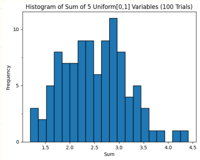
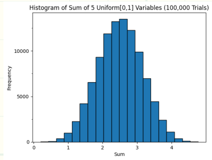
1) If x_1, x_2, x_3, x_4 and x_5 are random variables with a uniform distribution of similar range, what is the distribution of $x_1+x_2+x_3+x_4+x_5$? Why?

the distribution of the sum of 5 uniformly distributed variables is a Gaussian as shown in the plots. As the number of tests increases, the Shape is more obviously Gaussian





Summing random variables is a random process, Which results in a normal distribution

2) If X follows a Gaussian distribution with $\mu = 2$ and $\sigma = 7$, what is the probability of a value above 9? Above 15? What is the upper limit on how large X can be?

$P(x>9)$:

$\tau = (9-7)/2 = 1$

$P(\tau{=}1) = 0.6827 \rightarrow$ double sided

$\underline{\underline{P(x \geq 9) = (1-0.6827)/2 = 0.15865}}$

for $X>9$ it would be slightly less than the number above

$P(X>15)$:

$\tau = (15-7)/2 = 4$

$P(\tau{=}4) = 0.9999366$

$\underline{\underline{P(x \geq 15) = (1-0.9999366)/2 = 0.000032}}$

for $X>15$ it would be slightly less than the number above

X limit:

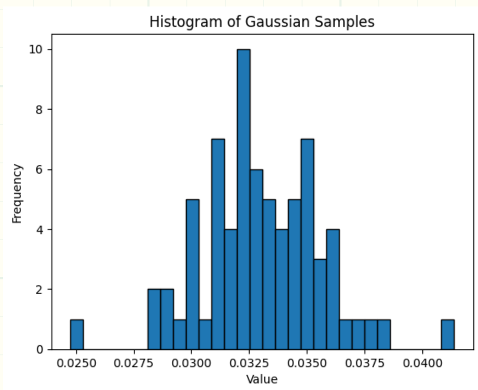
with a Gaussian distribution, the limits of X are $\pm\infty$, but the probability near those extremes is extremely small.

3) Six dozen MLB baseball seam heights were measured resulting in a mean height of 33/1000 of an inch and a standard deviation of 3/1000 of an inch. By randomly choosing samples from a Gaussian distribution with those parameters, estimate the range of seam heights for 72 samples. What is the probability of a 20/1000 of an inch seam height?

$N = 72$

$\mu = 33/1000$

$\sigma = 3/1000$



generated histogram with 72 samples

$\underline{\underline{h_{min} = 25/1000 \text{ in.} \quad , \quad h_{max} = 41/1000 \text{ in.}}}$

$P(20/1000)$:

counting a 20/1000 in. inseam as any baseball with inseams $19.5 < 1000h \leq 20.5$

$\tau(20.5) = |(20.5-33)/3| = 4.17$, $\tau(19.5) = |(19.5-33)/3| = 4.5$

$P(\tau{=}4.17) = (1-0.9999696)/2 = 0.0000152$, $P(\tau{=}4.5) = (1-0.9999931)/2 = 0.0000035$

$\underline{\underline{P(19.5 < 1000h \leq 20.5) = 0.0000152 - 0.0000035 = 0.0000117}}$

4) You measure the diameter of 6 parts resulting in the tabulated value in mm. Compute the mean and the standard deviation.

4.1
4.5
3.9
4.5
3.8
4.2

$$\bar{X} = \frac{4.1 + 4.5 + 3.9 + 4.5 + 3.8 + 4.2}{6} = 4.2$$

$$S = \sqrt{\frac{(4.1-4.2)^2 + (4.5-4.2)^2 + (3.9-4.2)^2 + (4.5-4.2)^2 + (3.8-4.2)^2 + (4.2-4.2)^2}{5}} = 0.3$$

5) Visually estimate the standard deviation of the data shown in Fig. 3.13.

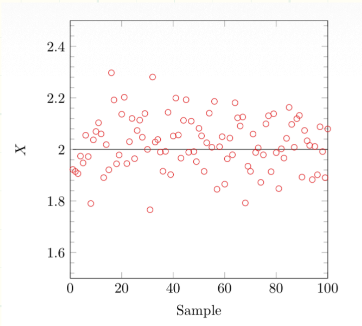


Figure 3.13

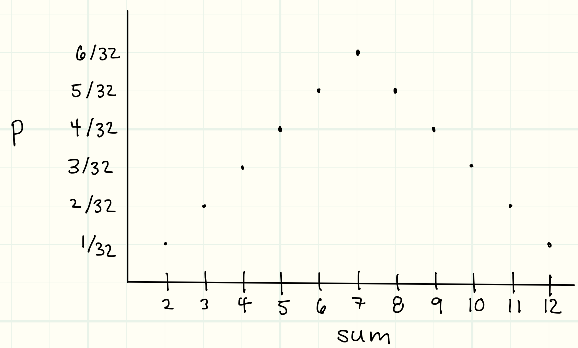
$\bar{X} = 2$

it looks like 95% of the data is within ±2, which would be 2σ, assuming a Gaussian distribution. The standard deviation would be half of the 2σ value

$S = 1$

6) Consider the PDF of the sum of the roll of two dice. Sketch the PDF. In what ways is it similar or different than a Gaussian PDF?

PDF of the sum of two dice



Comparison to Gaussian PDF

similar: symmetrical about center
probability decreases away from center

different: discrete points instead of a continuous function
does not extend to ± ∞
not bell curve shape → linear probability drop off

7) Consider the distribution of any quantity you encounter everyday, such as the height of adults. Does this quantity follow a Gaussian PDF? Why or why not?

going with adult height:
this property does not follow a Gaussian distribution.
the PDF of adult male height roughly follows a Gaussian distribution and the PDF of adult female height roughly follows a Gaussian distribution, but they don't have the same mean and standard deviation, so when you combine them to get the PDF for all adult height it is no longer Gaussian

8) Estimate the PDF of a sinusoid by sampling uniform random values of x between 0 and 1 in $X = \sin(2\pi x)$. Plot the histogram of X and compare it to Fig. 3.9.

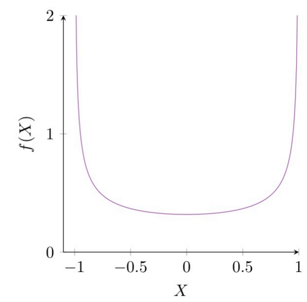
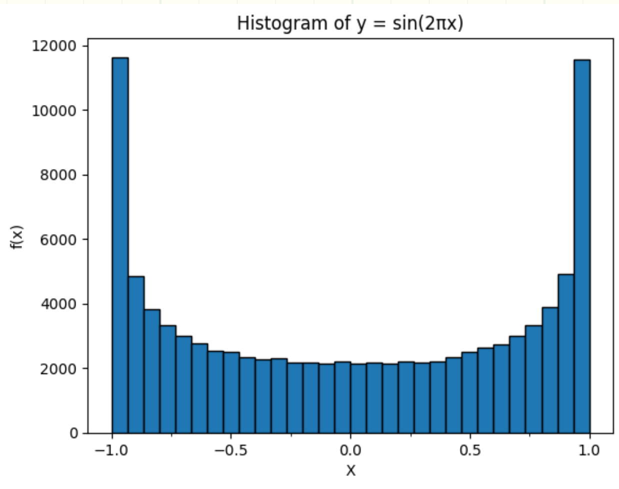
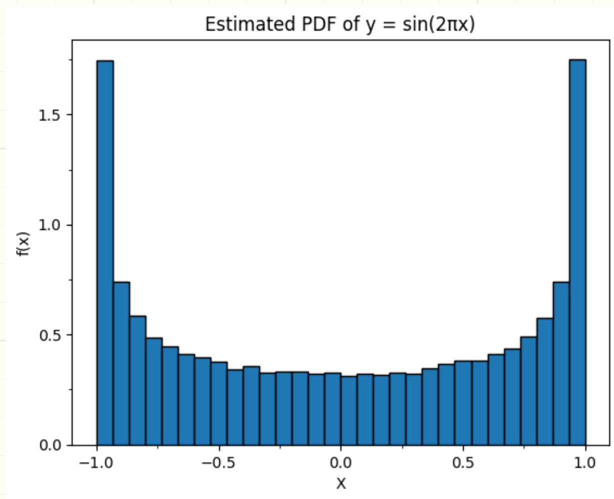


Figure 3.9: The Probability Density Function (PDF) of a sine wave.





I sampled 100,000 values of x and calculated $X = \sin(2\pi x)$ and grouped the result into 30 bins

the left plot is the histogram and the right is the estimated PDF calculated by the matplotlib hist function

both follow the same shape as fig. 3.9