combination Type II

~~& selection~~ (r identical objects in n ~~distinct boxes with repetition~~)

We often deal with problems on counting, where one has to make r selections from n-type of objects with repetition freely allowed.

This Problem can also be described in the following way using analogy of identical objects & distinct boxes.

It is possible to distribute r identical objects in n distinct boxes, with no restriction put on the no. of objects a box contain is,

$$\boxed{(n+r-1)C_{n-1} \quad \text{i.e.} \quad C(n+r-1,\, n-1).}$$

Ex 1) Ten balls are picked from a pile of red blue & white balls. Find how many such selections contain less than 5 red balls.

→ The no. of ways to select 10 balls from a pile of red, blue & white balls is equivalent to distributing 10 identical objects into 3 distinct boxes. $\{ n = 3,\ r = 10 \}$.

∴ No of ways of selection $= (n+r-1)C_{n-1}$

$= (3+10-1)C_{3-1}$

$= 12C_2 = \frac{12!}{2!\,10!} = 66$ ways

(2) (ii) The no. of ways to select 5 red ball is, $\{ r=5, n=3 \}$

$(n+r-1)C_{n-1} = (3+5-1)C_{3-1} = 7C_2$

$= \frac{7!}{2!\,5!} = \underline{\underline{21}}$

$\therefore$ Hence the no. of ways to select 10 balls from a pile of red, blue & white balls, so that each selection contains less than 5 red balls is $66-21 = \underline{\underline{45}}$ ways

Ex. In how many ways can one distribute 10 apples among 4 children.

$\rightarrow$ Consider apple as identical objects & the children corresponds to distinct boxes.

$\therefore$ No. of ways to distribute 10 apples among 4 children is, $\{ n=4, r=10 \}$.

$= (n+r-1)C_{(n-1)} = (4+10-1)C_{(4-1)}$

$= 13C_3 = \frac{13 \times \overset{2}{\cancel{12}} \times 11 \times \cancel{10!}}{\cancel{3} \times 2 \times 1 \times \cancel{10!}}$

$= 22 \times 13 = \underline{\underline{286}}$ ways

Ex. How many non negative integer sol^n are there in the equation
$x+y+z+u+v=10,000$?

→ The Problem is that of distributing 10,000 identical objects in 5 distinct boxes where there is no restriction on the no. of objects the box may contain.

$n=5, \quad r=10,000.$

$\therefore$ No. of sol^n $= (5+10,000-1)\, C_{5-1}$

$= 10,004\, C_4$ ways.

$= 4.1708 \times 10^{14}$ ways

Ex In how many ways can 5 balls be selected from 8 identical red balls 4 8 identical white balls.

→ The Problem is that of distributing 5 identical objects in two distinct boxes corresponding to their colours.

$\cancel{k=5} \quad \cancel{r=5}. \quad r=5 \quad n=2.$

$\therefore$ No. of ways to make required selection $= (5+2-1)\, C_{(2-1)}$

$= 6C_1$

$= 6$ ways

Combinations of objects not all different

The total no. of combinations which can be made of n different objects taken some or all at a time is,

$$^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n = 2^n - 1.$$

1) In order to pass MCA first semester examination minimum marks have to be secured in each of the 7 subject. In how many cases can a student fail.

→ The possibility of fail or pass in each subject can be considered in 2 ways.

∴ Possibility of pass or fail in all the 7 subject can be 2^7 ways.

But this include the case in which student passes in all 7 subjects.

Hence,

required no. of combinations in which the student can fail are,

$2^7 - 1 = \underline{\underline{127}}$ ways.

2) There are 15 true or false questions in an exam. In how many ways can a student answer the exam if he or she can also choose not to answer some of them.

⑤

→ If the student attemps all the 15 que, then he or she can do so in 2^{15} ways.

But since he or she can choose not to answer some of them, the correct solution is $2^{15} - 1 = 32768 - 1$

$= \underline{32767}$ ways

<u>Ex</u> A man has 10 friends. In how many ways can he go to dinner with 1 or more of them.

→ since he has to select some or all of his 10 friends,

the no. of ways is $\cancel{2^{15}}$ $\underline{\underline{2^{10} - 1}}$ ways

$= 1024 - 1$

$= \underline{\underline{1023}}$ ways

<u>Ex</u> A bit is either 0 or 1. A byte is a sequence of 8 bits. Then find no. of bytes.

→ Total no. of bytes $= \underline{\underline{2^8}} = \underline{\underline{256}}$ ways.