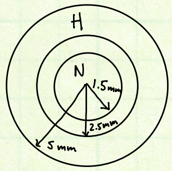
Problem: Axial Conduction in a Concentric Pipe Heat Exchanger

Nitrogen gas at $T_{n,in}$ = 300 K and atmospheric pressure is cooled by helium gas at $T_{h,in}$ = 240 K and atmospheric pressure in a concentric tube heat exchanger that utilizes N_{tube}= 30 tubes in parallel. The mass flow rate of the nitrogen is m_n = 9.5 g/s and the mass flow rate of helium is m_h = 1.5 g/s. The specific heat for nitrogen and helium are 1038 J/kg-K and 5193 J/kg-K, respectively. The heat exchanger is made of copper with pipe wall thickness of δ = 1 mm and length L = 0.3 m. The inner pipe has an inner diameter of D_{in} = 0.3 cm. The outer pipe (which is well-insulated on its outside surface) has an inner diameter of D_{out} = 1.0 cm. The helium flows through the annulus and the nitrogen through the center tube in a counter-flow configuration. (Assume: the average heat transfer coefficient is 211 W/m^2-K for nitrogen and 231 W/m^2-K for helium).

assume : $k_w = 397\ W/m\cdot K$



$L = 0.3\ m$, $N_{tubes} = 30$, parallel

$\dot{m} = \dot{m}/N_{tube}$

$\dot{C}_h = \dot{m}_n c_{p_n}/N_{tube} = 0.329\ \frac{W}{K}$, $\dot{C}_c = \dot{m}_n c_{p_n}/N_{tube} = 0.26\ W/K$, $\dot{C}_{min} = 0.26\frac{W}{K}$, $C_R = 0.79$

using ε-NTU method

a) Estimate the outlet temperatures of the helium and nitrogen and the heat transfer rate assuming there is **no axial conduction**.

$$\dot{q}_{max} = \dot{C}_{min}(T_{h,in} - T_{c,in}) = 0.26\ W/K\,(300\,K - 240\,K) = 15.6\ W$$

$$R_w = \frac{\ln(r_2/r_1)}{2\pi L k_w} = \frac{\ln(2.5/1.5)}{2\pi(0.3m)397\ W/m\cdot K} = 0.000683\ \frac{K}{W}$$

$$UA = \left[\frac{1}{(hA)_c} + R_w + \frac{1}{(hA)_n}\right]^{-1} = \left[\frac{1}{231 W/m^2K\cdot\pi\cdot 0.005m\cdot 0.3m} + 0.000683\frac{K}{W} + \frac{1}{211 W/m^2K\cdot\pi\cdot 0.003m\cdot 0.3m}\right]^{-1} = 0.385\ W/K$$

$$NTU = \frac{UA}{\dot{C}_{min}} = \frac{0.385\frac{W}{K}}{0.26\frac{W}{K}} = 1.481$$

$$\varepsilon = \frac{1 - \exp[-NTU(1-C_R)]}{1 - C_R\exp[-NTU(1-C_R)]} = \frac{1-\exp(-1.481(1-0.79))}{1-0.79\exp(-1.481(1-0.79))} = 0.635$$

$$\underline{\dot{q} = \varepsilon\,\dot{q}_{max} = 9.9\ W/\text{tube}}$$

$$\dot{q} = \dot{C}_h(T_{h,i} - T_{h,o}) = \dot{C}_c(T_{c,o} - T_{c,i})$$

$$T_{h,o} = T_{h,i} - \dot{q}/\dot{C}_h = 300\,K - 9.9\,W/0.329\,W/K = \underline{269.9\ K} \longleftarrow T_{h,o}$$

$$T_{c,o} = T_{c,i} + \dot{q}/\dot{C}_c = 240\,K + 9.9\,W/0.26\,W/K = \underline{\underline{278.1\ K}} \longleftarrow T_{c,o}$$

$$\dot{q}_{tot} = \dot{q}\cdot N_{tube} = 9.9\,W\cdot 30 = \underline{297\ W} \longleftarrow \dot{q}_{tot}$$

b) Calculate the axial conduction parameter and determine whether axial conduction is a concern for this heat exchanger. If so, use both the **Low-λ model** and the **Temperature Jump model** to include axial conduction in the estimates of the heat transfer rate and outlet temperatures (Note: you can ignore the axial conduction within the outer pipe).

$$R_{AC} = \frac{L}{k A_c} = \frac{0.3\,m}{397 W/m\cdot K\cdot\pi((0.0025m)^2 - (0.0015m))^2} = 60.13\ \frac{K}{W}$$

$$\lambda = \frac{1}{R_{AC}\dot{C}_{min}} = \frac{1}{60.13 K/W\cdot 0.26 W/K} = 0.064$$

there will be axial conduction

low λ:

$$\dot{q}_{in} = \dot{q}_{NAC} - \dot{q}_{AC} = \varepsilon\dot{q}_{max} - \frac{1}{R_{AC}}(T_{h,i} - T_{c,i}) = 9.9\,W - \frac{1}{60.13 K/W}(60\,K) = \underline{8.9W}$$

$$T_{h,o} = T_{h,i} - \dot{q}/\dot{C}_h = 300\,K - 8.9W/0.329W/K = \underline{\underline{272.95\ K}} \longleftarrow T_{h,o}$$

$$T_{c,o} = T_{c,i} + \dot{q}/\dot{C}_c = 240\,K + 8.9W/0.26W/K = \underline{\underline{274.23\ K}} \longleftarrow T_{c,o}$$

$$\dot{q}_{tot} = \dot{q}_{in}\cdot N_{tube} = 8.9W\cdot 30 = \underline{\underline{267\ W}} \longleftarrow \dot{q}_{tot}$$

$\left.\begin{matrix} T_{h,o} \\ T_{c,o} \\ \dot{q}_{tot}\end{matrix}\right\}$ low λ

Temperature Jump:

$$T^*_{h,in} = \frac{R_{AC}\dot{C}_c\dot{C}_h T_{h,in} + \dot{C}_c T_{c,in} + \dot{C}_h T_{h,in}}{R_{AC}\dot{C}_c\dot{C}_h + \dot{C}_c + \dot{C}_h} = \frac{60.13\frac{K}{W}\cdot 0.26\frac{W}{K}\cdot 0.329\frac{W}{K}\cdot 300K + 0.26\frac{W}{K}\cdot 240\,K + 0.329\frac{W}{K}\cdot 300K}{60.13\frac{K}{W}\cdot 0.26\frac{W}{K}\cdot 0.329\frac{W}{K} + 0.26\frac{W}{K} + 0.329\frac{W}{K}} = 297.28\ K$$

$$T^*_{c,in} = T^*_{h,in} - R_{AC}\dot{C}_h(T_{h,in} - T^*_{h,in}) = 297.28\,K - 60.13\frac{K}{W}\cdot 0.329\frac{W}{K}(300K - 297.28K) = 243.44\ K$$

$$\dot{q}_{TJ} = \frac{(T^*_{h,in} - T^*_{c,in})}{R_{AC}} = \frac{(297.28\,K - 243.44\,K)}{60.13\frac{K}{W}} = \underline{0.9\ W}$$

$$\dot{q} = \dot{q}_{NAC} - \dot{q}_{TJ} = 9W$$

$$T_{h,o} = T_{h,i} - \dot{q}/\dot{C}_h = 300\,K - 9.0W/0.329W/K = \underline{\underline{272.64\ K}} \longleftarrow T_{h,o}$$

$$T_{c,o} = T_{c,i} + \dot{q}/\dot{C}_c = 240\,K + 9.0\,W/0.26W/K = \underline{\underline{274.61\ K}} \longleftarrow T_{c,o}$$

$$\dot{q}_{tot} = \dot{q}_{in}\cdot N_{tube} = 9.0\,W\cdot 30 = \underline{\underline{270\ W}} \longleftarrow \dot{q}_{tot}$$

$\left.\begin{matrix} T_{h,o} \\ T_{c,o} \\ \dot{q}_{tot}\end{matrix}\right\}$ TJ