a = 414 b = 662

→ Find gcd by applying Euclidean algorithm.

b > a

b = 662, a = 414.

Apply division algorithm.

662 = 414 × 1 + 248

414 = 248 × 1 + 166

248 = 166 × 1 + 82.

166 = 82 × 2 + [2] ← gcd

82 = 2 × 41 + 0

∴ gcd(414, 662) = 2.

(2) a = 595, b = 252

→ 595 = 252 × 2 + 91

252 = 91 × 2 + 70

91 = 70 × 1 + 21.

70 = 21 × 3 + [7]

21 = 7 × 3 + 0

[gcd = 7]

a, b → d.

d = gcd(a, b) = ax + by.

Find integers x & y such that

$$6x + 10y = 2$$

→ I) gcd of 6 & 10.

$10 = 6 \times 1 + 4$ — (i)

$6 = 4 \times 1 + \boxed{2}$ — (ii)

$4 = 2 \times 2 + 0$ — (iii)

gcd = 2

To find x & y.

Consider step (ii)

$2 = 6 - 4 \times 1$

from (i)

$2 = 6 - (10 - 6 \times 1) \times 1$

$2 = 6 - 10 \times 1 + 6 \times 1$

$2 = 6 \times 2 - 10 \times 1$

$2 = 6x + 10y$

$x = 2 \quad y = -1$

Find integers x & y
such that

$$128x + 58y = 2.$$

→ I) we find gcd of (128, 58).

$128 = 58 \times 2 + 12$ — (i)

$58 = 12 \times 4 + 10$ — (ii)

$12 = 10 \times 1 + 2$ — (iii) ✓

$10 = 2 \times 5 + 0$ — (iv)

gcd (128, 58) = 2.

II) To find x & y.

From (iii)

$2 = 12 - 10 \times 1$.

From (ii)

$2 = 12 - (58 - 12 \times 4) \times 1$

$2 = 12 - 58 \times 1 + 12 \times 4$.

$2 = 12 \times 5 - 58 \times 1$.

from (i)

$2 = (128 - 58 \times 2) \times 2 - 58 \times 1$

$2 = 128 \times 2 - 58 \times 4 - 58 \times 1$

$2 = 128 \times 2 - 58 \times 5$

gcd = $128x + 58y$

$x = 2, \; y = -5$

gcd $(a, b) = ax + by$ ✓ ∴