1.2-3 (1-2 in text) Figure P1.2-3 illustrates a plane wall made of a thin (th_w = 0.001 m) and conductive (k = 100 W/m-K) material that separates two fluids, A and B. Fluid A is at T_A = 100°C and the heat transfer coefficient between the fluid and the wall is $\bar{h}_A$ = 10 W/m^2-K while fluid B is at T_B = 0°C with $\bar{h}_B$ = 100 W/m^2-K.

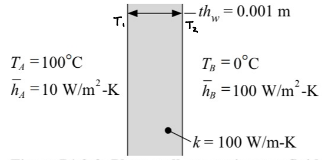

Figure P1.2-3: Plane wall separating two fluids

a.) Draw a resistance network that represents this situation and calculate the value of each resistor (assuming a unit area for the wall, A = 1 m^2).
b.) If you wanted to predict the heat transfer rate from fluid A to fluid B very accurately, then which parameter (e.g., th_w, k, etc.) would you try to understand/measure very carefully and which parameters are not very important? Justify your answer.

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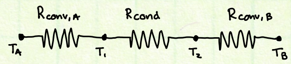
a) assuming no contact resistance and no radiation/radiation resistance

$$R_{conv,A} = \frac{1}{\bar{h}_A A_{s,A}} = \frac{1}{10 \frac{W}{m^2 K} \cdot 1 m^2} = 0.1 \frac{K}{W}$$

$$R_{cond} = \frac{th_w}{k A_c} = \frac{0.001 m}{100 W/m{\cdot}K \cdot 1 m^2} = 0.00001 \frac{K}{W}$$

$$R_{conv,B} = \frac{1}{\bar{h}_B A_{s,B}} = \frac{1}{100 W/m^2 K \cdot 1 m^2} = 0.01 \frac{K}{W}$$



b) Checking the effect of 10% measurement uncertainty for $\bar{h}_A$, $\bar{h}_B$, k, th_w on $R_{conv,A}$, R_{cond}, $R_{conv,B}$

$R_{conv,A}$ Vary $\bar{h}_A$: min / max : 0.0909 / 0.1111

R_{cond} Vary k: 0.00000909 / 0.0000111

vary th_w: 0.000009 / 0.000011

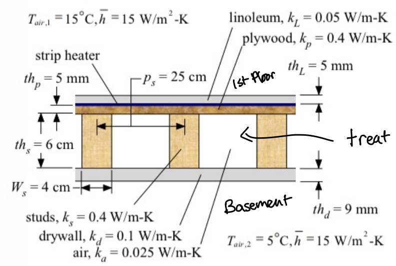
Vary k, th_w: 0.00000818 / 0.00001222

$R_{conv,B}$ Vary h_B: 0.00909 / 0.0111

Because the magnitude of $R_{conv,A}$ is so much larger than $R_{conv,B}$ and R_{cond}, variations in $\bar{h}_A$ will have a larger effect on the heat transfer than the other variables. R_{cond} has a small magnitude, but because it depends on two measured values, the stacking uncertainties can have a proportionally large effect on its value. But accurate measurements of $\bar{h}_A$ will have the largest impact on the system calculations.

1.2-12 (1-5) You have decided to install a strip heater under the linoleum in your bathroom in order to keep your feet warm on cold winter mornings. Figure P1.2-12 illustrates a cross-section of the bathroom floor. The bathroom is located on the first story of your house and is W = 2.5 m wide x L = 2.5 m long. The linoleum thickness is th_L = 5.0 mm and has conductivity k_L = 0.05 W/m-K. The strip heater under the linoleum is negligibly thin. Beneath the heater is a piece of plywood with thickness th_P = 5 mm and conductivity k_P = 0.4 W/m-K. The plywood is supported by th_s = 6.0 cm thick studs that are W_s = 4.0 cm wide with thermal conductivity k_s = 0.4 W/m-K. The center-to-center distance between studs is p_s = 25.0 cm. Between each stud are pockets of air that can be considered to be stagnant with conductivity k_a = 0.025 W/m-K. A sheet of drywall is nailed to the bottom of the studs. The thickness of the drywall is th_d = 9.0 mm and the conductivity of drywall is k_d = 0.1 W/m-K. The air above in the bathroom is at $T_{air,1}$ = 15°C while the air in the basement is at $T_{air,2}$ = 5°C. The heat transfer coefficient on both sides of the floor is $\bar{h}$ = 15 W/m²-K. You may neglect radiation and contact resistance for this problem.

$A = 2.5\,m \times 2.5\,m = 6.25\,m^2$



treat as solid, no convection

Figure P1.2-12: Bathroom floor with heater.

a.) Draw a thermal resistance network that can be used to represent this situation. Be sure to label the temperatures of the air above and below the floor ($T_{air,1}$ and $T_{air,2}$), the temperature at the surface of the linoleum (T_L), the temperature of the strip heater (T_h), and the heat input to the strip heater ($\dot{q}_h$) on your diagram.

b.) Compute the value of each of the resistances from part (a).

c.) How much heat must be added by the heater to raise the temperature of the floor to a comfortable 20°C?

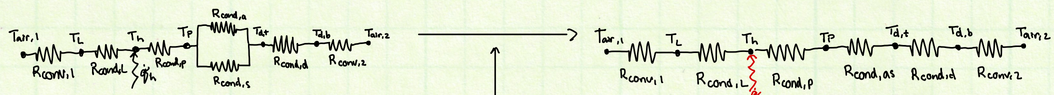
d.) What physical quantities are most important to your analysis? What physical quantities are unimportant to your analysis?

e.) Discuss at least one technique that could be used to substantially reduce the amount of heater power required while still maintaining the floor at 20°C. Note that you have no control over $T_{air,1}$ or $\bar{h}$.

$A_s = 0.04\,m \cdot 2.5\,m \cdot 10 = 1\,m^2$

$A_{air} = 0.21 \cdot 2.5 \cdot 10 = 5.25\,m^2$

a)



combine $R_{cond,a}$ & $R_{cond,s}$ to an effective resistance, $R_{cond,as}$

b)

$$R_{conv,1} = \frac{1}{\bar{h}A} = \frac{1}{15\,W/m^2K \cdot 6.25\,m^2} = 0.010667\,\frac{K}{W}$$

$$R_{cond,L} = \frac{t_L}{k_L A} = \frac{0.005\,m}{0.05\,W/m\cdot K \cdot 6.25\,m^2} = 0.016\,\frac{K}{W}$$

$$R_{cond,P} = \frac{t_P}{k_P A} = \frac{0.005\,m}{0.4\,W/m\cdot K \cdot 6.25\,m^2} = 0.002\,\frac{K}{W}$$

$$R_{cond,a} = \frac{t_s}{k_a A_a} = \frac{0.06\,m}{0.025\,W/m\cdot K \cdot 5.25\,m^2} = 0.45714\,\frac{K}{W}$$

$$R_{cond,s} = \frac{t_s}{k_s A_s} = \frac{0.06\,m}{0.4\,W/m\cdot K \cdot 1\,m^2} = 0.15\,\frac{K}{W}$$

$$\rightarrow \frac{1}{R_{cond,as}} = \frac{1}{R_{cond,a}} + \frac{1}{R_{cond,s}} \qquad \therefore\ R_{cond,as} = \frac{1}{\frac{1}{0.45714} + \frac{1}{0.15}}\,\frac{K}{W} = \underline{0.11294\,\frac{K}{W}}$$

$$R_{cond,d} = \frac{t_d}{k_d A} = \frac{0.009\,m}{0.1\,W/m\cdot K \cdot 6.25\,m^2} = 0.0144\,\frac{K}{W}$$

$$R_{conv,2} = \frac{1}{\bar{h}A} = R_{conv,1} = 0.010667\,\frac{K}{W}$$

c)

$$\frac{(T_L - T_{air,1})}{R_{conv,1}} = \frac{(T_h - T_L)}{R_{cond,L}}$$

$$T_h = \frac{R_{cond,L}}{R_{conv,1}}(T_L - T_{air,1}) + T_L = \frac{0.016}{0.010667}(20-15)°C + 20°C = 27.5°C$$

assuming steady state, $\dot{q}_h = \dot{q}_{out} = \dfrac{\Delta T_{left}}{R_{tot,left}} + \dfrac{\Delta T_{right}}{R_{tot,right}}$

$R_{tot,left} = R_{conv,1} + R_{cond,L} = 0.026667\,\frac{K}{W}$

$R_{tot,right} = R_{cond,p} + R_{cond,as} + R_{cond,d} + R_{conv,2} = 0.14\,\frac{K}{W}$

$$\dot{q}_h = \frac{(27.5°C - 15°C)}{0.026667\,\frac{K}{W}} + \frac{(27.5°C - 5°C)}{0.14\,\frac{W}{K}} = 468.75\,W + 160.71\,W$$

$$\underline{\underline{\dot{q}_h = 629.46\,W}}$$

d) small resistances in series with large resistances and large resistances in parallel with small resistances are unimportant.

by this logic, the studs and air layer is most important because it has the largest effective resistance (accounts for most of $R_{tot,right}$), and the studs are more important because $R_{cond,s} < R_{cond,a}$. $R_{cond,p}$ is very small, so the plywood is not very important. otherwise the resistances are pretty similar in magnitude so the other properties are of similar importance.

e) if air temperature and convection coefficient can't be modified, and the floor is kept at 20°C, then the heat flux upward will remain the same. to decrease the heat lost down through the floor into the basement we need to decrease the temperature difference between the heater strip and basement or increase $R_{tot,right}$. Decreasing the thickness of the linoleum or replacing it with a more conductive material would decrease the required heater strip temperature, while keeping the floor surface at the desired temperature.