11-9. The following table is an actual velocity profile measured through a turbulent boundary layer on a rough surface made up of 1.27-mm balls packed in a dense, regular pattern. There is no pressure gradient or transpiration. The fluid is air at 1 atm and 19°C; and $u_\infty = 39.7$ m/s, $\delta_2 = 0.376$ cm, $Re_{\delta_2} = 9974$, $c_f/2 = 0.00243$. The distance y is measured from the plane of the tops of the balls. The objective of this problem is to analyze these data in the framework

y, cm	u, m/s	y, cm	u, m/s
0.020	12.94	0.660	27.38
0.030	14.08	1.10	31.10
0.051	15.67	1.61	34.15
0.081	17.31	2.12	37.21
0.127	19.24	2.82	39.37
0.191	20.87	3.58	39.68
0.279	22.68		
0.406	24.54		

of the rough-surface theory developed in the text. What is the apparent value of k_s? Of Re_k? What is the roughness regime? What is the apparent value of κ? How does the wake compare with that of a smooth surface? Do the data support the theory?

The use of the plane of the tops of the balls as the origin for y is purely arbitrary. Feel free to move the origin if this will provide a more coherent theory.

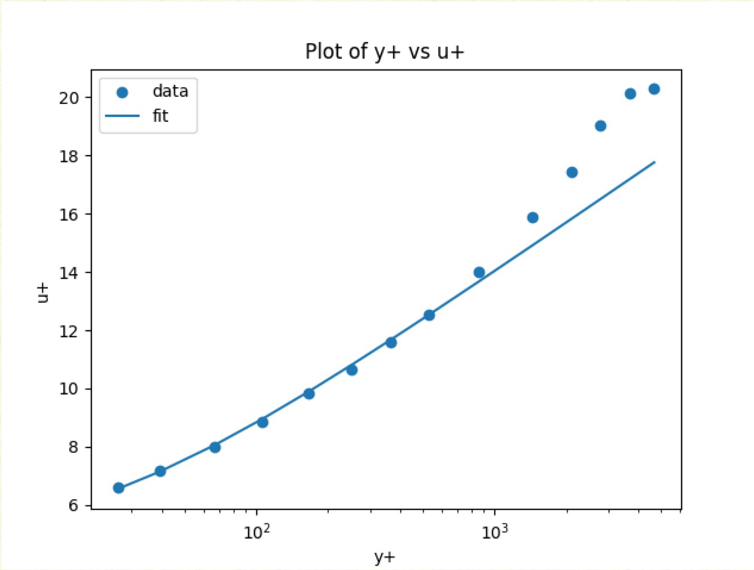
$$Re_k = \frac{u_\tau k_s}{\nu}$$

$$u^+ = \frac{\bar{u}}{u_\tau} = \frac{\bar{u}/u_\infty}{\sqrt{C_f/2}}, \quad u_\tau = \sqrt{\frac{C_f u_\infty^2}{2}}$$

$$y^+ = \frac{y u_\tau}{\nu}$$

fit function:

$$u^+ = \frac{1}{\kappa}\ln\left(\frac{32.6\, y^+}{Re_k} + 1\right) = \frac{1}{\kappa}\ln\left(32.6\,\frac{y+C}{k_s} + 1\right)$$



Results:

5 points included in wake

$y_{offset} = 0.139$ mm

$k_s = 0.808$ mm

$Re_k = 105.6$

the roughness regime is fully rough

for a smooth case, the C_f would be lower. Trying to fit the curve with $C_f/2$ halved results in a much worse fit and a lower Re_k. This happens because C_f is no longer independent of Re.

The data fits the theory well. This can be seen by the close match of the data and fit lines except in the wake region.

12-17. TEXSTAN *analysis of the turbulent thermal boundary layer over a flat plate with constant surface temperature and with zero pressure gradient*: Choose a starting x-Reynolds number of about 2×10^5 (corresponding to a momentum Reynolds number of about 700) and pick fluid properties that are appropriate to air, evaluated at a free-stream temperature of 300 K. Use constant fluid properties and do not consider viscous dissipation. The geometrical dimensions of the plate are 1 m wide (a unit width) by 3.0 m long in the flow direction, corresponding to an ending Re_x of about 2.9×10^6 (a momentum Reynolds number of about 5400). Let the velocity boundary condition at the free stream be 15 m/s and let the energy boundary conditions be a free-stream temperature of 300 K and a constant surface temperature of 295 K. The initial velocity and temperature profiles appropriate to the starting x-Reynolds number (fully turbulent boundary-layer profiles) can be supplied by using the *kstart*=3 choice in TEXSTAN. For a turbulence model, choose the mixing-length turbulence model with the Van Driest damping function (*ktmu*=5) and choose the variable turbulent Prandtl number model (*ktme*=3) corresponding to Eq. (12-7).

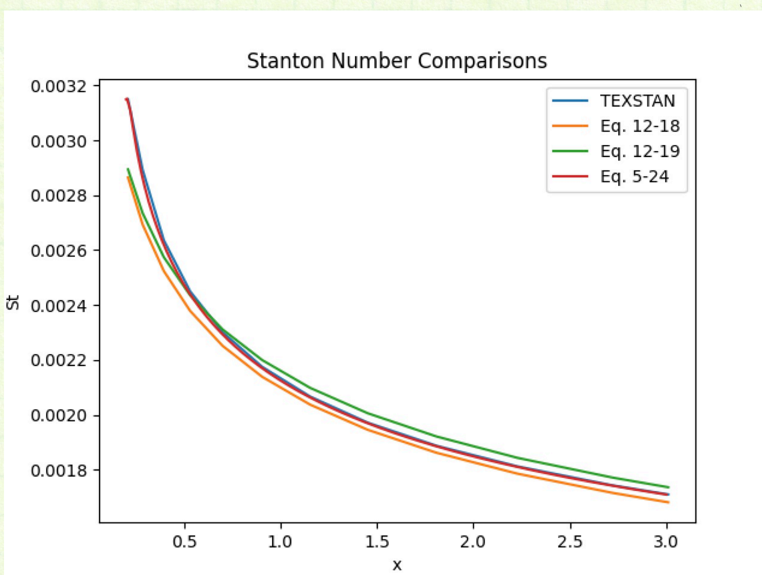
Calculate the boundary layer flow and compare the Stanton number results based on x Reynolds number and enthalpy thickness Reynolds number with the results in the text, Eqs. (13-18) and (13-19). Calculate the Stanton number distribution using energy integral Eq. (5-24) and compare with the TEXSTAN calculations. Feel free to investigate any other attribute of the boundary-layer flow. For example, you can investigate the thermal law of the wall, comparing to Fig. 13-9.

Comparing Stanton Numbers from Texstan to book equations

12 - 18) $St\,Pr^{0.4} = 0.0287\,Re_x^{-0.2}$

12 - 19) $St\,Pr^{0.5} = 0.0125\,Re_{\delta_2}^{-0.25}$

5 - 24) $St = \dfrac{d\Delta_2}{dx} = \dfrac{\dot{q}_0''}{\rho u_\infty c\,(T_0 - T_\infty)}$



- All equations match decently well, but eq. 5-24 from the integral equation matches the texstan results best with all values nearly overlapping

$T^+ = \dfrac{(T_s - \bar{T})\sqrt{\tau_s/\rho}}{\dot{q}_s''/\rho c}$, $y^+ = \dfrac{y u_\tau}{\nu} = \dfrac{y u_\tau \rho}{\mu}$, $u_\tau = \dfrac{\tau_s}{\rho} = \sqrt{\dfrac{C_f U_\infty^2}{2}}$, $\tau_s = \dfrac{\rho C_f U_\infty^2}{2}$

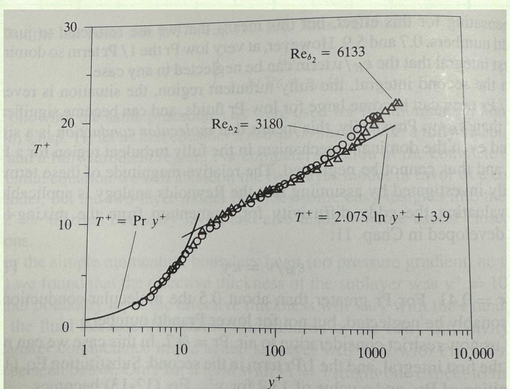
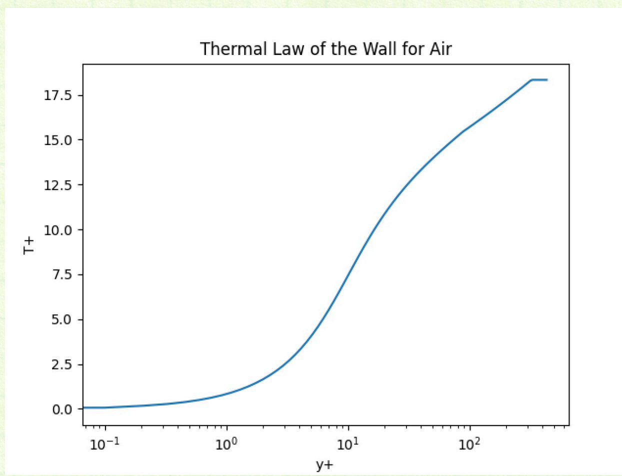
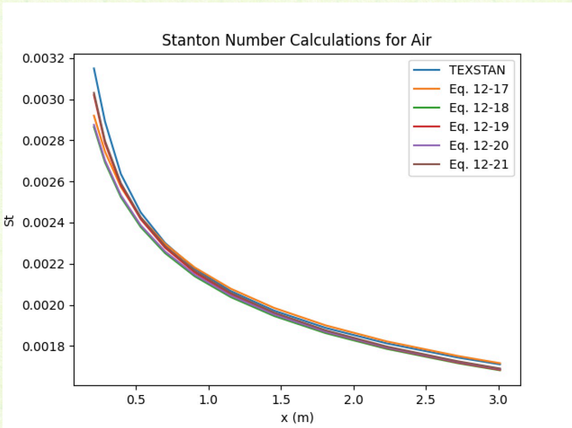


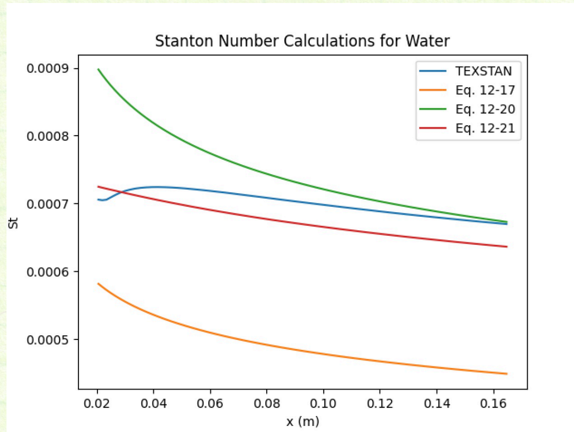
Figure 12-9 A thermal law of the wall for air: equilibrium thermal boundary layers; enthalpy thickness Reynolds numbers are indicated.

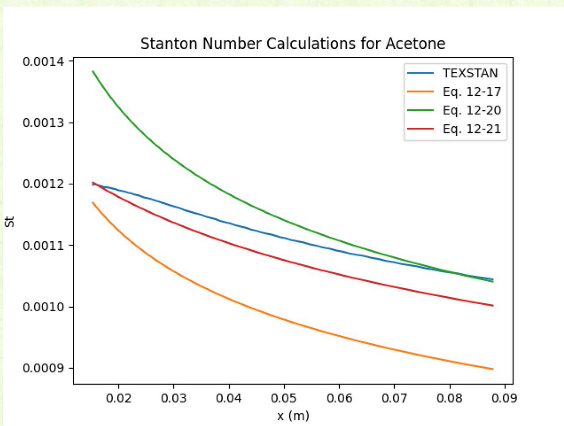


The figure recreated from Texstan values on the right matches figure 12-9 well, but the data does not extend as far into the wake. The shape of the curve looks very similar, with the T^+ values looking to be close

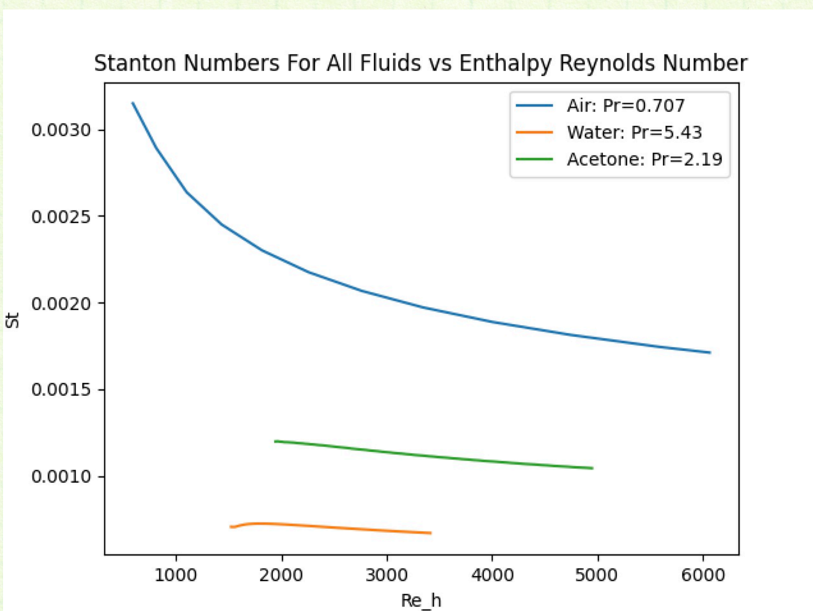
12-18. TEXSTAN *analysis of the turbulent thermal boundary layer over a flat plate with constant surface temperature and with zero pressure gradient*: This is an extension to Prob. 12-17 to investigate the effect of Prandtl number on heat transfer. Examine fluids ranging from gases to light liquids and compare to Fig. 13-13. Compare Eqs. (13-17) through (13-21) with the TEXSTAN results.







As in the previous question, all equations work well for air. However, the equations have noticeably worse performance for the liquids, acetone and water. The performance of the equations for liquids are consistent for both with the same equations over or under predicting in the same regions.



This plot shows the same trends as figure 12-13, in that St decreases with increasing Pr or increasing $Re_{\Delta_2} = Re_h$