Ex.s based on Function & Relation

Ex. Let R be a relation on Q, defined by
$R = \{(a,b) \mid a, b \in Q \ \& \ a-b \in Z\}$
Show that R is an equivalence relation.

→ Given,
$R = \{(a,b) \mid a, b \in Q \ \& \ a-b \in Z\}$.

i) Let $a \in Q$ then $a - a = 0 \in Z$
$\therefore (a,a) \in R$
So R is Reflexive.

ii) $(a,b) \in R \Rightarrow a - b \in Z$
ie $a-b$ is an integer.
$\Rightarrow -(a-b)$ is an integer.
$\Rightarrow b-a$ is an integer.
$\Rightarrow (b,a) \in R$.
Thus, $(a,b) \in R \Rightarrow (b,a) \in R$.
$\therefore R$ is symmetric.

iii) $(a,b) \in R$ & $(b,c) \in R$.
$\Rightarrow a-b \in Z$ & $b-c \in Z$
ie $(a-b)$ is an integer & $(b-c)$ is an integer.
$\Rightarrow \{(a-b)+(b-c)\}$ is an integer
$\Rightarrow a-c$ is an integer
$\Rightarrow (a,c) \in R$
Thus, $(a,b) \in R$ & $(b,c) \in R \Rightarrow (a,c) \in R$.
$\therefore R$ is Transitive.

Thus R is reflexive, symmetric & transitive
$\therefore R$ is an equivalence Relation.

Ex. Show that the relation " is congruent to" on the set of all triangles in a plane is a equivalence relation.

→ Let S be the set of all triangles in a plane.

Then the congruence relation on S is,

i) Reflexive, since $\Delta \cong \Delta$, for every $\Delta \in S$

ii) Symmetric, since $\Delta_1 \cong \Delta_2 \Rightarrow \Delta_2 \cong \Delta_1$ for all $\Delta_1, \Delta_2 \in S$

iii) Transitive, since $\Delta_1 \cong \Delta_2$ & $\Delta_2 \cong \Delta_3 \Rightarrow \Delta_1 \cong \Delta_3$ for all $\Delta_1, \Delta_2, \Delta_3 \in S$.

Hence the given relation is an equivalence relation.

Ex. If $(x+1, y-2) = (3, 1)$ find the values of x & y.

→ Since we have,

$$(x+1, y-2) = (3, 1)$$

$$\Rightarrow x+1 = 3 \quad , \quad y-2 = 1$$

$$\boxed{x = 2} \quad \boxed{y = 3}$$

Ex. If $A = \{1, 2\}$ find $A \times A$.

→ given $A = \{1, 2\}$ $\qquad \{1, 2\} \times \{1, 2\}$

$$\therefore A \times A = \{(1,1), (1,2), (2,1), (2,2)\}.$$

Ex. If $A \times B = \{(3,2), (3,4), (5,2), (5,4)\}$ Find A & B.

→ clearly, we have,

A = set of all first components of $A \times B$

$\therefore A = \{3, 5\}$.

B = set of all second components of $A \times B$

$B = \{2, 4\}$.

Ex. If A & B are two sets given in such a way that $A \times B$ contains 6 elements. If three elements of $A \times B$ are $(1,3)$, $(2,5)$ & $(3,3)$ find its remaining elements.

→ Given $(1,3)$, $(2,5)$ & $(3,3)$ are in $A \times B$.

$n(A \times B) = 6$

$\therefore A = \{1, 2, 3\}$ & $B = \{3, 5\}$

$\therefore A \times B = \{(1,3)\ (1,5)\ (2,3)\ (2,5), (3,3)\ (3,5)\}$.

Hence remaining elements of $A \times B$ are, $(1,5)$ $(2,3)$ & $(3,5)$.

Ex. Express $\{(x,y) \mid x^2 + y^2 = 25$, where x & $y \in W\}$ as a set of ordered pairs.

→ we have, $x^2 + y^2 = 25$

$\therefore x = 0, y = 5 \Rightarrow 0^2 + 5^2 = 25$ $\quad x = 4, y = 3 \Rightarrow 4^2 + 3^2 = 25$

$x = 3, y = 4 \Rightarrow 3^2 + 4^2 = 25$ $\quad x = 5, y = 0 \Rightarrow 5^2 + 0^2 = 25$

$\therefore$ The given set $= \{(0,5), (3,4), (4,3), (5,0)\}$

Ex. $A = \{1, 2, 3\}$ $B = \{2, 4, 6\}$ Show that R

$R = \{(1,2)\ (1,4)\ (3,2), (3,4)\}$ is a relation

from A to B find, i) domain R

ii) codomain R iii) Range

→ Given,

$R = \{(1,2), (1,4), (3,2), (3,4)\}$.

As $R \subseteq A \times B$, $\Rightarrow$ R is relation from A to B.

$D(R) = \{1, 3\}$, $\text{codomain}(R) = \{2, 4, 6\}$

Range of $R = \{2, 4\}$ {second component of R}

Ex. Let $A = \{1, 2, 3, 4, 5\}$ $B = \{1, 4, 5\}$.

let R be a relation from A to B

Such that, $(x, y) \in R$ if $x < y$

i) List elements of R ii) Find domain, codomain & range.

→. $A = \{1, 2, 3, 4, 5\}$ $B = \{1, 4, 5\}$.

$R = \{(1,4)\ (1,5), (2,4)\ (2,5)\ (3,4)\ (3,5)\ (4,5)\}$

$\text{Domain}(R) = \{1, 2, 3, 4\}$

$\text{codomain}(R) = \{1, 4, 5\} = B$

$\text{Range}(R) = \{4, 5\}$.

Ex. Find the domain of real valued function

$$f(x) = \frac{x^2+2x+1}{x^2-8x+12}$$

→ Given, $f(x) = \dfrac{x^2+2x+1}{x^2-8x+12}$

$f(x)$ is not defined when,

$$x^2-8x+12=0$$

$$\Rightarrow (x-6)(x-2)=0$$

$$x=2. \text{ or } x=6.$$

Domain of $f = R - \{2,6\}$.

Ex. Find the domain and Range of the real valued function of real variable

$$f(x) = \frac{x-2}{2-x}$$

→ We have,

$$f(x) = \frac{x-2}{2-x}$$

$f(x)$ is not defined, when ~~for~~

$2-x=0$ ie $x=2$

$\therefore$ Domain of $f = R - \{2\}$.

Also, $y = f(x) = \dfrac{x-2}{2-x} = \dfrac{x/2}{-(x/2)} = -1$

$$\Rightarrow y=-1$$

$\therefore$ Range of f $= \{-1\}$.

Ex. Functions f, g, h are defined on a set,

$X = \{1, 2, 3\}$ as,

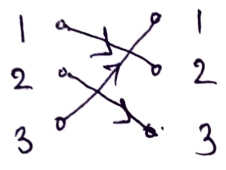
$f = \{(1,2), (2,3), (3,1)\}$ $\quad$ $h = \{(1,1), (2,2), (3,1)\}$

$g = \{(1,2), (2,1), (3,3)\}$

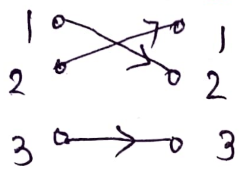
Find gof, fog. Are they equal.

Also find $fogoh$ & $fohog$.

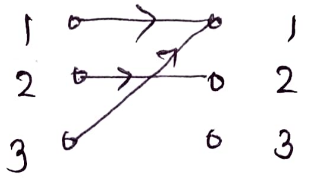
→



f

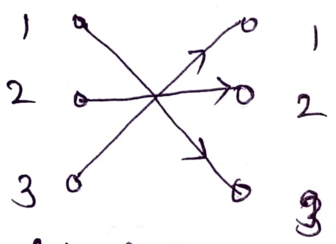


g



h

i) fog is



$1 \to 2 \to 3$, $\quad 2 \to 1 \to 2$

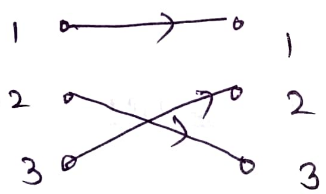
$\therefore 1 \to 3$ $\quad \therefore 2, 2.$

$3 \to 3 \to 1$

$\therefore 3 \to 1$

$fog = \{(1,3)\,(2,2), (3,1)\}$

ii) gof is,



$1 \to 2 \to 1$ $\quad 2 \to 3 \to 3$

$\therefore 1 \to 1.$ $\quad 2 \to 3$

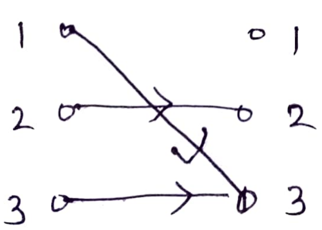
$3 \to 1 \to 2$

$\therefore 3 \to 2$

$gof = \{(1,1), (2,3), (3,2)\}$

$fog \neq gof$

iii) $fogoh = (fog)oh.$



$1 \to 1 \to 3$ $\quad 2 \to 2 \to 2$

~~1,3~~ $1 \to 3$ $\quad 2 \to 2$

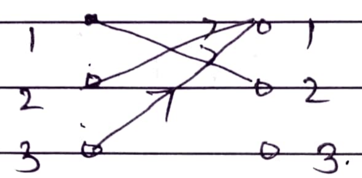
$3 \to 1 \to 3$

$3 \to 3$

$\therefore fogoh = \{ (1,3), (2,2), (3,3) \}$

iv) $fohog$

hog is,



$1 \to 2 \to 2$

$1 \to 2$

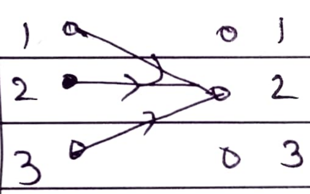
$3 \to 3 \to 1$

$3 \to 1$

$2 \to 1 \to 1$

$2 \to 1$

$fo(hog)$



~~$2 \to 1 \to 2$~~

$2 \to 1 \to 2$

$(2,2)$

$1 \to 2 \to 2$

$\therefore (1,2)$

$3 \to 1 \to 2$

$(3,2)$

~~Ex~~

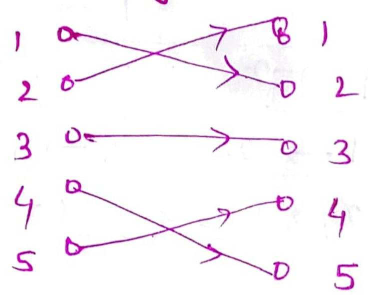
Ex. Let $A = \{a, b, c, d\}$ $B = \{s, t, u\}$ $C = \{l, m, n\}$.
Obtain the composition of the following functions $f: A \to B$ $g: B \to C$.
where,

$$f = \{(a,s)\ (b,t),\ (c,u),\ (d\ t)\}$$
$$g = \{(s,m)\ (t,l)\ (u,n)\}.$$

$\to$ ~~f~~ $a \to s \to m$, $b \to t \to l$, $c - u \to n$, $d \to t \to l$
$\therefore a \to m$ $\therefore b \to l$ $c \to n$ $d \to l$

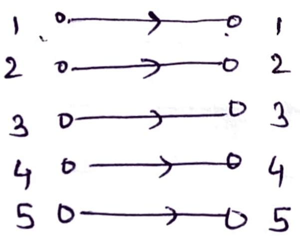
$$g \circ f = \{(a,m), (b,l), (c,n), (d,l)\}.$$

Ex. Let $A = \{1, 2, 3, 4, 5\}$ $g: A \to A$ is as shown in fig.



Determine the composition, $g \circ g$, $g \circ (g \circ g)$
Determine whether each is one to one or onto f^n.

$\to$ $g \circ g$

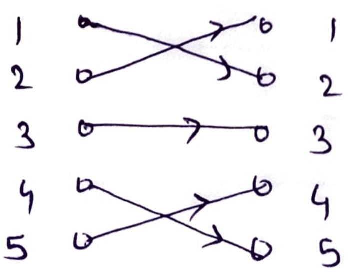


$1 \to 2 \to 1$ $\quad$ $2 \to 1 \to 2$, $\quad$ $3 \to 3 \to 3$
$1 \to 1$ $\quad$ $2 \to 2$ $\quad$ $3 \to 3$

$4 \to 5 \to 4$ $\quad$ $5 \to 4 \to 5$
$4 \to 4$ $\quad$ $5 \to 5$

$g \circ g$ is one one & onto.

$g \circ (g \circ g)$



$1 \to 1 \to 2$ $\quad$ $2 \to 2 \to 1$
$1 \to 2$ $\quad$ $2 \to 1$

$3 \to 3 \to 3$ $\quad$ $4 \to 5 \to 5$
$3 \to 3$ $\quad$ $4 \to 5$
$5 \to 4 \to 4$
$5 \to 4$

$g \circ (g \circ g)$ is onto & one to one f^n.

Ex. let $f(x) = x+2$ $g(x) = x-2$ & $h(x) = 3x$

for $x \in R$ where R = set of real no.s

Find gof, fog, fof, gog, foh, hog, hof, $fohog$.

→

i) $gof(x) = g\{f(x)\} = g(x+2)$

$= (x+2)-2 = x$.

ii) $fog(x) = f\{g(x)\} = f\{(x-2)\} = (x-2)+2 = x$.

iii) $fof(x) = f\{f(x)\} = f\{(x+2)\} = (x+2)+2$

$= x+4$

iv) $gog(x) = g\{g(x)\} = g\{(x-2)\} = (x-2)-2$

$= x-4$

v) $foh(x) = f\{h(x)\} = f\{3x\} = (3x)+2$

vi) $hog(x) = h\{g(x)\} = h\{x-2\} = 3(x-2)$.

$= 3x-6$

vii) $hof(x) = h\{f(x)\} = h\{x+2\} = 3(x+2)$

vii) $fohog(x) = foh(g(x)) = foh(x-2)$

$= foh(x-2) = f(3(x-2))$

$= f(3x-6)$

$= (3x-6)+2$

$= 3x-4$

Ex. Let $f(x) = 2x+3$, $g(x) = 3x+4$
$h(x) = 4x$ for $x \in R$ where R = set of real no.s
Find gof, fog, foh, hof, goh.

→ i) $gof(x) = g[f(x)] = g(2x+3)$

$$= 3(2x+3)+4$$
$$= 6x+13.$$

ii) $fog(x) = f[g(x)] = f[3x+4]$

$$= 2(3x+4)+3$$
$$= 6x+11$$

iii) $foh(x) = f[h(x)]$

$$= f(4x)$$
$$= \cancel{f}\ 2(4x)+3$$
$$= 8x+3$$

iv) $hof(x) = h[f(x)]$

$$= h(2x+3)$$
$$= 4(2x+3) = 8x+12.$$

v) $goh(x) = g[h(x)]$

$$= g(4x)$$
$$= 3(4x)+4$$
$$= 12x+4$$

Ex. If $f(x) = x^2+1$ & $g(x) = x+2$ are function from R to R, where R is the set of real no. find fog & gof

→ i) $fog(x) = f\{g(x)\}$

$= f(x+2)$

$= (x+2)^2+1 = x^2+4x+5$

ii) $gof(x) = g\{f(x)\}$

$= g\{x^2+1\}$

$= (x^2+1)+2$

$= x^2+3$.