5) Antisymmeteric relation :-

Let R be a relation on a set A. Then R is an antisymmeteric iff aRb & bRa $\Rightarrow$ a = b for a, b $\in$ A i.e (a, b) $\in$ R, (b, a) $\in$ R $\Rightarrow$ a = b.

Thus R is an antisymmetric if we never have both aRb & bRa except when a = b.

An equivalent def^n of antisymmetric relation R is, If $a \neq b$ then either $a \not R b$ or $b \not R a$. This is sometimes useful to verify whether a given relation is antisymmetric.

Ex. 1) Let A = $\mathbb{R}$. Let R be the relation $\leq$.
Then $a \leq b$ & $b \leq a \Rightarrow a = b$.
Hence $\leq$ is an antisymmetric relation.

2) The relation in N defined by 'x divides y' is antisymmeteric because x divides y & y divides x implies x = y.

Note It is evident that a relation R on a set A is an antisymmeteric if $R \cap R^{-1} \subseteq I_A$ where I_A denotes the identity relation on A.

6) Transitive Relation :-

A relation R on a set A is transitive iff for all $a, b, c \in R$, aRb & $bRc \Rightarrow aRc$

ie $(a,b), (b,c) \in R \Rightarrow (a,c) \in R$.

It follows that a relation R is not transitive if there exist an elements $a, b, c \in A$ such that aRb & bRc but $a \not R c$.

If such element a, b, c do not exist, then R is transitive.

Ex. 1) Let A :- set of triangles & let R be the relation of being congruent. Then for triangles $a, b, c \in A$, aRb & $bRc \rightarrow aRc$. Hence R is transitive.

2) Let A be set of people & let R be the relation of being "brother of" Then a is brother of b & b is brother of c implies a is brother of c Hence R is transitive.

3) Let $A = N$ = the set of natural no.s & let

$$R = \{(a,b), a, b \in N / a+b \text{ is an odd no.}\}.$$

Then R is not transitive since $(1,2)$ & $(2,1) \in R$ but $(1,1) \notin R$.

4) Let N be the set of all natural nos let a relation R on N defined by x is a divisor of y where $x, y \in N$. Then

a) R is reflexive, since every natural no. is divisor of itself b) R is antisymmetric, since x divides y & y divides $x \Rightarrow x = y$ c) R is Transitive.

Equivalence Relation.

A relation R on a set A is called an equivalence relation (denoted by ~) if and only if following three conditions hold.

1) R is reflexive
2) R is Symmetric
3) R is Transitive.

Or

A binary relation R on a set A is called as equivalence relation if it is reflexive, symmetric & transitive.

Ex 1) Let $A = \{a, b, c\}$ be any set of no.s The relation R on the set A is defined by 'is equal to' is an equivalence relation because it is reflexive since every element in A is equal to itself. It is symmetric as $a = b \Rightarrow b = a$. Also $a = b$ & $b = c$ then $a = c$ so is transitive also.

2) A is set of triangles & R is 'similarity' of triangles. Then R is equivalence relation.

3) A is set of students & R is the relation of "logical equivalence" is also equivalence relation.

4) A is set of lines in a plane & R is the relation of lines being 'parallel' is also equivalence relation.

Note

1) If R_1 & R_2 are equivalence relations on set A then $R_1 \cap R_2$ is an equivalence relation.

2) If R_1 & R_2 are equivalence relations, it is not necessary that $R_1 \cup R_2$ is also an equivalence relation.

Matrix Representation of a Relation.

Let, $A = \{a_1, a_2 --- a_m\}$

$B = \{b_1, b_2 --- b_n\}$.

be finite sets containing respectively m & n elements.

Let R be a relation from A to B.

By def^n $R \subseteq A \times B$.

Hence R can be represent by $m \times n$ matrix $M_R = [m_{ij}]$ which is defined as follows

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i b_j) \in R \\ 0 & \text{if } (a_i b_j) \notin R. \end{cases}$$

The matrix M_R is called as the matrix of R.

Ex. Let $A = \{a, b, c, d\}$ & $B = \{1, 2, 3\}$.

$R = \{(a,1)\ (a,2)\ (b,1)\ (c,2)\ (d,1)\}$.

then

$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

Graphical Representation of a Relation.

If A is finite set & R is a relation on A it is possible to represent R pictorically by means of a graph.

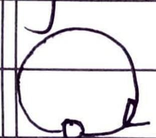
The elements of A are represented by points or circles, called as nodes or vertices.

If aRb, this is indicated by drawing an arc from a to b with an arrowhead pointing in the direction $a \to b$.

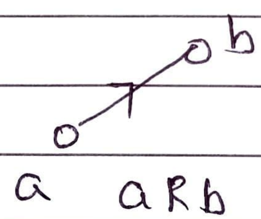
If aRa, this is shown by drawing loop around a. These arcs or loops are called as edges of the graph.

The resulting graph is called as directed graph or digraph of R.

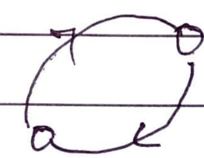
The various types are illustrated in the following Fig.



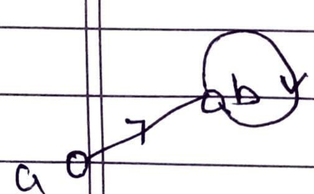
aRa



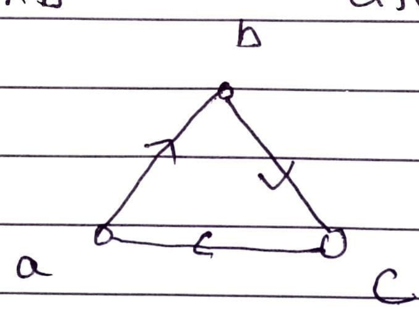
aRb



aRb & bRa



aRb & bRb



aRb & bRc & cRa