Problem 1-6

1. You are to analyze the uncertainty of A where $A = \frac{BC^4}{\pi D}$. Under what circumstance would TSM and MCM return similar results?

if B, C, and D are uncorrelated,

$\theta_B = \frac{C^4}{\pi D}$, $\theta_C = 4\frac{BC^3}{\pi D}$, $\theta_D = -\frac{BC^4}{\pi D^2}$

TSM: $u_A^2 = u_B^2\left(\frac{C^4}{\pi D}\right)^2 + u_C^2\left(\frac{4BC^3}{\pi D}\right)^2 + u_D^2\left(-\frac{BC^4}{\pi D^2}\right)^2$

MCM and TSM would give a similar result if all variables (B, C, and D) are resampled every time

2. Which variable in 1) would you want to have the smallest relative uncertainty?

C. Looking at $\theta_C = 4\frac{BC^3}{\pi D}$, the factor of 4 out front makes variations in C more impactful. This also makes sense intuitively as variations in C would be amplified by the 4th power.

3. List advantages of MCM compared to TSM

- MCM is inexpensive to compute
- MCM can deal with non-Gaussian error distributions
- MCM does not require computation of derivatives, which is helpful for functions with lots of variables or difficult derivatives

4. Consider the equation $y = 1.0 - 0.2x + 0.01x^2 + z^{1/2}$. Determine the uncertainty in $y\,(x=1, z=1)$ for 2% uncertainty in x and 4% uncertainty in z. Assume no correlated errors

$u_x = 0.02x = 0.02$, $u_z = 0.04z = 0.04$

$\theta_x = -0.2 + 0.02x$, $\theta_z = 1/2\sqrt{z}$

$$u_y^2 = (0.02)^2(-0.2 + 0.02(1))^2 + (0.04)^2(1/2(\sqrt{1}))^2$$

$$u_y^2 = 0.000413$$

$$\underline{\underline{u_y = 0.0203}}$$

5. thermal cameras find temperature by measuring the radiative flux from a surface and assuming a known value for the emissivity, ε, of the surface:

$$T_s = \left(\frac{q''}{\varepsilon\sigma}\right)^{1/4}$$

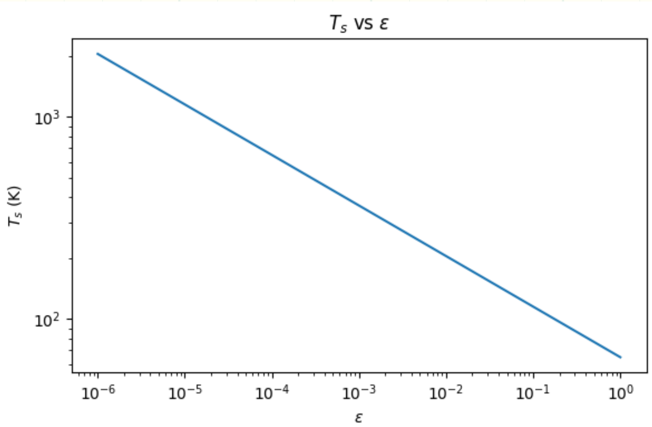
where q'' is the radiative flux and σ is the Stefan-Boltzmann constant $5.67 \times 10^{-8}\ W/m^2K^4$. Assuming no uncertainty in q'' or σ, find the uncertainty in surface temperature as a function of the u_ε for various values of ε between 0 and 1.

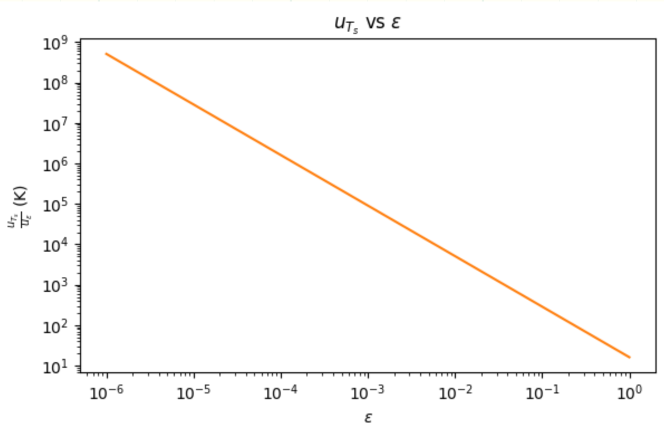
$$u_{T_s}^2 = u_\varepsilon^2\left(\left(\frac{q''}{\sigma}\right)^{1/4}\left(-\frac{1}{4\varepsilon^{5/4}}\right)\right)^2$$

because u_{T_s} is only a function of one uncertainty we can remove the square and divide by u_ε to see how u_{T_s} changes as we vary ϵ

$$\left|u_{T_s}/u_\varepsilon\right| = \left|\left(\frac{q''}{\sigma}\right)^{1/4}\left(-\frac{1}{4\epsilon^{5/4}}\right)\right|$$

I set $q'' = 1$ and used $\sigma = 5.67E{-8}$ and plotted T_s and u_{T_s} for 100 values of ϵ ranging from 0.000001 to 1.0 ($\epsilon = 0$ is undefined)





both plots are linear on a log-log plot, but the uncertainty plot has a much steeper slope.

As $\epsilon \to 0$, u_{T_s} explodes

6. The coefficient of performance of a gas refrigerator is given by

$$COP = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{1}{\dot{Q}_H/\dot{Q}_L - 1}$$

The input power could be measured using a brake that produces a force F on an arm that is L long as $\dot{W}_{in} = FL2\pi\Omega$, where Ω is the rotation rate measured with a tachometer. Both heat transfer rates are determined by measuring the mass flow rate of air in the heat exchangers and the temperature change across the heat exchanger $\dot{Q} = \dot{m} C_p \Delta T$. The table below shows the uncertainties and nominal values of all variables. The 95% confidence uncertainties are shown in the table below. To minimize the uncertainty in COP, should you use the first or second formula for COP? Use TSM assuming no correlated errors. Be careful with units!

Variable:	value:	total relative uncertainty:
L	2 m	2%
F	7.5 N	2%
Ω	955 RPM	2%
ΔT_L	70°C	2%
ΔT_H	100°C	2%
$\dot{m}_H$	0.05 kg/s	2%
$\dot{m}_L$	0.05 kg/s	2%
C_p	1 kJ/kg K	2%

Variables and uncertainties in correct units

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L: 2 ± 0.04 m
F: 7.5 ± 0.15 N
Cp: 1000 ± 20.0 J/kg*K
m_H: 0.05 ± 0.001 kg
m_L: 0.05 ± 0.001 kg
Delta_T_H: 100 ± 2.0 K
Delta_T_L: 70 ± 1.4 K
Omega: 15.917 ± 0.31833 rev/s
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$$\dot{Q}_H = \dot{m}_H C_p \Delta T_H \quad , \quad \dot{Q}_L = \dot{m}_L C_p \Delta T_L$$

i) $$COP = \frac{1}{\dot{Q}_H/\dot{Q}_L - 1} = \left(\frac{\dot{m}_H \Delta T_H}{\dot{m}_L \Delta T_L} - 1\right)^{-1}$$

$$\theta_{\dot{m}_H} = -\left(\frac{\dot{m}_H \Delta T_H}{\dot{m}_L \Delta T_L} - 1\right)^{-2}\left(\frac{\Delta T_H}{\dot{m}_L \Delta T_L}\right)$$

$$\theta_{\Delta T_H} = -\left(\frac{\dot{m}_H \Delta T_H}{\dot{m}_L \Delta T_L} - 1\right)^{-2}\left(\frac{\dot{m}_H}{\dot{m}_L \Delta T_L}\right)$$

$$\theta_{\dot{m}_L} = -\left(\frac{\dot{m}_H \Delta T_H}{\dot{m}_L \Delta T_L} - 1\right)^{-2}\left(-\frac{\dot{m}_H \Delta T_H}{\dot{m}_L^2 \Delta T_L}\right)$$

$$\theta_{\Delta T_L} = -\left(\frac{\dot{m}_H \Delta T_H}{\dot{m}_L \Delta T_L} - 1\right)^{-2}\left(-\frac{\dot{m}_H \Delta T_H}{\dot{m}_L \Delta T_L^2}\right)$$

$$u_{COP} = \sqrt{u_{\dot{m}_H}^2 \theta_{\dot{m}_H}^2 + u_{\Delta T_H}^2 \theta_{TH}^2 + u_{\dot{m}_L}^2 \theta_{\dot{m}_L}^2 + u_{\Delta T_L}^2 \theta_{\Delta T_L}^2}$$

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u_COP = sqrt(1e-06 * 2.4198e+04 + 4.0 * 0.0060494 + 1e-06 * 2.4198e+04 + 1.96 * 0.012346) = 0.31111
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$$\underline{\underline{COP = 2.333 \pm 0.311}} \longrightarrow 95\% \text{ uncertainty}$$

ii) $$COP = \frac{\dot{m}_L C_p \Delta T_L}{FL2\pi\Omega}$$

$$\theta_{\dot{m}_L} = \frac{C_p \Delta T_L}{FL2\pi\Omega}$$

$$\theta_{C_p} = \frac{\dot{m}_L \Delta T_L}{FL2\pi\Omega}$$

$$\theta_{\Delta T_L} = \frac{\dot{m}_L C_p}{FL2\pi\Omega}$$

$$\theta_F = -\frac{\dot{m}_L C_p \Delta T_L}{F^2 L2\pi\Omega}$$

$$\theta_L = -\frac{\dot{m}_L C_p \Delta T_L}{FL^2 2\pi\Omega}$$

$$\theta_\Omega = -\frac{\dot{m}_L C_p \Delta T_L}{FL2\pi\Omega^2}$$

$$u_{cop} = \sqrt{u_{\dot{m}_L}^2 \theta_{\dot{m}_L}^2 + u_{Cp}^2 \theta_{Cp}^2 + u_{\Delta T_L}^2 \theta_{\Delta T_L}^2 + u_F^2 \theta_F^2 + u_L^2 \theta_L^2 + u_\Omega^2 \theta_\Omega^2}$$

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u_COP = sqrt(1e-06 * 2177.5 + 400.0 * 5.4436e-06 + 1.96 * 0.0011109 + 0.0225 * 0.096776 + 0.0016 * 1.3609 + 0.10134 * 0.021487) = 0.1143
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$$\underline{\underline{COP = 2.333 \pm 0.114}} \longrightarrow 95\% \text{ uncertainty}$$

<u>equation 2 gives a better uncertainty for COP</u>