1) You have measured a quantity X five times giving the following result: $X_1 = 10$, $X_2 = 12$, $X_3 = 9$, $X_4 = 1$, $X_5 = 14$.

If you wish to know the mean of X to within 2% (95% confidence), based on your initial sample, how many samples are required?

Using the full data:

$\bar{X} = 9.2$

$S_x = 4.97$

Using table 4.1 for Student's t:

$S_{\bar{x}} = \frac{4.97 \cdot 1.134}{\sqrt{5}} = 2.52$

to get the mean within ± 2% with 95% confidence

$S_{\bar{x}} = 1.96 \cdot \bar{X} \cdot 0.02 = 0.361$

$N = \left(\frac{S_x}{S_{\bar{x}}}\right)^2 = \left(\frac{4.97}{0.361}\right)^2 = 190$

2) What is the dynamic range of a 12-bit camera?

dynamic range $= 2^{N \text{ bits}} = 4096$

3) What is the dynamic range of a 12-bit DAQ board?

dynamic range $= 2^{N \text{ bits}} = 4096$

4) What is the dynamic range of a speedometer that goes to 120 mph in 5 mph increments?

dynamic range $= 120/5 = 24$

5) You make the five measurements below:
100
120
110
70
110
If you want to determine the mean of this variable to within 2% at 68% confidence, what is your best estimate for the number of samples that would be required?

$\bar{X} = 102$

$S_x = 19.235$

68% confidence $= 1\sigma$

$\bar{X} \cdot 0.02 = 2.04$

$S_{\bar{x}} = \frac{S_x}{\sqrt{N}}$

$N = \left(\frac{S_x}{S_{\bar{x}}}\right)^2 = \left(\frac{19.235}{2.04}\right)^2 = 89$

6) You are making pressure measurements with a sensor that has specified uncertainty of 0.5% of reading. For a given test condition, the readout of the sensor says 2000 psi, and based on the variation in the reading, you estimate that your random uncertainty is $r_P = 40$ psi. If you plan on acquiring the output of this sensor digitally, how many independent samples should you acquire to make the best use of your instrument and your time?

using 95% uncertainty, we want to minimize N for

$$1.96 S_{\bar{x}} = 2000\,\text{psi} \cdot 0.005 = 10\,\text{psi}$$

$$S_{\bar{x}} = \frac{S_x}{\sqrt{N}} = \frac{10\,\text{psi}}{1.96}$$

$$N = \left(\frac{1.96\, S_x}{10}\right)^2 = \left(\frac{1.96 \cdot 40}{10}\right)^2 = 61.5$$

$$\underline{\underline{N = 62}}$$

7)What is your voltage resolution if your voltage range is ±5V and the A-D converter has 8 bits?

dynamic range $= 2^8 = 256$

resolution $= 10\text{v} / 256 = 0.039\ \text{V}$

8)You have a bourdon-tube pressure meter, which is NIST traceable. You wish to calibrate a pressure transducer against it. This pressure transducer is the diaphragm type, with a full bridge strain-gage arrangement. The excitation voltage is ±10 V and an amplifier of gain 10 is used. You have taken the 21 data points shown in the table below in the order in which they are listed. Based on this data, discuss the severity of the following error sources:
a) Hysteresis
b) Noise
c) Zero Error
d) Sensitivity Error

P[psi]	Volts
0.0000	0.10000
2.0000	2.1100
4.0000	4.1000
6.0000	6.3500
8.0000	8.3500
10.000	10.300
8.0000	7.9500
6.0000	5.7200
4.0000	3.7500
2.0000	1.7000
0.0000	-0.02000
2.0000	2.1000
4.0000	4.2000
6.0000	6.3000
8.0000	8.4000
10.000	10.200
8.0000	7.9000
6.0000	5.8000
4.0000	3.7000
2.0000	1.7000
0.0000	-0.0100000

a) hysteresis seems to have a big impact on this data.
When increasing the pressure, the voltage overshoots a 1-1 fit, when decreasing the pressure, the voltage undershoots the fit. comparing the 6 psi data with the arrow indicating loading direction,

↑ 6.35, 6.3
↓ 5.72, 5.8

b) noise seems to be the second largest error source for this data, behind hysteresis. When comparing data from the same load direction, this error tends to be < 0.1 V, while hysteresis can be on the order of 0.5 or 0.6 V.

c) zero error seems to have minimal impact on this calibration. the maximum error at zero is +0.1V and the other values are -0.01V and -0.02V

d) sensitivity does not seem to be a major concern of this calibration due to the nearly 1-1 fit of pressure and voltage in this range

9) If you have an uncertainty u_{X_1} from one error source and another u_{X_2} which is 4 times larger, what percentage of the overall uncertainty of X comes from u_{X_1} ? (Hint: compare the overall uncertainty with and without u_{X_1})

$$U_X = \sqrt{U_{X_1}^2 + U_{X_2}^2} = \sqrt{U_{X_1}^2 + (4U_{X_1})^2} = \sqrt{17}\, U_{X_1}$$

$$U_{X_2} = 4U_{X_1} = \sqrt{16}\, U_{X_1} \approx U_X$$

when including U_{X_2},

$U_X = \sqrt{17}\, U_{X_1}$ → U_{X_2} has a much larger impact than U_{X_1}

overall uncertainty percentages: $\boxed{U_{X_1}: 1/17 = 5.88\%}$

$U_{X_2}: 16/17 = 94.12\%$

10) If you have three error sources resulting in uncertainties $u_{X_1} = u_{X_2}$ from one error source and another u_{X_3} which is 4 times larger, what percentage of the overall uncertainty of X comes from u_{X_1} ?

$$U_X = \sqrt{U_{X_1}^2 + U_{X_2}^2 + U_{X_3}^2} = \sqrt{U_{X_1}^2 + U_{X_1}^2 + 16\, U_{X_1}^2} = \sqrt{18}\, U_{X_1}$$

inside the square root, U_{X_1} and U_{X_2} each contribute 1/18

Overall uncertainty percentages: $\boxed{U_{X_1} : 1/18 = 5.56\%}$

$U_{X_2} : 1/18 = 5.56\%$

$U_{X_3} : 16/18 = 88.89\%$

11) A young baseball player uses a bat sensor to measure the speed of the tip of the bat for all their swings. The file BatSpeedSS.txt contains 15 readings of the sensor. Find the player's average tip speed and the uncertainty of that speed to 95% confidence.
Repeat the exercise with the file BatSpeed.txt which contains a larger sample.

BatSpeedSS.txt :

$N = 15$

$\bar{V} = 50.3$ mph

$S_V = 1.34$ mph

$S_{\bar{V}} = 0.34$ mph

95% confidence = 1.96σ

$\bar{V} = 50.3 \pm 0.68$ mph

BatSpeed.txt

$N = 280$

$\bar{V} = 49.3$ mph

$S_V = 3.94$ mph

$S_{\bar{V}} = 0.24$ mph

$\bar{V} = 49.3 \pm 0.46$ mph