1) Hydrogen at 400K and 0.1ATM is flowing over a flat plate at 2 m/s. How large is the displacement and momentum thickness at 150 cm from the leading edge? [10]

$\mu = 1.0784 \times 10^{-5}\ kg/m\cdot s$, $\rho_{atm} = 0.0606\ kg/m^3$

$u_\infty = 2\ m/s$, $x = 1.5\ m$

1. Calculate density and kinematic viscosity

$P_{atm}\,\rho = P\,\rho_{atm}$

$\rho = \rho_{atm}\dfrac{P}{P_{atm}} = 0.1\,\rho_{atm} = 0.1 \cdot 0.0606\ kg/m^3$

$\rho = 0.00606\ kg/m^3$

$\nu = \dfrac{\mu}{\rho} = \dfrac{1.0784 \times 10^{-5}\ kg/m\cdot s}{0.00606\ kg/m^3} = 0.00178\ m^2/s$

2. Calculate Reynolds number

$Re_x = \dfrac{\rho u_\infty x}{\mu} = \dfrac{u_\infty x}{\nu}$

$Re_x = \dfrac{2\ m/s \cdot 1.5\ m}{0.00178\ m^2/s} = 1685.4$

3. Calculate displacement and momentum thickness

eqs. 9-17 & 9-18 : displacement and momentum thickness for laminar, constant property, constant u_∞ flow over an impermeable wall

9-17) $\delta_1 = 1.73\sqrt{\nu x/u_\infty}$ (displacement thickness)

9-18) $\delta_2 = 0.664\sqrt{\nu x/u_\infty}$ (momentum thickness)

based off

$\delta_1 = \int_0^\infty \left(1 - \frac{u\rho}{u_\infty \rho_\infty}\right) dy = \int_0^\infty (1 - \zeta')\,dy$

$\delta_2 = \int_0^\infty \frac{\rho u}{\rho_\infty u_\infty}\left(1 - \frac{u\rho}{u_\infty \rho_\infty}\right) = \int_0^\infty \zeta'(1 - \zeta')\,dy$

with ζ, ζ', ζ'' from table 9-1

$$\underline{\underline{\delta_1 = 1.73\sqrt{\frac{0.00178\ m^2/s \cdot 1.5\ m}{2\ m/s}} = 0.063\ m}} \longleftarrow \delta_1$$

$$\underline{\underline{\delta_2 = 0.664\sqrt{\frac{0.00178\ m^2/s \cdot 1.5\ m}{2\ m/s}} = 0.024\ m}} \longleftarrow \delta_2$$

2) Find an expression for the drag coefficient vs Re for a flat plate of length L and width W for uniform inflow and 0º angle of attack and laminar flow. Use the plate area for the reference area and assume all drag comes from shear [10]

$A = L \cdot W$

assuming constant velocity, u_∞, $Re_L = \frac{\rho u_\infty L}{\mu}$

$F_D = C_D A$ → assumes $\alpha = 0$

for laminar flow,

$$C_f = \frac{2\tau_s}{\rho u_\infty^2} = \frac{0.664}{Re_x^{1/2}}$$

$$C_{f_m} = \frac{1.328}{Re_x^{1/2}} = 2C_f = 2\left(\frac{2\tau_s}{\rho u_\infty^2}\right)$$

$$\frac{1.328}{Re_x^{1/2}} = \frac{4\tau_s}{\rho u_\infty^2}$$

$$\tau_{s,m} = 0.332\frac{\rho u_\infty^2}{Re_L^{1/2}}$$

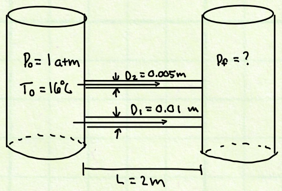
assuming all drag comes from shear at the surface

$F_D = \tau_{s,m} A \Rightarrow C_D = \tau_{s,m}$

$$C_D = 0.332\frac{\rho u_\infty^2}{Re_L^{1/2}}$$

C_D

3) [20]
Two air tanks are connected by two parallel circular tubes, one having an inside diameter of 1 cm and the other an inside diameter of 0.5 cm. The tubes are 2 m long. One of the tanks has a higher pressure than the other, and air flows through the two tubes at a combined rate of 0.00013 kg/s. The air is initially at 1 atm pressure and 16°C. Assuming that fluid properties remain constant and that the entrance and exit pressure losses are negligible, calculate the pressure differences between the two tanks.



Properties of air at 1 atm 16°C:

$\rho = 1.22\ kg/m^3$

$\mu = 1.793 \times 10^{-5}\ \frac{N \cdot s}{m^2}$, $\nu = 1.47 \times 10^{-5}\ m^2/s$

$Pr = 0.709$

assume constant properties

$\dot{m}_{tot} = 0.00013\ kg/s$

$\rho V = \frac{4\dot{m}}{\pi D^2}$

$\dot{m} = \rho \frac{\pi D^2}{4} V = \rho V D \left(\frac{\pi D}{4}\right)$

$Re_D = \frac{\rho V D}{\mu} = \frac{\dot{m}}{\mu}\frac{4}{\pi D}$

$Re_x = \frac{\rho V x}{\mu} = \frac{4\dot{m}x}{\pi D^2 \mu}$

if all flow was through the small tube (leads to highest Re)

$Re_D = \frac{4 \cdot 0.00013 kg/s}{\pi \cdot 1.793 \times 10^{-5} kg/m \cdot s \ 0.005 m} = 1846$ => the flow is laminar

$x_{fd} = D\frac{Re}{20} = 0.923\ m$ -> worst case , the flow is fully developed

Equations for friction factor and pressure drop

$C_f = \frac{16}{Re}$

$\Delta P = 2\bar{C}_{f_{app}} \rho V^2 \frac{x}{D}$

using C_f for $\bar{C}_{f_{app}}$ because the flow is fully developed

$\Delta P = 32 \frac{\mu}{D \rho V} \rho V^2 \frac{x}{D}$

$\Delta P = 32 \frac{\mu V x}{D^2} = 32\frac{\mu V L}{D^2}$

Relate the flow of the two tubes

$\dot{m}_1 + \dot{m}_2 = \dot{m}_{tot}$, $\rho_1 = \rho_2 = \rho$

$\rho(V_1 A_{c_1} + V_2 A_{c_2}) = \rho \dot{\forall}$, $\dot{\forall}$ = volume flow rate

$\Delta P_1 = \Delta P_2$

$$32\frac{\mu V_1 L}{D_1^2} = 32\frac{\mu V_2 L}{D_2^2}$$

$$V_1 = V_2 \frac{D_1^2}{D_2^2}$$

$$\left(V_2 \frac{D_1^2}{D_2^2}\right)\left(\frac{\pi D_1^2}{4}\right) + V_2\frac{\pi D_2^2}{4} = \frac{\dot{m}_{tot}}{\rho}$$

$$V_2 = \frac{\dot{m}_{tot}}{\rho}\frac{4}{\pi}\left(\frac{D_1^4}{D_2^2} + D_2^2\right)^{-1} = \frac{0.00013 kg/s}{1.22 kg/m^3}\frac{4}{\pi}\left(\frac{(0.01m)^4}{(0.005m)^2} + (0.005m)^2\right)^{-1}$$

$V_2 = 0.319\ m/s$ => $V_1 = 1.277\ m/s$

$$\Delta P = 32\frac{1.793 \times 10^{-5} kg/m \cdot s \cdot 0.319 m/s \cdot 2m}{(0.005m)^2}$$

$\underline{\underline{\Delta P = 14.65\ Pa}}$ ← ΔP

4) [20]

Consider fully developed, constant-property laminar flow between parallel planes with constant heat rate per unit of length and a fully developed temperature profile. Suppose heat is transferred to the fluid on one side and *out* of the fluid on the other at the *same rate*. What is the Nusselt number on each side of the passage? Sketch the temperature profile. Suppose the fluid is an oil for which the viscosity varies greatly with temperature, but all the other properties are relatively unaffected by temperature. Is the velocity profile affected? Is the temperature profile affected? Is the Nusselt number affected? Explain.

TEXSTAN can be used to confirm the Nusselt number result of this problem. Let the Reynolds number be 1000, select a Prandtl number, and pick fluid properties that are appropriate to the Prandtl number. You can choose geometrical dimensions and a mass flow rate for the channel to provide the required Reynolds number and a channel length of 20–40 hydraulic diameters. Use constant fluid properties and do not consider viscous dissipation for this part of the problem. The thermal boundary condition is constant heat flux, and you can choose an arbitrary value for one surface and the value for the other surface will be the same magnitude with opposite sign. This is a thermally fully developed problem, so the initial conditions will be a velocity profile that is hydrodynamically fully developed and a temperature profile that is thermally fully developed. Because this problem has asymmetrical thermal boundary conditions, choose the option in TEXSTAN that permits calculation from surface to surface.

$Nu = \frac{hD}{k}$

for parallel plates with constant heat rate:

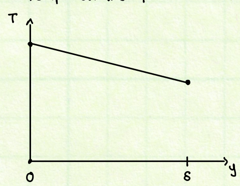
$Nu = 8.235$

assuming heat flows in from the bottom and out the top,

$Nu_{top} = -8.235$ ⟵ Nu

$\underline{\underline{Nu_{bottom} = 8.235}}$

temperature profiles: plate spacing = δ



$q''_{in} = -q''_{out} = h(T_0 - T_\delta)$

at each boundary $\frac{\partial T}{\partial y} = \frac{T_0 - T_\delta}{\delta}$

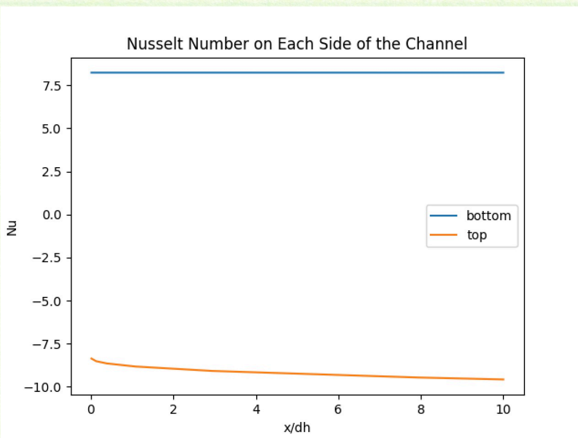
Varying viscosity

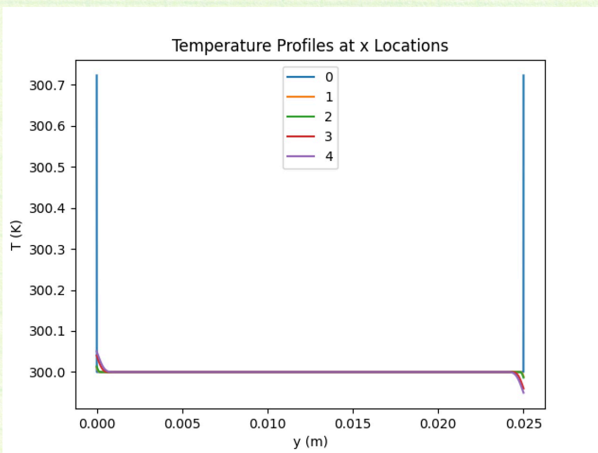
$0 = -\frac{dP}{dx} + \mu \nabla^2 u$

Velocity depends on viscosity so the velocity profile will change based on the temperature profile

due to the velocity profile changing based on the temperature, the temperature profile will also change because velocity impacts heat transfer, which changes the temperature profile

the velocity/viscosity changes will also impact h, which changes Nu





the Nusselt numbers match what is expected

fully developed temperature profile could not be set because the wall temperature is symmetric they would eventually develop to a straight diagonal line as shown above

5) [20]

TEXSTAN *analysis of the laminar thermal boundary layer over a flat plate with constant surface temperature and zero pressure gradient*: Choose a starting x-Reynolds number of about 1000 (a momentum Re of about 20) and pick fluid properties that are appropriate to fluids with a Prandtl number = 0.7, 1.0, and 5.0, evaluated at the free stream temperature. Use constant fluid properties and do not consider viscous dissipation. The geometrical dimensions of the plate are 1 m wide (a unit width) by 0.2 m long in the flow direction, corresponding to an ending Re_x of about 2×10^5 (a momentum Re of about 300). Let the velocity boundary condition at the free stream be 15 m/s and let the energy boundary conditions be a free stream temperature of 300 K and a constant surface temperature of 295 K. The initial velocity and temperature profiles appropriate to the starting x-Reynolds number (Blasius similarity profiles) can be supplied by using the *kstart*=4 choice in TEXSTAN.

For each Prandtl number calculate the boundary-layer flow and evaluate the concept of boundary-layer similarity by comparing nondimensional temperature profiles at several x locations to themselves for independence of x. Compare the Nusselt number results based on x-Reynolds number with Eq. (10-10). Convert the Nusselt number to Stanton number and compare the Stanton number results based on x-Reynolds number with Eq. (10-13) and based on enthalpy-thickness Reynolds number with Eq. (10-16). Calculate the Stanton number distribution using energy integral Eq. (5-24) and compare with the TEXSTAN calculations. Feel free to investigate any other attribute of the boundary-layer flow.

10-10: $Nu_x = 0.332\, Pr^{1/3} Re_x^{1/2}$

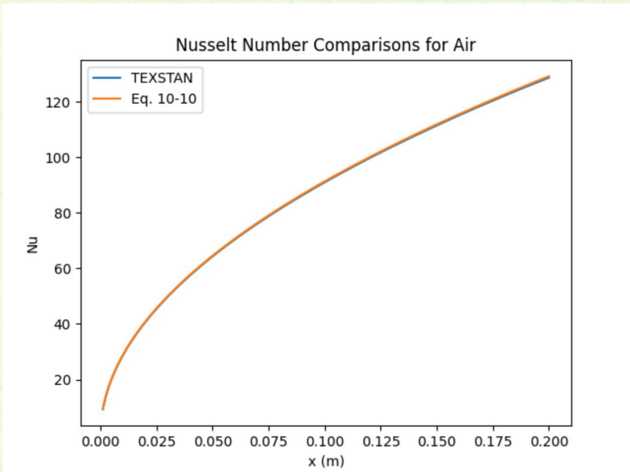
10-13: $StPr^{2/3} = 0.332 Re_x^{-1/2}$

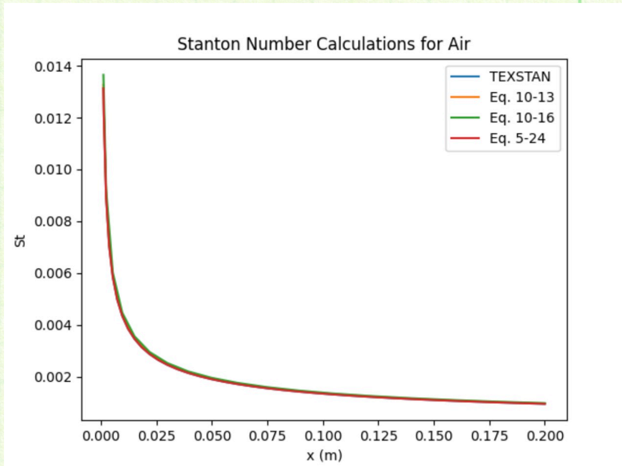
10-16: $StPr^{4/3} = 0.2204/Re_{\Delta_2}$

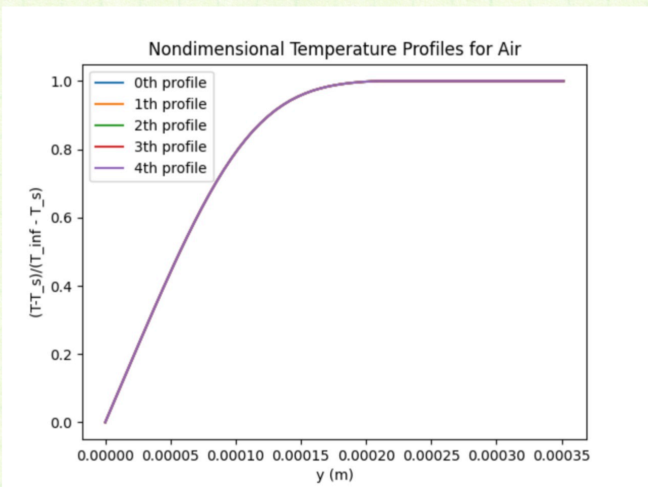
5-24: $St = \dfrac{d\Delta_2}{dx}$

5-24) $St = \dfrac{d\Delta_2}{dx} = \dfrac{\dot{q}_0''}{\rho u_\infty c\,(T_0 - T_\infty)}$

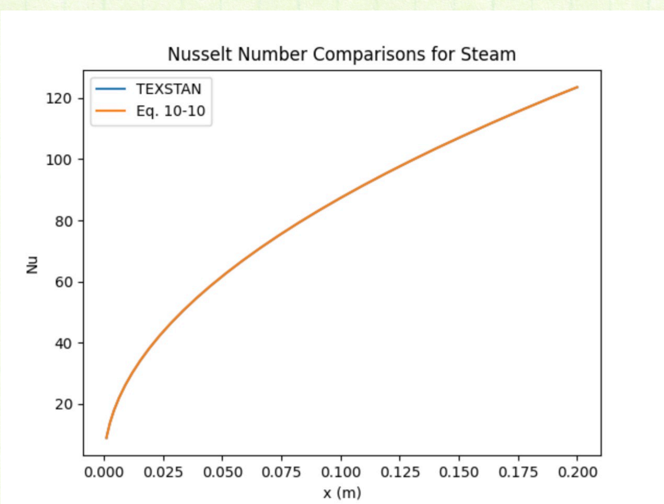
keeping the plate length the same for all fluids (Re start and end will be different)

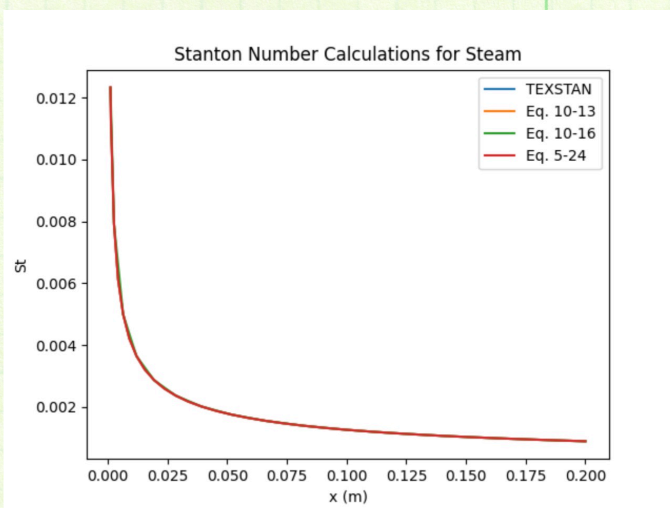


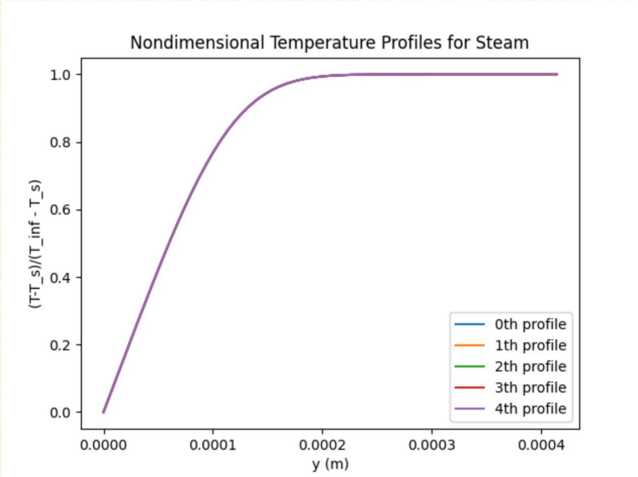


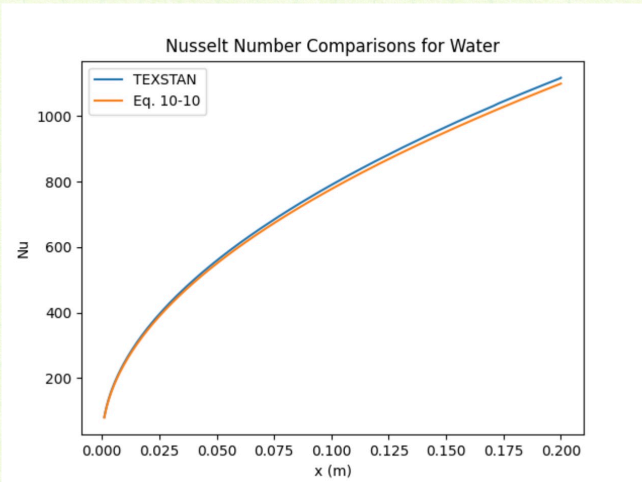


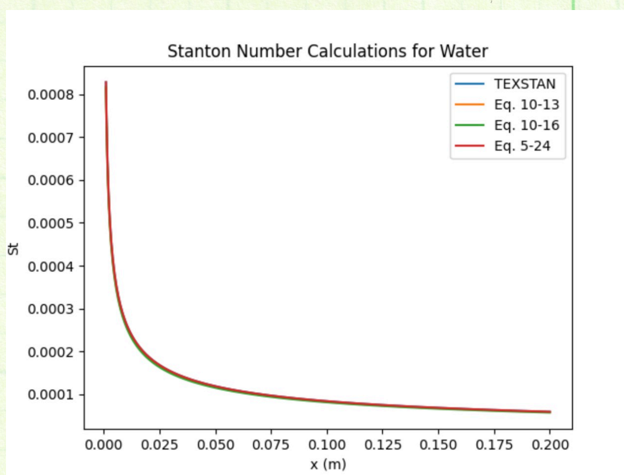
the curves are nearly indistinguishable

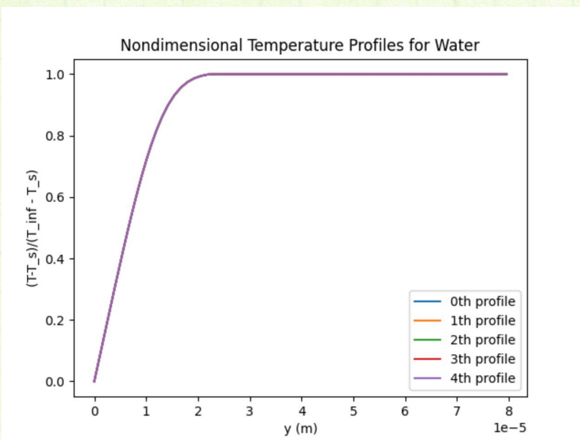




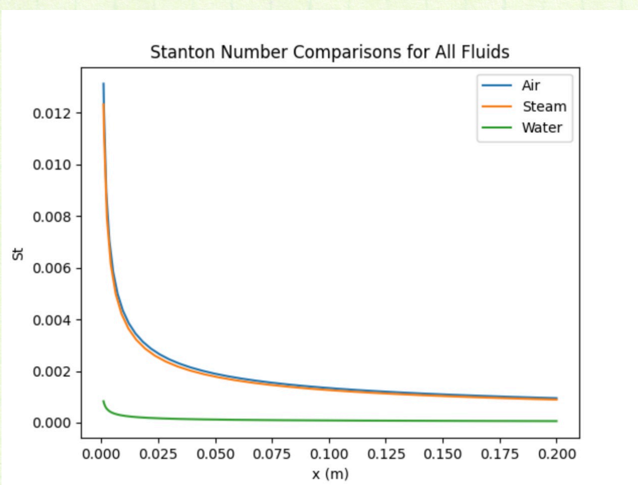


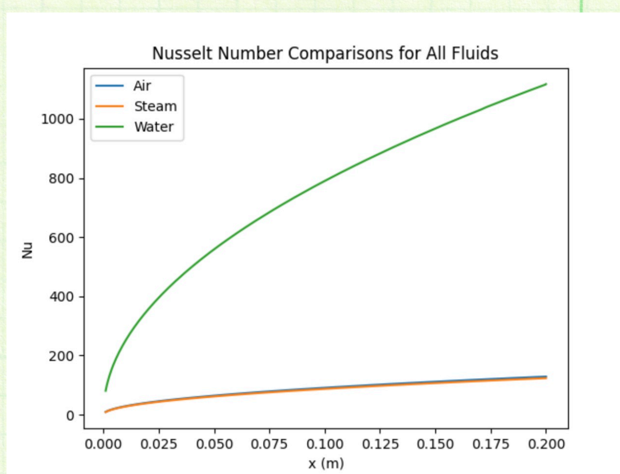






all equations have similar results for all fluids





Steam and air are very similar, but water has significantly higher Nu

[20]

TEXSTAN ***analysis of the turbulent momentum boundary layer over a flat plate with zero pressure gradient***: This problem is essentially a repeat of the previous problem, but choosing other turbulence models available in TEXSTAN. There exists a 1-equation model (*ktmu*=11) and four 2-equation (k-ε) models (*ktmu*=21,22,23,24). The initial velocity profile appropriate to the starting x-Reynolds number (a fully turbulent boundary layer profile), along with turbulence profiles for k (and ε) can be supplied by using the *kstart*=3 choice in TEXSTAN. Choose an initial free-stream turbulence of 2 percent. Note that by setting the corresponding initial free-stream dissipation (for the two-equation model) equal to zero, TEXSTAN will compute an appropriate value.

Calculate the boundary-layer flow and compare the friction coefficient results based on x Reynolds number and momentum thickness Reynolds number with the results in the text, Eqs. (11-20) and (11-23). Calculate the friction coefficient distribution using momentum integral Eq. (5-11) and compare with the TEXSTAN calculations. Feel free to investigate any other attribute of the boundary-layer flow. For example, you can calculate the mixing-length model results from the previous problem and compare in a manner similar to Fig. 11-7. Likewise you can investigate the law of the wall, comparing to Fig. 11-6.

Plots compare all equations for each model

11.20: $c_f/2 = 0.0125\,\mathrm{Re}_{\delta_2}^{-1/4}$

11.23: $c_f/2 = 0.0287\,\mathrm{Re}_x^{-0.2}$

5.11: $$c_f = \frac{\tau_0}{\frac{1}{2}\rho_\infty u_\infty^2} = 2\frac{d\delta_2}{dx}$$

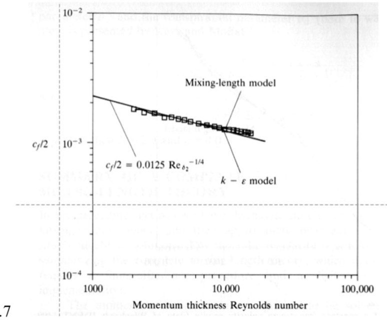
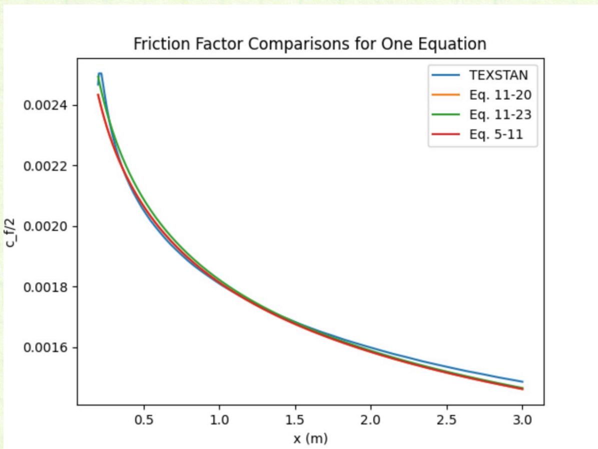
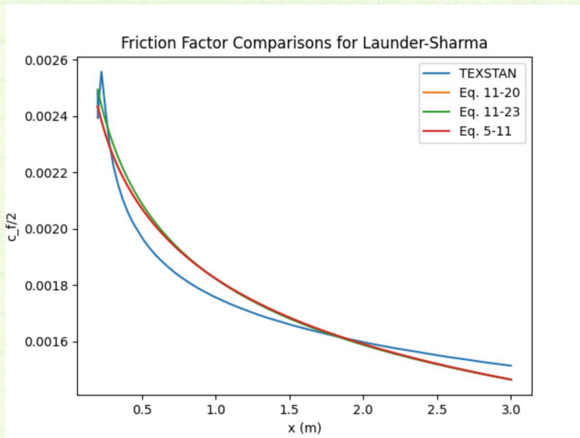
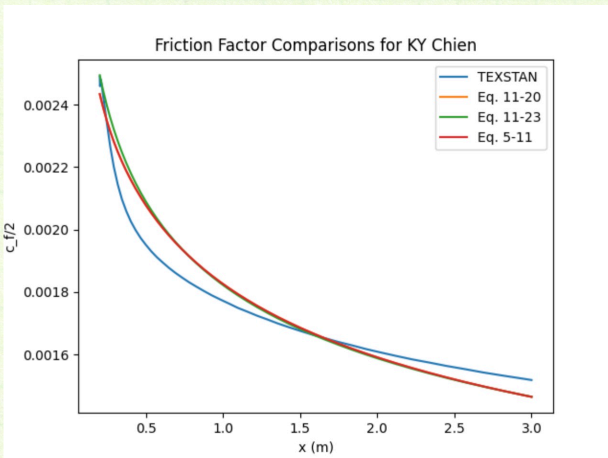
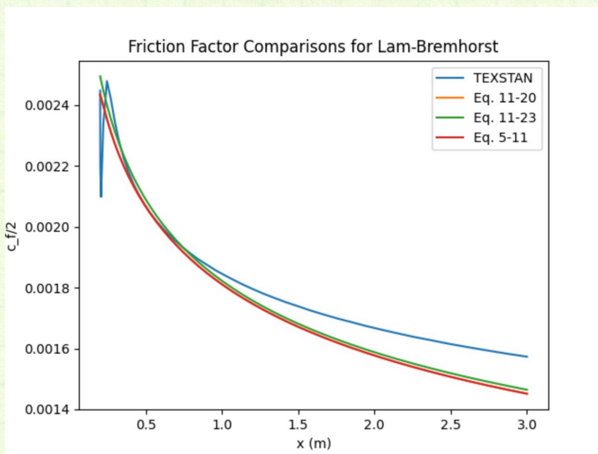


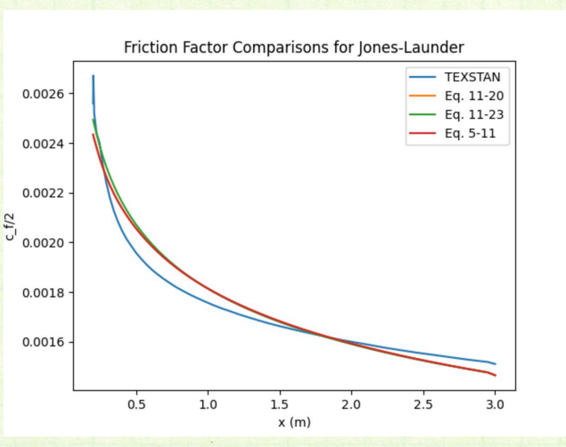
Fig 11.7

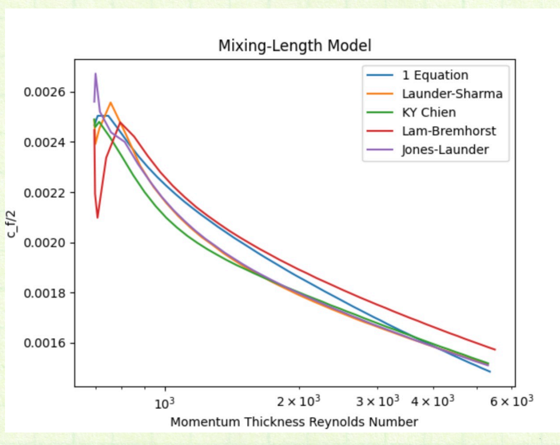












compare to fig. 11-7