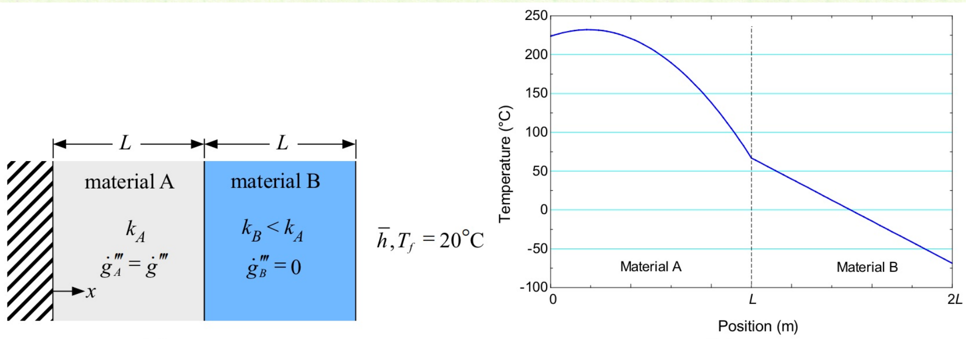
1.3-3 (1-7 in text) One of the engineers that you supervise has been asked to simulate the heat transfer problem shown in Figure P1.3-3(a). This is a 1-D, plane wall problem (i.e., the temperature varies only in the *x*-direction and the area for conduction is constant with *x*). Material A (from $0 < x < L$) has conductivity k_A and experiences a uniform rate of volumetric thermal energy generation, $\dot{g}'''$. The left side of material A (at $x = 0$) is completely insulated. Material B (from $L < x < 2L$) has *lower* conductivity, $k_B < k_A$. The right side of material B (at $x = 2L$) experiences convection with fluid at room temperature (20°C). Based on the facts above, critically examine the solution that has been provided to you by the engineer and is shown in Figure P1.3-3(b). There should be a few characteristics of the solution that do not agree with your knowledge of heat transfer; list as many of these characteristics as you can identify and provide a clear reason why you think the engineer's solution must be wrong.



(a) (b)

Figure P1.3-3: (a) Heat transfer problem and (b) "solution" provided by the engineer.

1) if the left boundary is insulated, the temperature gradient should be zero $\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$, in the provided solution $\frac{\partial T}{\partial x} > 0$ => there would be heat flow through the left boundary

2) at $x = L$, $k_A \left.\frac{\partial T}{\partial x}\right|_A = k_B \left.\frac{\partial T}{\partial x}\right|_B \Rightarrow \left.\frac{\partial T}{\partial x}\right|_A = \frac{k_B}{k_A} \left.\frac{\partial T}{\partial x}\right|_B$, $k_B < k_A \Rightarrow \frac{k_B}{k_A} < 1 \Rightarrow \left.\frac{\partial T}{\partial x}\right|_A < \left.\frac{\partial T}{\partial x}\right|_B$ in the provided solution, $\left.\frac{\partial T}{\partial x}\right|_A > \left.\frac{\partial T}{\partial x}\right|_B$

3) at $x = 2L$, the boundary condition is convection heat transfer to a 20°C environment, but $T(2L) \approx -70$°C and $\frac{\partial T}{\partial x} < 0$, which indicates heat is flowing out of material B to the environment. However, $T_\infty > T(2L)$ so by Newton's law of cooling, heat should be flowing from the environment to material B

A flow of liquid passes through a test section consisting of an L = 3 inch section of pipe with inner and outer radii, r_{in} = 0.75 inch and r_{out} = 1.0 inch, respectively. The test section is uniformly heated by electrical dissipation at a rate $\dot{g}''' = 1\text{x}10^7$ W/m^3 and has conductivity k = 10 W/m-K. The pipe is surrounded with insulation that is th_{ins} = 0.25 inch thick and has conductivity k_{ins} = 1.5 W/m-K. The external surface of the insulation experiences convection with air at T_∞ = 20°C. The heat transfer coefficient on the external surface is $\bar{h}_{out}$ = 20 W/m^2-K. A thermocouple is embedded at the center of the pipe wall. By measuring the temperature of the thermocouple, it is possible to infer the mass flow rate of fluid because the heat transfer coefficient on the inner surface of the pipe ($\bar{h}_{in}$) is strongly related to mass flow rate ($\dot{m}$). Testing has shown that the heat transfer coefficient and mass flow rate are related to according to:

$$\bar{h}_{in} = C\left(\frac{\dot{m}}{1[\text{kg/s}]}\right)^{0.8}$$

where C= 2500 W/m^2-K. Under nominal conditions, the mass flow rate through the meter is $\dot{m}$ = 0.75 kg/s and the fluid temperature is T_f = 18°C. Assume that the ends of the test section are insulated so that the problem is 1-D. Neglect radiation and assume that the problem is steady-state.

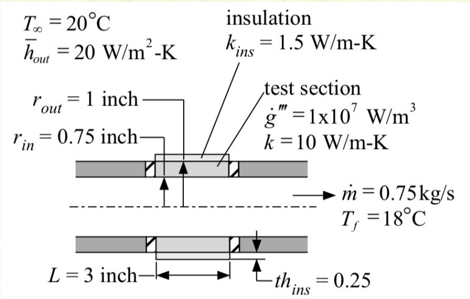


Figure P1.3-9: A simple mass flow meter.

a.) Develop an analytical model in EES that can predict the temperature distribution in the test section. Plot the temperature as a function of radial position for the nominal conditions.

$L = 0.0762\,m$, $r_{out} = 0.0254\,m$, $r_{in} = 0.01905\,m$, $t_{ins} = 0.00635\,m$, $r_{ins} = r_{out} + t_{ins}$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) + \frac{\dot{g}}{k} = 0$$

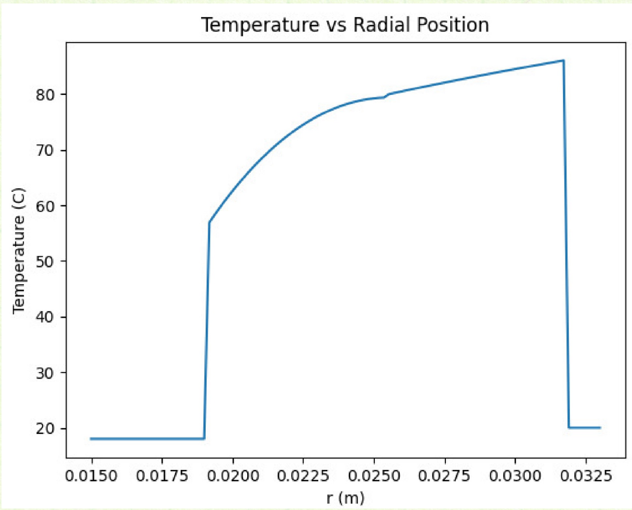
assuming symmetry : $\frac{\partial T}{\partial \phi} = \frac{\partial T}{\partial z} = 0$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{k} = 0$$

$$T(r) = -\frac{\dot{g}''' r^2}{4k} + C_1 \ln(r) + C_2$$

$$\frac{dT}{dr} = -\frac{\dot{g}''' r}{2k} + \frac{C_1}{r}$$

$$\dot{q} = -k\,2\pi r L\,\frac{dT}{dr}$$



BCs: $r = r_{in}$: convection to fluid = conduction in wall

$r = r_{out}$: convection in test section = conduction in insulatation

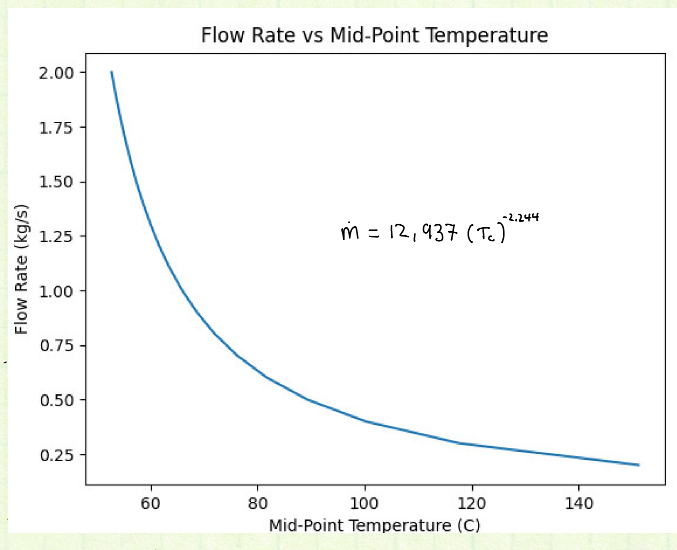
$r = r_{out} + t_{ins}$: conduction in insulation = convection to air

code below

$$T(r) \begin{cases} r < r_{in}: T = T_f \\ r_{in} \le r \le r_{out}: T = -\frac{\dot{g}''' r^2}{4k} + C_1 \ln r + C_2, & C_1 = 326.8,\ C_2 = 1441 \\ r_{out} < r \le r_{ins}: T = C_3 \ln r + C_4, & C_3 = 27.89,\ C_4 = 182.6 \\ r > r_{ins}: T = T_\infty \end{cases}$$

the apparant discontinuity at $r = r_{out}$ is due to rounding of C_1, C_2, C_3, and C_4 values

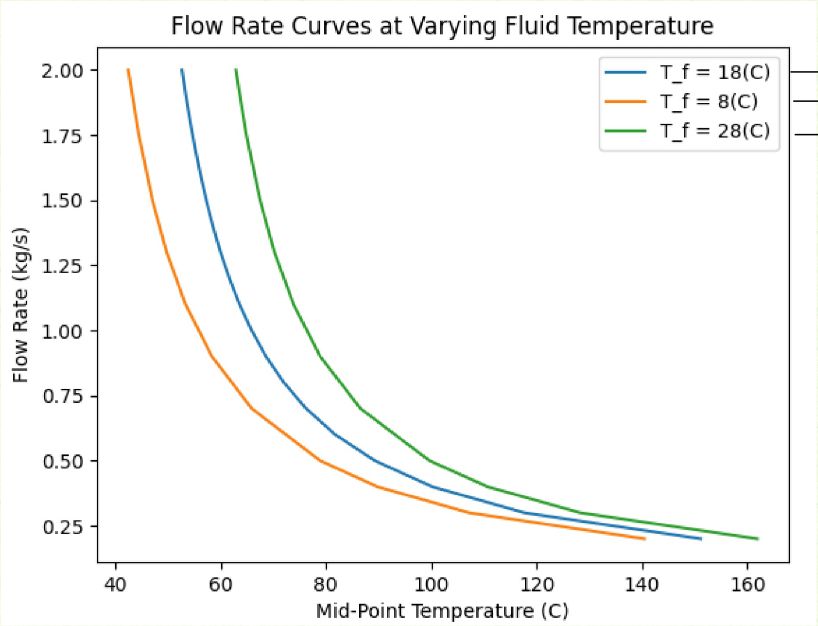
b.) Using your model, develop a calibration curve for the meter; that is, prepare a plot of the mass flow rate as a function of the measured temperature at the mid-point of the pipe. The range of the instrument is 0.2 kg/s to 2.0 kg/s.



$\dot{m} = 12,937\,(T_c)^{-2.244}$

a power fit was used as it provided the closest approximation

c.) Overlay on your plot from (b) the mass flow rate as a function of the measured temperature for $T_f = 8°C$ and $T_f = 28°C$. Is your meter robust to changes in T_f?
In order to improve the meters ability to operate over a range of fluid temperature, a temperature sensor is installed in the fluid in order to measure T_f during operation.



$\dot{m} = 12{,}937\,(T_c)^{-2.244}$

$\dot{m} = 2{,}500.7\,(T_c)^{-1.932}$

$\dot{m} = 43{,}164\,(T_c)^{-2.446}$

At low mass flow rates, the meter is robust, with fluid temperature having a minor effect on mid-point temperature. As the flow rate increases, this becomes less true. At the highest rated flow rate, 2.0 kg/s, the temperature varies by over 20°C

EES Code and Results

Equations Window

```
"Given"
r_in = 0.75[in]*convert(in, m)
r_out = 1.0[in]*convert(in, m)
L = 3[in]*convert(in, m)
t_ins = 0.25[in]*convert(in, m)
k_ins = 1.5[W/(m*C)]
k = 10[W/(m*C)]
g_dot = 1e7[W/m^3]
T_inf = 20[C]
h_out = 20[W/((m^2)*C)]
r_c = (r_in + r_out) / 2
r_ins = r_out + t_ins

"Nominal Conditions"
m_dot = 0.75 [kg/s]
C = 2500 [W/((m^2)*C)]
T_f = 18 [C]
h_in = C * (m_dot/1 [kg/s])^0.8

"Temperature at r_in"
T_r_in = -g_dot * (r_in)^2 / (4 * k) + c_1 * ln(r_in) + c_2

"Temperature at the center"
T_r_c = -g_dot * (r_c)^2 / (4 * k) + c_1 * ln(r_c) + c_2

"Temperature at r_out"
T_r_out = -g_dot * (r_out)^2 / (4 * k) + c_1 * ln(r_out) + c_2
T_r_out = c_3 * ln(r_out) + c_4

"Temperature at insulation edge"
T_r_ins = c_3 * ln(r_ins) + c_4

"Heat balance r_in"
-h_in * (T_r_in - T_f) = -k * (-g_dot * r_in / (2 * k) + c_1 / r_in)

"Heat balance r_out"
-k * (-g_dot * r_out / (2 * k) + c_1 / r_out) = -k_ins * c_3 / r_out

"Heat balance insulation edge"
-k_ins * c_3 / r_ins = -h_out * (T_r_ins - T_inf)
```

Solution

Main

Unit Settings: SI C kPa kJ mass deg

$C = 2500$ [W/((m^2)*C)]	$c_1 = 326.8$	$c_2 = 1441$	$c_3 = 27.98$
$c_4 = 182.6$	$\dot{g} = 1.000E+07$ [W/m^3]	$h_{in} = 1986$	$h_{out} = 20$ [W/((m^2)*C)]
$k = 10$ [W/(m*C)]	$k_{ins} = 1.5$ [W/(m*C)]	$L = 0.0762$ [m]	$\dot{m} = 0.75$ [kg/s]
$r_c = 0.02223$	$r_{in} = 0.01905$ [m]	$r_{ins} = 0.03175$	$r_{out} = 0.0254$ [m]
$T_f = 18$ [C]	$T_{inf} = 20$ [C]	$t_{ins} = 0.00635$ [m]	$T_{r,c} = 74.02$
$T_{r,in} = 56.41$	$T_{r,ins} = 86.1$	$T_{r,out} = 79.85$	

10 potential unit problems were detected. Check Units

Compilation time = 78 ms   Calculation time = 16 ms