7-3. Consider fully developed laminar flow of a constant-property fluid in a circular tube. At a particular flow cross section, calculate the total axial momentum flux by integration over the entire cross section. Compare this with the momentum flux evaluated by multiplying the mass flow rate times the mean velocity. Explain the difference, then discuss the implications for the last part of Prob. 7-2.

Axial Momentum Flux by Integration: fully developed, constant property, laminanar

$$\int_0^{2\pi}\int_0^{r} \rho\,(r\,u(r))\,(u(r))\,dr\,d\theta = \rho\int_0^{2\pi}\int_0^{r_0} r\left(2V\left(1-\frac{r^2}{r_0^2}\right)\right)\left(2V\left(1-\frac{r^2}{r_0^2}\right)\right)dr\,d\theta$$

$$= \rho 4V^2\int_0^{2\pi}\int_0^{r}\left(\frac{r^5}{r_0^4}-2\frac{r^3}{r_0^2}+r\right)dr\,d\theta$$

$$= \rho 4V^2\int_0^{2\pi}\frac{r_0^6}{6r_0^4}-\frac{r_0^4}{2r_0^2}+\frac{r_0^2}{2}\,d\theta$$

$$= 4\rho V^2\pi\left(\frac{r_0^6}{3r_0^4}-\frac{r_0^4}{r_0^2}+r_0^2\right) = \frac{4}{3}\rho V^2\pi r_0^2$$

Mass Flow Rate times Mean Velocity: $\dot{m}\cdot V = (\rho V\pi r_0^2)V = \rho V^2\pi r_0^2$

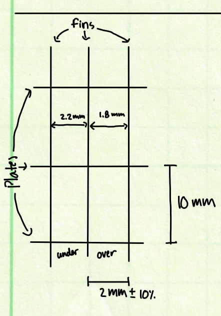
the two methods are different by a factor of $\frac{4}{3}$, with the integration method being larger

↳ explain more?

Implications on Problem 7.2: 7.2 → fluid density varies along flow direction, constant over cross section

in the calculations above, the density is constant, so the mass flowrate and mean velocity will be constant in the fully developed section of the pipe. With variable density, the mass flow rate will remain constant, which means the mean velocity will vary, so it can't be used in the $\dot{m}V$ equation. The $\frac{4}{3}$ factor difference would also impact the results, with the integral method providing a better estimation.

7-5. A particular heat exchanger is built of parallel plates, which serve to separate the two fluids, and parallel continuous fins, which extend between the plates so as to form rectangular flow passages. For one of the fluids the plate separation is 1 cm and the nominal fin separation is 2 mm. However, manufacturing tolerance uncertainties lead to the possibility of a 10 percent variation in the fin separation. Consider the extreme case where a 10 percent oversize passage is adjacent to a 10 percent undersize passage. Let the flow be laminar and the passages sufficiently long that an assumption of fully developed flow throughout is reasonable. For a fixed pressure drop, how does the flow rate differ for these two passages and how does it compare to what it would be if the tolerance were zero?



$$Re = \frac{4 r_h G}{\mu} \quad , \quad r_h = \frac{a \cdot b}{2(a+b)} \quad , \quad G = \dot{m}/A_c \quad \Rightarrow \quad Re = \frac{4 \rho V r}{\mu}$$

$$\Delta P = 4 \bar{f}_{app} \frac{\rho V^2}{2 g_c} \frac{x}{D_h} \quad , \quad g_c = 1 \quad , \quad D_h = \frac{2ab}{(a+b)} = 4 r_h$$

Nominal (2mm Separation): $a = 0.01\text{m}$, $b = 0.002\text{ m}$

$\alpha^* = \frac{b}{a} = \frac{10}{2} = 5$, $\frac{1}{\alpha^*} = 0.2$ from figure 6-4 (old textbook) $f\,Re = 19.2 \Rightarrow f = \frac{19.2}{Re}$

$$\Delta P = 4\left(\frac{19.2\, \mu\, 2(a+b)}{4 \rho V a b}\right) \frac{\rho V^2}{2} \frac{x}{D_h}$$

$$\frac{\Delta P}{x \mu} = \frac{19.2\,(a+b)^2 V}{2\,(ab)^2} = \text{constant for all cases}$$

Oversized (2.2 mm Separation): $a = 0.01\text{m}$, $b = 0.0022\text{ m}$

$\alpha^* = \frac{b}{a} = \frac{10}{2.2} = 4.55$, $\frac{1}{\alpha^*} = 0.22$, $f\,Re = 18.75$

$$\Delta P = 4\left(\frac{18.75\, \mu\, 2(a+b)}{4 \rho V a b}\right) \frac{\rho V^2}{2} \frac{x}{D_h}$$

$$\frac{\Delta P}{x \mu} = \frac{18.75\,(a+b)^2 V}{2\,(ab)^2}$$

Undersized (1.8 mm Separation): $a = 0.01\text{m}$, $b = 0.0018\text{ m}$

$\alpha^* = \frac{b}{a} = \frac{10}{1.8} = 5.56$, $\frac{1}{\alpha^*} = 0.18$, $f\,Re = 19.5$

$$\Delta P = 4\left(\frac{19.5\, \mu\, 2(a+b)}{4 \rho V a b}\right) \frac{\rho V^2}{2} \frac{x}{D_h}$$

$$\frac{\Delta P}{x \mu} = \frac{19.5\,(a+b)^2 V}{2\,(ab)^2}$$

Oversized vs Undersized:

$$\frac{18.75\,(0.01+0.0022)^2 V_o}{2\,(0.01 \cdot 0.0022)^2} = \frac{19.5\,(0.01+0.0018)^2 V_u}{2\,(0.01 \cdot 0.0018)^2}$$

$$\frac{V_o}{V_u} = \frac{19.5}{18.75} \frac{(0.01+0.018)^2}{(0.01+0.0022)^2} \frac{(0.01 \cdot 0.0022)^2}{(0.01 \cdot 0.0018)^2} = 1.45$$

Oversized vs Nominal:

$$\frac{V_o}{V_n} = \frac{19.2}{18.75} \frac{(0.01+0.002)^2}{(0.01+0.0022)^2} \frac{(0.01 \cdot 0.0022)^2}{(0.01 \cdot 0.0018)^2} = 1.20$$

Nominal vs Undersized:

$$\frac{V_n}{V_u} = \frac{19.5}{19.2} \frac{(0.01+0.018)^2}{(0.01+0.002)^2} \frac{(0.01 \cdot 0.002)^2}{(0.01 \cdot 0.0018)^2} = 1.21$$

7-9. TEXSTAN *analysis of laminar entry flow in a circular pipe*: Let the Reynolds number be 1000, and pick fluid properties that are appropriate to the Prandtl number of air or water. You can choose how to set up the TEXSTAN problem in terms of values for the geometrical dimensions and mean velocity for the pipe to provide the required Reynolds number and a pipe length equivalent to $x^+ = (x/D)/\mathrm{Re} = 0.3$. Note that the x^+ variable is the inverse of the Langhaar variable, used in Fig. 7-7. Use constant fluid properties, and note that the energy equation does not have to be solved. For initial conditions let the velocity profile be flat at a value equal to the mean velocity.

Use Eq. (7-20) to obtain the mean friction coefficient and use Eq. (7-21) along with the pressure drop data to obtain the apparent friction coefficient, then plot the local, mean, and apparent friction coefficient versus x^+ to show how the data approach the hydrodynamic fully developed values that are shown on Fig. 7-7 and in Shaw and London.[3] Confirm the hydrodynamic entrance length, and compare to Eq. (7-23). Plot the nondimensional velocity profiles at various x^+ locations and compare to Fig. 7-6 to demonstrate the concept of how the profiles evolve from a flat profile into hydrodynamically fully developed profile. Plot the absolute value of the pressure gradient versus x^+ to show how the gradient becomes constant beyond the hydrodynamic entrance region. Evaluate the ratio of centerline velocity to mean velocity and plot it versus x^+ to show how the ratio becomes a constant (= 2.0) beyond the hydrodynamic entrance region.

Properties and dimensions:

Change pipe length to: $(x/D_h)/Re_{D_h} = 0.1$

using water

$\rho = 1000\ \mathrm{kg/m^3}$, $P_r = 6.9$, $\mu = 0.89\times10^{-3}\ \frac{N\cdot s}{m^2}$, $C_p = 4182\ \mathrm{J/kgK}$

$L/D = 0.1 \cdot Re = 100$

$L = 10\,m$, $D = 0.1\,m$

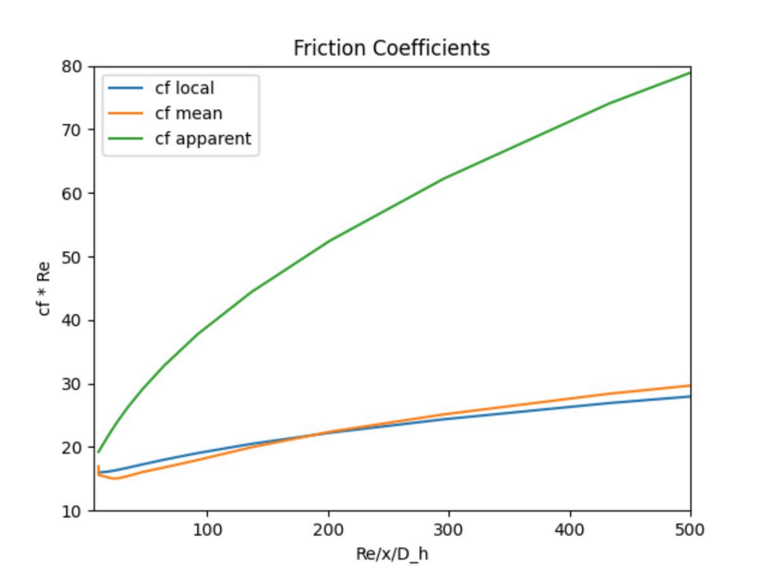
$$V = \frac{Re\,\mu}{\rho D} = \frac{1000 \cdot 0.9\times10^{-3}}{1000 \cdot 0.1}\ \frac{m}{s} = 0.009\ \frac{m}{s}$$

Equations:

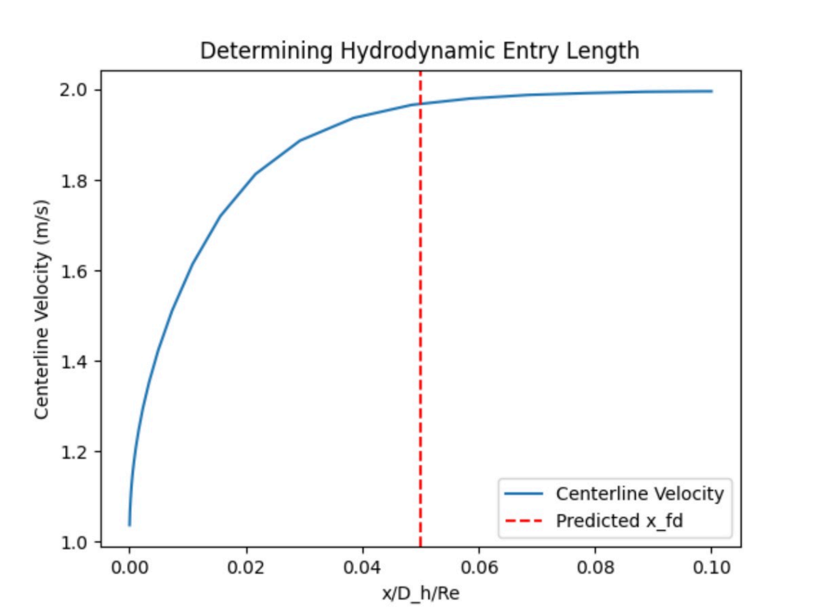
$$(7\text{-}20)\quad C_{f_m} = \frac{\frac{1}{x}\int_0^x \tau_s\,dx}{1/2\,\rho V^2}$$

$$(7\text{-}21)\quad \Delta P^* = \frac{\Delta P}{\frac{1}{2}\rho V^2} = 4\bar{C}_{f_{app}}\frac{x}{D}$$

$$(7\text{-}23)\quad \frac{x}{D} = \frac{Re}{20}$$

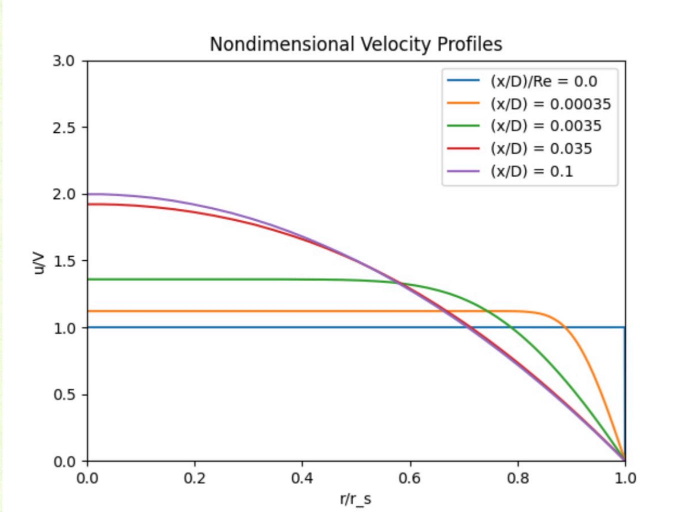


the local and apparent coefficients match expected values decently, but mean values are lower than expected



Predicted to be fully developed at $x/D_h/Re = 0.05$ from eq. 7-23

this matches the 99% of developed centerline velocity point



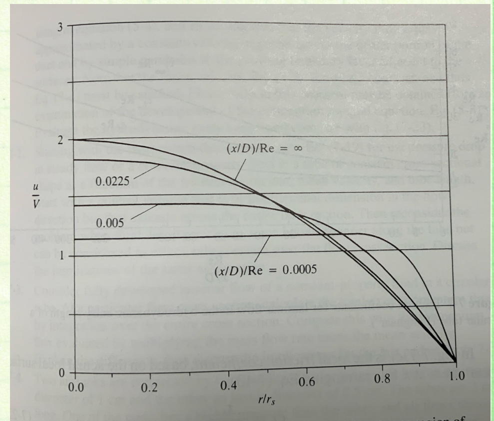
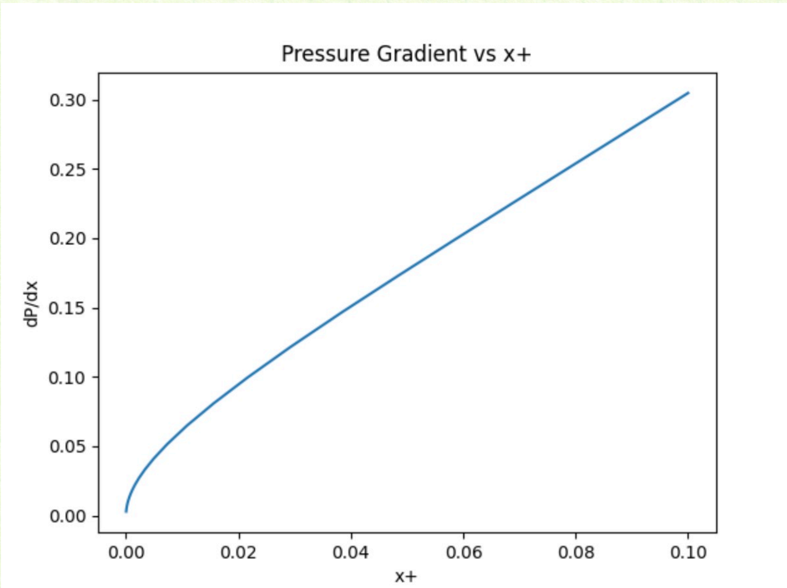
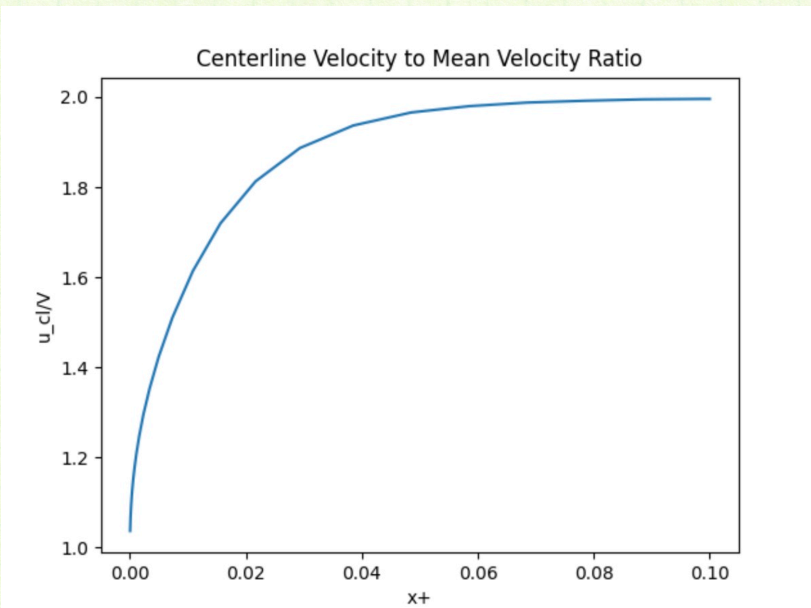


Figure 7-6 Axial velocity distribution in the hydrodynamic entrance region of a circular tube.

good match



linear in fully developed region



converges to 2

8-3. Consider a 0.6-cm (inside-diameter) circular tube that is of sufficient length that the flow is hydrodynamically fully developed. At some point beyond the fully developed location, a 1.2-m length of the tube is wound by an electric resistance heating element to heat the fluid, an organic fuel, from its entrance temperature of 10°C to a final value of 65°C. Let the mass flow rate of the fuel be 1.26×10^{-3} kg/s. The following average properties may be treated as constant:

$$\mathrm{Pr} = 10$$
$$\rho = 753 \text{ kg/m}^3$$
$$c = 2.092 \text{ kJ/(kg} \cdot \text{K)}$$
$$k = 0.137 \text{ W/(m} \cdot \text{K)}$$
$$\mu = 0.00065 \text{ Pa} \cdot \text{s}$$

Calculate and plot both tube surface temperature and fluid mean temperature as functions of tube length. What is the highest temperature experienced by any of the fluid?

TEXSTAN can be used to confirm the results of this problem. Use constant fluid properties and do not consider viscous dissipation. The thermal boundary condition is constant heat flux; use the value calculated from the problem analysis. This is an unheated starting length problem, so the initial conditions are a velocity profile that is hydrodynamically fully developed and a temperature profile that is flat at a value equal to the entry temperature.

given:

$L = 1.2\text{m}$

$T_e = 10°C$, $T_o = 65°C$

$\dot{m} = 1.26 \times 10^{-3}$ kg/s

find: $T_{s,tube}(L)$

$T_{m,fluid}(L)$

$T_{max,fluid}$

$$q = \dot{m} c_p \Delta T = 1.26\times10^{-3} \text{ kg/s} \cdot 2.092 \text{ kJ/kgK} \cdot 55\text{ K} = 145 \text{ W}$$

$$q'' = q/A = q/\pi D L = \frac{145\text{W}}{\pi \cdot 0.006\text{m} \cdot 1.2\text{m}} = 6410 \text{ W/m}^2$$

$$q'' = h(T_s - T_m)$$

$$V = \frac{\dot{m}}{\rho A} = \frac{4\dot{m}}{\rho \pi D^2} = \frac{4 \cdot 1.26\times10^{-3}\text{ kg/s}}{753 \text{ kg/m}^3\, \pi\, (0.006\text{m})^2} = 0.06 \text{ m/s}$$

$$Re = \frac{\rho V D}{\mu} = \frac{753 \text{ kg/m}^3 \cdot 0.06 \text{ m/s} \cdot 0.006\text{m}}{0.00065 \text{ Pa}\cdot\text{s}} = 417 \rightarrow \text{laminar}$$

$$Nu_D = 4.36 = \frac{hD}{k}$$

$$h = \frac{4.36k}{D} = \frac{4.36 \cdot 0.137 \text{ W/m}\cdot\text{k}}{0.006\text{m}} = 99.6 \text{ W/m}^2\text{k}$$

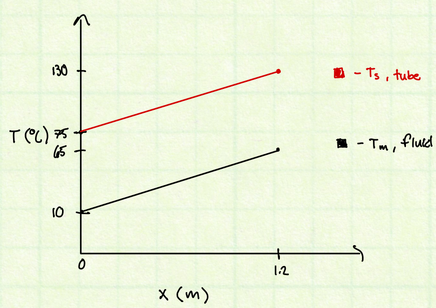
$$T_s - T_m = \frac{q''}{h} = \frac{6410 \text{ W/m}^2}{99.6 \text{ W/m}^2\text{k}} = 64.4 \text{ K}$$

at $x=L$, the fluid temperature at the wall is equal to $T_s(L) = T_m(L) + 64.4°C = 129.4°C$

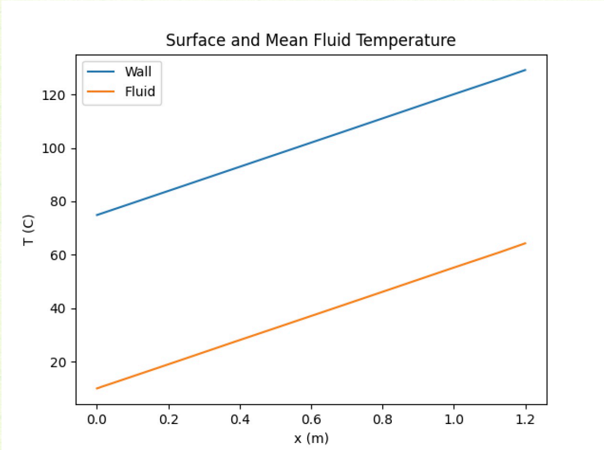
using th,cl from Texstan, the highest temperature of the centerline is 105°C

fluid: $T(x) = T_o + \frac{q'' \pi D x}{c \dot{m}} = 10°C + 6410 \text{ W/m}^2 \cdot \pi \cdot 0.006\text{m} / (2092 \text{ J/kg°C} \cdot 1.26\times10^{-3} \text{ kg/s}) \cdot x$

$$\underline{\underline{T(x) = 10°C + 45.84 \tfrac{°C}{m}\, x}}$$



using x=0 at the start of the heated section



from Texstan