HW2: Orthogonality

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1 Cosines

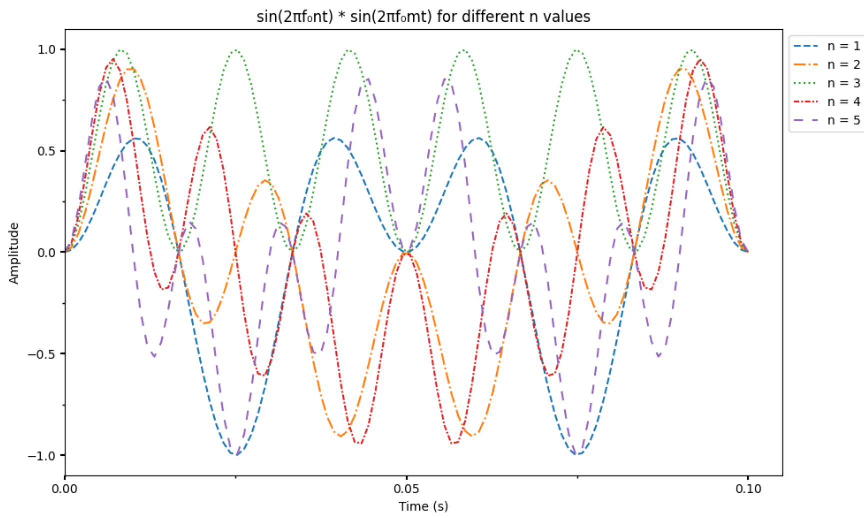

Figure 1: Plot of the product of $sin(2\pi m f_0 t)$ and $sin(2\pi n f_0 t)$ for m=3, n=1,2,3,4,5

$$\int_{t=0}^{T} cos(2\pi n f_0 t) * cos(2\pi m f_0 t) dt = 0 \tag{1}$$

$$\int_{t=0}^{T} cos(2\pi n f_0 t) * cos(2\pi n f_0 t) dt = \frac{1}{2 f_0} \tag{2}$$

I wrote the following code to do the numerical integrals. The example shown is for Eq. 1, but can be adapted by changing np.cos to np.sin in the necessary locations to match Eq. 3 and Eq. 5.

```
print("Cosines:")
for i in n:
    print(f"\nn = {i}")
    sum = 0
    for j in range(len(t) - 1):
        # Calculate the integral
        cos_i = (np.cos(2 * np.pi * m * f_0 * t[j]) * np.cos(2 * np.pi * i * f_0 * t[j]))
        cos_i1 = (np.cos(2 * np.pi * m * f_0 * t[j + 1]) * np.cos(2 * np.pi * i * f_0 * t[j + 1]))
        sum += (cos_i + cos_i1) / 2 * dt
        #print(f"t = {t[j]:.4f}, partial sum = {sum:.4f}")
    print(f"Integral of cos(2*pi*{m}*f0*t) * cos(2*pi*{i}*f0*t) over one period: {sum:.4f}")
```

In the cases where m = n, shown in Eq. 2 and Eq. 4, I calculated $\frac{1}{2f_0} = 0.05$. I then ran the integrals and checked that the values matched what was expected.

For Eqs. 1 and 2, the output was:

```
Cosines:

n = 1
Integral of cos(2*pi*3*f0*t) * cos(2*pi*1*f0*t) over one period: -0.0000

n = 2
Integral of cos(2*pi*3*f0*t) * cos(2*pi*2*f0*t) over one period: -0.0000

n = 3
Integral of cos(2*pi*3*f0*t) * cos(2*pi*3*f0*t) over one period: 0.0500

n = 4
Integral of cos(2*pi*3*f0*t) * cos(2*pi*4*f0*t) over one period: 0.0000

n = 5
Integral of cos(2*pi*3*f0*t) * cos(2*pi*5*f0*t) over one period: 0.0000
```

2 Sines

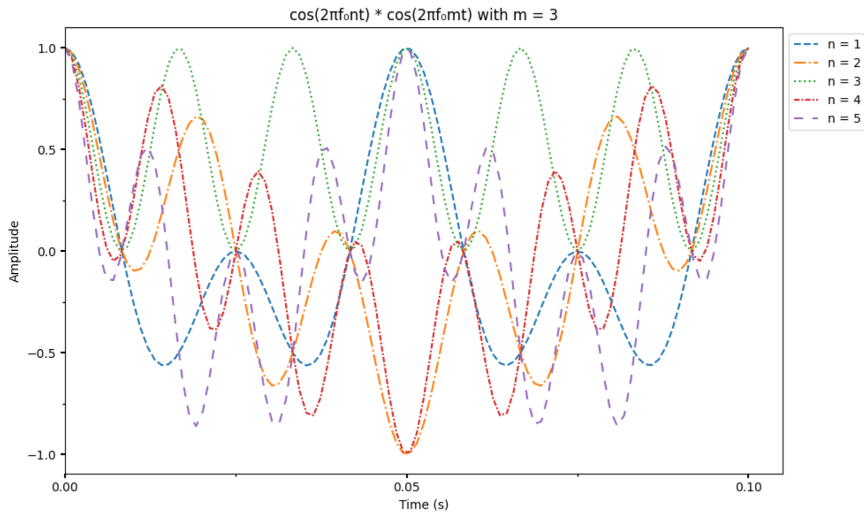

Figure 2: Plot of the product of $cos(2\pi m f_0 t)$ and $cos(2\pi n f_0 t)$ for m=3, n=1,2,3,4,5

$$\int_{t=0}^{T} sin(2\pi n f_0 t) * sin(2\pi m f_0 t) dt = 0 \tag{3}$$

$$\int_{t=0}^{T} sin(2\pi n f_0 t) * sin(2\pi n f_0 t) dt = \frac{1}{2 f_0} \tag{4}$$

For Eqs. 3 and 4, the output was:

```
Sines:

n = 1
Integral of sin(2*pi*3*f0*t) * sin(2*pi*1*f0*t) over one period: -0.0000

n = 2
Integral of sin(2*pi*3*f0*t) * sin(2*pi*2*f0*t) over one period: 0.0000

n = 3
Integral of sin(2*pi*3*f0*t) * sin(2*pi*3*f0*t) over one period: 0.0500

n = 4
Integral of sin(2*pi*3*f0*t) * sin(2*pi*4*f0*t) over one period: 0.0000

n = 5
Integral of sin(2*pi*3*f0*t) * sin(2*pi*5*f0*t) over one period: 0.0000
```

3 Sine and Cosine

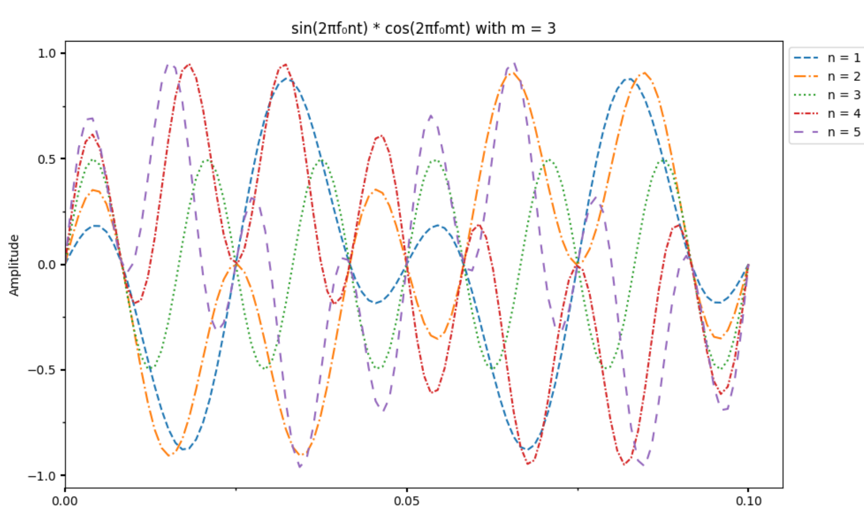

Figure 3: Plot of the product of $cos(2\pi m f_0 t)$ and $sin(2\pi n f_0 t)$ for m=3, n=1,2,3,4,5

$$\int_{t=0}^{T} sin(2\pi n f_0 t) * cos(2\pi m f_0 t) dt = 0 \tag{5}$$

And for Eq. 5, the output was:

```
Sine and Cosine:

n = 1
Integral of cos(2*pi*3*f0*t) * sin(2*pi*1*f0*t) over one period: -0.0000

n = 2
Integral of cos(2*pi*3*f0*t) * sin(2*pi*2*f0*t) over one period: -0.0000

n = 3
Integral of cos(2*pi*3*f0*t) * sin(2*pi*3*f0*t) over one period: 0.0000

n = 4
Integral of cos(2*pi*3*f0*t) * sin(2*pi*4*f0*t) over one period: 0.0000

n = 5
Integral of cos(2*pi*3*f0*t) * sin(2*pi*5*f0*t) over one period: -0.0000
```