and the following items :

Item 1 weight = 25 value = 80

Item 2 weight = 35 value = 90

Item 3 weight = 45 value = 120

Item 4 weight = 30 value = 70

use dynamic programming to solve the 0/1 Knapsack problem.

i\W	0	25	35	45	30	70
0	0	0	0	0	0	0
1	0	80	80	80	80	80
2	0	80	90	90	90	90
3	0	80	90	120	120	90
4	0	80	90	80	120	150

Max value = 150.

18. For a graph

A → BA , W - 1
A → CA W - 4
B → CB · W - 3
B → DB W - 2
B → EB W - 2
D → BD - W - 1
D → CD , W - 5
E → DE W - 3

use Bellman-Ford and solve it.

A → B = 1
A → C 4
B → C = 3
B → D = 2
B → E = 2
D → B = 1
D → C = 5
E → D = 3

V	A	B	C	D	E
d	0	α	α	α	α
P	-	-	-	-	-

①

V	A	B	C	D	E
d	0	1	4	α	α
p	-	A	A	-	-

②

V	A	B	C	D	E
d	0	-1	4	α	1
p	-	A	A	-	B

③

V	A	B	C	D	E
d	0	-1	4	3	1
p	-	A	A	E	B

(4)

V	A	B	C	D	E
d	0	-1	4	3	1
p	-	A	A	E	B

path	distance	shortest path
A	0	A
B	-1	A -> B
C	4	A -> C
D	3	A -> E -> D
E	1	A -> B -> C

19. Find the expected value of the sum of outcomes when rolling 3 four sided dice. show your calculation and reasoning.

Sum - 3 (1+1+1)

sum

4 = $\frac{3}{64}$ (1+1+2, 1+2+1, 2+1+1)

5 = $\frac{6}{64}$ (1+1+3, 1+2+2, 1+3+1, 2+1+2, 2+2+1, 3+1+1)

6 = $\frac{10}{64}$ (1+2+3, 1+3+2, 2+1+3, 2+2+2, 2+3+1, 3+1+2, (3+2+1, 1+4+1, 2+2+2, 2+3+1)

7 = 7 {1+3+3, 2+2+3, 2+3+2, 3+1+3, 3+2+2, 3+3+1, 1+4+2, 2+3+2, 2+4+1, 3+2+2, 3+3+1, 4+1+2)

8 = 12/64　11 = 3/64

9 = 10/64　12 = 1/64

10 = 6/64

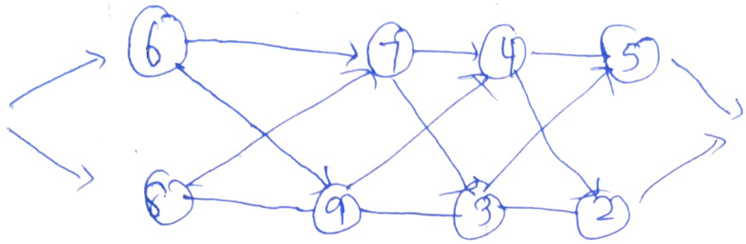
Σ (sum * probability)

$= \left(3 \times \frac{1}{64}\right) + \left(4 \times \frac{3}{64}\right) + \left(5 \times \frac{6}{64}\right) + \left(6 \times \frac{10}{64}\right) + \left(7 \times \frac{12}{64}\right) + 8\left(\frac{12}{64}\right) + \left(9 \times \frac{10}{64}\right)$

$+ \left(10 \times \frac{6}{64}\right) + \left(11 \times \frac{3}{64}\right) + \left(12 \times \frac{1}{64}\right)$

$= \frac{480}{64} = 7.5$

20. Calculate min. time for Line 1 : [6, 7, 4, 5].
Line 2 [8, 9, 3, 2] with transfer lines [4, 5, 6] 1 to 2
and [6, 5, 4] 2 to 1.



	1	2	3	4
$F_1[j]$	6	13	17	12
$F_2[j]$	8	7	20	22

	1	2	3	4
$L_1[j]$	1	1	1	1
$L_2[j]$	2	2	2	2

21. Keys {10, 20, 30] probabilities {0.2, 0.5, 0.3} do OBST.

$K = \{10, 20, 30\}$

$P = \{0.2, 0.5, 0.3\}$

$j - i = 3$

$3 - 0 = (0, 3)$

$\text{cost}(0,3) = \min \begin{cases} 2.1 \\ 1.5 \\ 1.1 \end{cases}$

	0	1	2	3
0	0	0.2	0.7	1.1
1		0	0.5	1.1
2			0	0.3
3				0

22. using 5 cities

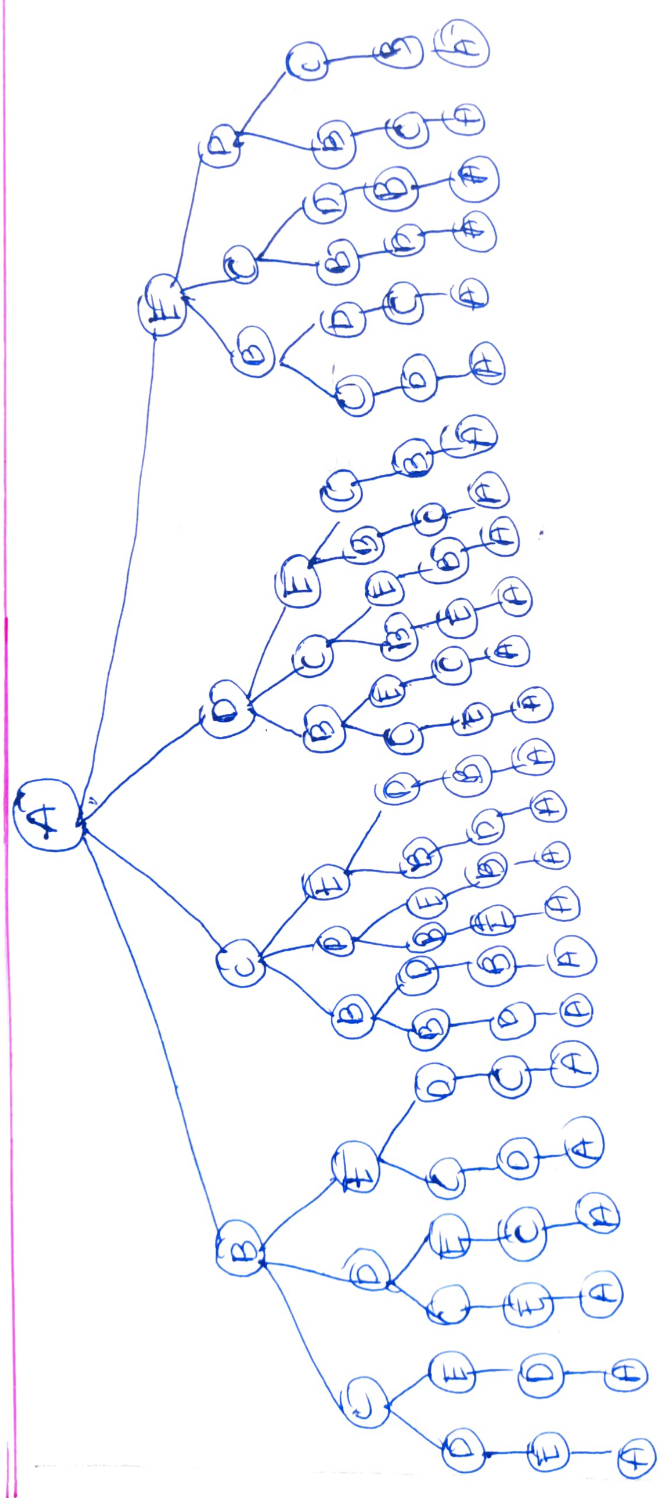
A [0, 14, 4, 10, 20]

B [14, 0, 7, 8, 7]

C [4, 7, 0, 12, 6]

D [10, 8, 12, 0, 15]

E [20, 7, 6, 15, 0]



23 Knapsack 0/1   weight = 50 units

I - 1   W = 10   V = 50
I - 2   W = 20   V = 70
I - 3   W = 30   V = 90
I - 4   W = 25   V = 60
I - 5   W = 15   V = 40.

	0	10	20	30	25	15	50
0	0	0	0	0	0	0	0
1	0	50	50	50	50	50	50
2	0	50	70	70	70	70	70
3	0	50	70	90	90	90	160
4	0	50	70	90	90	90	160
5	0	50	70	90	90	90	160

24. Bellman Ford

1 → 2 1   w = 4
1 → 3 1   w = 5
2 → 3 2   w = -2
3 → 4 3   w = 3
4 → 2 4   w = -10

1 → 2 - 4
1 → 3 - 5
2 → 3 - 3
4 → 2 - 10

V	1	2	3	4
d	0	α	α	α
p	-	-	-	-

v	1	2	3	4
d	0	4	5	∞
p	-	1	1	-

v	1	2	3	4
d	0	4	2	∞
p	-	1	2	-

v	1	2	3	4
d	0	4	2	5
p	-	1	2	3

v	D	path
1.	0	1
2	4	$1 \to 2$
3	2	$1 \to 2 \to 3$
4	5	$1 \to 2 \to 3 \to 4$

25. Roll six sided dice determine the no. of ways to get a sum of 18, ensuring that atleast one die show a 6.

$$x + x^2 + x^3 + x^4 + x^5 + x^6$$

$$x(1+x+x^2+x^3+x^4+x^5)$$

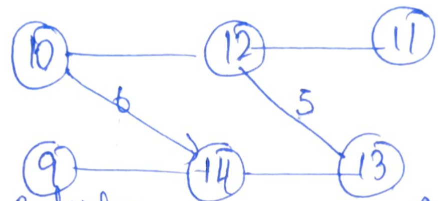
$$= \frac{x(1+x^6)}{1-x}$$

For six dice

$$\left(\frac{x(1-x^6)}{1-x}\right)^6 = x^6(1-x^6)^6(1-x)^{-6} = (x^9)$$

$$= 340$$

26. Given Line 1 [10, 12, 11] Line 2: [9, 14, 13].
Transfer lines [6, 5], by 2 units



Before Reduction

L_1	6	5
L_2	30	30

After Reduction

L_1	4	5
L_2	28	27

27. For Keys (8, 12, 16, 20, 24) with access possibilities {0.2, 0.05, 0.4, 0.25, 0.1} Determine OBST using DP.

(8, 12, 16, 20, 24)

(0.2, 0.05, 0.4, 0.25, 0.1)

j - i = 0

j - i = 1

0 - i = 2

2 - 0 = [0, 2]

3 - 1 = [1, 3]

4 - 2 = [2, 4]

5 - 3 = [3, 5]

	0	1	2	3	4	5
0	0	0.2	0.3	0.7	1.45	1.8
1		0	0.05	0.5	1.	1.3
2			0	0.4	0.9	1.2
3				0	0.25	0.05
4					0	0.1
5						0

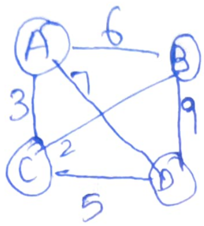
$$\text{Cost} = \min\left\{\begin{array}{l} \text{cost}(1,1) + \text{cost}(2,5) \\ \text{cost}(1,2) + \text{cost}(3,5) \\ \text{cost}(1,3) + \text{cost}(4,5) \\ \text{cost}(1,4) + \text{cost}(5,5) \end{array}\right\} + 0.8$$

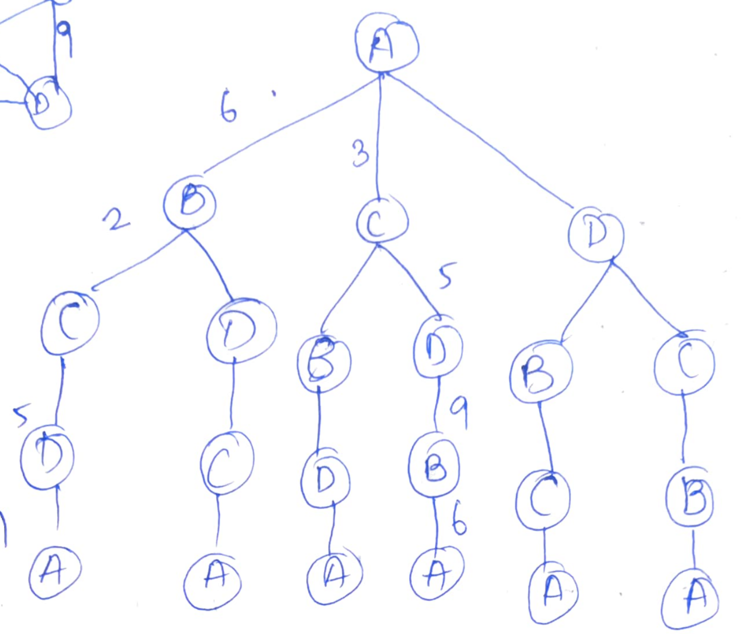
$$= \min\left\{\begin{array}{l} 2 \\ 1.3 \\ 1.4 \\ 1.8 \end{array}\right\} = 1.3$$

28. TSP 4 cities

A [0, 6, 3, 7]   C [3, 2, 0, 5]

B [6, 0, 2, 9]   D [7, 9, 5, 0]





dp

A - B - C - D - A - 20
A - D - C - B - A - 20   } min optimal path.

29. Knapsack 0/1 50 units

I_1 - w=10 v=60
I_2 - w=20 v=100
I_3 - w=30 v=120
I_4 - w=40 v=200.

v\w	0	10	20	30	40	50
0	0	0	0	0	0	0
1	0	60	60	60	60	60
2	0	60	100	160	160	160
3	0	60	100	120	180	180
4	0	60	100	120	200	260.

30. Bellman - Ford

A → BA , ω = 6

A → DA , ω = 7

B → CB , ω = 5

B → EB , ω = 4

B → DB , ω = 8

C → BC , ω = -2

D → CD , ω = -3

D → ED , ω = -9

E → FE , ω = 7

F → CF , ω = 2

V	A	B	C	D	E	F
d	0	∝	∝	∝	∝	∝
P	-	-	-	-	-	-

1

V	A	B	C	D	E	F
d	0	6	4	7	2	9
P	-	A	D	A	B	E

2

V	A	B	C	D	E	F
d	0	2	4	7	2	9
P	-	C	D	A	B	E

3

V	A	B	C	D	E	F
d	0	2	4	7	2	9
P	-	C	D	A	B	E

4

V	A	B	C	D	E	F
d	0	2	4	7	2	9
P	-	C	D	A	B	E

5

V	A	B	C	D	E	F
d	0	2	4	7	2	9
P	-	C	D	A	B	E

vertex	distance	Path
A	0	A
B	2	A-D-C-B
C	4	A-D-C
D	7	A-D
E	2	A-D-C-B-E
F	9	A-D-C-B-E-F