Problem 1

Figure 1 illustrates (in cross-section) a spherical cryogenic experiment that is placed at the center of a cubical enclosure (a box).

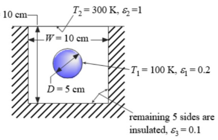

Figure 1: Spherical cryogenic experiment in a cubical enclosure.

The spherical experiment (surface 1) has diameter D = 5.0 cm and emissivity $\varepsilon_1 = 0.2$. The temperature of the experiment is maintained at T1 = 100 K. Each face of the cubical enclosure is W=10cm x W=10 cm. The top surface of the enclosure (surface 2) is maintained at T2 = 300 K and is black. The other five sides (surface 3) are insulated externally and have emissivity $\varepsilon_3 = 0.1$

a) If the view factor between the experiment and the top surface $F_{1\text{-}2} = 1/6$, what is the view factor between the top surface and the experiment ($F_{2\text{-}1}$)?

b) Draw and clearly label a resistance network that represents the radiation heat transfer problem.

c) Calculate the values of all of the resistances in your diagram from (b).

d) What is the net rate of heat transfer to the cryogenic experiment?

e) How would your answer to (d) change if the emissivity of the experiment were reduced (would the heat transfer to the cryogenic experiment increase, decrease, or stay the same)?

f) How would your answer to (d) change if the emissivity of the insulated sides were reduced (would the heat transfer to the cryogenic experiment increase, decrease, or stay the same)?

g) use the Integral Equation to repeat part **d)** of calculating the net rate of heat transfer to the cryogenic experiment

$A_1 = \pi D^2 = 0.007854\ m^2$

$A_2 = W^2 = 0.01\ m^2$

$A_3 = 5W^2 = 0.05\ m^2$

$\varepsilon = \frac{E}{E_b}$

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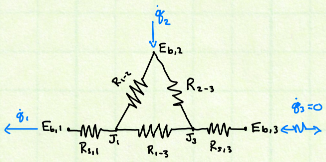
a) For the enclosed spherical experiment, $F_{1-2} + F_{1-3} = 1.0 \Rightarrow F_{1-3} = 1 - F_{1-2} = \frac{5}{6}$

$\& \ F_{1-2} A_1 = F_{2-1} A_2$

$$F_{2-1} = F_{1-2}\frac{A_1}{A_2} = \frac{1}{6}\cdot\frac{0.007854}{0.01} = 0.131$$

b)



c) View Factors:

$F_{1-1} = 0 = F_{2-2}$, $F_{1-2} = \frac{1}{6}$, $F_{1-3} = \frac{5}{6}$

$F_{3-1} = F_{1-3}\frac{A_1}{A_3} = \frac{5}{6}\,\frac{0.007854}{0.05} = 0.131 = F_{2-1}$

$F_{2-3} = 1 - F_{2-1} = 1 - 0.131 = 0.869$

$F_{3-2} = F_{2-3}\frac{A_2}{A_3} = \frac{0.869}{5} = 0.1738$

$F_{3-3} = 1 - F_{3-2} - F_{3-1} = 1 - 0.1738 - 0.131 = 0.6952$

F	1	2	3
1	0	$\frac{1}{6}$	$\frac{5}{6}$
2	0.131	0	0.869
3	0.131	0.1738	0.6952
i			

(column index j)

Resistances: $R_{s,i} = \frac{1-\varepsilon_i}{A_i\varepsilon_i}$, $R_{i,j} = \frac{1}{A_i F_{i-j}} = \frac{1}{A_j F_{j-i}}$

$R_{s,1} = \frac{1 - 0.2}{0.007854 m^2 \cdot 0.2} = 509.3\ /m^2$

$R_{s,3} = \frac{1 - 0.1}{0.05 m^2 \cdot 0.1} = 180\ /m^2$

$R_{1-3} = \frac{1}{0.05 m^2 \cdot 0.131} = 152.67\ /m^2$

$R_{1-2} = \frac{1}{0.01 m^2 \cdot 0.131} = 763.36\ /m^2$

$R_{2-3} = \frac{1}{0.01 m^2 \cdot 0.869} = 115.1\ /m^2$

d) $E_{b,i} = \sigma T_i^4$, $\sigma = 5.67\times10^{-8}\ W/m^2K^4$

$E_{b,1} = 5.67\times10^{-8} W/m^2K^4\ (100\ K)^4 = 5.67\ W/m^2$

$E_{b,2} = 5.67\times10^{-8} W/m^2K^4\ (300\ K)^4 = 459.27\ W/m^2$

$$\frac{E_{b,2} - E_{b,3}}{R_{2-3} + R_{s,3}} = \frac{E_{b,3} - E_{b,1}}{R_{s,3} + R_{1-3} + R_{s,1}}$$

$$E_{b,3}\left(\frac{1}{R_{s,3} + R_{1-3} + R_{s,1}} + \frac{1}{R_{2-3} + R_{s,3}}\right) = \frac{E_{b,2}}{R_{2-3} + R_{s,3}} + \frac{E_{b,1}}{R_{s,3} + R_{1-3} + R_{s,1}}$$

$E_{b,3} = 341.55\ W/m^2$

$$\dot{q}_1 = \frac{E_{b,2} - E_{b,1}}{R_{s,1} + R_{1-2}} + \frac{E_{b,3} - E_{b,1}}{R_{s,1} + R_{1-3} + R_{s,3}} = \frac{459.27 W/m^2 - 5.67 W/m^2}{509.3/m^2 + 763.36/m^2} + \frac{341.55 W/m^2 - 5.67 W/m^2}{509.3/m^2 + 152.67/m^2 + 180/m^2}$$

$\underline{\underline{\dot{q}_1 = 0.755\ W}}$

e) Try $\varepsilon_{1,new} = \varepsilon_1/2 = 0.1$

$R_{s,1} = \frac{1-0.1}{0.007854 m^2 \cdot 0.1} = 1145.91/m^2$, F_1, R_{1-2}, and R_{1-3} are unchanged

$E_{b,3} = 383.8\ W/m^2$

$\dot{q}_1 = 0.493\ W$

decreasing the emissivity of the experiment decreases the heat transfered to it

f) Try $\varepsilon_{3,new} = \varepsilon_3/2 = 0.05$

$R_{s,3} = 380/m^2$

$E_{b,3} = 290.9\ W/m^2$

$\dot{q}_1 = 0.63\ W$

decreasing the emissivity of the insulated walls also decreases the heat transferred to the experiment

g) $$\frac{\dot{q}''_T}{\varepsilon_i} - \sum_{j=1}^{N} \left(\frac{1}{\varepsilon_j} - 1\right) F_{ij}\, \dot{q}''_j = E_{bi} - \sum_{j=1}^{N} F_{ij} E_{bj}$$

$\dot{q}''_1 = E_{b1} - (E_{b2} F_{1-2} + E_{b3} F_{1-3})$

$\dot{q}''_2 = E_{b2} - (E_{b1} F_{2-1} + E_{b3} F_{2-3})$

$\dot{q}''_3 = E_{b3} - (E_{b1} F_{3-1} + E_{b2} F_{3-2})$ $\Rightarrow E_{b3} = 0 + (5.67 \cdot 0.131 + 459.27 \cdot 0.1738)\ W/m^2 = 80.56\ W/m^2$

$\dot{q}''_1 = 5.67\ W/m^2 - (459.27 \cdot \frac{1}{6} + 80.56 \cdot \frac{5}{6})\ W/m^2 = -138\ W/m^2$

$\dot{q}_1 = \dot{q}''_1 A_1 = -138\ W/m^2 \cdot 0.007854\ m^2 = -1.08\ W$

$\dot{q}_1 = 1.08 W$

Slightly larger than the result from the resistance diagram

Problem 2

A satellite orbits the earth at a height (above the surface of the earth) of H_{orbit} = 3.5E5 m. The diameter of the earth is D_{earth} = 1.29E7 m and the temperature of the earth is T_{earth} = 300 K. The distance between the earth and the sun is approximately R = 1.497E11 m. The sun has diameter D_{sun} = 1.39E9 m and the surface temperature of the sun is approximately T_{sun} = 5780 K. The earth and the sun can be considered black. The satellite is spherical with diameter D_{sat} = 1 m. The emissivity of the satellite surface is ε_{sat} = 0.5. There is q_{sat} = 100 W of power dissipation within the satellite that must be rejected from its surface through radiation.

Note: Using the dimensions above, the view factor between the satellite (surface 1) and the earth (surface 2) is $F_{1\text{-}2}$ = 0.3417; the view factor between the satellite (surface 1) and the sun (surface 3) is $F_{1\text{-}3}$ = 0.000005389. **The outer space has temperature of 3 K. Your interest is the radiation exchange between the satellite and the others only.** → assuming no radiation exchange w/ space

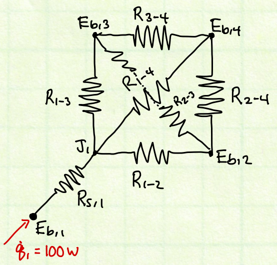
a) Draw the resistance network that represents this problem and estimate the steady state temperature of the satellite when it is on the day-side of the earth.

b) Estimate the steady state temperature of the satellite when it is on the night-side of the earth (no solar radiation).

c) Plot the steady-state day-side temperature as a function of the satellite emissivity. Explain the shape of your plot.

d) Overlay on your plot from (c) the steady-state day-side temperature as a function of the satellite emissivity if the satellite power dissipation were q_{sat} = 0 W. Explain the shape of your plot.

a)



Surfaces:

1 : satellite, T_1 = ? , ε_1 = 0.5 , A_1 = 3.14 m²

2 : earth , T_2 = 300K , ε_2 = 1.0 , A_2 = 5.228×10¹⁴ m²

3 : sun , T_3 = 5780K , ε_3 = 1.0 , A_3 = 6.07×10¹⁸ m²

~~4 : space , T_4 = 3 K , ε_4 = 1.0 (assumed)~~ →

View Factors:

F_{1-2} = 0.3417

a) F_{1-3} = 0.000005389 | b) F_{1-3} = 0

~~F_{1-4} = 1 - F_{1-2} - F_{1-3} = 0.658295~~ | ~~F_{1-4} = 0.6583~~ → not considered

$E_b = \sigma T^4$, $E_{b,2}$ = 459.27 W/m² , $E_{b,3}$ = 63,284,071.5 W/m²

$R_{i-j} = \frac{1}{A_i F_{ij}}$, R_{1-2} = 2.926 /m² , R_{1-3} = 185,563.2 /m² , $R_{s,1} = \frac{1-\varepsilon_1}{A_1 \varepsilon_1}$

$$\dot{q}_1 = \frac{E_{b,1} - E_{b,2}}{R_{s,1} + R_{1-2}} - \frac{E_{b,3} - E_{b,1}}{R_{s,1} + R_{1-3}} = E_{b,1}\left(\frac{1}{R_{s,1}+R_{1-2}} + \frac{1}{R_{s,1}+R_{1-3}}\right) - \frac{E_{b,2}}{R_{s,1}+R_{1-2}} - \frac{E_{b,3}}{R_{s,1}+R_{1-3}}$$

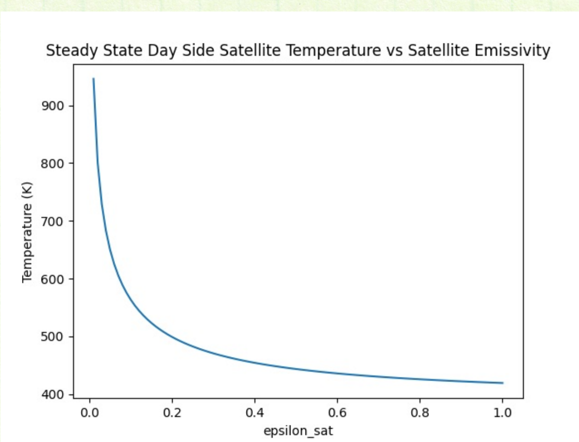
$$E_{b,1} = \left(\dot{q}_1 + \frac{E_{b,2}}{R_{s,1}+R_{1-2}} + \beta\frac{E_{b,3}}{R_{s,1}+R_{1-3}}\right) \Big/ \left(\frac{1}{R_{s,1}+R_{1-2}} + \beta\frac{1}{R_{s,1}+R_{1-3}}\right) , \quad \begin{array}{l}\beta = 1 \text{ if day-side} \\ \beta = 0 \text{ if night-side}\end{array}$$

$$T_1 = \sqrt[4]{E_{b,1}/\sigma}$$

$$\underline{\underline{T_{1,day} = 443.37\ K}}$$

b) $\underline{\underline{T_{1,night} = 350.11\ K}}$

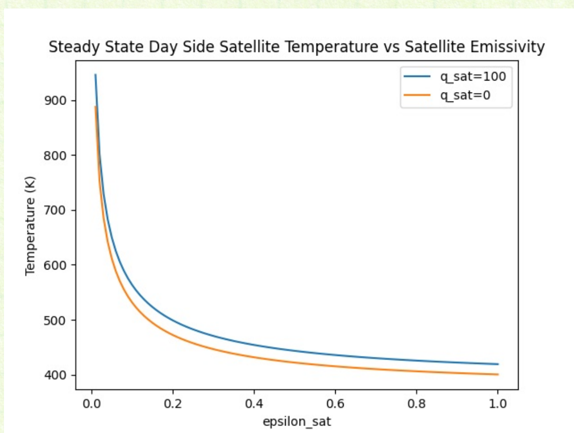
c)



$R_{s,1}$ is inversely proportional to ε

as $R_{s,1}$ increases, so does T_1 steady-state

d)



the steady state temperature is lower if the satellite doesn't have to reject heat from its electronics